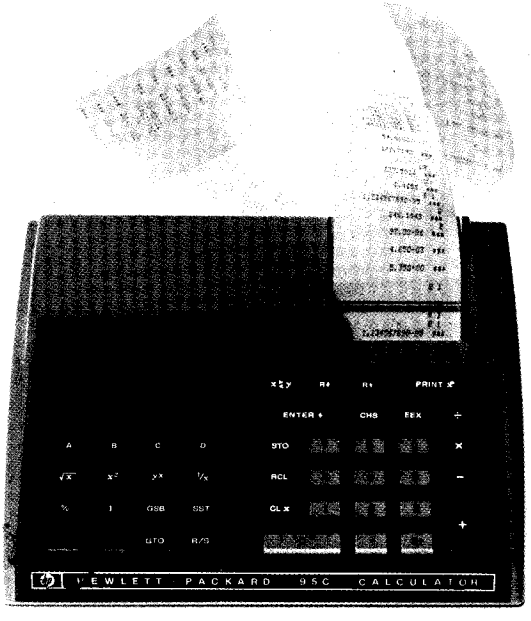
HEWLETT-PACKARD

HP-95C



APPLICATIONS

The program material contained herein is supplied without representation or warranty of any kind. Hewlett-Packard Company therefore assumes no responsibility and shall have no liability, consequential or otherwise, of any kind arising from the use of this program material or any part thereof.

PRINTED IN U.S.A.

INTRODUCTION

Howdy! This is *your* HP-95C Applications Book, your introduction to real world programs written in a number of fields including mathematics, statistics, finance, surveying, navigation and games. These programs demonstrate the diversity of uses for the HP-95C, and will give you immediate calculation aids for problems you encounter every day. You will also find the programs useful as guides to programming techniques and models for writing your own customized software. The comments accompanying each program will demonstrate the approach used to reach the solution and will help you follow what the programmer was thinking as you become an expert with your HP-95C.

Whether your interest lies in solving a particular problem in a specific area, or in learning more about the programming power of your calculator, we hope that this book will help you get the most from your HP-95C.

TABLE OF CONTENTS

A WORD ABOUT PROGRAM USAGE

This HP-95C Applications Book provides a description of the program, a set of instructions for using the program, and one or more example problems, each of which includes a list of the actual keystrokes required for its solution. Explanatory comments have been incorporated in each program listing to facilitate your understanding of the actual working of each program. Thorough study of a commented listing can help you to expand your programming repertoire since interesting techniques can often be found in this way.

The lists of keystrokes required to solve example problems indicate the resulting outputs. Those outputs indicated by *** are printed with the printer in MANUAL mode. Other outputs appear in the calculator display.

The completed User Instruction Form—which accompanies each program—is your guide to operating the programs in this Pac.

The form is composed of five labeled columns. Reading from left to right, the first column, labeled STEP, gives the instruction step number.

The INSTRUCTIONS column gives instructions and comments concerning the operations to be performed.

The INPUT-DATA/UNITS column specifies the input data, and the units of data if applicable. Data input keys consist of [0] to [9] and decimal point (the numeric keys), [EEX] *(enter exponent)*, and [CHS] *(change sign)*.

The KEYS column specifies the keys to be pressed after keying in the corresponding input data.

The OUTPUT-DATA/UNITS column specifies intermediate and final outputs and their units, where applicable.

The following illustrates the User Instruction Form for Quadratic Equation, the first program in this Book.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Key in coefficients and			
	display D.	c	[ENTER↑]	
		b	[ENTER↑]	
		a	[A]	(D)
3	If D ≥ 0, roots are real.			x_1
				x_2

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
4	If $D < 0$, roots are complex of			
	form $u \pm iv$.			u
				v

Step 1 requires you to key in the program. For Quadratic Equation, the program steps are labeled with "A" and an associated step number. To key this program for execution under A, switch the HP-95C to W/PRGM mode, press g CLEAR A, and key the steps in as listed. The choice of keying in the program under A is arbitrary, and may be changed to fit the user's needs, making the corresponding changes in the User Instructions shown on the previous page. Note also that some steps on the program listing require keystrokes not explicitly listed for entry in the program, e.g. A-024 LBL 1 is keyed in by three keystrokes f GTO (LBL) 1. (See the Owner's Handbook for a more detailed explanation for keying in programs).

Step 2 asks for the coefficients of the quadratic equation. Coefficient c is keyed in and followed by ENTER↑, coefficient b is keyed in and followed by ENTER↑, and coefficient a is keyed in and followed by the A key. The discriminant D is calculated and displayed with a pause. The calculator then determines if D is positive or negative, and automatically continues to step 3 or 4, respectively, and prints the roots of the quadratic equation.

ALGEBRA AND NUMBER THEORY

QUADRATIC EQUATION

The roots x_1, x_2 of $\quad ax^2 + bx + c = 0$

are given by $\quad x_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

If $\quad D = (b^2 - 4ac)/4a^2$

is positive or zero, the roots are real. In these cases, better accuracy may sometimes be obtained by first computing the root with the larger absolute value:

If $\quad -\dfrac{b}{2a} \geqslant 0, \qquad x_1 = -\dfrac{b}{2a} + \sqrt{D}$

If $\quad -\dfrac{b}{2a} < 0, \qquad x_1 = -\dfrac{b}{2a} - \sqrt{D}$

In either case, $\quad x_2 = \dfrac{c}{x_1\, a}$.

