

M HEWLETT-PACKARD CALCULATOR

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# Introduction

Programs for your 9820A Math Pac have been selected primarily from three sources: requests from you, our customers; programs you have submitted for the 9100 family of calculators; and some useful programs from previous calculator and larger computer systems' math libraries.

A new approach has been used for many of the programs. A few of them such as the Gaussian Quadrature program are multi-purpose. This particular program allows you to integrate a given function by any one of five different formulas enabling you to compare the integration results of several techniques without having to load and run another program or reenter data. Other programs have been designed to be easily used in conjunction with one another. For example, Fourier Series Coefficients for Unequally Spaced Data may be used with the General Function Plot program to produce several plots without having to reenter data. Still others perform tasks not indicated by the title, such as Real Matrix Inversion from which the (absolute value of the) determinant of a matrix may be obtained, and such as the Matrix Arithmetic programs which may be used to add, subtract, or find the dot product of vectors.

With the power of the new 9820A algebraic language and the ease and pleasure with which programs may be written, we believe you will be developing many programs and using techniques previously not possible on a desktop system. We encourage you to submit these to Hewlett-Packard so that we may share them with other users, either through the KEYBOARD, by adding them to the Program Catalog, or by including them in a future Pac. Your contributions, requests, and suggestions are our most important source for supplying you with user-oriented programs in the future. Program submittal forms and details for submitting programs may be found in the back of this Pac. Please use these forms when submitting programs.

We at Hewlett-Packard look forward to serving you.

Sincerely,

HEWLETT-PACKARD COMPANY

Homer Russell

Homer Russell

Product Development Engineer

Loveland, Colorado

# **Comments**

About 60 individual programs have been combined to form the 26 multi-purpose programs of your 9820A Math Pac. Selection of a particular option in one of the multi-purpose programs is done by pressing RUN PROGRAM a number of times to bring the different options into the display one at a time. When the desired option is displayed, pressing 1 RUN PROGRAM will cause the option being displayed to be selected. If by accident you pass by the option wanted, pressing RUN PROGRAM several more times will complete the display cycle and cause the display of the options to be repeated from the first.

As mentioned in the introduction, some of the programs perform functions in addition to those indicated in the title. Another example of this is the Eigenvalue program which will print out corresponding eigenvectors for most, but not all, problems. By considering a complex number to be the sole entry of a  $1 \times 1$  matrix, the Complex Matrix Arithmetic program can be used to add, subtract, or multiply complex numbers. Or the Complex Matrix Inversion program can be used in the same manner to find the reciprocal of a complex number.

When running a program and the calculator is stopped, waiting for data entry or displaying a message and you wish to abort the program, the calculator will probably be in an ENTER statement in which case pressing STOP will first be necessary to get out of the ENTER statement. You may press ERASE instead but remember that this will clear all memory – your program included.

At the top of the User Instructions accompanying each program is the system configuration for which the program was designed. The Internal Registers box that is checked indicates the minimum number of registers required for the program. ROM Blocks should be in the indicated slots, and slots shown blank should contain no blocks. Turn power OFF when inserting or extracting ROM Blocks. This is necessary so that the calculator may internally reconfigure codes assigned to keys.

The Math Pac has been laid out so that the Example page will be opposite the User Instructions for easy reference. To keep the User Instructions brief, the convention of underlining has been adopted to indicate key depression. For instance, "RUN PROGRAM" means the same as "Press RUN PROGRAM", and "GO TO 0 EXECUTE" means "Press GO TO 0 EXECUTE". Normally, each separate keystroke will be underlined, but in an expression like " $f(x) \longrightarrow Y$ " where the number of keystrokes required to define f(x) is only known by you, the user, only one line has been used. Occasionally redundancy is used for clarity by using the words, "key in" or "press" in conjunction with the underlining.

A Memory Allocation page has been included with each program. Most of the variables shown stored in particular registers have not been defined in the program writeup and have meaning only for the program writer. The primary purpose of the page is to show you what registers and flags have been used, indicating which ones are still available.

A pre-recorded set of magnetic cards for the 9820A Math Pac may be purchased under the part number 09829-70000 through your local Hewlett-Packard sales office.



With this program, matrices with real entries of any size up to  $5 \times 5$  (respectively,  $16 \times 16$  with the expanded internal memory) may be added or subtracted, and matrices up to  $4 \times 4$  (respectively, up to  $12 \times 12$ ) may be multiplied. However, the smaller that one matrix is, the larger the other one may be. For example, a  $16 \times 16$  matrix may be applied to a  $16 \times 1$  column matrix. In any case, the resultant matrix is printed but also remains in the calculator and may operate on or be operated on by a third matrix. Thus the program allows such computations as AB, BA, A+B, A-B, B-A, C(B-A), (B-A)C, ABC, and [C(B-A)+D]E, provided, of course, the operation in each instance is well-defined and matrix size limits are not exceeded. Since the resultant matrix remains in the calculator, data reentry is not required in performing a series of operations.

The actual size limitations are determined from the following relations:

$A = \frac{\text{Matrix Size}}{\text{mxn, B} = \text{mxn}}$	$\frac{\text{Operation}}{A+B \text{ or } A-B}$	$\frac{\text{Basic } 9820\text{A}}{\text{mn} \le 32}$	$\frac{\text{Expanded } 9820A}{\text{mn} \le 288}$
$A = m \times n, B = n \times p$	AB	mn+mp ≤ 32 (if A is entered first)	mn+mp ≤ 288 (if A is entered first)
		np +mp ≤ 32 (if B is entered first)	np+mp ≤ 288 (if B is entered first)

For example, if A is  $16 \times 16$  and B is  $16 \times 1$ , AB can be found on the basic 9820A providing B is entered first since:

$$np + mp = 16 \cdot 1 + 16 \cdot 1 = 32 \ (\le 32)$$

whereas

$$mn + mp = 16 \cdot 16 + 16 \cdot 1 = 272 \quad (>32)$$

showing that A cannot be entered first. (Note: The program always calls for matrix A to be entered first; so merely rename the matrices to correspond to this convention when necessary).

As a secondary function of the program, two vectors such as  $A = (a_1 \ a_2 \ a_3)$  and  $B = (b_1 \ b_2 \ b_3)$  may be added or subtracted by treating A and B as  $1 \times 3$  matrix and forming their product:

$$A \cdot B = AB = (a_1 \ a_2 \ a_3) \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

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PERIPHI	1 ERALS	2 3
	DA MARKED CARD READER	9862A PLOTTER 9864A DIGITIZER
9861	A TYPEWRITER	9863A PAPER TAPE READER
STEP	DISPLAY	INSTRUCTIONS
1.		ERASE LOAD EXECUTE
2.		Alternately insert magnetic cards and EXECUTE until NOTE 14 no longer
		appears, indicating completion of loading.
3.		(Optional) FIXED N n EXECUTE, or FLOAT N n EXECUTE.
4.		END RUN PROGRAM
5.	ROWS IN A?	m RUN PROGRAM m = number of rows in A
6.	A BY ROWS	aij RUN PROGRAM Row by row until A is entered; then RUN PROGRAM
		and A will be printed.
		Press the A and B keys in one of the following combinations (in Step 7 or 8)
		to designate the desired operation:
7.	A*B?	$\underline{A} \times \underline{B} = \underline{RUN \text{ PROGRAM}}$ and go to Step 11, or
		$\underline{A} + \underline{B}  \underline{RUN \text{ PROGRAM}}  \text{and go to Step 12, or}$
		A - B RUN PROGRAM and go to Step 12, or
		RUN PROGRAM to go to Step 8.
	D#4.0	D. A. DVIV PROGRAM - 1 - 4 G/- 0
8.	B*A?	B x A RUN PROGRAM and go to Step 9, or
		B + A RUN PROGRAM and go to Step 10, or
		B - A RUN PROGRAM and go to Step 10, or
		RUN PROGRAM to go to Step 7.
9.	ROWS IN B?	n RUN PROGRAM n = number of rows in B
<i>J</i> .	ROWD IN D:	n = number of rows in B
10.	B BY COLS.	bij RUN PROGRAM Column by column until B is entered; then go to
10.	D DT COLD.	Step 13.
		Step 10.
11.	COLS. IN B?	n PIIN DPOCPAM n - number of columns in P
11.	COLS. IN B?	$\underline{p}  \underline{RUN \ PROGRAM} \qquad p = number \ of \ columns \ in \ B$
12.	B BY ROWS	b <sub>ij</sub> RUN PROGRAM Row by row until B is entered.
14.	D DI KOWS	in the state of th
13.		The result is printed and then renamed A, replacing the old A in memory.
10.		Control returns to Step 7 and another operation may be performed with the
		new matrix A using a new incoming matrix B.

## **EXAMPLES**

1. Let

$$A = \begin{pmatrix} 3 & 2 & 4 \\ 1 & 6 & -1 \\ 1 & 2 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

then

$$AB = \begin{pmatrix} 9 \\ -7 \\ -1 \end{pmatrix}$$

BY ROWS

3.0000

2.0000 4.0000

1.0000

6.0000

-1.0000

1.0000 2.0000

9.0000

BY ROWS

1.0000

-1.0000

2.0000

A#B BY ROWS

9.0000

-7.0000

-1.0000

2. Show that  $(B-A)C = \begin{pmatrix} 0 & -3 \\ -4 & -3 \end{pmatrix}$ for  $A = \begin{pmatrix} -5 & -2 \\ 3 & 0 \end{pmatrix}$ ,  $B = \begin{pmatrix} 4 & 1 \\ 20 & 7 \end{pmatrix}$ , and  $C = \begin{pmatrix} 1 & -1 \\ -3 & 2 \end{pmatrix}$  by first forming B-A and

then multiplying by C.

A BY ROWS

-5.0000 -2.0000

3.0000

0.0000

BY COLS.

4.0000

20.0000

1.0000 7.0000

B - B

BY ROWS

9.0000 3.0000

17.0000

7.0000

BY ROWS

1.0000 -1.0000

-3.0000

2.0000

This becomes the new matrix A, replacing the old A in the calculator.

This is the next matrix in the sequence (= matrix C).

A\*B Final answer BY ROWS

> 0.0000 -3.0000

-4.0000

-3.0000

15

```
Ø :
ENT "ROWS IN A?"
*R(5*X) *CFG 13*0
+1": |--
1 #
ENT "A BY ROWS",
R(X+1+X) i JMP
FLG 13H
2:
(X-5)/R5→R2H
3:
0+X+R0;CFG 2;R5+
R3;R2+R4;SPC 2F
4 #
IF C=01PRT "
    FI '' |--
IF C=1%PRT "
   A-B"H
F. 1
IF C=-1; PRT "
    B-A"H
7 :
IF CFLG 4=2;PRT
        A*B" |
JMP 4F
a:
IF C=2;PRT "
   B#A"H
9:
IF (C=3)FLG 4;
PRT "
             A+B"
JMP 2H
10:
IF C=3;PRT "
   B+A" |-
11:
PRT "
         BY ROW
S"#SFC 1+YH
12:
PRT R((X+1+X)+5)
1Y(Y=R2)+1+Y1
SPC Y=1jJMP X=R2
R5H
13:
1+X+B;2+AF
14:
SFG 41CFG 131
ENT "A*B?",C%IF
FLG 13;CFG 13;
ENT "B*A?":CI
CFG 4% JMP FLG 13
=0|-
```

```
IF CFLG 4=2;ENT
"COLS.IN B?",R3;
JMP 3H
16:
IF C=2;ENT "ROWS
 IN 8?",R4;JMP 2
17:
R5+R4;R2+R3H
R0+1+R(0+R1)+
19:
IF R0(R1+1+R1)#1
IJMP 3H
20:
PRT "
IF FLG 41PRT "
   BY ROWS" (SPC
11JMP 2F
21:
PRT "
         BY COL
8."; SPC 1H
22:
IF FLG 4%ENT "B
BY ROWS", ZJJMP 2
23:
ENT "B BY COLS."
# Z |-
24:
PRT | SPC R1=R((
FLG 4=0)+3)+
25:
IF C=2;JMP 14F
26#
R2R(FLG 4=0)-R2+
RELG 4+5+RF
27:
IF 2>C!-Z+ZH
28:
Z+RB+ZH
29:
IF (C=3)(FLG 4=0
30#
ZARBH
31#
IF R((FLG 4=0)+3
)>R1#JMP -12H
32:
IF R(FLG 4+3)>R0
#JMP -14h
```

```
IF C#2#GTO 3H
34:
R2R5+Z10+XH
35:
IF FLG 4=0;R4→R5
JJMP 2H
36:
R3+R2H
27:
IF (X+1+X) > R2R5i
GTO 3H
38:
R(X+5+Z)+R(X+5);
JMP -1H
39:
0+A; IF FLG 2;
JMP 2H
40:
0 + R((A + 1 + A) + R2R5
+5) ; JMP A=R(FLG
4+4)R(FLG 4+2)F
41:
0+AiSFG 2h
4.2 :
IF (A+1+A)>R(3
FLG 4+2); JMP -11
43:
ZR((R2A-R2+R0)
FLG 4+(R2R0-R2+A
)(FLG 4=0)+5)+BH
44:
(R3A-R3+R1)FLG 4
+(R2R1-R2+A)(
FLG 4=0)+R2R5+5+
45#
B+RY→RYiJMP -3h
46:
END H
```

a series of operations.

With this program, matrices with complex entries of any size up to  $11 \times 11$  may be added or subtracted, and matrices up to  $8 \times 8$  may be multiplied. However, the smaller one matrix is, the larger the other one may be. For example, a  $30 \times 30$  matrix may be applied to a  $30 \times 1$  column matrix. In any case, the resultant matrix is printed but also remains in the calculator and may operate on or be operated on by a third matrix. Thus the program allows such computations as AB, BA, A+B, A-B, B-A, C(B-A), (B-A)C, ABC, and [C(B-A) + D]E, provided, of course, the operation in each instance is well-defined and matrix size limits are not exceeded. Since the resultant matrix remains in the calculator, data reentry is not required in performing

The actual size limitations are determined from the following relations:

Matrix Size	Operation	Expanded 9820A
$A = m \times n, B = m \times n$	$A \pm B$	mn ≤ <b>12</b> 9
$A = m \times n, B = n \times p$	AB	$mn + mp \le 129$ (If A is entered first)
		$\begin{array}{c} np + mp \leq 129 \\ (\text{If B is entered first}) \end{array}$

For example, if A is 11x11 and B is 11x1, AB can be found provided B is entered first since:

$$np + mp = 11 \cdot 1 + 11 \cdot 1 = 22 \ (\le 129)$$

whereas

$$mn + mp = 11 \cdot 11 + 11 \cdot 1 = 132 (> 129)$$

which is too large showing that A cannot be entered first. (Note: The program always calls for matrix A to be entered first; so merely rename the matrices to correspond to this convention when necessary).

As a secondary function of the program, two complex vectors such as  $C = (c_1 \ c_2 \ c_3)$  and  $C' = (c'_1 \ c'_2 \ c'_3)$  may be added or subtracted by treating C and C' as  $1 \times 3$  matrices. Their dot product  $C \cdot C'$  may be obtained by treating C as a  $1 \times 3$  matrix and forming their product:

treating C as a 1x3 matrix and
$$C \cdot C' = CC' = (c_1 \ c_2 \ c_3) / (c'_1) / (c'_2) / (c'_3)$$

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		173 X 429
PERIPHI	1 FRAIS	2 3
	DA MARKED CARD READER	9862A PLOTTER 9864A DIGITIZER
9861	A TYPEWRITER	9863A PAPER TAPE READER
STEP	DISPLAY	INSTRUCTIONS
1.		ERASE LOAD EXECUTE, then alternately insert magnetic cards and
		EXECUTE until completion of loading.
2.		(Optional) FIXED N n EXECUTE, or FLOAT N n EXECUTE.
3.		END RUN PROGRAM
4.	ROWS IN A?	$\underline{\mathbf{m}}$ RUN PROGRAM $\mathbf{m}$ = number of rows in A.
5.	A BY ROWS: RL	Rl a <sub>ij</sub> RUN PROGRAM Row by row until A is entered;
	A BY ROWS: IM	Im a <sub>ij</sub> RUN PROGRAM then RUN PROGRAM alone.
		Press the A and B keys in one of the following combinations (in Step 6 or 7)
		to designate the desired operation:
6.	A*B?	A x B RUN PROGRAM and go to Step 10, or
	•	$\underline{A} + \underline{B}  \underline{RUN  PROGRAM}  \text{and go to Step 11, or}$
		A - B RUN PROGRAM and go to Step 11, or
		RUN PROGRAM to go to Step 7.
7.	B*A?	B x A RUN PROGRAM and go to Step 8, or
		B + A RUN PROGRAM and go to Step 9, or
		B - A RUN PROGRAM and go to Step 9, or
		RUN PROGRAM to go back to Step 6.
8.	ROWS IN B?	$\underline{n}  RUN \ PROGRAM \qquad \qquad \underline{n} = number \ of \ rows \ in \ B.$
9.	B BY COLS: RL	Rl b <sub>ij</sub> RUN PROGRAM Column by column until B is entered;
	B BY COLS: IM	$\underline{\text{Im b}_{ij}}  \underline{\text{RUN PROGRAM}}  $ then go to Step 12.
10.	COLS. IN B?	$\underline{p}$ RUN PROGRAM $p = \text{number of columns in B.}$
11.	B BY ROWS: RL	Rl b <sub>ij</sub> RUN PROGRAM
	B BY ROWS: IM	Im b <sub>ij</sub> RUN PROGRAM Row by row until B is entered.
12.		The result is printed and then renamed A, replacing the old A in memory.
		Control returns to Step 6 and another operation may be performed with the
		new matrix A using a new incoming matrix B.

EXAMPL	ES	
Show that AB + C = $\begin{pmatrix} 37 + 26i \\ 39 - 4i \end{pmatrix}$ for		A By Rows
$A = \begin{pmatrix} 2 + 0i & 6 + 5i \\ 4 + 2i & 3 + 0i \end{pmatrix} , B = \begin{pmatrix} 2 - 3i \\ 6 - 0i \end{pmatrix} , and C = \begin{pmatrix} -3 + 2i \\ 7 + 4i \end{pmatrix}$	2i	2.0000 0.0000
(4+2i  3+0i), $(6-0i)$ , and $(7+4)$	41/	6.0000 5.0000
Note: Although not indicated in the User Instructions, if Im $a_{ij} = 0$ or Im $b_{ij} = 0$ , the program is set up to automatically make a 0 entry if RUN PROGRAM occurs alone when entry of either of these values is called for, thus saving the need to press the 0 key.		4.0000 2.0000 3.0000 0.0000
		B ROWS
		2.0000 -3.0000
		6.0000
		0.0000
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old A in the	he calculator.	40.0000 24.0000
		32.0000 -8.0000
This is the n	next matrix in	В
		Y ROWS
		-3.0000 2.0000
		7.0000
		4.0000
	Final answer	A+B Y ROWS
		37.0000 26.0000
		39.0000 -4.0000

# MEMORY ALLOCATION

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0 :
ENT "ROWS IN A?"
,R5;0→X+CH
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ČFG 13;ENT "A BY
ROWS: RL":R(X+1
→X2+4); IF FLG 13
IJMP 2H
0÷A;ENT "A BY RO
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±5)
3 *
XZR5+R2H
4:
0→X→R0;CFG 2;R5→
R3;R2+R4;SPC 2H
5 :
IF C≕01PRT "
A"
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IF C=1#PRT "
A-8"F
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(); a
IF CFLG 4=2;PRT
" A*B";
JMP 4H
<u></u>
IF C=21PRT "
8*A"F
10:
ÎF (C=3)FLG 4;
PRT A+B"
IJMP 2H
11
IF C=31PRT "
ar vestral
12: PRT "BY ROW
PRT " BY ROW
S"#SPC 1+YH
PRT_R((X+1+X)2+4
),R(2X+5);SPC 1;
SPC 2((Y(Y≭R2)+1
→Y)=1);JMP X=R2R
5 H
14:
1 + X + B   2 + A
15
SFG 4:CFG 13:
ENT "A*B?",C;IF
and the first of the fact that the first f

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FLG 13;CFG 13;
ENT "B*A?",C#
CFG 4#JMP FLG 13
= [7] [-
16:
IF CFLG 4=2;ENT
"COLS.IN B?",R3;
.IMP 3F
17:
IF C=2;ENT "ROWS
 IN 87", R4; JMP 2
18:
R5+R41R2+R3F
19:
RA+1+R(0+R1)+
29:
IF R0(R1+1+R1) #1
JUMP 3H
21#
PRT "
IF FLG 4; PRT "
  BY LOWS";SPC
1JJMP 2H
22:
PRT " BY COL
S."18FC 1H
23:
0+YFIF FLG 4#
ENT "B BY ROWS:
RL", Z, "B BY ROWS
: IM",Y;JMP 2H
24:
ENT "B BY COLS:
RL", Z, "B BY COLS
# IM", Yh-
25#
PRT Z,Y;SPC 1;
SPC 2(R1=R(3+(
FLG 4=0))}
26#
IF C=2/GTO 40H
2(R2R(FLG 4=0)-R
2+RFLG 4)+4+BF
28:
IF 2>C;-Z+Z;-Y+Y
29:
Z+RB+Z;Y+R(B+1)+
ΥH
30:
IF (C#3)(FLG 4=0
) #-Z+Z#-Y+YF
```

```
31:
Zarriyar (B+1) H
32:
IF R((FLG 4=0)+3
)>R1;GTO 20F
33:
IF R(FLG 4+3)>R0
#GTO 19H
34:
IF C#21GTO 44
354
2R2R5+Z10+XH
36:
IF FLG 4=0;R4+R5
SUMP 2H
371
R3+R2H
38#
IF (X+1+X)>2R2R5
igTo 4F
39:
R(X+5+Z)+R(X+5);
GTO -1H
40:
0+A; IF FLG 2;
JMP 2F
41:
0+R((A+1+A)+2R2R
5+5) | JMP A=2R(
FLG 4+4) R (FLG 4+
211
42:
0+AiSFG 2h
43:
IF (A+1→A)>R(3
FLG 4+2) (GTO 32H
44#
2(R2A-R2+R0)FLG
4+2(R2R0-R2+A)(
FLG 4=0)+4+XH
45:
2(R3A-R3+R1)FLG
4+2(R2R1-R2+A)(
FLG 4=0)+2R2R5+4
+BH
46:
ZRX-YR(X+1)+RB+R
B;YRX+ZR(X+1)+R(
B+1) +R(B+1) #GTO
-3-
47:
END H
```

This program will invert up to a  $9 \times 9$  matrix  $A = (a_{ij})$ ,  $a_{ij}$  real, on the basic 9820A and up to an  $18 \times 18$  matrix with the expanded internal memory  $^1$ . A modified Gauss-Jordan elimination technique is used and newly computed elements overlay the original matrix, saving storage space. In addition, a row pivot search is utilized to increase the accuracy.

After the inverse  $A^{-1}$  is obtained, a checking option is provided to invert  $A^{-1}$ , without keying in its elements, to obtain A again <sup>2</sup>. Comparison with the original A gives an indication of the inversion accuracy, part of the error due to the first inversion and part due to reinversion. This procedure is particularly recommended when the absolute value of the determinant,  $|\det A|$ , is small; this value may be obtained from register R0 after  $A^{-1}$  is printed.

<sup>&</sup>lt;sup>1</sup> To invert a 7x7, 8x8, or 9x9 matrix (respectively, 18x18 for the expanded internal memory), store END in line 45 leaving out the remaining lines. This provides the additional required storage area by eliminating the checking option.

 $<sup>^2</sup>$  The option only applies to matrices up to  $6 \times 6$  in size (respectively, up to  $17 \times 17$  for the expanded internal memory).

ROM BLOCKS		INTERNAL REGISTERS
		X 173 429
1 PERIPHERALS	•	2 3
	RKED CARD READER [	9862A PLOTTER 9864A DIGITIZER
9861A TYPE	EWRITER [	9863A PAPER TAPE READER
STEP	DISPLAY	INSTRUCTIONS
1.		ERASE LOAD EXECUTE
2.		Alternately insert magnetic cards and <u>EXECUTE</u> until NOTE 14 no longer appears, indicating completion of loading.
3.		(Optional) FIXED N n EXECUTE, or FLOAT N n EXECUTE as desired.
4.		If the matrix to be inverted is $7 \times 7$ or larger (or $18 \times 18$ for the expanded internal memory), GO TO 4 5 EXECUTE END STORE. This will annihilate the checking option portion of the program, giving the needed memory for the larger matrix.
5.		END RUN PROGRAM
6. SIZI	≘?	$\underline{n}  \underline{RUN \ PROGRAM} \qquad \qquad n = number \ of \ rows$
7. A(I,	J) ?	$\frac{a_{ij}}{a_{ij}}$ RUN PROGRAM (by rows) i, j = 1,2,,n. After all data is entered, the display blanks and calculations proceed. When completed, the elements of the inverse matrix $A^{-1}$ are printed by rows. $ \det A $ is in register R0 after printing.
8. CHE	ECK?	Only displayed and applicable for matrices less than $7x7$ in size (respectively, $18x18$ for the expanded internal memory). RUN PROGRAM to run the checking option and then return to Step 6 for a new problem. Otherwise, or if the matrix is larger than or equal to $7x7$ (respectively, $18x18$ ), return to Step 5 for a new problem.
DET	$\Gamma A = 0$	Displayed only when the program senses matrix A as being singular. No further processing can be done. Return to Step 5 for a new problem.

#### **EXAMPLES**

1.	Invert the third	order
	Hilbert Matrix	

$$A = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{pmatrix}$$

(13 /2 /3)	INVERSE OF
INVERSE OF	6.0000 5.0000
1,0000 .5000	9.0000 2.0000
. 3333	IS
.5000 .3333 .2500	0606 .1515
.3333 .2500 .2000	.2727 1818
IS	RE-INVERSION
9.0000	ΙS
-36.0000 30.0000	6.0000 5.0000
-36.0000 192.0000 -180.0000	9.0000 2.0000

$$A = \begin{pmatrix} 6 & 5 \\ 9 & 2 \end{pmatrix}$$

$$A = \begin{pmatrix} 6 & 8 \\ 3 & 4 \end{pmatrix}$$

TRUCCOCC NO

ROLL OF	inverse ur
6.0000	6.0000
5.0000	8.0000
9.0000	3.0000
2.0000	4.0000
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0606	6.66666667E 10
.1515	-1.333333333E 11
.2727	-5.000000000E 10
1818	1.000000000E 11

RE-INVERSION

The matrix A is singular and has determinant det A = 0. However, machine round-off caused calculation of the determinant to be det A = 6.000000000 E-11, thus allowing continuation of processing and giving the above erroneous results. This example indicates the worth of either examining the value | det A | in register R0 to be certain it is not approximately zero, or of reinverting the result. In this case, reinversion catches the error and displays
"DET A = 0". This example was specially selected and such occurrences can be expected to be rare. However, the user should be aware of the possibility. Since checking is so easy and desirable anyway, it was decided in writing the program not to abort calculations for all matrices with small determinants. This would in many cases eliminate matrices with perfectly valid inverses, and would unduly burden the user to rescale the data prior to entry.

Hilbert matrices are often use to test the accuracy of inversion techniques because they are ill-conditioned (a very small change in the coefficients produces a very large change in the solution).

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RE-INVERSION IS

# **MEMORY ALLOCATION**

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0:	22:	44:
SPC 8;ENT "SIZE?	. C→RBH	PRT R(ARC+X);
";A;CFG 1+	23:	GTO -4-
1 =	0+2H	45:
PRT " INVERSE	24:	a÷C;DSP "CHECK?"
OF";0+8H	IF (Z+1+Z)=B#	# SFG 1#STP H
2:	JMP 2H	voru ivoir r 46:
B+1>8;SPC 1F	25:	8
3	R(AC+Z)/R(AC+B)>	SPC 1;PRT " RE- INVERSION"H
Ö÷CH-	R(AC+Z) F	inversion e 47:
4 1	26:	``.' IF (C+1→C)>A;
Ċ+1→CH	IF Z⊭A;JMP -2H	GTO +8F
5 #	27:	48:
Ent "A(I.J)?".R(	1/R(AC+B) +R(AC+B	
AB+C) #PRT H	)	RCIJMP -1H
6.4	20:	49:
IF A#C;JMP -2H	a÷Y⊢	**
	291	50:
IF AFBIJMP -5H	ĪF (Y+1→Y)>A;	X+1+X;JMP RX=YH
8 :	GTO 9H	51:
_ 0 → 8 ; 1 → R 0 <del> </del>	30:	-RX+RX;IF X=C;
9:	IF Y=C#JMP -1H	JMP -3F
IF (8+1→8)>A;	31:	52:
GTO 36H	0+ZH	~~. X→Y≬Ø→ZH
10:	32:	53:
0-C+X+YH	IF (Z+1→Z)=B;	IF (Z+1+Z)>A;
11:	JMP 2H	JMP -4-
Y+1+Y; IF B=1;	33:	54:
JMP 5F	R (AY+Z) -R (AC+Z)R	R(AC+Z)→R0;R(AX+
12:	(AY+B)→R(AY+Z)H	2) >R(AC+2);R0>R(
0 + Z <del> -</del>	34:	AX+Z);JMP -1F
13:	IF Z#AJJMP -2H	55:
IF (Z+1+Z)>B-1;	35:	0+ZH
JMP 3F	-R(AY+B)R(AC+B)+	56:
1 4 8	R(AY+B);GTO -6H	 IF (Z+1→Z)>A;
IF Y=RZiJMP 5H	36:	GTO 8H
15:	0+C;SPC 1;PRT "	
JMP -2H	IS"H	
16#	37:	58:
R(AY+B)+RB;IF 0>	C+1→C;SPC 1H	Y+1+Y;JMP RY=ZH
RB:-RB+RB+	38:	59:
17:	IF (C>A)(FLG 1=0	IF Y=ZįJMP -3H
IF RB&X#JMP 2H	);JMP 7 <b>h</b>	60:
18:	39:	0 + C H
RB+X;Y+CH	0→Z;IF C>A;GTO 0	61:
19:	<del> </del>	C+1+C;R(AC+Z)+R0
IF Y≓A∮GTO -8H	40:	JR(AC+Y)⇒R(AC+Z)
20:	Z+1+Zh	;R0÷R(AC+Y);JMP
Γ(XX)RØ⇒RØ;IF C≠	41:	C=AH
0:JMP 2h	IE Z>A;JMP -4F	62:
21:	421	RZ÷RYįJMP -6H
DSP "DET A=0";	0 + X F	63:
STP H	don	END H
	X+1+X;JMP RX=ZH	

This program will invert up to a 5 x 5 matrix  $A=(a_{ij})$ ,  $a_{ij}$  complex, on the basic 9820A and up to a 12 x 12 matrix on the 9820A having the additional 256 internal registers. A modified Gauss-Jordan elimination technique is used and newly computed elements overlay the original matrix saving storage space. In addition, a row pivot search is utilized to increase the accuracy.

After the inverse  $A^{-1}$  is obtained, a checking option is provided to invert  $A^{-1}$ , without keying in its elements, to obtain A again<sup>2</sup>. Comparison with the original A gives an indication of the inversion accuracy, part of the error due to the first inversion and part due to reinversion. This procedure is recommended when possible. (Refer to the Real Matrix Inversion program for a discussion of the effect when the magnitude of the determinant is very small. This program does not include calculation of the determinant).

<sup>&</sup>lt;sup>1</sup> To invert a  $3 \times 3$ ,  $4 \times 4$ , or  $5 \times 5$  matrix (respectively,  $12 \times 12$  for the expanded 9820A), store END in line 47 of the program, leaving out the remaining lines. This provides the additional required storage area by eliminating the checking option.

<sup>&</sup>lt;sup>2</sup> The option only applies to  $1 \times 1$  and  $2 \times 2$  matrices (respectively,  $1 \times 1$ ,  $2 \times 2$ ,  $3 \times 3$ ,..., and  $11 \times 11$  for the expanded 9820A).

ROM BL	OCKS	INTERNAL REGISTERS
	1	2 3
PERIPHI 986	ERALS DA MARKED CARD READER	9862A PLOTTER 9864A DIGITIZER
_	A TYPEWRITER	9863A PAPER TAPE READER
STEP	DISPLAY	INSTRUCTIONS
1.		ERASE LOAD EXECUTE, then alternately insert magnetic cards and
		EXECUTE until completion of loading.
2.		If the matrix is $3 \times 3$ , $4 \times 4$ , or $5 \times 5$ (or $12 \times 12$ for the expanded 9820A),
		GO TO 4 7 EXECUTE END STORE. This will annihilate the checking
		option portion of the program, giving the needed memory for the larger
		matrix.
3.		(Optional) FIXED N n EXECUTE, or FLOAT N n EXECUTE as
		desired.
4.		END DIN DDOCDAM
<b>T.</b>		END RUN PROGRAM
5.	SIZE?	n RUN PROGRAM n = number of rows
6.	RL $A(I, J)$ ?	$ \frac{\text{Rl a}_{ij}}{\text{Im a}}  \frac{\text{RUN PROGRAM}}{\text{PROGRAM}}  \text{by rows} \qquad i, j = 1, 2,, n $
	IM $A(I,J)$ ?	* $\underset{*}{\text{Im }} a_{ij}$ RUN PROGRAM $\underset{*}{\text{by rows}}$ 1, j = 1,2,n
		After all data is entered, the display blanks and calculations proceed. When
		completed, the elements of the inverse matrix A-1 are printed by rows (real
		parts first with imaginary parts below them).
7.	CHECK?	Only displayed and applicable for 1x1 or 2x2 matrices (respectively, 1x1,
		2 x 2,, 11 x 11 for the expanded 9820A). RUN PROGRAM to run the
		checking option and then return to Step 5 for a new problem. Otherwise, or
		if the matrix is larger than 2x2 (respectively, 11x11), return to Step 4 for
		a new problem.
	DET A = 0	Displayed only when the program senses matrix A as being singular. No
	DEI II-0	further processing can be done. Return to Step 4 for a new problem.
		intener processing can be done. Return to step 4 for a new problem.
		*NOTE: If Im $a_{ij} = 0$ , RUN PROGRAM alone will automatically make the 0
		entry, saving the need to press the 0 key.

## **EXAMPLES**

# 1. Invert and check the 2x2 matrix

$$A = \begin{pmatrix} 1+i & 0+2i \\ -1+i & 1-i \end{pmatrix}$$

INVERSE OF

1.0000 1.0000

0.0000 2.0000

-1.0000

1.0000

1.0000

-1.0000

IS

.1000 -.3000

-.2000

-.4000

.1000 -.3000

.3000

.1000

RE-INVERSION

IS

1.0000

1.0000

0.0000

2.0000

-1.0000

1.0000

1.0000

-1.0000

# 2. Invert the 3x3 Hilbert matrix

A = 
$$\begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{pmatrix}$$
 Here, all imaginary parts are 0.

Hilbert matrices are often used to test the accuracy of inversion techniques as they are ill-conditioned (a very small change in the coefficients produces a very large change in the solution).

INVERSE OF	IS
1.0000	9.0000
0.0000	0.0000
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0.0000	0.0000
.3333	30.0000
0.0000	0.0000
.5000	-36.0000
0.0000	0.0000
.3333	192.0000
0.0000	0.0000
.2500	-180.0000
0.0000	0.0000
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.2000	180.0000
0.0000	0.0000

# MEMORY ALLOCATION

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IF A=B;JMP -5H
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9 #
IF (B+1+B)>A;
GTO 39H
10:
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JMP 5F
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IF (Z+1+Z)>B-1;
JMP 3E
14:
IF Y=RZ;JMP 5H
15:
JMP -2H
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171
IF ROCXIJMP 2H
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IF Y#A;GTO -8H
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IF C#0#JMP 2h
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21:
DSP "DET A=0";
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IF (Y+1+Y)>A#
GTO 9H
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0+ZH
33#
IF (Z+1+Z)=B;
JMP 3H
241
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35:
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)R(AY+B)-R(X+B)R
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36#
IF Z#A;JMP -3H
37:
R(X+B)R((AC+B+X)
+AA)-R(AY+B)RX+X
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38:
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0+C;SPC 1;PRT "
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C+1+C;SPC 2H
41:
IF (C)A)(FLG 1=0
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421
0+Z;IF C>A;GTO 0
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43:
IF (Z+1+Z)>A;
JMP -3H
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JMP R(X+1+X)=ZH
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PRT R(ARC+X+B),R
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JMP -3H
47:
0 + C : DSP "CHECK?"
ISFG LISTP H
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49#
IF (C+1+C)>A;
JMP 9H
50:
IF R(C+Y) 40;-RC+
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51:
0+XF
52:
JMP R(X+1+X)=YF
53:
-RX+RX(IF X=C)
JMP -3H
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X \rightarrow Y = 0 \rightarrow Z +
55#
IF (Z+1+Z)>A;
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	PROGRAM LISTING	
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# N SIMULTANEOUS LINEAR EQUATIONS WITH REAL COEFFICIENTS IN N UNKNOWNS, N $\leq$ 9, N $\leq$ 18

This program solves a system of up to nine simultaneous linear equations with real coefficients in nine unknowns (up to eighteen with the expanded internal memory):

It is assumed that the determinant det  $(a_{ij}) \neq 0$ , i, j = 1, 2, ..., n. If this is not the case, the message DET(AIJ) = 0" is printed and displayed and the program is terminated.

The program uses a modified Gauss-Jordan method. A row pivot search is performed to maximize the accuracy of the results.

ROM BLOCKS	INTERNAL REGISTERS  X 173
1 PERIPHERALS 9860A MARKED CARD READER [ 9861A TYPEWRITER [	2 3  9862A PLOTTER
STEP DISPLAY	INSTRUCTIONS
1.	ERASE LOAD EXECUTE
2.	Alternately insert magnetic cards and <u>EXECUTE</u> until NOTE 14 no longer appears, indicating completion of loading.
3.	(Optional) FIXED N n EXECUTE, or FLOAT N n EXECUTE as desired.
4.	END RUN PROGRAM
5. HOW MANY EQS.?	n RUN PROGRAM
6. A(I, J)?	$\begin{array}{lll} \underline{a_{ij}} & \underline{\text{RUN PROGRAM}} & \text{(by rows)} & \underset{1}{\overset{1}{\leq}} i \overset{\leq}{\leq} n \\ 1 \overset{\leq}{\leq} j \overset{\leq}{\leq} n+1 \\ \\ \text{After all $a_{ij}$ have been entered, calculations begin automatically and continue} \\ \text{until $x_1, x_2, \ldots, x_n$ are found and printed out starting with $x_1$. Control returns to Step 5 automatically for a new case.} \end{array}$
DET(AIJ) = 0	This message will be displayed and printed when the coefficient matrix on the left-hand side of the equation has zero determinant. The system cannot be processed. For a new case, return to Step 4.

# EXAMPLE

n = 5

$$2.1x_{1} - 3.6x_{2} + 5.8x_{3} - 7.0x_{4} - 1.2x_{5} = 12.8$$

$$3.7x_{1} + 5.1x_{2} - 7.8x_{3} - 9.5x_{4} - 3.5x_{5} = 9.8$$

$$1.8x_{1} + 2.2x_{2} + 6.5x_{3} + 3.2x_{4} - 2.5x_{5} = 15.9$$

$$-6.6x_{1} - 7.7x_{2} - 3.1x_{3} + 7.4x_{4} + 9.4x_{5} = 11.6$$

$$3.9x_{1} - 8.7x_{2} + 4.3x_{3} - 6.9x_{4} + 3.4x_{5} = 17.5$$

## NO. EQUATIONS

5.00000

## COEFFICIENTS

2.10000 -3.60000 5.80000 -7.00000 -1.20000 12.80000 3.70000 5.10000 -7.80000 -9.50000 -3.50000 9.80000 1.80000 2.20000 6.50000 3.20000 -2.50000 15.90000 -6.60000 -7.70000 -3.10000 7.40000 9.40000 11.60000 3.90000 -8.70000 4.30000 -6.90000 3.40000 17.50000

## SOLUTION

-.95180 24.10513 11.94184 -10.14219 32.23400

## MEMORY ALLOCATION

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10:
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PRT " DET(AIJ)
=0 ";SPC 8;STP H
12:
0+0+
13:
C+1+C; IF C>A;
GTO +2h
14:
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-A+C)+R(AY-A+C);
Z+R(AB-A+C);GTO
-- 1 |--
15:
\mathbb{B} + \mathbb{C} \vdash
16:
C+1+C; IF C>A;
GTO +2H
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R(AB-A+C)/R(AB-A
+B) +R(AB-A+C) #
GTO -1H
18:
0 + C +
19:
C+1+C; IF C=A;
GTO 6H
20#
IF B=C#GTO -1H
21:
Bayh
22:
Y+1→Y; IF Y>A;
GTO -3H
23#
R(AC-A+Y)-R(AC-A
+8)R(AB-A+Y) +R(A
C-B+Y) \in GTO -1 \vdash
24:
0→B;SPC 2;PRT "
   SOLUTION";
SPC 1H
25:
B+1+B; IF B=A;
GTO 0H
26:
PRT R(AB) (GTO -1
27:
END -
```



# N SIMULTANEOUS LINEAR EQUATIONS WITH REAL COEFFICIENTS IN N UNKNOWNS, N $\leq$ 17, N $\leq$ 36

This program solves a system of up to seventeen simultaneous linear equations with real coefficients in seventeen unknowns (up to thirty-six with the expanded internal memory):

The technique used  $^1$  is identical to the 9810A MATH PAC version  $^2$  and involves processing the elements as they are entered to reduce the coefficient matrix A in the matrix equation AX = Y to the identity matrix.

A system for which the determinant  $\det(a_{ij}) = 0$   $i, j = 1, 2, \ldots, n$  cannot be solved by the program and will cause "NOTE 10 IN 19" to be displayed, indicating an internal division by zero.

It is assumed that the equations are initially arranged so that  $a_{11} \neq 0$  or the same note as above will be displayed. If det  $(a_{ij}) \neq 0$  and  $a_{11} \neq 0$ , there is still a remote possibility that "NOTE 10 IN 19" may be displayed because of reduction of one of the divisors to zero. In this case, a rearrangement of the original equation (still maintaining  $a_{11} \neq 0$ ) will usually alleviate the problem.

When possible, use of Program I-5 is recommended in lieu of this one to take advantage of the pivot search for increased accuracy and to eliminate the limitation mentioned in the previous paragraph. The primary purpose of this program is to allow solving a large system of equations that could not otherwise be handled by a desktop calculator.

<sup>&</sup>lt;sup>1</sup> Based on a 9100 program submitted to Hewlett-Packard by Pavo Petricevic of the Food and Agriculture Organization of the United Nations, Rome, Italy, in turn based upon his paper, "A Method for Solving Simultaneous Linear Equations on Electronic Computers with Small Core Storage", presented at the Symposium on Use of Electronic Computers, in Belgrade, December, 1964.

<sup>&</sup>lt;sup>2</sup> Implemented by Dr. Jack Walden of the Calculator Labs at Hewlett-Packard.

ROM BI	LOCKS	INTERNAL REGISTERS   X 173   429
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	IA TYPEWRITER	
STEP	DISPLAY	ERASE LOAD EXECUTE
_,		
2.		Alternately insert magnetic cards and EXECUTE until NOTE 14 no longer appears, indicating completion of loading.
3.		(Optional) <u>FIXED N n EXECUTE</u> , or <u>FLOAT N n EXECUTE</u> as desired.
4.		END RUN PROGRAM
5.	N	n RUN PROGRAM
6.	DATA	$\frac{a_{ij}}{1}  \underline{\text{RUN PROGRAM}} \qquad \text{(by rows)} \qquad \frac{1}{1} \leq i \leq n \\ 1 \leq j \leq n+1$ Processing occurs as elements are entered. A slightly longer "wait time" is required at the end of each row. After the last entry, final processing occurs and $x_1, x_2, \ldots, x_n$ are printed in order and control is returned to Step 5 for a new case.
	NOTE 10 IN 19	Occurs when a division by zero is encountered. The program requires that $a_{11} \neq 0$ and det $(a_{ij}) \neq 0$ i, j = 1,2,n. If the note still occurs, try rearranging the equations (maintaining $a_{11} \neq 0$ ) and return to Step 4.

## EXAMPLE

n = 5

$$2.1x_{1} - 3.6x_{2} + 5.8x_{3} - 7.0x_{4} - 1.2x_{5} = 12.8$$

$$3.7x_{1} + 5.1x_{2} - 7.8x_{3} - 9.5x_{4} - 3.5x_{5} = 9.8$$

$$1.8x_{1} + 2.2x_{2} + 6.5x_{3} + 3.2x_{4} - 2.5x_{5} = 15.9$$

$$-6.6x_{1} - 7.7x_{2} - 3.1x_{3} + 7.4x_{4} + 9.4x_{5} = 11.6$$

$$3.9x_{1} - 8.7x_{2} + 4.3x_{3} - 6.9x_{4} + 3.4x_{5} = 17.5$$

- 2.10000 -3.60000 5.80000 -7.00000 -1.20000 12.80000 3.70000 5.10000 -7.80000 -9.50000 -3.50000 9.80000 1.80000 2.20000 6.50000 3.20000 -2.50000 15.90000 -6.60000 -7.70000 -3,10000 7.40000 9.40000 11.60000 3.90000 -8.70000 4.30000 -6.90000 3.40000 17.50000
- -.95180 24.10513 11.94184 -10.14219 32.23400

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1 :
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A+1+A;0+R0F
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                         R4+YH
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3 #
                         RZ-R3RY+RXH
PRT ;SPC (R0+1+R
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01)BH
                         1+X+X;1+Y+Y;1+Z+
4.:
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                         IF B+1-A=C#1+C+C
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IF R0>A;GTO 13H
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74
                         30:
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8 :
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GSB 39H
                         RY+RXH
9:
                         32:
GTO 11; IF A-1#C;
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GSB 40H
                         33:
10:
                         IF B+1-A#C;1+C+C
JMP -2H
                         JJMP -2H
11:
                         34:
R1+R3H
                         IF B>A;GTO 21H
12:
                         35#
R5+1+R5;GTO 2H
                         6+XISPC 2F
13:
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IF A=1;JMP 6H
                         PRT RXIJMP (1+X+
14:
                         X)=8+6H
GSB 38H
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15:
                         SPC 8;GTO 0H
G88 39H
                         38:
                         1+C;R5+Y;R4+Z;
16:
GTO 18; IF A-1#C;
                         RET H
GSB 40H
                         391
17:
                         R1-RZRY+R1;RET +
JMP -2H
18:
                         1+2+2;1+0+0;8+2-
R5+1+R5H
                         A+Y+YIRET H
19:
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R1/R3+RX;1+X+X;
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IF ROSBIGTO 2H
20:
IF A>1;JMP 2H
21:
X+R4;GTO 1H
22 #
1+C+R2;R4+A-1+R4
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```



# N SIMULTANEOUS LINEAR EQUATIONS WITH COMPLEX COEFFICIENTS IN N UNKNOWNS, N $\leq$ 4, N $\leq$ 11

This program solves a system of up to four simultaneous linear equations with complex coefficients in four unknowns (up to eleven with the expanded internal memory):

It is assumed that the determinant det  $(c_{ij}) \neq 0$ ,  $i,j=1,2,\ldots,n$ . If this is not the case, the message "DET C(IJ) = 0" is printed and displayed and the program is terminated.

The program uses a modified Gauss-Jordan method. A row pivot search is performed to maximize the accuracy of the results.

ROM BL	ocks	INTERNAL REGISTERS
		▼ 173
PERIPHE	1 =RAIS	2 3
	A MARKED CARD READER	9862A PLOTTER 9864A DIGITIZER
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STEP	DISPLAY	INSTRUCTIONS
1.	DISTER	ERASE LOAD EXECUTE
1.		ERASE LOAD EAECUTE
		All and the importance and conditional EVECUTE until NOTE 14 no longer
2.		Alternately insert magnetic cards and EXECUTE until NOTE 14 no longer
		appears, indicating completion of loading.
3.		(Optional) FIXED N n EXECUTE, or FLOAT N n EXECUTE as
		desired.
4.		END RUN PROGRAM
5.	HOW MANY EQS.?	n RUN PROGRAM
6.	RL C(IJ)?	$\frac{\text{Rl } c_{ij}}{\text{RUN PROGRAM}} \qquad \qquad 1 \leq i \leq n$
	IM C(IJ)?	* Im $c_{ij}$ RUN PROGRAM (by rows) $1 \le j \le n+1$
	•	After all data is entered, calculations proceed automatically. The answers
		are printed in order: $z_1, z_2, \ldots, z_n$ (real part of $z_i$ first, imaginary part
		of $z_i$ below it). Return to Step 5 for a new case.
		of $z_i$ below it). Return to step o for a new case.
	DEM C(II) 0	This message will be displayed and printed when the coefficient matrix on the
	DET $C(IJ) = 0$	
		left-hand side of the equation has zero determinant. The system of equations
		cannot be processed. For a new case, return to Step 4.
		· · · · · · · · · · · · · · · · · · ·
		* If Im $c_{ij} = 0$ , RUN PROGRAM alone will cause the program to automati-
		cally enter a zero, saving the need to press the "0" key.
		carry enter a zero, saving the need to press the or key.

#### EXAMPLE

Solve the following system of three equations in three unknowns:

$$(2 + 8i) z_1 + (-3 + 4i) z_2 + (4 + 6i) z_3 = 0 - 5i$$

 $(7 + i)z_1 + (9 - 8i)z_2 + (3 + 2i)z_3 = 5$ 

 $6z_1 + (8 + 7i)z_2 + (1 + i)z_3 = 4 - 5i$ 

NO. EQUATIONS

3.00000

COEFFICIENTS

2.00000

8.00000

-3.00000

4.00000

4.00000 6.00000

0.00000

-5.00000

7.00000 1.00000

9.00000

-8.00000

3.00000 2.00000

5.00000

0.00000

6.00000 0.00000

8.00000 7.00000

1.00000

1.00000

4.00000

-5.00000

## SOLUTION

1.80375  $Rl z_1$ -.10366 Im  $z_1$ 

-.45468  $Rl z_2$ 

.22895  $Im z_2$ 

-2.30466 -.72228  $R1 z_3$ 

 $Im z_3$ 

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IF A>(C+1+C);
JMP -2H
12:
IF X=0%SPC 1%
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13:
0+CH
14:
IF (C+1+C)>A#
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22:
X+R(AB+C+AA-2A);
JMP -3H
23:
0+CH
24:
IF (C+1+C) = Ai
GTO 7H
25:
IF B=C;JMP -1H
26:
Bayh
IF (Y+1+Y)>A;
JMP -3H
28:
R(AC+Y+AA-2A)-R(
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29#
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R(AC-A+Y)+R(AC+B
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AR-A+Y) +R/AC-A+Y
) |--
31:
JMP -4H
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32:

0+B;SPC 2;PRT "

SOLUTION";

SPC 1H

33:

IF (B+1+B)=A;

GTO 0H

34:

PRT R(AB) * R(AB-A

+AA);SPC 1;JMP -

1H

35:

END H
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# CHARACTERISTIC EQUATION

This program uses the method of Danilevsky to determine the coefficients in the characteristic equation det  $(A - \lambda I) = 0$  for a given nxn matrix A with real entries,  $n \le 9$  (respectively,  $n \le 17$  with a 9820A having the additional 256 internal registers). Since this is a polynomial equation in  $\lambda$ , Program II-2 may then be used to find the roots  $(\lambda)$  which are the eigenvalues of A.

A pivotal condensation scheme is incorporated in the program which increases the accuracy of the results.

Reference:

V.'N. Faddeeva, <u>Computational Methods of Linear Algebra</u> (New York: Dover Publications, Inc., 1959), pp 166-177.

ROM BLOCKS	INTERNAL REGISTERS
PERIPHERALS  9860A MARKED CARD READER	2 3 3
9861A TYPEWRITER	9863A PAPER TAPE READER
STEP DISPLAY	INSTRUCTIONS
1.	ERASE LOAD EXECUTE
2.	Alternately insert magnetic cards and <u>EXECUTE</u> until NOTE 14 no longer appears, indicating completion of loading.
3.	(Optional) FIXED N $\underline{n}$ EXECUTE, or FLOAT N $\underline{n}$ EXECUTE as desired.
4.	END RUN PROGRAM
5. N?	n RUN PROGRAM
6. DATA	$\frac{a_{ij}}{A}$ RUN PROGRAM (by rows) $1 \le i,j \le n$ After all elements of the matrix A have been entered, calculation of the coefficients of the characteristic polynomial proceeds automatically. Actually, partial or complete factoring of the characteristic polynomial may occur during the calculations. When this happens, the coefficients of each polynomial in the factorization will comprise the output rather than the coefficients of their product (i. e., the characteristic polynomial). A space will separate the sets of coefficients, and the first coefficient in each set is always 1.

#### **EXAMPLES**

Because of the inherent nature of the algorithm used, the characteristic polynomial is sometimes output in partially or fully factored form as shown in the second example.

$$\begin{array}{ccc} \mathbf{1.} & \mathbf{A} & = & \begin{bmatrix} \mathbf{1} & & \mathbf{2} \\ \mathbf{3} & & \mathbf{2} \end{bmatrix} \end{array}$$

$$0 = \det (A - \lambda I) = \det \begin{bmatrix} 1 - \lambda & 2 \\ 3 & 2 - \lambda \end{bmatrix} = \lambda^2 - 3\lambda - 4$$

The values printed out are 1, -3, -4, and no factoring occurs.

MATRIX

1.0000

3.0000 2.0000

COEFFICIENTS

1,0000 -3.0000

-4.0000

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & -5 \\ 0 & 1 & -2 \end{bmatrix}$$

$$0 = \det (A - \lambda I) = (\lambda^2 + 0\lambda + 1)(\lambda - 3)$$

In this example, the answer appears in partially factored form.

#### MATRIX

3.0000

0.0000

0.0000

0.0000

2.0000 -5.0000

0.0000

1.0000

-2.0000

#### COEFFICIENTS

1.0000

0.0000

1.0000

1.0000

-3.0000

## MEMORY ALLOCATION

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2:
                            20:
SPC 1;B+1+B+
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                            JMP 4F
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8#
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0+C+Zh
                           JMP -6H
9:
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IF (C+1+C)>B-1;
                           Z+R(AB-2A+C);
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                           JMP -7-
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IF Z>(R(AB-A+C)+
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Y)Y:JMP -1H
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YY+Z;C+R0;JMP -2
                           28:
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12:
                           +R(AB-A+B+Y)+RZ;
IF Z=0;B+C;JMP 1
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91-
                           291
13:
                           IF (B-1+B)>1#
1+Y; IF R0=B-1;
                           GTO 8H
JMP 4h
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Y-A+B-1) +R(AY-A+
                           PRT 1H
R0);Z+R(AY-A+B-1
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                           SPC 8;GTO 0H
JMP (Y+1+Y) > XH
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                           GTO 7; IF (B-1+X)
0+0-
                           =1;JMP -5h
                           36:
                           END H
```



# EIGENVALUES OF A MATRIX WITH REAL ENTRIES

For a given nxn matrix  $A = (a_{ij})$ ,  $a_{ij}$  real,  $n \le 8$ , this program will find the eigenvalues of A. If during the internal processing no factorization of the characteristic polynomial of A occurs, an eigenvector corresponding to each eigenvalue will also be printed. It is expected that for almost all practical applications, the eigenvectors will be obtained.

The method of Danilevsky¹ is used to obtain the characteristic polynomial of A which is then factored by the Newton-Raphson method² to get the roots of the characteristic equation which are the eigenvalues of A. Three check values are printed after the eigenvalues and give a measure of the accuracy of the results as the check values should all have the same value. If no factorization of the characteristic polynomial occurs, eigenvectors will be printed next, one for each of the preceding eigenvalues. The algorithm is such that the trailing coefficient of each eigenvector is normalized to 1.

References:

<sup>&</sup>lt;sup>1</sup> V. N. Faddeeva, <u>Computational Methods of Linear Algebra</u>, (New York: Dover Publications, Inc., 1959), pp. 166-176.

<sup>&</sup>lt;sup>2</sup> John M. McCormick, Mario G. Salvadori, <u>Numerical Methods in FORTRAN</u>, (New Jersey: Prentice-Hall, Inc., 1964), p. 196.

ROM BLOCKS	INTERNAL REGISTERS
	173 🔀 429
PERIPHERALS  9860A MARKED CARD READER  9861A TYPEWRITER	2 3  9862A PLOTTER 9864A DIGITIZER 9863A PAPER TAPE READER
STEP DISPLAY	INSTRUCTIONS
1.	ERASE LOAD EXECUTE
2.	Alternately insert magnetic cards and <u>EXECUTE</u> until NOTE 14 is no longer displayed, indicating completion of loading.
	g
3.	(Optional) FIXED N n EXECUTE, or FLOAT N n EXECUTE
4.	END RUN PROGRAM
5. MATRIX SIZE?	n RUN PROGRAM for an nxn matrix
6. ENTER DATA	a <sub>ij</sub> <u>RUN PROGRAM</u> row by row
7.	After all matrix elements have been entered, the display blanks while calculations are made. The eigenvalues are printed, real part first with imaginary part under it, followed by the three check values SUM (= sum of the eigenvalues), P1 (= coefficient of $\lambda^{n-1}$ in the characteristic equation), and TRACE (= trace, i.e., sum of the diagonal elements, of the original matrix), all of which should be approximately equal in value. If no factorization of the characteristic polynomial occurs internally during the calculations, an eigenvector corresponding to each eigenvalue will also be printed, and in the same order (i.e., the first eigenvector printed corresponds to the first eigenvalue printed, etc.). If $Z=(z_1,\ z_2,\dots,z_n)$ is an eigenvector, $z_1$ is printed first, then $z_2$ , etc., and $z_n$ will have the value 1. The real part of $z_i$ is printed with the imaginary part of $z_i$ under it. In any case, control returns to Step 5 for a new problem.
	Occasionally the root finding routine in unable to converge. Changing the convergence tolerance 1E -9 in line 52 may or may not enable convergence. In some cases, tightening this tolerance will give results with improved accuracy. Whatever value is used, the time to find a root should not exceed about two minutes.

#### EXAMPLE

Find the eigenvalues (and some corresponding eigenvectors) for the matrix

$$A = \begin{pmatrix} 4 & 2 & 7 \\ 0 & 6 & 5 \\ 1 & 8 & 3 \end{pmatrix}$$

#### MATRIX

4	4	$\odot$	0	Ø	0	0
2	11	Ø	Ø	0	0	0
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0.00000 6.00000 5.00000 1.00000

8.00000 3.00000

#### EIGENVALUES

.00000 Ist eigenvalue x		08151 00000	1st eigenvalue	λ
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-2.56114 2nd eigenvalue  $\lambda_2$ -.00000

11.47963 3rd eigenvalue  $\lambda_3$ .00000

SUM 13.00000 .00000

Check 13.00000 numbers

TRACE 13.00000

## EIGENVECTURS

-2.60621 an eigenvector for  $\mathbf{z}_{2}$ -.00000 the 1st eigenvalue

1.00000  $\mathbf{Z}_3$ 0.00000

-.88886  $\mathbf{z}_1$ -.00000

-.58403 an eigenvector for  $\mathbf{z}_{2}$ -.00000 the 2nd eigenvalue

1.00000

 $\mathbf{Z}_3$ 0.00000

1.17986  $\mathbf{z}_1$ .00000

> .91247 an eigenvector for  $\mathbf{z}_{2}$ .00000 the 3rd eigenvalue

1.00000  $\mathbf{Z}_3$ 0.00000

For

$$Z = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} ,$$

the relation

$$AZ = \lambda_i Z$$
 holds,

where  $\lambda_i$  is one of the three eigenvalues and Z is the corresponding eigenvector.

## MEMORY ALLOCATION

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JMP (Y+1+Y)>XF

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17:
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IF (C+1+C)>X#
JMP 8H
19:
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20:
IF (Y+1+Y)>B-1;
JMP 4H
21:
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C=1 i RR4/R (AB-A+B
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23:
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24:
IF C=B-1;Z+R6;
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Z+R(AB-2A+C+20);
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Z) \vdash
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0+YH
28:
RZ+R(Z+N+1)+RZ
+1+Z;JMP (Y+1+Y)
> X - P +
29#
1 + Y | AB - A + B + 19 + C +
30:
IF Y=B-1;AB-A+Y+
20+Z;-RZ/RC+RZH
31:
GTO -1H
32:
1/RC+RCH
33:
IF (B-1+B)>1;
GTO 8H
34:
1 + 0 -
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351
-1-RIAC-A+C+19+R
11); (R(R11+1) > R6
1+R13+R13#0+R7H
361
IF ((X-C+1+R9)+1)
+2)=2;GTO 56H
37:
1111AA+21+#121+Z
*R0)*Z*R1)*Z*R2)
+Z+R3F
38:
N-Z-RR1-RR3-R19;
-1+RR0+RR2H
391
Z+1+Z)>R9h
40:
0+R6; IF ((1+R7)+
R19+R19)>X-C+1;
GTO 58H
41:
1+YH
421
R(R11+Y)+(R((R0+
Y + Z = 1 + R4 + R6 - (R
((R1+Y+R8)-1)+R5
)RZ+RZH
43:
R(R12+Y)+R4R7+R5
R6+RR8H
44:
IF (Y+1+Y) (R9#
GTO -2H
451
1 + 1/1-
461
R(R0+Y)+(R((R2+Y
+2)-1)+R4)R6-(R(
(R3+Y+R8)-1)+R5)
RZ+RZH
47:
R(R1+Y)+R4R7+R5R
A+RRSH
48:
IF R9>(Y+1+Y);
GTO -2H
49.
(R(R2+R9-1)+Z)Z+
(R(R3+R9-1)+Y)Y+
R10H
```

Colour pay may he		- mil B	m. a.e
		671	841
	R6-((R(R0+R9)+R8	IF (Y+1+Y)=A;	END H
	)Z+(R(R1+R9)+R4)	GTO +4H	
	Y)/R10+R6H	68:	
	51:	R((R3+A+2C+Z)-1)	
	R7+(R8Y-R4Z)/R10	- → MH	
1	→R7F		
	52:	(R((R1-Y+R4)+1)+	
		B)X-(R((R0-Y+R5)	
÷			
à	9;GTO 41H	+1) +R6)RZ+RR4H	***
1	53:	70:	
	R9-1+R9H	BRZ+R6X+RR5;GTO	
3	54:	-3-	
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3 3		A÷Y⊢	
1	R(R0+Y) +R(R11+Y)	72:	
	(R(R1+Y)→R(R12+Y	0 ÷ R 4 + R 5 + X H	
Y .	);JMP (Y+1+Y)>R9		
		73:	
		IF (X+1+X)>A;	
1	56:	JMP 3H	
1	SPC 1;PRT R6.R7H	74:	
	57:	(R(A(A-Y+1)+X+20	
	(R6+R((R3+A+2R19	) → Z ) R (R1-A+X)+R4	
	+Z)-1))+R15+R15;	→ R 4 F	
	(R7+RZ)+R16+R16;	75:	
i i	GTO 40H	ZR(R0-A+X)+R5+R5	
1		GTO -2H	
Š			
	IF C=1;SPC 1;	76#	
	JMP 3H	IF (Y-1→Y)=0;	
	59:	JMP 5H	
	IF (8-1+X)=1;	77:	
1	GTO 34H	R4+R(R1-Y);R5+R(	
į.	60:	R0-Y)+	
	GTO 7H	78:	
	61:	iF (R(AA+8A-Y+25	
1	PRT "SUM"		
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8	P1",R13,"","	79:	
i.	TRACE"+R14	$R(R1+Z) \Rightarrow R(R1-Y);$	
	#SPC 2H	R4+R(R1+Z)+	
à	62#	80:	
1	IF FLG 1(A#1)!	R(R0+Z)+R(R0-Y);	
3	GTO 0H	R5+R(R0+Z);GTO 7	
3	63:	2 <b>h</b>	
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	IF (C+1+C)>A;	-Y);SPC 1;JMP 0+	
*	GTO ØH	83:	
c 6	66:	PRT "	
1	0+RR0+Y;1+RR1H	";sPC 1;GTO 65H	
		***	



#### POLYNOMIAL ARITHMETIC

With this program, two polynomials P(X) and Q(X) with real coefficients may be added, subtracted, multiplied or divided. For the basic 9820A the size constraints are:

Addition: deg  $P(X) + deg Q(X) + max (deg P(X), deg Q(X)) \le 55$ 

Subtraction: deg P(X) + deg Q(X) + max (deg P(X), deg Q(X))  $\leq 55$ 

Multiplication:  $deg P(X) + deg Q(X) \le 27$ 

Division:  $deg P(X) \le 27$ 

For example, two polynomials of degree 18 may be added or subtracted and two polynomials of degree 13 may be multiplied. Any polynomial P(X) of degree  $\leq 27$  may be divided by a non-zero polynomial of the same or lesser degree; attempt to divide by a polynomial of greater degree causes the message "deg  $Q > \deg P$ " to be displayed.

ROM BL	OCKS	INTERNAL REGISTERS  X 173 429
o E D I D LI E	1	2 3
PERIPHE 9860	IA MARKED CARD READER	9862A PLOTTER 9864A DIGITIZER
9861	A TYPEWRITER	9863A PAPER TAPE READER
STEP	DISPLAY	INSTRUCTIONS
1.		ERASE LOAD EXECUTE
2.		Alternately insert magnetic cards and EXECUTE until NOTE 14 is no longer
2.		displayed, indicating completion of loading.
3.		(Optional) FIXED N n EXECUTE, or FLOAT N n EXECUTE as
		desired.
4.		END
5.		RUN PROGRAM
		a n
6.	P(X) COEFF'S?	$\frac{a_i}{a_0} = \frac{\text{RUN PROGRAM}}{\text{RUN PROGRAM}}  \text{for } i = 0, 1, \dots, n \text{ and } P(X) = a_0 + a_1 X + \dots + a_n X^n.$
		$\underline{\text{RUN PROGRAM}}$ after entering all $a_i$ .
7.	Q(X) COEFF'S?	$\frac{b_i}{a}$ RUN PROGRAM for $i = 0, 1,, m$ and $Q(X) = b_0 + b_1 X + + b_m X^m$ .
		RUN PROGRAM after entering all b <sub>i</sub> .
8.	P(X)*Q(X)?	RUN PROGRAM to cycle between these four displays;
	P(X)/Q(X)?	1 RUN PROGRAM to select the option being displayed.
 	P(X)+Q(X)?	
	P(X)-Q(X)?	
9.		The coefficients of the resultant polynomial $H(X) = h_0 + h_1 X + + h_k X^k$ and,
		in the case of division, the remainder polynomial $R(X) = r_0 + r_1 X + + r_l X^l$ ,
		are printed out in order: $h_0$
		$h_1$
		÷
		$^{ m h}_{ m k}$
		$\mathbf{r_0}$
		$\mathbf{r_1}$
		:
		$\mathbf{r_{l}}$
		For a new case, return to Step 5.
		•

## **EXAMPLES**

$$P(X) = 4 + 1X + 3X^{2}$$
  
 $Q(X) = 2 + 7X$ 

#### INPUT

(Must be reentered for each case below)

P(X) COEFFICIENTS 4.0000  $\mathbf{a}_{\mathsf{o}}$ 1.0000  $a_1$ 3.0000  $\mathbf{a}_{\mathbf{z}}$ Q(X) COEFFICIENTS  $b_{\mathfrak{0}}$ 2.0000 7.0000

 $b_1$ 

.0204

.4286

3.9592

R(X) COEFFICIENTS

 $h_0$ 

 $\mathbf{r}_{0}$ 

## OUTPUT

Addition	Subtraction				
H(X)=P(X)+Q(X)	H(X) = P(X) - Q(X)				
H(X) COEFFICIENTS	H(X) COEFFICIENTS				
$egin{array}{lll} 6.0000 & h_0 \\ 8.0000 & h_1 \\ 3.0000 & h_2 \end{array}$	2.0000 h <sub>0</sub> -6.0000 h <sub>1</sub> 3.0000 h <sub>2</sub>				
Multiplication	Division				
H(X)=P(X)*Q(X)	P=QH+R				
H(X) COEFFICIENTS	H(X) COEFFICIENTS				

8.0000 30.0000

13.0000

 $h_0$ 

 $h_1$ 

hz  $h_3$ 

## **MEMORY ALLOCATION**

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ENT "P(X) COEFF'
89",R(X+1+X); [F
FLG 13=0;PRT ;
JMP 0H
2:
0 → Y; CFG 13 | SPC 2
FPRT "
             Q(X)
"#GSB 19h
3 :
FNT "Q(X) COEFF"
S?",R(Y+1+Y+X);
IF FLG 13=0;PRT
jump øh
4 #
CFG 13;0→A;ENT
P(X) * Q(X) ?", Z;
IF FLG 13=0;SPC
3;PRT " H(X)=P(X
) *Q(X)"; JMP 8H
5:
CFG 13;CFG 1;
ENT "P(X)/Q(X)?"
, Z ; IF FLG 13=0;
SFG 11JMP 15H
6.5
CFG 13%ENT "P(X)
+Q(X)?",Z;1+Z;
IF FLG 13=0;SPC
3!PRT " H(X)=P(X
)+Q(X)";JMP 3F
7 :
CFG 13;ENT "P(X)
-Q(X)?",Z;-1+Z;
IF FLG 13;JMP -3
ļ....
8#
SPC 3;PRT " H(X)
= P(X) - Q(X) " + 
9:
IF Y> (X+C) # Y+CH
10:
IF (A+1+A)>C#
JMP 18H
11:
(A \le Y) (A \le X) ((ZR (A
+X) +B) +RA) + (A>Y)
RA+(A)\times)B+R(A+X+
Y) #GTO 10H
```

```
X+Y-1+C+
13#
0+R((A+1+A)+C+1)
FUMP A=CH
14:
0 + A F
15:
IF (A+1→A)>X#
.IMP 13-
164
A+AH
17#
IF (B+1+B)>Y#
JMP -2H
18:
A+B+C+Z;RAR(B+X)
+RZ+RZ;GTO 17h
19:
PRT " COEFFICIE
MTS"; SPC 1; RET H
20:
IF (X-Y+1→B)≤0;
DSP "DEG Q>DEG P
"#JMP 14F
21#
SPC SIPRT "
P=OH+R"h
22:
Y-1+ZH
23:
R(Z+B)/R(X+Y)+R(
X+Y+B);0+AF
24:
A+B+C+IF (A+1+A)
<Z:RC-R(X+Y+B)R(
X+A) +RCIJMP 0H
25:
IF (B-1+B)>0;
JMP -2H
26:
IF (Z>0)(RZ=0);Z
-1+2 # JMP 0 h
27:
X-Y+1+CH
28#
1→A;SPC 3;PRT "
     H(X)"; GSB 1
C) |...
29:
PRT R(X+Y+A);
JMP (A+1+A)>CH
```

```
30:

IF FLG 1=0:JMP 4

H

31:

1+0:SPC 2:PRT "

R(X)":GSB 1

9L

32:

IF Z=0:PRT Z:

JMP 2L

33:

PRT RA:JMP (A+1+

A) 2 ZL

34:

SPC 8L

35:

END L
```



This program\* finds all roots for polynomials of the form  $a_0 + ib_0 + (a_1 + ib_1)z + (a_2 + ib_2)z^2 + \ldots + (a_n + ib_n)z^n = 0$  for  $n \le 8$  on the basic 9820A and  $n \le 72$  on the expanded 9820A.

Roots are found using the method of steepest descent detailed in the reference. Once a root is found, the polynomial is reduced by synthetic division and another root is extracted. The final root is determined algebraically. The algorithm is very accurate and very stable; it will virtually always find the roots. Multiple roots will be found at some slightly reduced accuracy (see EXAMPLES), and because of the synthetic division, very high order polynomials may show some loss in accuracy as more roots are found; but, in general, the program will find "normally" spaced roots accurate to better than 6 decimal places.

Reference: J. B. Moore, "A Convergent Algorithm for Solving Polynomial Equations", <u>Journal</u> of the Association for Computing Machinery, Volume 14, Number 2 (April, 1967).

<sup>\*</sup> Written by Larry Choice of Burr-Brown Research Corporation, Tucson, Arizona, with some step-saving modifications by Hewlett-Packard to allow higher order polynomials. The program is based on a FORTRAN program written by Dr. Jack Walden of Hewlett-Packard Calculator Labs, using the algorithm in the following reference:

#### REMARKS

- 1. Two convergence factors are used in the program (in lines 20 and 24) but experience has shown that these should not be changed: doing so has either reduced the accuracy or increased the computation time. The values used seem to be optimum for a 12 digit floating point machine. If after 50 iterations (line 21 of the program) no root is found, the value 88 is printed and processing halts. The user may press  $25 \longrightarrow R4$  RUN PROGRAM and obtain 25 more iterations. The other convergence factor is associated with too large a step size in the direction of a root. If 10 successive quarterings of the step size (line 23 of the program) does not adequately reduce the step size, the value 99 is printed and processing halts. This would be a very rare occurrence.
- 2. Execution time varies with the order of the polynomial. For low order polynomials ( $n \le 5$ ) about 10 to 15 seconds are required to find each root. A higher order polynomial such as  $x^{60} + 1 = 0$  will require about 5 minutes for the first roots, and this time decreases as the order of the polynomial is reduced by the synthetic division. Repeated roots and closely spaced roots take longer to find.
- 3. The coefficients  $a_i$  and  $b_i$ , i = 0, ..., n are stored according to the formulas:

$$a_{i} \longrightarrow R(8+i)$$
  
 $b_{i} \longrightarrow R(9+n+i)$   
 $i = 0, 1, ..., n$ 

To correct a coefficient after it has been entered, use one of the above formulas to locate the appropriate "R" register and store the correct value of the coefficient, e.g.,  $\underline{a_1} \longrightarrow \underline{R} \ \underline{9} \ \underline{EXECUTE}$  and then continue entering the rest of the coefficients in the normal manner following the User Instructions.

ROM BL	оскѕ	INTERNAL REGISTERS
		X 173  429
PERIPHE	1 ERALS	2 3
	DA MARKED CARD READER	9862A PLOTTER 9864A DIGITIZER
9861	A TYPEWRITER	9863A PAPER TAPE READER
STEP	DISPLAY	INSTRUCTIONS
1.		ERASE LOAD EXECUTE
2.		Alternately insert magnetic cards and EXECUTE until NOTE 14 no longer
		appears, indicating completion of loading.
3.		(Optional) FIXED N n EXECUTE, or FLOAT N n EXECUTE as
		desired.
4.		END RUN PROGRAM
5.	N	n RUN PROGRAM
		3.
6.	A	$ \frac{\mathbf{a_k}}{\mathbf{b_k}}  \frac{\text{RUN PROGRAM}}{\text{PUN PROCRAM}} \left( \begin{array}{c} \text{for } \mathbf{k} = 0, 1, \dots, n \text{ and} \\ \mathbf{P(z)} = \mathbf{a_0} + \mathbf{ib_0} + (\mathbf{a_1} + \mathbf{ib_1})\mathbf{z} + \dots + (\mathbf{a_n} + \mathbf{ib_n})\mathbf{z}^n \end{array} \right) $
7.	В	K RON PROGRAM
		After all coefficients have been entered, the calculator stops with the value
		n+1 in the display. The purpose of this stop is to allow the input data to be
		checked and manually corrected by overstoring the correct value in the
		proper register prior to going to Step 9. Refer to the REMARKS section for locating registers.
		101 locating registers.
9.		RUN PROGRAM
		The roots are printed, real part first, imaginary part under it, and
		numbered using a counter that goes from n to 1. For a new case,
		return to Step 4.
		*NOTE: If $b_k = 0$ , RUN PROGRAM alone will cause a 0 to be entered
		automatically for $b_k$ , saving an extra key stroke.

## **EXAMPLES**

# 1. Find the roots of

$$P(z) = (0 + 2i) \ + \ (2 - 3i)z \ + \ (-3 + 1i)z^2 \ + \ (1 + 2i)z^3 \ + \ (2 - 3i)z^4 \ + \ (-3 + 1i)z^5 \ + \ (1 + 0i)z^6$$

0.00000 0.00000 1.00000 2.00000 -3.00000 -3.00000 1.00000 1.00000 4.00000 -3.00000 -3.00000 -3.00000		2. Find the roots $P(z) = 105 + 10$	of $5z + 45z^2 + 10z^3 + z^4$
6.00000 1.00000 0.00000		1(2) - 100 + 10	0.00000
6.00000			105.00000 0.00000
.59090 .86603	Real Part Imaginary Part		1.00000 105.00000
5.00000 .50000 86603	:		0.00000 2.00000 45.00000 0.00000
4.00000 1.00000 .00000			3.00000 10.00000 0.00000
3.00000 .00000 -1.00000			4,09000 1,00000 0.00000
2.00000 -1.00000			Roots:
90999 1.99999 2.99999 .99999		Real Part Imaginary Part : :	4.00000 -2.89621 .86723 3.00000 -2.89621 86723 2.00000 -2.10379 2.65742 1.00000

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20:
IF R0R041E-10;
TF R1R1(1E-10)
JMP 15H
21:
IF R4>50;PRT 88;
STP -
22:
R(11+2C) +R6;R(12
+3C) +R7;GTO 7H
23#
IF R5410;R0/4+R0
#R1/4→R1#GTO 15H
24:
IF YYK1E-8; IF ZZ
61E-8;JMP 11H
25#
PRT 99;STP H
26#
(2R((2)X)C+11)Y
)Y/4+(R(12+3C)+Z
) Z + Z +
27:
YR((8+2C+X+A)+1)
-ZRA+R(A+2)+
28#
YR((A+C+1+A)+1)-
ZRA+R(A+2)F
29:
IF (X+1+X)≤C;
JMP -2H
30:
0+X+Y+Zh
31:
(-R(((9+C+X+B)+C
+1+A)+C+1)+R2)RB
+R(8+X)RA+Y+YH
32:
Z-R(8+X)R2+RARB+
21-
33:
IF (X+1+X) (C)
JMP -2H
34:
YY+ZZ+R2;RET H
25:
SPC JPRT COR(11+
2C),R(12+3C)F
361
(-1 + X -
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```
(R(11+2C) \rightarrow A)R(9+
X)-R(12+3C+Z)(R(
(9+C+X+Y)+1)+B)+
R(8+X) +R(8+X) H
98:
RZR(9+X)+AB+RY+R
Y |--
39:
IF 0≤(X-1→X);
JMP -2H
401
R((8+X)+C)+ZF
41:
R(X+1)+RXIR((X+0)
+A)+2)+RAH
42:
IF (X+1+X)-74C;
JMP -1-
43:
Z+R(7+C); IF (C-1
→C) #1; GTO 4F
44#
TR11+X1X+R9R9+ZH
45:
0+C)X+R8R9)/Z,(R
8X-R9C)/Z#SPC 9H
46:
END H
```



# POLYNOMIAL EVALUATION, NORMALIZATION, OR CONSTRUCTION

Three options are available in this program:

- 1. Evaluate a polynomial  $P(z) = c_0 + c_1 z + \ldots + c_n z^n$ , deg  $P(z) \le 29$ , where the  $c_k$  and z may be real or complex. The notation  $c_k = Rl(c_k) + i \operatorname{Im}(c_k)$  and  $z = Rl(z) + i \operatorname{Im}(z)$  is used where  $\operatorname{Im}(c_k)$  and  $\operatorname{Im}(z)$  are taken as 0 in the real case.
- 2. Normalize the coefficients  $c_i$  of  $P(z) = c_0 + c_1 z + \ldots + c_n z^n$ , deg  $P(z) \le 29$ , by dividing each  $c_i$  by  $c_n$ . Both real and complex values are allowed for the  $c_i$ .
- 3. Construct the coefficients  $c_0$ ,  $c_1$ , . . . ,  $c_n$  of P(z) given the roots  $r_1$ ,  $r_2$ , . . .  $r_n$  of P(z),  $n \le 29$ . In general, the root  $r_k$  is given by  $r_k = \text{Rl}(r_k) + i \, \text{Im}(r_k)$  with  $\text{Im}(r_k)$  specified as 0 when  $r_k$  is real.

ROM BL	ocks	INTERNAL REGISTERS
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PERIPHI	1 ERALS	3
	DA MARKED CARD READER	9862A PLOTTER 9864A DIGITIZER
9861	A TYPEWRITER	9863A PAPER TAPE READER
STEP	DISPLAY	INSTRUCTIONS
1.		ERASE LOAD EXECUTE
2.		Alternately insert magnetic cards and EXECUTE until NOTE 14 is no
		longer displayed, indicating completion of loading.
		ionger uniproject, indicating compression or remaining.
3.		(Optional) FIXED N n EXECUTE, or FLOAT N n EXECUTE as
0.		desired.
		desifed.
4.		END
4.		END
_		DAM DOCO M
5.		RUN PROGRAM
c		DATA DECCE AND As a sale between these three displaces. 1. DATA DECCE AND As
6.	EVALUATE P(Z)?	RUN PROGRAM to cycle between these three displays; 1 RUN PROGRAM to
	NORMALIZE P(Z)?	select the option being displayed. Go to Step 11 if constructing $P(z)$ .
	CONSTRUCT P(Z)?	
	4.5	Rl(c <sub>i</sub> ) RIIN PROGRAM
7.	RL(C)	$for i = 0, 1,, n \text{ where } P(z) = c_0 + c_1 z + + c_n z^n$
8.	IM(C)	Miles i' RUN PROGRAM )
		After all ci have been entered, RUN PROGRAM. If normalizing, the norma-
		lized coefficients are printed and control is returned to Step 6. If evaluating
		P(z), proceed with Step 9.
9.	RL(Z)	RI(z) RUN PROGRAM
10.	$\mathbf{IM}(\mathbf{Z})$	* Im(z) RUN PROGRAM
		P(z) is printed for the given $z$ and control is returned to Step 9. Another $z$
		may then be processed or control returned to Step 6 by RUN PROGRAM.
11.	RL(R)	$\frac{\text{Rl}(\mathbf{r}_i)}{\text{RUN PROGRAM}}$ for $i = 1, 2,, n$ where $\mathbf{r}_1, \mathbf{r}_2,, \mathbf{r}_n$ are the roots
12.	IM(R)	* $\frac{\text{Rl}(\mathbf{r_i})}{\text{Im}(\mathbf{r_i})}$ $\frac{\text{RUN PROGRAM}}{\text{RUN PROGRAM}}$ for $i = 1, 2,, n$ where $\mathbf{r_1}, \mathbf{r_2},, \mathbf{r_n}$ are the roots of $\mathbf{P}(\mathbf{z})$ .
		After all r <sub>i</sub> have been entered, RUN PROGRAM. The coefficients
		$c_{\scriptscriptstyle 0},c_{\scriptscriptstyle 1},\ldots,c_{n}$ of $P(z)$ are printed in order (real part first with imaginary
		part underneath) and control is returned to Step 6.
:		
		* NOTE: If $Im(c_i)$ , $Im(z)$ , or $Im(r_i)$ is 0, no entry need be keyed in;
		RIIN PROGRAM only and 0 will be assumed for the imaginary entry.

Evaluate $P(z) = 5 - 3z + 6z^{2} \text{ at } z = 8$	Normalize $P(z) = 5 - 3z + 6z^2$	Construct $P(z)$ from $r_1 = 2$ , $r_2 = 3$ , $r_3 = -7$
(2) 0 02   02 10 2 0	1 (2) = 0 - 02 + 02	$\Gamma(2)$ from $\Gamma_1 = 2$ , $\Gamma_2 = 3$ , $\Gamma_3 = -7$
P(Z) COEFFICIENTS	P(Z) COEFFICIENTS	GIVEN ROOTS OF P(Z) ARE
5.0000 0.0000	5.0000 0.0000	2.0000 0.0000
-3.0000 0.0000	~3.0000 0.0000	3.0000 0.0000
6.0000 0.0000	6.9000 0.0000	-7.0000 0.0000
AT Z = 8.0000 0.0000 P(Z) IS 365.0000 0.0000	NORMALIZED P(Z) COEFFICIENTS .8333 0.0000 5000 0.0000 1.0000	CONSTRUCTED P(Z) COEFFICIENTS 42.0000 0.0000 -29.0000 0.0000 2.0000 1.0000 0.0000
Evaluate $P(z) = (6-5i) + (4+2i)z + (-7+i)z^2$ at $z = 6+3i$	Normalize $P(z) = (6-5i) + (4+2i)z + (-7+i)z^2$	Construct $P(z) \text{ from } r_1 = 3 + 6i, r_2 = 7 + 4i$
P(Z) COEFFICIENTS	P(Z) COEFFICIENTS	GIVEN ROOTS OF P(Z) ARE
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Й: Ø→A→CF " CFG 13; CFG 1; FNT "EVALUATE P( Z)?",Z;IF FLG 13 =0;SFG 1;JMP 12h 2: CFG 13; ENT "NORM ALIZE P(Z)?",Z;IF FLG 13=0;JMP 111-2: CFG 13; ENT "CONS TRUCT P(Z)?",Z; IF FLG 13; JMP -2 4. : PRT " GIVEN RO OTS";" OF P(Z) ARE"H 5: 0+R0+R1H (C+B)+1+C+1+R(2C);0+R(2C+1)+ 7 : CFG 13%ENT "RL(R )",X;IF FLG 13; SPC SIPRT " -00NSTRUCTED" #JMP 2 ;:-- <u>|</u>---8 : 0 - Y ; ENT "IM(R)" , Y;SPC 1;PRT X,YH 9: R(2B+2)+A\*R(2B+3) + 2 -10: R(2B) - XA + YZ + R(2B)+2)+ 11: R(2B+1)-YA-XZ+R(28+3)+ 12: GTO 6; IF 04(B-1+ B);GTO 9H 13: GSB 16H 14: CFG 13; ENT "RL(C )",X+R(2C+2);IF FLG 137JMP 3H

0 + Y ; ENT "IM(C)", Y; Y+R(2C+3); SPC 1; PRT X, Y; C+1+C; JMP -1H 164 PRT " P ( 7 ) " " COEFFICIENTS "#RET H 17: IF FLG 1=0;JMP 7 .... 18: CFG 13; ENT "RL(Z )", XIIF FLG 13; JMP 14h 19: 0 → Y ; ENT "IM(Z)", Y;SPC 3;PRT AT Z ="18PC 1; PRT X,YH 28# SPC 2#PRT " P(Z) IS";SPC 1H 21: 0+A+B;C+ZH 221 R(2Z)+AX-BY+R0;R (2Z+1)+BX+AY+B0+A;JMP(Z-1+Z)=AH 23: PRT A, B; JMP -5H 241 XX+YY+ZH 25: IF (A+1+A)>C# JMP 3H 26#  $R(2A) \rightarrow B + R(2A + 1) \rightarrow$ RØH 271 (XB+YR0)/Z→R(2A) ; (XR0-YB)/Z→R(2A +1) #GTO 25h 28: SPC 3;PRT " NO RMALIZED"H 29: GSB 16H 30: 1+8-

31: SPC 1;PRT R(2A); R(2A+1);A+1+A; JMP A>CH 32: SPC 8;GTO 0H 33: END H



# POLYNOMIAL INTEGRATION, DIFFERENTIATION, OR ORIGIN SHIFT

For a given polynomial P(X) with real coefficients, this program can be used to find the coefficients of the polynomial  $\int P(X)dX$ ,  $\frac{d}{dX}P(X)$ , or P(X-a) where a is a real number specified by the user.

With the basic 9820 memory size of 173 R-registers, the size limitations are:

Polynomials P(X) with complex coefficients can be handled by writing P(X) in the form  $P(X) = Q(X) + iH(X) \text{ where } Q(X) \text{ and } H(X) \text{ are polynomials with real coefficients, and then applying the program to each of } Q(X) \text{ and } H(X) \text{ separately since } \int P(X) = \int Q(X) + i \int H(X), \frac{d}{dX} P(X) = \frac{d}{dX} Q(X) + i \frac{d}{dX} H(X), \text{ and } P(X-a) = Q(X-a) + iH(X-a).}$ 

ROM BL	оскѕ	
		X 173 A29
PERIPHI	1 ERALS	
9860	DA MARKED CARD READER	9862A PLOTTER 9864A DIGITIZER
9861	A TYPEWRITER	9863A PAPER TAPE READER
STEP	DISPLAY	INSTRUCTIONS
1.		ERASE LOAD EXECUTE
2.		Alternately insert magnetic cards and EXECUTE until NOTE 14 is no longer displayed, indicating completion of loading.
3.		END
4.		RUN PROGRAM
5.	P(X) COEFF'S?	$\underline{a_i}  \underline{\text{RUN PROGRAM}}  i = 0, 1, \dots, n  \text{where}  P(X) = a_0 + a_1 X + \dots + a_n X^n.$ $\underline{\text{RUN PROGRAM}}  \text{after all } a_i  \text{have been entered.}$
6.	INTEGRATE P(X)? DIFF. P(X)? SHIFT P(X)?	RUN PROGRAM to cycle between these three displays; 1 RUN PROGRAM to select the option being displayed.
		Bypass the next step when integrating or differentiating.
7.	P(X-A) FOR $A = ?$	<u>a</u> <u>RUN PROGRAM</u> where a is the shift value.
8.		The coefficients of the resultant polynomial $b_0 + b_1 X + \ldots + b_k X^k$ are now printed in the order $b_0, b_1, \ldots, b_k$ . To run a new problem, return to Step 4.

```
1. Integrate P(X) = 5 + 3X + 9X^2 + 8X^3
                                                     P(X)
                                                 COEFFICIENTS
                                                           5.0000
                                                           3.0000
                                                           9.0000
                                                           8,0000
                                                  INTEGRAL OF
                                                      P(X)
                                                  COEFFICIENTS
      Answer: 5X + 1.5X^2 + 3X^3 + 2X^4
                                                          0.0000
                                                          5.0000
                                                          1.5000
                                                          3.0000
                                                          2.0000
2. Differentiate P(X) = 2 + 3X + X^2 + 6X^3
                                                     P(X)
                                                 COEFFICIENTS
                                                          2.0000
                                                          3.0000
                                                          1.0000
                                                          6.0000
                                                 DERIVATIVE OF
                                                 P(X)
COEFFICIENTS
      Answer: 3 + 2X + 18X^2
                                                         3.0000
                                                        2.0000
18.0000
3. Find P(X-4) if P(X) = 5 + 8X
                                                 P(X)
COEFFICIENTS
                                                          5.0000
                                                          8.0000
                                                     A=
                                                        4.0000
                                                    P(X-A)
                                                COEFFICIENTS
      Answer: -27 + 8X
                                                       -27.0000
8.0000
```

### MEMORY ALLOCATION

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```
0:
0+X;1+BH
1:
CFG 131GSB 12H
2:
ENT "P(X) COEFF"
S?",R(X+1→X);IF
FLG 13=0;PRT ;
JMP 0H
3:
CFG 13; ENT "INTE
GRATE P(X)?",Z;
IF FLG 13=0;JMP
12-
4:
CFG 13; ENT "DIFF
. P(X)?",Z;IF
FLG 13=0; JMP 15H
5 1
CFG 13; ENT "SHIF
T P(X)?",Z;IF
FLG 13;JMP -2H
F. #
ENT "P(X-A) FOR
A=?",AH
7 :
SPC 3;PRT "
 A=";SPC 1;PRT
A;SPC 2;PRT "
  P(X-A)"IGSB 14
8:
X+C+YH
9 #
IF C>B;R(C-1→C)-
AR(C+1) +RC; JMP 0
10:
IF X>(B+1+B);
JMP -2H
11:
0+X%JMP 11H
12:
PRT "
           P(X)"
#GSB 14h
13:
RET H
14:
PRT " COEFFICIE
NTS"; SPC 1; RET H
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```
15:
SPC SIPRT "
               ΤN
TEGRAL OF " # GSB 1
21
16:
(1+Z)+X+Y;0+RYH
17:
IF (B+1+B)>Y;
IMP SH
18:
R(B-1)/Z+R(X+B);
Z+1+Z;GTO -1F
19:
SPC SEPRT " DER
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20:
X-1+Y:0+ZH
21:
Z+1+ZR(B+1)Z+R(
X+B); JMP (B+1+B)
> Y +
22:
IF (0+B)=Y#PRT Y
JUMP 2H
23:
PRT R((B+1+B)+X)
JMF B=YH
24:
SPC SH
25:
END H
```



# SIMULTANEOUS ORDINARY DIFFERENTIAL EQUATIONS Runge-Kutta (Gill Method)

This program solves the following system of simultaneous first-order ordinary differential equations:

$$\frac{dy_1}{dx} = f_1(x, y_1, y_2, ..., y_n)$$

$$\frac{dy_2}{dx} = f_2(x, y_1, y_2, \dots, y_n)$$

$$\vdots$$

$$dy_n$$

 $\frac{dy_n}{dx} = f_n(x, y_1, y_2, \dots, y_n)$ 

with initial conditions  $x_0$ ,  $y_1(x_0)$ ,  $y_2(x_0)$ ,...,  $y_n(x_0)$  specified by the user.

The program may also be used to solve an equation of the form:

$$\frac{d^{m}y}{dx^{m}} = f\left(x, y, \frac{dy}{dx}, \frac{d^{2}y}{dx^{2}}, \dots, \frac{d^{m-1}y}{dx^{m-1}}\right)$$

with given initial conditions  $y(x_0), \frac{dy}{dx}(x_0), \dots, \frac{d^{m-1}}{dx^{m-1}}(x_0)$  by rewriting the equation as a system of firstorder equations as above.

### References:

Ralston and Wilf, Mathematical Methods for Digital Computers, Vol. 1 (New York: John Wiley and Sons, Inc., 1960), p. 115.

Carnahan, Luther, Wilkes, Applied Numerical Methods, (New York: John Wiley and Sons, Inc., 1969), pp. 363-366.

### REMARKS

- 1. The number of equations, n, that can be solved simultaneously depends on the amount of programming required for each equation since there is a trade-off between program length and data storage. Of the 87 registers available, 3n + 8 are needed for data, leaving 87 (3n + 8) registers for programming the n equations at about 8 keystrokes per register. For example, with the basic calculator memory, the maximum number of equations is 20 which allows about 8 keystrokes to define each equation. A more reasonable number is 10 equations which allow an average of about 39 keystrokes for each equation.
- 2. To define the equations, the user must first formulate expressions for  $dy_1/dx$ ,  $dy_2/dx$ ,...,  $dy_n/dx$  obtaining as needed the values x,  $y_1$ ,...,  $y_n$  from register X, R8, R9,..., R(7+n) respectively. Then, beginning in program line 28 and proceeding for as many lines as are needed, the user keys in and stores as program lines the expressions:  $dy_1/dx \longrightarrow R(8+n)$ ,  $dy_2/dx \longrightarrow R(9+n)$ ,...,  $dy_n/dx \longrightarrow R(7+2n)$  where n refers to the number of equations. Registers R0 and Z are available as temporary storage areas.

ROM BLO	ocks	INTERNAL REGISTERS
<u></u>		X 173  429
PERIPHE	RALS	2
9860	A MARKED CARD READER	9862A PLOTTER 9864A DIGITIZER
9861	A TYPEWRITER	9863A PAPER TAPE READER
STEP	DISPLAY	INSTRUCTIONS
1.		ERASE LOAD EXECUTE
2.		Alternately insert magnetic cards and EXECUTE until NOTE 14 is no
		longer displayed, indicating completion of loading.
3.		(Optional) FIXED N n EXECUTE, or FLOAT N n EXECUTE as
0.		desired.
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4.		GO TO 2 8 EXECUTE and then using as needed register X for x,
		R8 for $y_1$ , R9 for $y_2$ ,, and R(7+n) for $y_n$ , key in:
		$\frac{dy_1/dx}{}$ $$ $R(8+n)$ $STORE$ $\frac{dy_2/dx}{}$ $$ $R(9+n)$ $STORE$
		$\frac{\mathrm{dy_n/dx}}{\mathrm{dx}} \ \ \underline{\hspace{1cm}} \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ $
5.		END RUN PROGRAM
6.	HOW MANY EQS?	n RUN PROGRAM
7.	START AT X = ?	x <sub>0</sub> RUN PROGRAM
8.	END AT $X = ?$	x <sub>max</sub> RUN PROGRAM
9.	DELTA X = ?	Δx RUN PROGRAM
10	PRINT INTERVAL?	p RUN PROGRAM for printing every pth increment in x.
10.	PRINT INTERVAL:	p RUN PROGRAM for printing every pen increment in x.
		$y_i(x_0)$ RUN PROGRAM $i=1,2,,n$ . Calculations proceed automatically
11.	INITIAL VALUES?	
		after the last initial value is entered. After the results are printed, proceed
		to Step 12.
12.	NEW ENDING X?	xmax RUN PROGRAM if results are desired for a new xmax larger than
		the previous one. After these additional results are obtained, control
		returns to Step 12.
1		

Solve  $d^2y/dx^2 = x^3 + 7x^2 - 14x + 40 - y + dy/dx$  for  $0 \le x \le .5$ ,  $\Delta x = .01$  and printing every fifth one, subject to the initial conditions

$$y(0) = 20$$

$$\frac{dy}{dx}(0) = 0$$

The solution may be obtained by first reducing the equation to a set of simultaneous equations using the following substitutions:  $y_1 = y$ ,  $y_2 = dy_1/dx = dy/dx$ .

This gives  $dy_1/dx = y_2$ ,  $dy_2/dx = x^3 + 7x^2 - 14x + 40 - y_1 + y_2$  with initial conditions  $y_1(0) = 20$ ,  $y_2(0) = 0$ . A solution to this set of two simultaneous equations will yield the solution to the original equation:  $y = y_1$  is the relation giving the desired result, and the printout of the  $y_2$  values may be ignored.

The Memory Allocation page shows that  $y_1$  is in R8,  $y_2$  is in R9,  $dy_1/dx$  is to be stored in R10, and  $dy_2/dx$  is to be stored in R11.

### Key in:

### Printout:

27: "F"H 28: R9+R10H 29: XXX+7XX-14X+40-R 8+R9+R11H 30: RET H 31: END H	INPUT DATA 2.0000 0.0000 .5000 .0100 5.0000 20.0000	Number of Equations Starting x Ending x Delta x Print Interval  y <sub>1</sub> (0) y <sub>2</sub> (0)
	RESULTS	
	.0500	x
	20.0251 1.0075	$y_1(x)$ $y_2(x)$
	.1000	x
	20.1010 2.0300	$y_1(x)$ $y_2(x)$
	.1500	x
	20.2284 3.0675	$y_1(x)$ $y_2(x)$
	.5000	: x
	22.6250 10.7500	$y_1(x)$ $y_2(x)$

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A:
                          16:
                          TISP "NEW ENDING
ENT "HOW MANY EQ
S.?",B,"START AT
                          X?" (SPC 6H
 X= ?".X."END AT
                          17:
 X= ?" | R2H
                          STP -
1 :
                          18:
ENT "DELTA X= ?"
                          Z+R2;GTO 6H
,R1, "PRINT INTER
                          19:
VAL?" , R4H
                          GSB "F"H
2 a
                          20:
17+01+28+Y#PRT "
                          ((8+C)+B+A+R7)+B
   INFUT DATA";
                          + Y |-
SPC 1; PRT B, X, R2
                          21:
.R1.R4%SPC 2h
                          RARI>RAKI>AI
3:
                          TF 28>A-8;GT0 +0
0+A+
                          ļ....
4. :
                          22#
A+1+A;0+R(Y+A);
                          1+A+
ENT "IMITIAL VAL
                          23#
UES?";R(C+A);
                          RC+R6RR7-R6R3RY+
PRT JUMP A=BH
                          RT.
5 #
                          24:
1+R5;SPC 2;PRT "
                          RY-3R6R3RY+2R6RR
    RESULTS";
                          7FLG 2+RYH
SPC 1H
                          25:
6:
                          (((A+1)A)+7+C)+B
SFG 2;.5+R6;2+R3
                          →R7)+B→Y; IF A≤B;
#GSB 19H
                          GTO 23H
7 :
                          26:
R1/2+X+X11-r.5+R
                          RET H
6;1+R3;GSB 19H
                          27:
8:
                          " F " |--
1+7.5+R6;GSB 19F
                          28:
9 :
                          END H
R1/2+X+X;CFG 2;1
76+R6;2+R3;GSB 1
91-
10:
IF R5#R4;R5+1→R5
#GTO 15H
11:
PRT X; SPC 1;8+C;
P-A-
12:
PRT R(C+A)+
13:
A+1+A; IF B>A;
GTO 12H
14:
SPC 211+R5H
15:
IF R2>X;GTO 6H
```



# ROMBERG QUADRATURE FOR $\int_{a}^{b} f(x) dx$

The Romberg method for obtaining successive approximations of a definite integral  $\int_{a}^{b} f(x) dx$  is con-

sidered to be a very elegant and efficient technique. Wilf<sup>2</sup> states that the method virtually eclipses the Newton-Cotes formulas on every score, and that only the Gaussian quadrature formulas can be considered to be on a comparable level. Further comparisons of the different methods together with a description of the Romberg method may be found in references 1 and 2.

Basically, the technique consists of generating a table of values called the Romberg tableau, the columns of which (and, eventually the rows) converge to the value of the integral assuming no roundoff error.

The tableau looks like:

$$T_0^0$$
 $T_0^1$   $T_1^0$ 
 $T_0^2$   $T_1^1$   $T_2^0$ 
 $T_0^3$   $T_1^2$   $T_2^1$   $T_3^0$ 

where the elements are generated from the recursion relation  $T_m^{\ k} = \frac{4^m \, T_{m-1}^{\ k+1} - T_{m-1}^{\ k}}{4^{m-1}}$  and the recursion

is initiated using the trapezoid sums (that is, successive approximations to the integral using repeated interval halving and the Trapezoid Rule) given in the first column.

The program is based on a FORTRAN version<sup>1</sup> and includes a convergence check on the trapezoidal sums since for some functions these will converge faster than other columns in the Romberg table. In addition, the improvements suggested by McCalla have been incorporated in this program: a recursion formula for successive trapezoidal sums has been included to reduce computations<sup>4</sup>, and a modification has been made to make the basic Romberg algorithm less sensitive to the accumulation of rounding errors<sup>3</sup>.

The user has the option to print the complete tableau or to designate a tolerance and have only the final estimate of the integral printed (when two successive elements of a column are within the designated tolerance). In either case, the column elements are flashed in the display as they are calculated so the user may view the convergence.

### References:

<sup>&</sup>lt;sup>1</sup> Thomas Richard McCalla, <u>Introduction to Numerical Methods and FORTRAN Programming</u>, (New York: John Wiley and Sons, Inc., 1967), pp. 287-297.

<sup>&</sup>lt;sup>2</sup> Ralston and Wilf, <u>Mathematical Methods for Digital Computers</u>, Volume II, (New York: John Wiley and Sons, Inc., 1967), pp. 133-143.

<sup>&</sup>lt;sup>3</sup> A. M. Krasun and W. Prager, "Remark on Romberg Quadrature", <u>Communications of the ACM</u>, Vol. 8, No. 4 (April, 1965), pp. 236-237.

<sup>&</sup>lt;sup>4</sup> Peter Henrici, <u>Elements of Numerical Analysis</u>, (New York: John Wiley and Sons. Inc., 1964), pp. 259-262.

ROM BL	OCKS	INTERNAL REGISTERS
		X 173
PERIPHI		
	A TYPEWRITER	9862A PLOTTER 9864A DIGITIZER
	A TYPEWRITER	
STEP	DISPLAY	INSTRUCTIONS
1.		ERASE LOAD EXECUTE
2.		Alternately insert magnetic cards and EXECUTE until NOTE 14 no longer
		appears, indicating completion of loading.
		(O (C. A)) DIVIED N. DIVIDOVINI A DI OATI N. A EVERYITE
3.		(Optional) FIXED N n EXECUTE, or FLOAT N n EXECUTE
		and set degrees or radians, if desired.
4.		GO TO 3 5 EXECUTE
7.		GO TO 5 5 EXECUTE
5.		Using register X to represent the variable x in the expression for $f(x)$ , key
		in the following:
		f(x) — Y STORE RETURN STORE END STORE.
		<u>· · · · · · · · · · · · · · · · · · · </u>
6.		END RUN PROGRAM
7.	A	a RUN PROGRAM a = lower limit
	В	b RUN PROGRAM b = upper limit
8.	PRINT TABLEAU?	RUN PROGRAM to omit tableau printout and go to Step 9, or
		1 RUN PROGRAM to get tableau printout and go to Step 10.
9.	EPSILON	$\underline{\varepsilon}$ RUN PROGRAM $\varepsilon$ = accuracy tolerance for the value of the integral.
		RUN PROGRAM alone in this step will automatically enter a default value
		of $\varepsilon = 10^{-11}$ .
10.		Calculated tableau elements are flashed in the display. If the tableau print
		option has been selected, the tableau is printed, row by row, until <u>STOP</u> is
		pressed. Otherwise, the tableau is not printed and calculations proceed until
		the estimate of the integral is within the designated tolerance and printed.
		Control returns to Step 7 in this case.

1. Using  $\varepsilon = 10^{-6}$ , verify that  $\int_{1}^{2} x^{-2} dx = .5$ 

### Program Steps

### Results

```
34: INTERVAL
"F"⊢
35: 2.00000000 00 00
1/XX→Y⊢
36:
RET ⊢ INTEGRAL
37: 0F F(X) IS
END ⊢ 5.00000014E-01
```

2. Print the Romberg tableau for  $\int_{1}^{2} x^{-2} dx$ 

### Romberg Tableau

The arrows indicate the actual order of printout.

1.000000000E 00 2.0000000000E 00 ROMBERG TABLEAU 6.2500000000E-01 5.347222222E-01 5.089937642E-01 5.089937642E-01 5.022708503E-01 5.022708503E-01 5.004176115E-01 5.004176115E-01 5.005691701E-01 5.000298790E-01 5.000040302E-01 5.000019226E-01 5.001423846E-01

INTERVAL

5.000356019E-01 5.000001227E-01 5.000000014E-01 5.000000001E-01 5.000000000E-01 5.000000000E-01

5.000019434E-01 5.000000810E-01 5.000000183E-01 5.000000109E-01

### MEMORY ALLOCATION

		REGISTERS
Α	f a	
В	b, $R_0^{n-1}$ , K	en e
C	n	en de la companya de La companya de la companya del companya de la companya de la companya del companya de la companya del la companya de la
X	<b>X</b>	and the second of the second o
Ŷ	f(x), Temporary	and the second s
	Available to User	
Z	Available to User	
0	Available to User	
1	<b>h</b>	
2	δ	
3	Lim	
4	j, n max	
5	<b>ε</b>	
6	$T_1$	
7	$R_1$	
8	$T_2$	
9	$R_2$	
	•	
	•	
	•	
		FLAGS
		0
		1 Branching Indicator
	the second secon	2 Print Indicator
		* <b>4</b> .
		5
		6 6 6
		8
		· · · · · · · · · · · · · · · · · · ·
		10 :
		11
		13 Option Indicator

14 15

```
SPC SICFG 1/CFG
2; CFG 13; ENT A, B
....
1 :
PRT " INTERVA
L",A,B;SPC 2H
2:
ENT "PRINT TABLE
AU?",Z;IF FLG 13
#JMP 2H
3#
SFG 2; PRT " ROMB
ERG TABLEAU";
JMP 3H
4. ;
1E-11+R5;ENT "EP
SILON", R5H
PRT "
        INTEGRA
       OFF(X) I
8" h
6.
((B \rightarrow X) - A \rightarrow R \perp) \times A \rightarrow R
21GSB "F"H
7 :
YaB;Aax;GSB "F"H
8:
1Y+B)R1/2+R6H
9#
1 + C + R3 H
10:
R1/2+R1;0+B;1+R4
---
11:
A+(2R4-1)R1+X;
GSB "F"H
12:
Y+8+8#IF (R4+1+R
13:
2R18+R(2C+5)+ZF
14:
((2+(R(2C+4)+B))
/2+R(2C+6)+Z)-B+
BH
15:
DSP R(2C+4); IF
FLG 21PRT R(2C+4
);GTO +3F
16:
IF 0>8;-8+8h
```

```
171
IF R5>BIPRT Zi
GTO OH
18:
IF RISR2;C+R4;
GTO +3F
19:
C+1+C; IF FLG 2;
SPC 1H
20:
2R3+R3:GTO -10H
21:
1+(C-2)FLG 1+C;
SEG 11-
22#
0+21C+8H
29:
4Z+3+Zh
24:
(R(2B+7)+Y)+(Y-R)
(2B+5))/Z+YH
25:
IF B>1#GTO +3H
261
R7+(Z-1)(R6-R7)/
22+R6;Y+R7F
27:
GTO -8; IF R4>(C+
1+C);GTO -5H
281
Y+R(2B+5)+
29:
I((R(2B+3)+Y)+R(
28+2))/2+R(28+4)
+X)-Y+YE
30:
DSP R(28+2); IF
FLG 21PRT R(28+2
) # GTO +3 H
31:
IF 0>Y;-Y;Y
32:
IF R5>Y;PRT X;
GTO OH
331
B-1+8;GTO -10F
244
35:
END H
```

M HEWLETT PACKARD

MEWLETT PACKARD

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For a given function f(x), this program will approximate  $\int\limits_a^b f(x)\,dx$  or  $\int\limits_a^\infty f(x)\,dx$  by the 8, 12, 16, 20, or 24-point Gauss-Legendre quadrature formula.

Derivation of the formulas may be found in either reference, and an extensive table of constants is given in reference 2 for orders other than 8, 12, 16, 20, or 24

References:

<sup>&</sup>lt;sup>1</sup> Brice Carnahan, H. A. Luther, James O. Wilkes, Applied Numerical Methods (New York: John Wiley and Sons, Inc.), pp. 101-112.

<sup>&</sup>lt;sup>2</sup> A. H. Stroud, Don Secrest, Gaussian Quadrature Formulas, (Prentice-Hall, Inc., 1966).

### REMARKS

- 1. Since a large number of constants must be keyed in and stored, it is recommended that they then be recorded on magnetic cards and retained for later use by  $\underline{RECORD}$   $\underline{"}$   $\underline{D}$   $\underline{A}$   $\underline{"}$  ,  $\underline{R}$   $\underline{8}$   $\underline{4}$  where 84 refers to the highest data register containing one of the constants.
- 2. If the user wishes to use only one or two of the formulas, only those related constants need be keyed in and stored in the designated registers. However, comparing the integral estimates using several of the formulas has the advantage of giving the user some indication of convergence and accuracy of the individual estimates.

### Constants Required

	20-Point Formula	
Store in registers R5 through R12	9.931285992E-01 9.639719273E-01 9.122344283E-01 8.391169718E-01 7.463319065E-01 6.360536807E-01 5.10860887E-01 2.277858511E-01 7.652652113E-02	Store in registers R41 through R60
	1.761400714E-02 4.060142980E-02 6.267204833E-02	
Store in registers R13 through R24	8.327674158E-02 1.019301198E-01 1.181945320E-01 1.316886384E-01 1.420961093E-01 1.491729865E-01 1.527533871E-01	
	24-Point Formula	
	9.951872200E-01 9.747285560E-01 9.382745520E-01 8.864155270E-01 8.200019860E-01 7.401241916E-01 6.480936519E-01	
Store in registers R25 through R40	5.454214714E-01 4.337935076E-01 3.150426797E-01 1.911188675E-01 6.405689286E-02 1.234122980E-02 2.853138863E-02 4.427743882E-02 5.929858492E-02 7.334648141E-02 8.619016153E-02 9.761865210E-02 1.074442701E-01 1.15505681E-01 1.258374563E-01 1.279381953E-01	Store in registers R61 through R84
	Store in registers R13 through R24  Store in registers	Store in registers R5 through R12  Store in registers R13 through R24  Store in registers R14 Store in registers R15 through R24  Store in registers R17 Store in registers R18 Store in registers R18 Store in registers R19 Store in registers R25 Store in registers

ROM BLOCK	S	INTERNAL REGISTERS				
	1	2 3				
PERIPHERALS  9860A MARKED CARD READER 9862A PLOTTER 9864A DIGITIZER  9860A MARKED CARD READER						
9861A TY	'PEWRITER	9863A PAPER TAPE READER				
STEP	DISPLAY	INSTRUCTIONS				
1.		ERASE LOAD EXECUTE				
2.		Alternately insert the program magnetic cards and EXECUTE until NOTE 14 is no longer displayed, indicating completion of loading.				
3.		LOAD "DA" EXECUTE				
4.		Alternately insert the data magnetic cards and <u>EXECUTE</u> until NOTE 14 is no longer displayed, indicating completion of loading.				
5.		(Optional) <u>FIXED N n EXECUTE</u> , or <u>FLOAT N n EXECUTE</u> as desired.				
6.		GO TO 1 7 EXECUTE				
7.		Using register X to represent the variable x in the expression for $f(x)$ , key in the following: $\underline{f(x)} \longrightarrow \underline{Y}  \underline{STORE}  \underline{RETURN}  \underline{STORE}  \underline{END}  \underline{STORE}$				
8.		END RUN PROGRAM				
9. A		a RUN PROGRAM				
10. B		$\underline{b}  \underline{RUN \ PROGRAM} \qquad \text{For } \int_{a}^{\infty} f(x)  dx, \text{ use } b = 1E \ 99.$				
11. 8,	12, 16, 20, or 24?	n RUN PROGRAM where n is one of the displayed values and refers to the n-point Gauss-Legendre integration formula.				
12.		The estimate of the integral is printed for the chosen n and control returns to Step 11. Another n may then be selected or control may be transferred to Step 9 for selection of a new interval by RUN PROGRAM alone.				

1. Evaluate  $\ln t = \int_{-\infty}^{t} \frac{1}{x} dx$  for t = 5 using the

Output

8-point formula and then the 12-point formula. Here  $f(x) = \frac{1}{x}$  and the following program lines are required.

INTERVAL 1.000000000E 00 5.000000000E 00

8-Point Formula

INTEGRAL 0F F(X) IS 1.609437439E 00

16: "F"+ 17: 1/X+YF INTEGRAL OF F(X) IS 1.609437912E 00 18:

12-Point Formula

RET H 19: EHD H

> This last value corresponds exactly with ln 5 for the number of digits shown.

Mathematics 2. (This example requires the Trigonometric Block). Evaluate the gamma function

 $\int e^{-X} x^{\alpha-1} dx \quad \text{for} \quad \alpha = 1.8 \text{ using}$ 

each of the formulas.

Output

INTERVAL 0.000000000E 00 1.000000000E 99

INTEGRAL OF F(X) IS 9.318350473E-01

8-Point Formula

12-Point Formula

16-Point Formula

16: "F"-17: EXP (-X)X\*.8\*YH 184

Program Lines

INTEGRAL OF F(X) IS 9.313784467E-01 RET H 19: END H

INTEGRAL OF F(X) IS 9.310731317E-01

INTEGRAL OF F(X) IS 9.313880955E-01

20-Point Formula

INTEGRAL OF F(X) IS 9.313838718E-01

24-Point Formula

The values are seen to be converging to the true value  $\Gamma(1.8) = 9.313837710 \text{ E}-01.$ 

### **MEMORY ALLOCATION**

ш-э	REGISTERS	ege and it is to take the second	
Α :	Lower Limit		n for all characters and the service of the service
	Upper Limit		**************************************
В	Pointer 1		000 o para - 1984 - Additional delica esta esta esta esta esta esta esta est
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	X	Name	
Υ	f(x)		- (ISBN 1-2579997) - NORTH CONTROL OF THE AND
Z	Available to User	-	
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War wa 9	Sum		00/44/4 (4/44-4/44/4)
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	0:     CFG 1; SFG 14;     ENT A, BH 1:     PRT " INTERVA L", A, B; SPC 2; IF     B=1E99; A+1+B;     SFG 1H 2:     (B+A) /2+R1; (B-A)     /2+R2H 3:     CFG 13; ENT "8, 12     ,16, 20, OR 24", C/ 2+R4; IF FLG 13;     GTO 0H 4:     IF (8(C=8)+18(C=12)+32(C=16)+50(C=20)+72(C=24)+R 3)=0; GTO -1H 5:     R3-R4+CH 6:     0+R0H 7:     INTEGRA L", " OF F(X) I S", R0R2; SPC 8;     GTO -4H 8:     R2RC+R1+X; IF FLG 1; 2/(1+RC)-1+A+XH 9:     GSB "F"H 10:     IF FLG 1; 4Y/(1+R C)(1+RC)+YH 11:     R(C+R4)Y+R0+R0H 12:     -R2RC+R1+X; IF FLG 1; 2/(1-RC)-1+A+XH 13:     GSB "F"H 14:     IF FLG 1; 4Y/(1-R C)(1-RC)+YH 15:     R(C+R4)Y+R0+R0; IF FLG 1; 4Y/(1-R C)(1-RC)+YH IF FLG 1; 4Y/(1-R C)(1-R C)(1-RC)+YH IF FLG 1; 4Y/(1-R C)(1-R C)(1-RC)+YH IF FLG 1; 4Y/(1-R C)(1-R C)(1-		
	GTO -8H	; \$ \$	



#### NUMERICAL INTEGRATION WITH EQUALLY SPACED BASE POINTS NEWTON-COTES CLOSED FORMULAS

Let  $x_0, x_1, \ldots, x_n$  be equally spaced points each h units apart at which corresponding values  $f(x_0), f(x_1), \ldots, f(x_n)$  of a function f are known. Using only this information, with no explicit expression for f itself,

$$\int_{\mathbf{x}_n}^{\mathbf{x}_n} f(\mathbf{x}) d\mathbf{x}$$

may be approximated using one of the Newton-Cotes closed integration formulas. This program allows selection of any one of the first six of these formulas:

1. 
$$\int_{x_0}^{x_1} f(x) dx \approx \frac{h}{2} [f(x_0) + f(x_1)]$$
 (Trapezoidal Rule)

2. 
$$\int_{x_0}^{x_2} f(x) dx \approx \frac{h}{3} [f(x_0) + 4f(x_1) + f(x_2)]$$
 (Simpson's Rule)

$$3. \int_{x_0}^{x_3} f(x) \, dx \approx \frac{-3h}{8} \left[ f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3) \right] \qquad \text{(Simpson's Second Rule)}$$

4. 
$$\int_{X_0}^{X_4} f(x) dx \approx \frac{2h}{45} \left[ 7f(x_0) + 32f(x_1) + 12f(x_2) + 32f(x_3) + 7f(x_4) \right]$$

5. 
$$\int_{x_0}^{x_5} f(x) dx \approx \frac{5h}{288} \left[ 19f(x_0) + 75f(x_1) + 50f(x_2) + 50f(x_3) + 75f(x_4) + 19f(x_5) \right]$$

$$6. \int_{X_0}^{X_6} f(x) dx \approx \frac{h}{140} \left[ 41f(x_0) + 216f(x_1) + 27f(x_2) + 272f(x_3) + 27f(x_4) + 216f(x_5) + 41f(x_6) \right]$$

The program is designed so that formula k may be used m times to successively integrate over m adjacent intervals provided that mk+1 pairs  $(x_i, f(x_i))$  are known. A running sum is kept of the integral over these intervals and, at the user's option, may be output after the last point of any interval has been processed.

Several formulas have been included to allow the user to match one of the formulas to the number of available data points. Also, more than one formula may apply in which case the user has the option of running the program for each formula to compare results.

Reference:

Brice Carnahan, H. A. Luther, James O. Wilkes, <u>Applied Numerical Methods</u> (New York: John Wiley and Sons, Inc., 1969), pp. 70-75.

ROM BL	OCKS	INTERNAL REGISTERS
		X 173 A29
PERIPHE		_ ·
9860	A MARKED CARD READER	9862A PLOTTER 9864A DIGITIZER
9861	A TYPEWRITER	9863A PAPER TAPE READER
STEP	DISPLAY	INSTRUCTIONS
1.		ERASE LOAD EXECUTE
2.		Alternately insert magnetic cards and EXECUTE until NOTE 14 is no
6		longer displayed, indicating completion of loading.
3.		(Optional) $\underline{\text{FIXED N}}$ $\underline{\text{n}}$ $\underline{\text{EXECUTE}}$ , or $\underline{\text{FLOAT N}}$ $\underline{\text{n}}$ $\underline{\text{EXECUTE}}$ as
		desired.
4.		END RUN PROGRAM
5.	TRAPEZOID RULE?	RUN PROGRAM to cycle between these 6 displays; 1 RUN PROGRAM to
	SIMPSON'S RULE?	select the option being displayed.
	SIMPSON RULE 2?	
	NEWTON-COTES 4?	
	NEWTON-COTES 5?	
	NEWTON-COTES 6?	
6.	X0?	x <sub>0</sub> RUN PROGRAM
7.	Y0?	y <sub>0</sub> RUN PROGRAM
8.	X INCREMENT?	h RUN PROGRAM
9.	NEXT Y?	yi RUN PROGRAM or
		RUN PROGRAM to print the running sum of the integral. A request to print
		(i.e., $\underline{\text{RUN PROGRAM}}$ alone) at other than the end of an interval will be ig-
		nored, and more values of y must be entered. However, it is easy to recog-
		nize when the end of an interval is reached: a series of dashes will be
		printed. To run a new case, STOP and go to Step 4.
		NOTE: In Step 9, when the end of an interval has been reached, the
		x-increment size h may be manually changed by $\underline{h} \longrightarrow \underline{C} $ EXECUTE
		prior to entering the next y.

#### EXAMPLES

h = .25
2
$\int f(x) dx = 16.58$
0

#### Increment Change

X	Y		
$(x_0) = 0$	(y <sub>0</sub> ) 2	)	
. 25	2.8		
. 50	3.8	}	h = .25
. 75	5.2	- 1	
1.00	7.0	J	
1.50	12.1	Ì	h = .5
2.00	20	}	n = .5
	$\int_{0}^{2} f(x) dx = 1$	6.62	

SIMPSON'S RULE

X 0 Y 0 0.0000 2.0000

INCREMENT

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INTEGRAL X0 TO LAST X 16.5833

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**y** 1  $\mathbf{x}_{2}$ 

 $\mathbf{X}_{3}$ У3

 $X_4$ У4

1.2500

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SIMPSOM'S RULE

XØ ΥÕ 0.0000 2.0000

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> .7500 5.2000  $\mathbf{x}_3$ У3 1.0000 X4 7.0000 y 4

1.5000

 $X_5$ 12.1000  $y_5$ 2.0000  $\mathbf{x}_{6}$ 

**y**6

20.0000

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                         CFG 13;ENT "NEXT
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                         Y?",RB;GTO +1;
CFG 13; ENT "SIMP
                         IF FLG 13 GTO +0
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                         FIF FLG 15GTO 26
IF FLG 13=012+A;
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                         GTO 11H
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                         C(R0+4R1+R2)/3+Y
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                         GTO 11H
4:
                         18:
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                         3178+7+7-
IF FLG 13=0;5+A;
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ES 5";GTO 7H
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                         2017(R0+R4)+32(R
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                         END -
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# MODEL 20

## NUMERICAL INTEGRATION WITH UNEQUALLY SPACED BASE POINTS, INTERPOLATION, FIRST AND SECOND DERIVATIVES, AND CURVE THROUGH POINTS USING SPLINE FUNCTIONS

This program will determine a curve s(x) passing through a given set of data points  $(x_1, y_1)$ ,  $(x_2, y_2)$ , ...,  $(x_n, y_n)$ , where the  $x_i$  are distinct but need not be equally spaced, and then print the value of the integral,

 $\int_{-\infty}^{x_n} s(x)dx$ , of this curve. The user then has the following options:

- 1. Plot the points and the curve s(x) through them (with automatic data scaling), and a set of labeled axes.
- 2. Interpolate. For a given  $x, x_1 \le x \le x_n$ , print all of the following (in this order):
  - x value of x
  - s''(x) value of the second derivative of the curve at x
  - s'(x) value of the first derivative of the curve at x
  - s(x) value of the curve, s(x), at x

Up to 17 points are allowed on the basic 9820A and up to 59 on the 9820A having the additional 256 internal registers.

The curve used to fit the given points is a third-degree natural spline function and derives its name from the fact that such a curve approximates the behavior of a mechanical spline used by draftsmen to draw a smooth curve through a set of points. The curve has the properties of having continuous first and second derivatives, and of being the "smoothest" curve through the given points in the sense that

$$\sigma_k = \int_{x_1}^{x_n} [s^k(x)]^2 dx \text{ is made as small as possible for } k = 1, 2. \text{ In fact, } s(x) \text{ is the unique curve with these}$$

properties and is a piecewise function given by a polynomial of degree at most 3 in each of the intervals  $[x_i, x_{i+1}]$ , in general by a different polynomial in each such interval. These polynomial segments making up the curve s(x) join smoothly at the juncture points  $x_i$  in that the two polynomials which meet at a given  $x_i$  have the same ordinate and the same first and second derivatives.

The process to determine s(x) involves the solution of a set of simultaneous linear equations which is iteratively carried out by the method of successive overrelaxation. The accuracy tolerance,  $\epsilon$ , to within which the parameters  $s''(x_i)$  are to be found in this iteration is specified by the user.

For a derivation of the equations and the flow chart upon which this program is based, see Greville's article in reference 1. Reference 2 goes more into the theory of spline functions in general with applications in various other areas.

For users not having a plotter but wishing to make use of the other options in the program, an easy editing of Pass 2 of the program makes this possible: simply eliminate lines 0 through 12 of Pass 2, renumber the remaining lines starting with 0, and ignore Steps 7 and 8 of the User Instructions.

References: <sup>1</sup> Ralston and Wilf, <u>Mathematical Methods for Digital Computers</u>, Volume II (New York: John Wiley and Sons, Inc., 1967), pp. 156-168.

<sup>&</sup>lt;sup>2</sup> T. N. E. Greville, <u>Theory and Applications of Spline Functions</u> (New York: Academic Press, 1969).

ROM BL	ocks	INTERNAL REGISTERS  Peripheral Control I*  X 173 429
	1	2 3
PERIPH	ERALS OA MARKED CARD READER	9862A PLOTTER 9864A DIGITIZER
	A TYPEWRITER	9863A PAPER TAPE READER
STEP	DISPLAY	INSTRUCTIONS
1.		ERASE LOAD EXECUTE, then alternately insert magnetic cards for
		Pass 1 and EXECUTE until completion of loading.
		END DAW PROCESS
2.		END RUN PROGRAM
9	m o	$ε$ RUN PROGRAM $ε$ = error tolerance (e. g., $10^{-3}$ or $10^{-6}$ ).
3.	E?	$\varepsilon$ RUN PROGRAM $\varepsilon$ = error tolerance (e.g., $10^{-3}$ or $10^{-6}$ ).
4.	X	$x_1$ RUN PROGRAM ) data point entry (where $x_1 < x_2 < < x_n$ ).
<b></b> .	Y	$x_1 = \frac{x_1}{y_1} = \frac{\text{RUN PROGRAM}}{\text{RUN PROGRAM}}$ data point entry (where $x_1 < x_2 < < x_n$ ).
	1	1 KON PROORAW 1 KON TROORAW alone after all data is entered.
		After calculations are completed and the integral is printed, go to Step 5.
		incor curculations are compressed and one integral to printed, go to trop of
5.	PASS 2	LOAD EXECUTE, then alternately insert magnetic cards for Pass 2 and
		EXECUTE until completion of loading.
6.		END RUN PROGRAM
0.		END INTERCONTAIN
7.	XMIN?	If no plot is to be made, RUN PROGRAM and go to Step 9.
		·
8.	XMIN?	Xmin RUN PROGRAM \ plot parameters, where
	XMAX?	$\underline{x_{\text{max}}}$ RUN PROGRAM $x_{\text{min}} \leq x_{\text{i}} < x_{\text{max}}$ $i = 1,, n$
	YMIN?	$y_{\min}$ RUN PROGRAM $y_{\min} \le y_i < y_{\max}$ $i = 1,, n$
	YMAX?	y <sub>max</sub> RUN PROGRAM 200 points are plotted (as specified
		in line 2, Pass 2).
9.	EVALUATE AT X=?	x RUN PROGRAM. The following are printed in order:
		x value of x
		s''(x) second derivative at x
		s'(x) first derivative at $x$
		s(x) ordinate at $x$
		Control returns to Step 9 for another x.
		*Not required if modifications noted at bottom of the previous page are made.

#### **EXAMPLES**

1. Using a tolerance  $\epsilon = 10^{-3}$ , use the program to fit a curve s(x) through the following points and print the 14

value of the integral  $\int_{-\infty}^{14} s(x)dx$ :

x	у
1	2
2.5	12
4 7 9	6
7	10
	10.5
10	8.3
14	1

If a plotter is available, plot the curve using:

$$x_{min} = 1$$
  $y_{min} = 0$   
 $x_{max} = 15$   $y_{max} = 12$ 

2. The program may be of aid in lofting associated with boat designing. Using a tolerance  $\varepsilon=10^{-3}$ , plot half of the top view of a boat hull through the points:

$$\begin{array}{c|cccc} x & y \\ \hline 0 & 2 \\ 3 & 2.5 \\ 7 & 2 \\ 10 & 0 \\ \end{array}$$

Use

$$x_{min} = 0$$

$$y_{\min} = 0$$

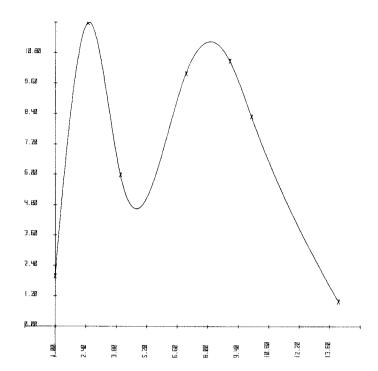
$$x_{max} = 15$$

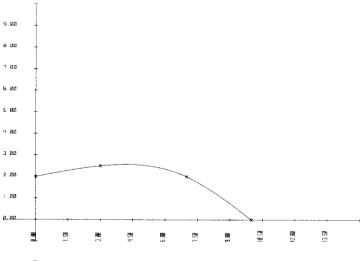
$$y_{\text{max}} = 10$$

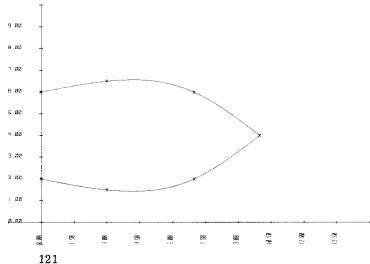
Then find the ordinates, y, corresponding to x=5 and x=8.

12	00000 13886 10 <b>00</b> 0 52773	x s''(x) s'(x) s(x)
:	00000 14258 63102 45215	x s''(x) s'(x) s(x)

Note: The full top view may be obtained as shown to the right by running the program twice: first using the above x values with the y values shifted up 4 units, then running the program using the same x values and 4-y with y as in the tables.







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#### Pass 1 Й: ENT "E?",R(0→A); CFG 13H 1 # ENT XIIF FLG 13; JMP 2H 2: ENT Y;PRT X→R(2) A+1+A)-1)+Y+R(2A);SPC 1;JMP -1H 3: $A \rightarrow R (4A + 1) \rightarrow R (5A) \rightarrow$ B#4(2-13)+Ch 4 : (R(2(B+1+B)+2)-R(2B))/(R(2B+1)-R $(2B-1) \rightarrow R((3A+B+Z$ 1-A))→RZ;JMP B>1 E : (2(RZ-R(Z-1))/(R(Z-1-A)+R(Z-A))+R(Z+A))3/2+R(Z+2All-6.8 IF A-1>B;GTO -2H 7 : 0+X12+8H 8: C(((R((4A+B+Z)+ $1) \rightarrow Y) - R(Z-1)) / (1$ +R(Z-2A)/R(Z-2A-1))-Y)/2-RZ+R(Z+A)) >YH 9: IF $\Gamma(YY) > X$ $\Gamma(YY)$ $+ \times +$ 10: RZ+Y+RZ; IF (8+1+ B) #A (GTO -2H 11: IF ROSX;GTO -4H 12: 0 + C | 1 + B | 13: (R((4A+B+Z)-2A)+Y)(R(2B)+R(2B+2) )/2-YYY(RZ+R(Z+1 ))/24+C+C;JMP A= (B+1+B)+14: 1 + B H

```
15:
(R((4A+B+Z)+1)-R
Z)/R(Z-2H+Z)+RZ;
JMP (B+1+B)=AF
16:
PRT "
          INTEGRA
L",C$SPC 2h
17:
DSP "PASS 2"F
18:
END -
```

```
Pass 2
0:
CFG 1; CFG 13;
ENT "XMIN?",X%
IF FLG 13 JMP 13
ļ....
1 :
FLT 2; FXD 2; ENT
"XMAX?",C,"YMIN?
",Y>B,"YMAX?",Zh
2:
(C-X)/200≯R0F
34
SCL 1.1X-.10,0,1
.1Y-.1Z,Z#AXE X,
Y . . 1 (C-X) . . 1 (Z-Y
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4. ;
LTR 1.1X-.1C,8,2
11; PLT B; JMP (B+
.1(Z-Y) +B)>.95(Z
-Y)+Y+
5.5
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6.1
LTR B,1.1Y-.1Z,2
12;PLT B;JMP (B+
.1(C-X)+B)>.95(C
-X)+Xh
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8:
LTR R(2B-1)-X,R(
28)-Y,211;PLT "X
";JMP (B+1+B)>AH
9:
R(1+B)+XF
10:
GSB +5h
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```
11:
PLT X,Y; IF (X+R0
→X) <R(2A-1) | JMP
-1-
12:
PEN H
13:
SFG 1→B∮ENT "EVA
LUATE AT X=?",X;
GSB +2h
14:
JMP -1H
15:
IF BOATRET H
16:
GTO +1; IF A=B;
GTO +6; IF (X-R(2
(H-1) \rightarrow Z) = 0 \% JMP 2 + 1
17:
IF (0)(X-R(2B-1)
*Y))+(04(X-R(2B+
1) > 2) ) | 8 + 1 > 8 |
JMP -2h
18:
R(4A+B)+YR(2A+B)
⇒C:IF FLG 1;PRT
X & C.E.
19:
(R(4A+B)+R(4A+B+
1)+C)/6+CH
20:
R(2B) + YR(3A + B) + Y
ZC+YH
21:
IF FLG 1; PRT R(3
A+B)+(X-R(2B-1)+
Z)C+Z(X-R(2B-1))
R(2A+B)/6,Y|SPC
1 -
22#
RET H
23:
END -
```



#### CURVE FITTING BY CHEBYSHEV POLYNOMIALS

This program¹ fits a least-squares curve to a set of given data points  $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$  where the  $x_i$  lie in an interval (a, b) and are equally spaced. The user specifies a degree m and the program outputs the coefficients  $a_0, a_1, \ldots, a_m$  of a polynomial  $P(x) = a_0 + a_1x + \ldots + a_mx^m$  passing near or through each input point.

The program determines the  $a_i$  by considering P(x) as a linear combination of Chebyshev polynomials  $T_i(x)$ ,  $P(x) = c_0 T_0(x) + c_1 T_1(x) + \ldots + c_m T_m(x)$  and applying the least-squares criterion to the expression

$$S = \sum_{i=1}^{n} \left( y_i - P(x) \right)^2 \text{ to give a system of simultaneous equations } \frac{\partial S}{\partial c_j} = 0, j = 0, 1, \dots, m \text{ from which the } c_i$$

can be determined. Calculation of the  $c_i$  is facilitated by using the orthogonality properties of the Chebyshev polynomials and evaluating P(x) at special points

$$\overline{x}_i = \frac{\cos(2i-1)\pi}{2(m+1)}$$
 within the interval (-1, 1) to force off-diagonal elements to be zero in this system of equa-

tions. Corresponding values  $\overline{y}_i$  are needed in the system of equations in order to be able to solve for the  $c_i$ . The program obtains these by applying a linear transformation to the  $x_i$  to bring the  $x_i$  within the interval (-1,1) and then using these transformed values and applying the Lagrange interpolation formula to the  $\overline{x}_i$  to obtain the  $\overline{y}_i$ . The system of equations is then easily solved for the  $c_i$ . The program then applies a linear transformation to  $\overline{x}$  to change the expression  $P(\overline{x}) = c_0 T_0(\overline{x}) + c_1 T_1(\overline{x}) + \ldots + c_m T_m(\overline{x})$  to the form  $P(x) = a_0 + a_1 x + \ldots + a_m x^m$  over the original interval (a, b).

If a plotter is available, provisions are made to use the program in conjunction with the General Function Plot, program III-7, to plot P(x) without any data reentry as shown in the Example.

Currently the program is set up for a 9820A with 429 total registers. However, the program may still be used on a 9820A with 173 registers by segmenting the program into three parts: part 1 consisting of lines 0 through 19, part 2 consisting of lines 20 through 35, and part 3 consisting of lines 36 through 56. Renumber the lines starting from 0 for each part, add an END statement as the terminal line, and run each as a separate program. First run part 1, and when calculations are completed (indicated by a non-blank display) load part 2 and run it by LOAD EXECUTE END RUN PROGRAM and then run part 3 the same way. DO NOT press ERASE prior to loading part 2 or part 3.

If n denotes the number of data pairs, m the polynomial degree specified, and R the number of available R-registers, then problem size is determined by the relation  $2n + 3m \le R-3$ . With 173 registers, this gives  $2n + 3m \le 78$  and with 429 registers, this gives  $2n + 3m \le 217$ . Reduce these numbers by 6 if the Peripheral Control I block is used.

<sup>&</sup>lt;sup>1</sup> Written by Frank Yockey of Hewlett-Packard Calculator Labs, based on the FORTRAN program given in the following reference:

Shan S. Kuo, Numerical Methods and Computers, (Palo Alto: Addison-Wesley, 1965), pp. 228-238.

ROM BL	OCKS	Mathematics (If plotting) INTERNAL REGISTERS
		Trigonometrie Peripheral Control I 173 X 429 1
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PERIPHI 986	EHALS DA MARKED CARD R	EADER
_	A TYPEWRITER	9863A PAPER TAPE READER
		3000A TAILE TAIL HEADEN
STEP	DISPLAY	INSTRUCTIONS
1.		ERASE LOAD EXECUTE
2.		Alternately insert magnetic cards and EXECUTE until NOTE 14 no longer
		appears, indicating completion of loading.
		11
,		(Outle all EWED V EWEGVER BY OAR V EWEGVER
3.		(Optional) <u>FIXED N</u> <u>n</u> <u>EXECUTE</u> , or <u>FLOAT N</u> <u>n</u> <u>EXECUTE</u>
4.		<sup>2</sup> END RUN PROGRAM
5.	DEGREE?	$\underline{m}$ RUN PROGRAM $\underline{m}$ = polynomial degree desired
6.	X1?	$x_1$ RUN PROGRAM $x_1 = $ first x value
7.	DELTA X?	$\Delta x$ RUN PROGRAM $\Delta x = x$ increment
''	DEETH A.	101/ 11/Oddan 4x - x increment
		V
8.	Υ?	$\frac{y_i}{n}  \underline{\text{RUN PROGRAM}} \qquad \qquad i = 1, 2, \dots, n$
		When all $y_i$ have been entered, press RUN PROGRAM. The display blanks
		while calculations are made and the coefficients $a_0, a_1, a_2, \ldots, a_m$ in the
		expression $P(x) = a_0 + a_1x + + a_mx^m$ are then printed in order. Return
		to Step 4 for a new problem. <sup>3</sup>
		<sup>1</sup> Only 173 registers are required if the program is segmented as described
		at bottom of previous page.
		·
!		<sup>2</sup> If plotting of the data results is desired. Call
		<sup>2</sup> If plotting of the data points is desired, follow procedure I shown in the
		example. Do this prior to Step 4 of the instructions above.
		<sup>3</sup> If plotting of the polynomial is desired, follow procedure II shown in the
		example.

#### **EXAMPLE**

Use the program to fit a parabola through the points

x	у	X	у
1	2	6	11
2	5	7	11
3	7	8	10
4	8	9	9
5	10	10	8

Input: degree = 2  $x_1 = 1$ delta x = 1

Output

COEFFICIENTS

0 - 6000 -1 - 5601

1.0000 3.8159

2.0000 -.2910

#### I. DATA PLOTTING

For plotting the input data, load the program then press:

 $\begin{array}{c|cccc} x_{\min} & \longrightarrow & A & \text{EXECUTE} \\ \hline x_{\max} & \longrightarrow & B & \text{EXECUTE} \\ \hline y_{\min} & \longrightarrow & C & \text{EXECUTE} \\ \hline y_{\max} & \longrightarrow & Y & \text{EXECUTE} \\ \end{array}$ 

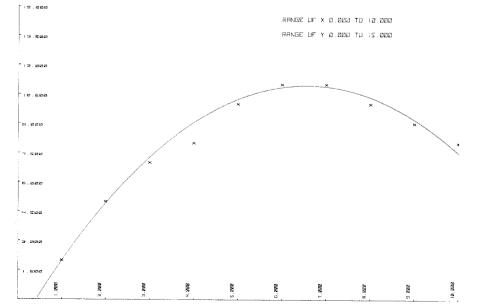
For this example, use:  $x_{min} = 0$ ,  $x_{max} = 10$ ,  $y_{min} = 0$ ,  $y_{max} = 15$ .

Modify line 2 as shown below and insert the new line 3 (key it in and press INSERT STORE).

2: ENT "Y?", RZ; GTO +2; IF FLG 13=0; PRT X, RZ; SPC 1; X +R9+X; Z+1+Z+R1; JMP 1H 3: SCL A, B, C, Y; LTR X-R9-.0032(B-A), R(Z-1)-.0032(Y-C), 111; PLT "X"; JMP -1H 4: SPC 1; PRT " COE FFICIENTS"; SPC 1

#### II. POLYNOMIAL PLOT

To plot the polynomial, <u>END EXECUTE</u> and load the General Function Plot, program III-7, (DO NOT PRESS ERASE). Key in the below function and then run the program using the same  $\mathbf{x}_{\min}$ ,  $\mathbf{x}_{\max}$ ,  $\mathbf{y}_{\min}$ , and  $\mathbf{y}_{\max}$  as above and no. of points = 100.



25:
"F"+
26:
R(R7+(R6-1+R5))+
YH
27:
YX+R(R7+(R5-1+R5))+
1)+Y; JMP R5=0H
28:
RET H
29:
END H

III-6	,	MEMO	RY ALLOCAT		escannes convertourous	
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6	m+1	And the state of t		2,000	449	
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ENT "DEGREE?", Z+
1 → R6, "X1?", R8 → X,
"DELTA X?", R9;
CFG 13;TBL 2h
1 :
PRT "
            DEGREE
"#SPC 1#PRT Z#
SPC 21PRT
  X";"
SPC 1;12+R0+ZH
2:
ENT "Y?", RZ; IF
FLG 13=0;PRT X,R
Zispc 1;x+R9+X;Z
+1+Z+R1#JMP 0H
34
SPC 1;PRT " COE
FFICIENTS"; SPC 1
4 :
(((R1-12)R7)+R(1
*A)*R2)*R6*R3)*R
6+84-
5 4
COS ((\pi/2)(2A-1)
ZR6) >R(R6-A+R2);
.IMP (A+1+A)) R6F
F. :
R7-1+A;2/A+B;-1+
RRIJR1+CH
7 :
RC+B\rightarrow R(C+1) [JMP
(C+1+C)=R2-1F
8:
0 + A + B +
(i) !!
IF R(R2+A) > R(R1+
B);GTO "150"H
10:
"151";(R(R2+A)-R
(R1+B-1+C))/(R(C
+1)-RC)+YH
11:
IF 8>11GTO +2F
Y(R(R0+B+C)-R(C-
1))+R(C-1)+R(R4+
A);GTO "157"H
13:
IF R7(B+1;GTO -1
```

```
14:
-Y(Y-1)(Y-2)R(R0
+R-2+C1/6+(YY-1)
(Y-2)R(C+1)/2+Zh
15:
Z - (Y + 1) (Y - 2) YR (C
+2)/2+Y(YY-1)R(C
+31/6+R(R4+A)+
16:
"157"!A+1+A+
17:
IF R64A;GTO +3H
18:
IF R(R2+A) \le R(R1+
B);GTO "151"-
19:
"150"; IF (B+1+B)
≤R7-1;GTO -10H
20:
0+AH
21:
0+Y;0+B;IF A=1;
GTO "265"h
22:
IF A>1;GTO "270"
---
23:
Y+R(R4+B)+Y;JMP
(B+1+B) > R6-1H
24:
GTO "275" H
25:
"265";Y+R(R2+B)R
(R4+B)+YiJMP (B+
1+81) R6+1h
26#
GTO "275" H
27:
"270";1+RR1H
28:
R(R2+B) \rightarrow R(R1+1);
2+CH
29:
2R(R2+B)R(R1+C-1
)-R(R1+C-2)+R(R1
+C);JMP (C+1+C)>
AH
30:
Y+R(R4+B)R(R1+A)
+Y$IF (B+1+B)≤R6
-1#GT0 -2h
```

```
"275";2Y/R6+R(R3
+A); [F (A+1+A) < R
6-1;GTO -10H
32:
RRS/2+RRSH
331
(R8+C)+R9(R7-1)+
B |--
24:
(B+C)/2+R10;(B-C
)/2+R11H
35:
RR3+RR4;R(R3+1)+
R(R4+1);0+Ch
36:
IF R6<2;GTO "597
37#
0 + R (R1+C) + R (R0+C
) → R (R4+C+2) # JMP
(C+1+C) > R6-1F
38:
39:
COS (#C/2) + RR0 1 1
+B+
40:
2R(R1+B-1)-R(R0+
B) +R(R0+B) | JMP (
B+1+B)>Ch
41:
0+81-
42:
R(R4+B)+R(R3+C)R
(R0+B) \Rightarrow R(R4+B) \dagger R
(R1+B)+AiR(R0+B)
B ) |-
43:
IF (B+1+B) < C;
GTO -1H
44:
GT0 -5H
45#
"597"H
461
RR4+RR0;1+CH
471
0 → R (R0+C) # JMP (C
+1+C)=R6h
481
1+0-
```

```
49:
"580";R(R0+C)+R(
R4+C)/R11+C+R(R0
+C);1+A+R5+B;C+1
* Y |--
50:
B+X;XR5+R5;R5R11
↑C+Z;A(Y-X)+AF
51:
R(R0+C-B)+A*R10*
B*(-1)*B*R(R4+C)
/Z+R(R0+C-B); IF
(B+1+B)<C;GTO -1
---
52:
IF (C+1+C) #R6;
GTO "580"H
59#
0 + A H
54#
PRT A,R(RO+A);
SPC :JMP (A+1+A)
=R6H
55:
R0+R7;SPC 8H
56:
END H
```

#### GENERAL FUNCTION PLOT

This program plots a given function over a specified range and then, at the user's option, draws a set of axes and labels them. An automatic scaling feature is included which automatically calculates the range of y for the designated range of x, the plot then being made over this range. Or, if the user chooses, the range of y may be designated in the input. This would usually be done when the user wants the labeled values along the y-axis to come out in even increments.

<sup>&</sup>lt;sup>1</sup> Written by Frank Yockey of Hewlett-Packard Calculator Labs.

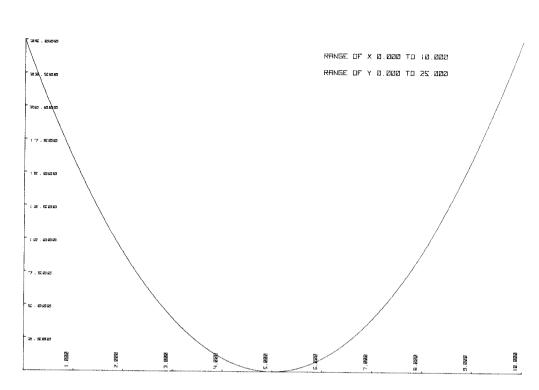
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PERIPHI	ERALS	
_	DA MARKED CARD READER	X 9862A PLOTTER 9864A DIGITIZER
	A TYPEWRITER	9863A PAPER TAPE READER
STEP	DISPLAY	INSTRUCTIONS
1.		ERASE LOAD EXECUTE
2.		Alternately insert magnetic cards and EXECUTE until NOTE 14 no longer
		appears, indicating completion of loading.
3.		Set radians, if desired.
4.		GO TO 2 6 EXECUTE
5.		Using register X to represent the variable x in the expression for $f(x)$ ,
		key in: $\underline{f(x)} \longrightarrow \underline{Y} \underline{STORE} \underline{RETURN} \underline{STORE} \underline{END} \underline{STORE}$
6.		END RUN PROGRAM
7.	X MIN?	<sup>x</sup> min RUN PROGRAM
8.	X MAX?	x <sub>max</sub> RUN PROGRAM
9.	NO. OF POINTS?	n RUN PROGRAM
10.	Y MIN?	ymin RUN PROGRAM (or RUN PROGRAM alone for auto-scale).
11.	Y MAX?	y <sub>max</sub> RUN PROGRAM
		The function is now plotted. If auto-scale has been selected, there will be a slight delay prior to the plot while the minimum and maximum y values are calculated by the program.
12.	AXES?	RUN PROGRAM to get axes or  1 RUN PROGRAM to return to Step 7 to plot the same function over a new range.
		If axes have been designated, they will be plotted and labeled after which control returns to Step 7.

#### **EXAMPLES**

1. Plot  $y = (x-5)^2$  with  $x_{min} = 0$ ,  $x_{max} = 10$ , number of points = 100, and letting the program calculate  $y_{min}$  and  $y_{max}$ .

#### Program Lines:

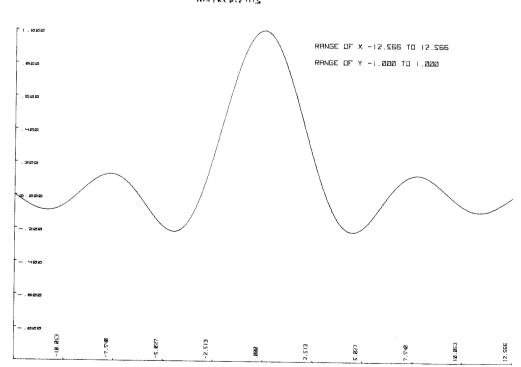
```
25:
"F"+
26:
(X-5)(X-5)+Y+
27:
RET +
28:
END +
```



2. Plot  $y = \sin x/x$  with  $x_{\min} = -4\pi$ ,  $x_{\max} = 4\pi$ ,  $y_{\min} = -1$ ,  $y_{\max} = 1$ , number of points = 200, and with the calculator set in radians. (This example requires the Trigonometric ROM block).

#### Program Lines:

25: "F"⊢ 26: SIN X/X→Y⊢ 27: RET ⊢ 28: END ⊢



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*i*4 : ENT "XMIN?", R0, " XMAX?" RI "NO. O F POINTS?".R2H 1: CFG 13; ENT "YMIN ?".B!GTO +1!IF FLG 13=0;EMT "YM AX?";AJJMP 7H 2: R0+X;(R1-R0)/R2+ ZIGSB "F"H 2: Y⇒A→B¦GTO +4F 4 : GSB "F"H 5 : IF Y>AiY+AH 6: IF YSBFY+BH 7 :  $IF (X+Z+X) \le R1$ GT0 -3H F1 11 SCL RO.RI.B.A; XMIN" PRT " XMAX",R0 \*RI#SPC + 9 : PRT " YMIN" ymax",B, AFSPC 8H 10: R0+X1(R1-R0)/R2+ 111 G88 "F"H 12: PLT X,Y; IF (X+Z+ X) (R1;GTO -1F 13: PEN JOFG 13JENT "AXES?";Z!IF FLG 13=0;GTO 0H 14: SCL 0,10,0,10; AXE 0:0:1:1h 15: FLT 2H

16: B+CI-.1+XIIF ((A -B)/10+Z){1E2# FXD 011F Z491 FXD 3; IF Z (1E-2; FLT -17: IF (1+X+X) < 10; LTR .1, X, 1111 PLT C+Z+CiGTO +0 ---18: LTR 6,9,211H 19: PLT "RANGE OF Y "FPLT BFPLT " TO "IPLT AF FLT i-.1+XiR0+Ci IF ((R1-R0)/10+Z JE1E2;FXD 0;IF Z 49;FXD 3;IF Z41E -2;FLT -21: IF (1+X+X) < 10; LTR X, 1, 112; PLT C+Z+C;GTO +0 ---22: LTR 6,9.5,211H 23: PLT "RANGE OF X ";PLT ROJPLT " T O "FPLT RIH 24: GTO ØH 25: · F · 26# END H

# HEWLETT · PACKARI

### LAGRANGE INTERPOLATION FOR UNEQUALLY SPACED DATA

If n+1 data points  $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$  are given, this program will evaluate at any  $x_j$  the unique polynomial  $P(x) = a_0 + a_1x + \ldots + a_nx^n$  of degree n which passes through the given n+1 points,  $n \le 60$ .

As shown in the reference, P(x) may be expressed in the form

$$P(x) = \sum_{k=0}^{n} \left[ \prod_{\substack{i=0\\i\neq k}}^{n} \left( \frac{x-x_i}{x_k-x_i} \right) \right] y_k$$

known as the Lagrange interpolation formula, and this is the form used in the program to find  $P(x_j)$  for a given  $x_j$ .

In the Example section is shown an example of extrapolating intermediate values of  $\ln x$  using several values obtained from a table covering the interval of interest.

Reference:

Shan S. Kuo, Numerical Methods and Computers (Reading, Massachusetts: Addison Wesley Publishing Company, 1965).

	1 RALS MARKED CARD READER TYPEWRITER	2 3  9862A PLOTTER
STEP	DISPLAY	INSTRUCTIONS
1.		ERASE LOAD EXECUTE
2.		Alternately insert magnetic cards and <u>EXECUTE</u> until NOTE 14 no longer appears, indicating completion of loading.
3.		(Optional) <u>FIXED N n EXECUTE</u> , or <u>FLOAT N n EXECUTE</u> as desired.
4.		END RUN PROGRAM
	GIVEN X VALUE? GIVEN Y VALUE?	$\frac{x_i}{y_i}  \frac{\text{RUN PROGRAM}}{\text{RUN PROGRAM}} \qquad \text{for } i = 0, 1, \dots, n$ After all of the given points have been entered, RUN PROGRAM and go to Step 7.
7. 1	EVALUATE AT X= ?	X RUN PROGRAM  The value of x entered and the corresponding interpolated value $y = P(x)$ will be printed and control returned to Step 7 for another x. To run a new problem, STOP and return to Step 4.

#### EXAMPLE

Using the following values of  $\ln x$  obtained from a table, find the value of  $\ln x$  at the points,  $x=1.981,\ 1.9825,\ 1.985,\ and\ 1.987.$ 

#### Given Values

x	ln x
1.980	. 6830968447
1.982	. 6841064359
1.983	. 6846108495
1.986	. 6861225656

The answers agree with the actual values of  $\ln x$  to the full 10 digits printed.

GIVEN VALUES ARE 1.980000000E 00 6.830968447E-01 1.982000000E 00 6.841064359E-01 1.983000000E 00 6.846108495E-01 1.986000000E 00 6.861225656E-01 INTERPOLATED VALUES ARE 1.981000000E 00 6.836017677E-01 1.982500000E 00 6.843586745E-01 1.985000000E 00 6.856189141E-01 1.987000000E 00 6.866259636E-01

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6 :
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PRT X","
      Y" : X : C i
GTO -2H
7 #
0+2;1+Yh
8:
IF (Z+1+Z) = HFC+Y
R(2B)+C;GTO -2H
9:
IF Z=B;GTO -1F
10:
Y(X-R(2Z-1))/(R(
2B-1)-R(2Z-1))+Y
#GTO -2H
11:
END H
```



### FOURIER SERIES COEFFICIENTS FOR UNEQUALLY SPACED DATA POINTS

This program  $^1$  calculates the Fourier Series coefficients  $a_n$  and  $b_n$  that represent a periodic time function f(t) represented by discrete x, y pairs of data points over a period T. The x coordinate values have to be entered in ascending order but need not be equally spaced. The Fourier Series representation of f(t) is:

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{2\pi nt}{T} + b_n \sin \frac{2\pi nt}{T}$$

where

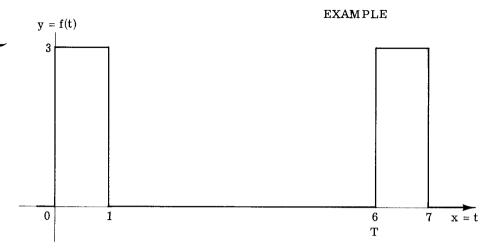
$$\begin{array}{lll} a_0 & = & \frac{1}{T} \int\limits_{t_0}^{t_0 + T} f(t) \, dt \\ \\ a_n & = & \frac{2}{T} \int\limits_{t_0}^{t_0 + T} f(t) \, \cos \frac{2\pi \, nt}{T} \, dt & n = 1, 2, 3, \dots \\ \\ b_n & = & \frac{2}{T} \int\limits_{t_0}^{t_0 + T} f(t) \, \sin \frac{2\pi \, nt}{T} \, dt & n = 1, 2, 3, \dots \end{array}$$

The program evaluates the coefficients by numerically integrating using a parabolic fit through each grouping of three consecutive points. Execution time is dependent upon the number of pairs of data points and the number of coefficients desired. Also, if n denotes the highest coefficients  $a_n$ ,  $b_n$  desired and k the number of data pairs, problem size is subject to the constraint  $n + k \le 116$  (or  $\le 113$  if the Peripheral Control I block is in the calculator). For example, 100 data pairs could be entered and 16 values of  $a_n$  and  $b_n$  obtained.

If a plotter is available, the General Function Plot program, III-7, may be used in conjunction with this program to produce an approximating Fourier Series plot, an f(t) frequency spectrum plot, and a phase spectrum of f(t) plot. The necessary programming is included in the Example section.

<sup>&</sup>lt;sup>1</sup> Based on a FORTRAN program written by Ivar Larson, Hewlett-Packard Calculator Marketing, New Products Development.

ROM BLOCKS	(If plotting) INTERNAL REGISTERS
1	Peripheral Control I 173 X 429
PERIPHERALS  9860A MARKED CARD RE	ADER
9861A TYPEWRITER	9863A PAPER TAPE READER
STEP DISPLAY	INSTRUCTIONS
1.	ERASE LOAD EXECUTE
2.	Alternately insert magnetic cards and <u>EXECUTE</u> until NOTE 14 no longer appears, indicating completion of loading.
3.	(Optional) <u>FIXED N n EXECUTE</u> , or <u>FLOAT N n EXECUTE</u> as desired.
4.	END RUN PROGRAM
5. FINAL N?	$\underline{n}$ RUN PROGRAM indicating the highest coefficients $a_n$ , $b_n$ desired.
6. X? Y?	$\frac{x_i}{y_i} \frac{\text{RUN PROGRAM}}{\text{RUN PROGRAM}} \qquad \text{Data pair entry with } x_1 < x_2 < x_3 < \dots$ $\frac{y_i}{y_i} \frac{\text{RUN PROGRAM}}{\text{RUN PROGRAM}}. \qquad \text{The a}_n \text{ and b}_n \text{ will be calculated and printed. Execution time depends on n and the number of points entered. For another case, return to Step 4.}$



Using the x, y pairs below selected from the above rectangular wave form, find the first 10 values of  $\mathbf{a}_n$  and  $\mathbf{b}_n$  of the Fourier Series representing this wave form.

(Note: If plotting is to be done, be sure to place the Peripheral Control I block in place prior to Step 1 in the User Instructions).

#### Input Data

Final n = 10

X	у	x	у	x	у	х	у
0	1.5	. 8	3	2.5	0	5	0
. 1	3	.9	3	2.75	0	5.2	0
. 15	3	1	1.5	3	0	5.4	0
. 2	3	1.1	0	3.5	0	5.8	0
. 25	3	1.2	0	3.7	0	5.85	0 -
. 3	3	1.3	0	3.9	0	5.9	0
. 4	3	1.5	0	4	0	6	1.5
. 5	3	1.7	0	4.25	0		
.6	3	1.9	0	4.5	0		
. 7	3	2	0	4.8	0		

The data pairs must be entered such that each succeeding x increases in value. About 5 minutes of calculation time is required for this example. Since the data points and coefficients  $\mathbf{a}_n$ ,  $\mathbf{b}_n$  are stored, the General Function Plot program, III-7, may be used to produce the following plots:

Approximating Fourier Series Plot Frequency Spectrum of f(t) Plot Phase Spectrum of f(t) Plot

The function routines "F" used in the General Function Plot program are shown on the next page.

COE	FFICIENTS	
	N AN BN	
	0.000000000 .50000000	$\mathbf{a}_{0}$
	1.000000000 .82623580 .47682981	$a_1$ $b_1$
	2.00000000 .41197091 .71317318	a <sub>2</sub> b <sub>2</sub>
	3.000000000 .000000000 .63073598	a <sub>3</sub> b <sub>3</sub>
	4.000000000 20361381 .35201543	•
	5.00000000 16140575 .09246861	
	6.000000000 .00000000 00072307	
	7.00000000 .11229009 .06417231	
	8.000000000 .09651357 .16664684	
	9.00000000 .00000000 .19360269	
	10.00000000 07365751 .12756916	a 10 b 10

#### EXAMPLE (Continued)

CAUTION: DO NOT press <u>ERASE</u> in Step 1 of the User Instructions of the General Function Plot program. Instead, press <u>END</u> <u>EXECUTE</u> and then proceed with the program loading and remaining User Instructions.

1. Approximating Fourier Series Plot Load the General Function Plot program and key in this function:

#### 25: "F"L 26: IF FLG 6=0;SFG 6 1688 +5H R6+R5;0+R10F 28: IF (R5+1+R5)>R7; R10+R18+Y; RET H 99: R(2R9+29+R5-R6) COS (2R5mX/R8)+R 10+R10H 30: R(2R9+R7+30+R5-2 R6)SIN (2R5mX/R8 )+R10+R10;GTO -2 311 0 - R 1 0 F

IF (R10+1→R10)>R

LTR R(28+2R10)-. 0032(R1-R0),R(29 +2R10)-.0032(A-B

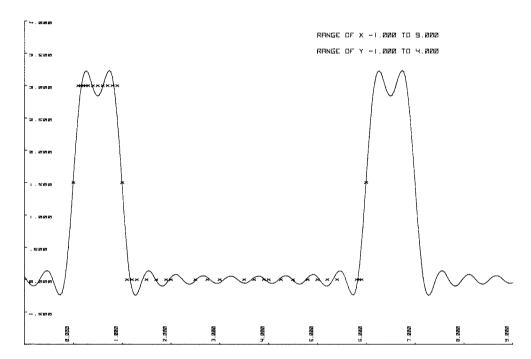
PLT "X";GTO -2H

9;RET + 33;

) + 111H 34:

#### x<sub>min</sub> = -1 x<sub>max</sub> = 9 no. of points = 300 y<sub>min</sub> = -1 y<sub>max</sub> = 4

Input Data



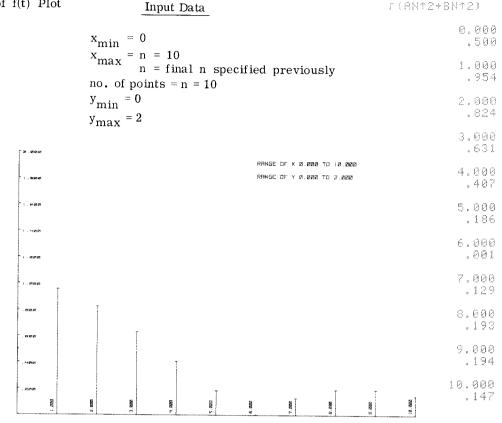
For the next two plots, load the General Function Plot program. Key in the function and the following changes for proper x axis labeling.

#### Changes

#### EXAMPLE (Continued)

#### 2. Frequency Spectrum of f(t) Plot

#### 



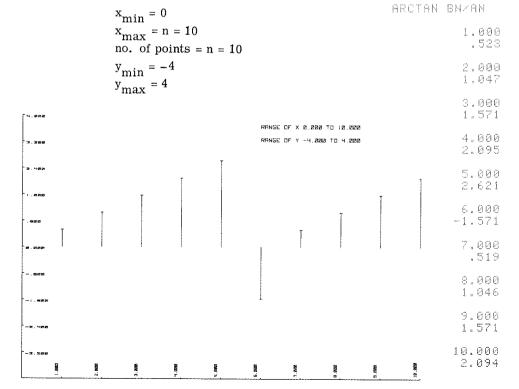
#### 3. Phase Spectrum of f(t) Plot

#### 25: "F"-261 PEH 71F FLG 5=01 07R107Y;SFG 5; SFG 14;PRT " AR CTAN BN/AN";SPC IFRET H R10+1+R10;R(2R9+ R10+30+R7-R6)+R1 1;R(2R9+R10+29)+ R12H 28: ATH (R11/R12)+(( R11>0)-(0>R11))( 0>R12)π→YE 29: PLT X+.03,Y;PLT X,Y;PLT X-.03,Y; PLT X,Y;PRT X,Y; SPC 1;0+Y;RET H

30:

END +

#### Input Data



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16:
(R11+R14)/2+R8;
GTO +2h
17:
R12+R6;R13+R7;R1
4+88+
18:
0+R15H
19:
2R6X+R7+R16;2R6C
+R7+R17H
20:
R9+R12;R10+R13;R
11+R14h
21:
IF R15+R0#0;GTO
+21-
221
R6(XXX-CCC)/3+R7
(XX-CC)/2E+R8(X-C
)+R18+R18#GTO +1
31-
23:
IF R0>0;SFG 1H
24:
((R15+R0)/R2+R19
0 C+R20H
25:
((R15+R0)/R2+R19
) C+R20+
26#
2m(R20-INT R20)+
RZOH
27:
2m(XR19-INT (XR1
9)) >R21F
28:
((2mR19+R22)R22+
R19)R22+R22H
29:
(R4R19-2R6)/R22+
Z:(R3R19-2R6)/R2
2+R25;SIN R20+R2
6-
30:
R16(COS R21+R29)
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R27)/R19-R25R26+
R23H
31:
R16R28/R19-ZR29-
R17R26/R19+R25R2
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321
FLG 1+R15+2A+29+
21-
33:
R23+RZ+RZ+
24:
R24+R(Z+R1-R0+1)
*R(Z+R1-R0+1)h
35:
IF R15+R0=R1;R15
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36:
TF B#A-1;GTO -32
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37:
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38#
PRT 0+Z,R18/(R2+
R8) +R18; SPC 1; (R
1+R7)-(R0+R6)+1+
BIA+R9H
99:
IF (Z+1+Z)>B-1
SPC SIJMP 3H
40:
PRT 2+R0,2R(2A+Z
+29+C)/R2+RC,2R(
B+C)/R2+R(B+C)F
41:
SPC 1;JMP -2F
42#
END H
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# PARTIAL SUM OF AN INFINITE SERIES/ PARTIAL PRODUCT OF AN INFINITE PRODUCT

This program will evaluate sums or products of the form

$$\sum_{n=k}^{N} f(n, x) \qquad \text{or} \qquad \prod_{n=k}^{N} f(n, x)$$

and, in particular, sums or products of the form

$$\sum_{n=k}^{N} \ g(n) \qquad \qquad \text{or} \qquad \qquad \prod_{n=k}^{N} \ g(n)$$

#### REMARKS

1. After loading the program, the expression  $f(n, x) \rightarrow Y$  (or  $g(n) \rightarrow Y$ ) is keyed in and stored beginning in program line 30 and continuing for as many lines as required to define the function. When keying in the expression, substitute C for n and X for x, the storage locations of n and x respectively. Register R1 contains  $(-1)^n$  for  $n = k, \ldots, N$  so that in the expression for f(n, x) or g(n),

$$(-1)^n$$
 may be replaced by R1  
 $(-1)^{n\pm 1}$  may be replaced by -R1  
 $(-1)^{2n}$  may be replaced by R1R1  
 $(-1)^{2n\pm 1}$  may be replaced by -R1R1

If storage registers or flags are needed, refer to the MEMORY ALLOCATION page for availability. Flags 2, 3, and 4 are available to the user and are cleared by the program each time a new sum or product is formed starting at the lower limit.

- 2. An option is provided in the User Instructions to enter an epsilon. If entered, summation of a series will continue until  $|f(n, x)| \le \varepsilon$  or n = N, the upper limit, whichever occurs first. When a product is being formed, the test is for  $|f(n, x)-1| \le \varepsilon$  and n = N. Similarly for g(n). The test can essentially be made to depend only upon  $\varepsilon$  by specifying a very large N. The test can be made to depend only upon N by not specifying an epsilon, in which case calculations will continue until n = N.
- 3. When "UPPER LIMIT?" is displayed, the user may set flag 1 prior to keying in N, RUN PROGRAM for displaying the sum or product as it is being formed, giving a visual check for convergence or divergence.
- 4. After an upper limit has been selected, calculations proceed until the end results are printed. Calculations may be interrupted by pressing the SFG\*key, causing the value of x (if there is one) and the current value of n to be printed together with the sum or product to that point. Calculations then resume and this process may be repeated any number of times independent of whether or not intermediate sums or products are being displayed.
- 5. After the end results are printed and if no  $\epsilon$  has been specified, "UPPER LIMIT?" is again displayed and a new upper limit N' may be specified. If N < N', calculations will continue from n = N + 1 saving recalculation time up to n = N.

<sup>\*</sup> SFG refers to the SET/CLEAR FLAG N key.

ROM BL	OCKS	INTERNAL REGISTERS  X 173 429
PERIPHE	1 ERALS	2 3
9860	OA MARKED CARD READER	9862A PLOTTER 9864A DIGITIZER
9861	A TYPEWRITER	9863A PAPER TAPE READER
STEP	DISPLAY	INSTRUCTIONS
1.		ERASE LOAD EXECUTE
2.		Alternately insert magnetic cards and <u>EXECUTE</u> until NOTE 14 is no
		longer displayed, indicating completion of loading.
3.		GO TO 3 0 EXECUTE and key in and store the function being considered:
		$f(n, x) \longrightarrow Y$ (or $g(n) \longrightarrow Y$ ) STORE RETURN STORE END STORE
		using register X to represent x and register C to represent n, as needed.
4.		END RUN PROGRAM
5.	SUM?	RUN PROGRAM to cycle between these two displays; 1 RUN PROGRAM to
	PRODUCT?	select the option being displayed.
6.	EPSILON?	ε RUN PROGRAM or
:		RUN PROGRAM if there is no ε.
7.	VALUE OF X?	x RUN PROGRAM or
		RUN PROGRAM if there is no $x$ (g(n) case).
8.	LOWER LIMIT?	k RUN PROGRAM
9.	UPPER LIMIT?	* <u>SFG</u> <u>1</u> <u>EXECUTE</u> (optional) for display of intermediate results and then
		N RUN PROGRAM or
		RUN PROGRAM to return to Step 6 for a new case.
10.		* <u>SFG</u> while the program is running to print x (if there is one), the current
		value of n, and the sum or product up to that n. Calculations continue auto-
		matically.
		After final answers are printed, control returns to Step 9 unless an epsilon
		has been specified in which case control returns to Step 6.
		* SFG refers to the SET/CLEAR FLAG N kev.

#### **EXAMPLES**

10  $\sum_{n=1}^{m \text{ otherwistics}} x^n \text{ for } x = .5 \text{ and no epsilon. If the } \frac{1}{2} x^n \text{ for } x = .5$ 

 $X \nmid C - Y$  may be keyed in for the function. If not, choose a register not used by the program, R3 for instance. Manually store the value 1 in R3 after loading the program. Then  $x^n$  may be formed as follows:

#### Program Lines

Results

X N .50000 10.00000

,99902

2. Evaluate the sum  $\sum_{n=1}^{N} \frac{(-1)^n}{n}$  and terminate when  $\left|\frac{(-1)^n}{n}\right| \le 10^{-2}$ . When the display requests the upper limit, use N =  $10^9$ , for example.

Program Lines

Results

N 100.00000 SUM -.68817

3. Evaluate the product  $10! = \prod_{n=1}^{10} n$ 

## Program Lines

Results

10.000 PRODUCT 3628800.000

# **MEMORY ALLOCATION**

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```
0 :
CFG 13; CFG 5;
ENT
    "SUM?",Z;IF
FLG 13=0 ! SFG 5 !
GTO 2H
1 #
CFG 13%ENT "PROD
UCT?",Z;IF FLG 1
31GTO 0H
2:
-1+R0h
ENT "EPSILON?",R
0 h
4 :
CFG 131CFG 61
ENT "VALUE OF X?
",X;IF FLG 13=0;
SFG 6H
5:
ENT "LOWER LIMIT
?",R2;GSB 24H
6.5
CFG 13%CFG 0%
CFG 1; ENT "UPPER
LIMIT?", A; IF
FLG 13%GTO 2H
7 :
IF C>A;GSB 24H
8:
GSB "F"H
9 :
IF FLG 5;8+Y+8;
GTO +2H
10:
BY+BH
1 1 :
IF FLG 1:DSF BH
12:
IF FLG 0:CFG 0:
GSB 17H
13#
-R1+R11(FLG 5=0)
-Y+Z; IF 0Z+Z+Z
14:
GTO +2; IF (C=A)+
(Z(R0);GSB 17H
15:
C+1+C;GTO 6;IF 0
KROSGTO 2H
16:
C+1+C;GTO 8H
```

```
17:
IF FLG 63PRT "
      \times " \vdash-
18#
PRT "
               H '' |--
19:
IF FLG 6; PRT XH
20:
PRT C#SPC 1H
21:
IF FLG 5; PRT "
      SUM"; JMP 2F
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231
PRT BISPC LIPRT
...
-";SPC 1;RET H
24#
CFG 2:CFG 3:CFG
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25:
(1→R1)-FLG 5→B#R
2+C+Z; IF 0>Z; -Z+
Z H
26:
IF (Z=0)+(Z=2);
RET H
27:
TF Z/2#Z/2-.5+1E
11-1E11;-1+R1F
28:
RET H
29:
" F " |--
20:
END H
```

This program uses the principle of interval-halving to find real roots of an equation f(x) = 0 in

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a closed interval [a, b] where the equation may be algebraic (e.g.,  $5x^4 - 3x + 1 = 0$ ), rational (e.g.,  $x^{3/2} + \sqrt{x} - 2 = 0$ ), or transcendental (e.g.,  $3\cos x - 4x = 0$ ).

The user specifies the continuous, real function f, an interval [a, b], an accuracy tolerance  $\epsilon$ , and

The user specifies the continuous, real function f, an interval [a,b], an accuracy tolerance  $\varepsilon$ , and a search increment delta x (= $\Delta x$ ). The program then begins at the left of the interval and compares the functional values at the ends of the interval  $[a, a + \Delta x]$ . If f(a) and  $f(a + \Delta x)$  are of opposite sign, this interval will be searched for a root. Otherwise, or even after a root is found, the program proceeds in the same manner with the interval  $[a + \Delta x, a + 2\Delta x]$ , etc. At most one root will be found by the program for each of these small intervals and the program will examine  $N = INT \frac{(b-a)}{\Delta x}$  of these intervals.

Reference:

Royce Beckett and James Hurt, <u>Numerical Calculations and Algorithms</u> (New York: McGraw-Hill Book Company, 1967), pp. 46-52.

ROM BLOCKS	INTERNAL REGISTERS
	X 173 A29
1 PERIPHERALS	2 3
9860A MARKED CARD READER	9862A PLOTTER 9864A DIGITIZER
9861A TYPEWRITER	9863A PAPER TAPE READER
STEP DISPLAY	INSTRUCTIONS
1.	ERASE LOAD EXECUTE
2.	Alternately insert magnetic cards and <u>EXECUTE</u> until NOTE 14 no longer appears, indicating completion of loading.
3.	(Optional) FIXED N $\underline{n}$ EXECUTE, or FLOAT N $\underline{n}$ EXECUTE as desired.
4.	GO TO 2 1 EXECUTE and key in and store the function being considered using register X to represent the variable x and storing the resulting function value in Y: $\underline{f(x)} \longrightarrow \underline{Y} \underline{STORE} \underline{RETURN} \underline{STORE} \underline{END} \underline{STORE}.$
5.	END RUN PROGRAM
6. START AT X = ?	a RUN PROGRAM
7. END AT X = ?	b RUN PROGRAM
8. DELTA X = ?	Δx RUN PROGRAM
9. EPSILON = ?	$\underline{\epsilon}$ RUN PROGRAM or RUN PROGRAM alone and $\epsilon=10^{-10}$ will be assumed.
	The search begins automatically and roots are printed as they are found together with a reevaluation of the function at the roots. Control returns to Step 6 for designation of a new interval and the same function. For a new function, <u>STOP</u> and go to Step 4.

### **EXAMPLES**

1. Find the roots of  $x^3-8x^2+5x+14=0$  in the interval  $[-10,\ 10]$  using  $\Delta x=1$  and  $\epsilon=10^{-6}$ .

Key in these program lines:

Results:

```
20:
"F"-
                                                          ROOTS OF F(X)
ON THE INTERVAL
21:
XXX-8XX+5X+14+YF
22:
                                                              -10.00000000
                                                              10.00000000
RET H
                                                                 ARE
END H
                                                                 F(X)
                                                               -1.00000000
                                                                0.00000000
                                                                X
F(X)
                                                                2.00000000
                                                                0.00000000
                                                                 F(X)
                                                                7.000000000
                                                                0.00000000
                                                           END OF SEARCH
```

2. Find the roots of  $x^{5/2}$  -  $2\sqrt{x}$  = 0 in the interval [0, 20] using  $\Delta x$  = 1 and  $\epsilon$  = 10<sup>-6</sup>.

Key in these program lines:

Results:

```
20:
"F"H
                                                         ROOTS OF F(X)
ON THE INTERVAL
21:
T(XXXXX) -2TX+YF
                                                               0.00000000
22:
RET +
                                                              20.00000000
23:
END H
                                                                ARE
                                                                F(X)
                                                               0.00000000
                                                               0.00000000
                                                                F(X)
                                                               1.41421366
                                                                .00000032
                                                          END OF SEARCH
```

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В	b					
С	f(a)					
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",R0F
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F(X)"," ON THE I
MTERVAL"; SPC 1;
PRT A,R2; SPC 2;
PRT "
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#SPC H
3#
A+X;GSB "F"+
4.:
Yach
.....
A+R1+BH
6:
IF C=0;GTO 17h
7 :
IF B>R2;GTO 19H
8 :
Baxh
9#
GSB "F"H
10:
IF 06(Y+R3)C;R3+
C:B+A:GTO 5H
11:
B+R4H
12:
(A+R4)/2+XF
13:
GSB "F"H
14:
IF (Y \le R0)(Y > -R0)
FRT "X"
        F(X)",X
,Y;SPC 1;R3+C;B+
A#GTO 5H
15:
IF 0>CY;X+R4;
GTO -3H
16:
X⇒A;Y⇒C;GTO -4F
17:
              Х" э
PRT "
        F(X)":X:
Y;SPC 1;B⇒AF
```

```
18:
IF R2>B; GTO 3H
19:
SPC 1; PRT " END
OF SEARCH"; SPC
8; GTO ØH
20:
"F"H
21:
END H
```



Most methods to compute Bessel functions such as  $J_n(x)$  and  $Y_n(x)$  have certain restrictions. For instance, the power series representation for  $J_{n}\left(x\right)$  converges slowly and loses accuracy due to roundoff for values of x greater than 10. Asymptotic expansions of  $J_n(x)$  and  $Y_n(x)$  are applicable when x is large relative to n. Neither of these two standard techniques are useful when both n and x are large. A method developed by J. C. P. Miller overcomes these restrictions and provides a simple algorithm to calculate  $J_n(x)$ ,  $Y_n(x)$ , and many other Bessel functions over the entire range x > 0,  $n \ge 0$ , n an integer. This program<sup>2</sup> uses the technique developed by Miller to compute any of the following Bessel functions of integer order:

Function	Name	Symbol Used in Program
$J_{n}(x)$	Bessel function of the first kind	JN(X)
$Y_n(x)$	Bessel function of the second kind	YN(X)
$I_n(x)$	Modified Bessel function of the first kind	IN(X)
$j_n(x)$	Spherical Bessel function of the first kind	JSN(X)
$y_n(x)$	Spherical Bessel function of the second kind	YSN(X)
$i_{n}(x)$	Modified Spherical Bessel function of the first kind	ISN(X)
$k_{n}(x)$	Modified Spherical Bessel function of the third kind	KSN(X)

As an example of the method, the modified Bessel function of the first kind,  $I_n(x)$ , is obtained from the descending recurrence relation

$$I_{p-1} = (2p/x) I_{p} + I_{p+1}$$

with normalization by

$$I_0 + 2 \sum_{m=1}^{\infty} I_m = e^X$$

<sup>&</sup>lt;sup>1</sup> Generation of Bessel Functions on High Speed Computers, Irene Stegun and Milton Abramowitz, Mathematical Tables and Other Aids to Computation, Volume 11, 1957, pp. 255-257.

<sup>&</sup>lt;sup>2</sup> Based on a very complete, accurate, and efficiently programmed set of 9100 Bessel function programs submitted to Hewlett-Packard by J. Robert Cooke, Associate Professor, New York State College of Agriculture, Cornell University.

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9861	A TYPEWRITER	9863A PAPER TAPE READER
STEP	DISPLAY	INSTRUCTIONS
1.		ERASE LOAD EXECUTE
2.		Alternately insert magnetic cards and <u>EXECUTE</u> until NOTE 14 no longer appears, indicating completion of loading.
3.		(Optional) FIXED N $\underline{n}$ EXECUTE, or FLOAT N $\underline{n}$ EXECUTE as desired.
4.		END RUN PROGRAM
5.	N?	$\underline{n}  \underline{RUN \ PROGRAM} \qquad \qquad n = order$
6.	X?	$\underline{x}  \underline{RUN \ PROGRAM} \qquad \qquad x = argument$
7.	JN(X)?	RUN PROGRAM to cycle between these 7 displays;
	YN(X)?	1 RUN PROGRAM to select the function displayed.
	IN(X)?	
	JSN(X)?	
	YSN(X)?	
	ISN(X)?	
	KSN(X)?	
		After the selected function is evaluated and the results printed, control
		returns to Step 5 for a new case.

### **EXAMPLES**

Verify the following values selected from Bessel function tables:

	Symbol in Program
$J_4(2) = 3.399571981 E - 2$	JN(X)
$I_5(2) = 9.825679323 E - 3$	IN(X)
$j_3(.1) = 9.5185 E - 6$	JSN(X)
$i_1(3.7) = 3.99283865$	ISN(X)
$k_2(1.6) = .80213693$	KSN(X)
$y_2(4.5) = .10491554$	YSN(X)
$Y_0(10) = 5.567116730 E - 2$	YN(X)
$Y_2(100) = 7.683686678 E - 2$	YN(X)

In the last example above, x is large relative to n and for such cases, Program IV-2, Bessel Function II, using an asymptotic expansion will find the answer much faster. The time required to find a result using this program should not exceed about 75 seconds. If no result is obtained in that time, the program is unable to converge, as in the case of  $i_{50}(1)$ .

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```
15:
IF B=C;R3+R1;IF
1E13>(Z-2(0)Z)Z+
7) # 10+10 (C(1)
FLG 1+2;GTO 11H
16:
IF B=0;GTO 21H
17:
IF (R0=0+R0)+
FLG 3;R3+A+A;IF
FLG 2;R3R6/B+R5+
R51-R6+R6H
18:
IF (FLG 1=0)(
FLG 3=0); 5+8+8+
19:
28(R3+(2(FLG 1=0
)(FLG 4=0)-1)R2X
/2B)/X+Z;R3+R2;Z
→R3F
20:
B-1-.5(FLG 1=0)(
FLG 3=0) +B;GTO 1
51-
21:
IF (FLG 1=0)(
FLG 3=0);GTO 25H
221
2A+R3+ZH
23:
IF FLG 3;Z/EXP X
+ZH
24#
GTO 28H
25:
TF FLG 4=0;EXP X
+Z;(Z-1/Z)/2+Z;
GTO 27h
261
SIN X+ZH
27:
R3X/Z+ZH
R1/Z+A;R3/Z+B;R5
/Z→R5;IF 0>R2;-R
2+R2H
29:
IF 0>Z;-Z+ZH
30:
IF (Z>1E98)+(R2>
1E98);-10-10(C41
)FLG 1+Z;GTO 11H
31:
TF FLG 21GTO 45H
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```
32:
IF FLG 1; PRT "JN
(X)"E
33:
IF FLG 3; PRT "IN
(X)"H
341
IF FLG 4; PRT "JS
N(X)"-
35:
IF (FLG 1=0)(
FLG 3=0)(FLG 4=0
);PRT "ISN(X)"H
36#
GTO 53H
37:
0+41
38:
Y+1+YF
39:
IF Y>C#0→Z#GTO +
31-
40:
IF Y=C;B+A; n+Z;
GTO +2h
41:
(2Y+1)B/X-A+ZB+
A;Z-2FLG 6Z+B;
GTO 38H
421
IF FLG 61008 (nY
+Z)A+AH
43:
IF FLG 5; PRT "YS
N(X)";GTO 53H
44:
PRT "KSN(X)";
GTO 53H
451
X/2+XF
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649)B+4R5)/n+ZH
471
(AZ-1/nX)/B+A;Z+
F: |--
48#
IF R4=018+AH
49:
IF R441;GTO 52H
50:
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	PROGRAM LISTING	
51: AY/X-B+Z;A+B;Z+A ;JMP (Y+1+Y)=R4H 52: PRT "YN(X)"H 53: PRT A;SPC 1;GTO 0H		
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This program 'uses the method of asymptotic expansion' to find, for a given x and real number r with  $x >> r \ge 0$  (x very much larger than r), any one of the following Bessel functions of fractional order:

Function	Name	Symbol Used in Program
$J_{\mathbf{r}}$ (x)	Bessel function of the first kind	JR (X)
$Y_{r}(x)$	Bessel function of the second kind	YR (X)
$I_r$ (x)	Modified Bessel function of the first kind	IR (X)
$K_{r}(x)$	Modified Bessel function of the second kind	KR(X)

An accuracy check is built into the program so that if the result obtained is not accurate to within one unit in the tenth significant figure, as is usually the case for small arguments, an error bound,  $\epsilon$ , will also be printed. The actual answer will be between the answer A that is printed and  $A - \varepsilon$ . For small values of the argument, say  $x \le 10$ , the asymptotic expansion is truncated when the terms are not monotonically decreasing.

<sup>&</sup>lt;sup>1</sup> Based on a select set of 9100 Bessel function programs submitted to Hewlett-Packard by J. Robert Cooke, Associate Professor, New York State College of Agriculture, Cornell University.

<sup>&</sup>lt;sup>2</sup> Handbook of Mathematical Functions, Milton Abramowitz and Irene Stegun, National Bureau of Standards, 7th printing, 1968, Chapter 9.

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PERIPH	ERALS	
	OA MARKED CARD READER	9862A PLOTTER 9864A DIGITIZER
9861	A TYPEWRITER	9863A PAPER TAPE READER
STEP	DISPLAY	INSTRUCTIONS
1.		ERASE LOAD EXECUTE
2.		Alternately insert magnetic cards and <u>EXECUTE</u> until NOTE 14 no longer appears, indicating completion of loading.
3.		(Optional) <u>FIXED N n EXECUTE</u> , or <u>FLOAT N n EXECUTE</u> as desired.
4.		END RUN PROGRAM
5.	R?	r RUN PROGRAM r = order
6.	X?	x RUN PROGRAM x = argument
7.	JR(X)?	RUN PROGRAM to cycle between the 4 displays;
	YR(X)?	1 RUN PROGRAM to select the function displayed.
	IR(X)?	
	KR(X)?	
		After the selected function is evaluated and the result printed, control returns
		to Step 5 for a new case.

# **EXAMPLES**

Verify the following values selected from Bessel function tables:

 $J_{2.5}$  (4) = .4409

 $Y_0$  (10) = .0556711673

 $Y_2 (100) = .076836867$ 

 $I_{\frac{1}{3}}(9.8) = 2315.5$ 

 $I_0$  (225) = 1.384583 E96

 $K_1$  (13) = 8.0786 E - 7

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# MEMORY ALLOCATION

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Ø :
TBL 2; TBL 6; SFG
14; ENT "R?", Z*2+1
C: "X?"; XH
1 #
PRT "
        R",Z,"
X",XH
2:
CFG 13;ENT "JR(X
19".7:IF FLG 13=
0;SFG 1;GTO 6H
3:
CFG 13%ENT "YR(X
)?",Z;;F FLG 13=
MISEG 21GTO 6H
4:
CFG 13; ENT "IR(X
)?",Z;IF FLG 13=
0;SFG 3;GTO 6H
5
CFG 13;ENT "KR(X
)?",Z;IF FLG 13;
GTO 2H
F. :
0+Y+R0;1+A+B+Z+R
11
7 :
(Z/B+Z)*(CC-(2B-
1)(28-1))/8X+Z+R
2-
8:
IF 0>Z%-Z+ZH
9:
IF Z>A;Z+A;FRT "
ERROR IS", R2;
GTO 16H
10:
Z+HH
11:
COS ((FLG 1+FLG
2) #INT (8/2) ) R2+
21-
12:
IF R0=0+R0;Z+Y+Y
#6T0 +2H
13:
Z+R1+R1+
14:
R+1+8F
15:
IF A>1E-10;R2+Z;
GTO 7H
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16:
r(2/πX) + Zh
17:
X - (C + 1) \pi / 4 + C +
18#
ZSIN C+R0F
19:
ZCOS C+CH
20:
CR1-YR0+AF
21:
YC+R1R0+BH
22:
EXP X+C;r(n/2X)+
21-
23:
C(R1-Y) \times \Gamma(2\pi X) \rightarrow X
24:
Z(R1+Y)/C→YH
25#
IF FLG 1;PRT "JR
(X)",A;GTO 29H
26#
IF FLG 2;PRT "YR
(X)",B;GTO 29H
27:
IF FLG 3;PRT "IR
(X)",X;GTO 29H
28:
PRT "KR(X)",YH
29:
SPC 11GTO 0H
30:
END -
```

1.	Initial Submission Revision
2.	Equipment required:
3.	Program Title:
4.	Program Description and Application:
5.	Contributor's Name:
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