If $D < 0$, the roots are complex, being

$$u \pm iv = \frac{-b}{2a} \pm \frac{\sqrt{4ac - b^2}}{2a}\, i$$

A-000 LBLA		A-021 GTO1	
A-001 ENT↑		A-022 +	$x_1 = -\dfrac{b}{2a} + \sqrt{D}$
A-002 R↓		A-023 GTO2	
A-003 ÷		A-024 LBL1	
A-004 2		A-025 X≷Y	$x_1 = -\dfrac{b}{2a} - \sqrt{D}$
A-005 ÷		A-026 -	
A-006 CHS	$-\dfrac{b}{2a}$	A-027 LBL2	
A-007 ENT↑		A-028 PRTX	x_1
A-008 X²	$b^2/4a^2$	A-029 1/X	
A-009 R↓		A-030 R↓	
A-010 R↓		A-031 ×	
A-011 X≷Y		A-032 PRTX	$x_2 = \dfrac{c}{x_1\, a}$
A-012 ÷	c/a	A-033 SPC	
A-013 STO		A-034 RTN	
A-014 -		A-035 LBL3	Complex roots.
A-015 PSE	D	A-036 CHS	
A-016 X<0		A-037 √X	$v = \sqrt{-D}$
A-017 GTO3	If $D < 0$, roots are complex.	A-038 X≷Y	
A-018 √X		A-039 PRTX	u
A-019 X≷Y		A-040 X≷Y	
A-020 X<0	If $-b/2a < 0$, GTO 1	A-041 PRTX	v
		A-042 SPC	

REGISTERS

0 c/a	1	2	3
4	5	6	7

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Key in coefficients and			
	display D.	c	ENTER↑	
		b	ENTER↑	
		a	A	(D)
3	If D ≥ 0, roots are real.			x_1
				x_2
4	If D < 0, roots are complex of			
	form u ± iv.			u
				v

Example 1:

Find the roots of $x^2 + x - 6 = 0$.

Keystrokes: **Outputs:**

6 CHS ENTER↑ 1 ENTER↑ 1 A →

6.25 (D)
-3.00 *** (x_1)
2.00 *** (x_2)

Example 2:

Solve the quadratic equation $2x^2 - 3x + 5 = 0$.

Keystrokes: **Outputs:**

5 ENTER↑ 3 CHS ENTER↑ 2 A

-1.94 (D)
0.75 *** (u)
1.39 *** (V)

Thus $x_{1,2} = 0.75 \pm 1.39i$.

COMPLEX OPERATIONS

This program allows for chained calculations involving complex numbers. The four operations of complex arithmetic (+, −, ×, ÷) are provided, as well as several of the most used functions of a complex variable z ($1/z$, z^n, and $z^{1/n}$). Functions and operations may be mixed in the course of a calculation to allow evaluation of expressions like $z_3/(z_1 + z_2)$, z^{-2}, $z_1z_2^3$, etc., where z_1, z_2, z_3 are complex numbers of the form a + ib.

Equations:

Let $z_k = a_k + ib_k = r_k e^{i\theta k}$, $\quad k = 1, 2$

Let the result in each case be u + iv.

$$z_1 + z_2 = (a_1 + a_2) + i(b_1 + b_2)$$

$$z_1 - z_2 = (a_1 - a_2) + i(b_1 - b_2)$$

$$z_1z_2 = r_1r_2\, e^{i(\theta_1 + \theta_2)}$$

$$z_1/z_2 = \frac{r_1}{r_2}\, e^{i(\theta_1 - \theta_2)}$$

$$1/z = \frac{1}{r}\, e^{-i\theta}, \qquad z \neq 0$$

$$z^n = r^n e^{in\theta} \qquad (n = \text{integer})$$

$$z^{1/n} = r^{1/n}\, e^{i\left(\frac{\theta}{n} + \frac{360k}{n}\right)}, k = 0, 1, \ldots, n-1 \qquad (n = \text{integer})$$

(All n roots will be output and temporarily stored during the calculation of $z^{1/n}$, k = 0, 1, ..., n−1; at the end of the calculation, the final root will be stored.)

Remarks:

The logic system for this program may be thought of as a kind of Reverse Polish Notation (RPN) with a stack whose capacity is two complex numbers. Let the bottom register of the complex stack be ξ and the top register τ. These are analogous to the X, Y- and Z, T- registers in the calculator's own four-register stack.* A complex number z_1 is input to the ξ-register by the keystrokes a_1 ENTER↑ b_1 A. Upon input of a second complex number z_2 (as a_2 ENTER↑ b_2 A), z_1 is moved to τ and z_2 is placed in ξ. The previous contents of τ are lost.

Functions operate on the ξ-register, and the result is left in ξ; τ is unchanged. Arithmetic operations involve both the ξ- and τ- registers; and result of the

operation is left in ξ and τ is unchanged.

*Each register of the complex stack must actually hold two real numbers: the real and the imaginary part of its complex contents. Thus, it takes two of the calculator's registers to represent one register in the complex stack. In this discussion we speak of the complex stack registers as though they were each just one register.

A-000 LBLA	Input a↑b
A-001 R 3	
A-002 S 1	Last b → R_1 (b_1)
A-003 R↓	
A-004 S 3	Present b → R_3 (b_2)
A-005 R↓	Last a → R_0 (a_1)
A-006 R 2	
A-007 S 0	Present a → R_2 (a_2)
A-008 R↓	
A-009 S 2	
A-010 RTN	- - - - - - - - - - - - - -
A-011 LBL1	
A-012 R 0	Add (+)
A-013 S+2	
A-014 R 2	
A-015 PRTX	$a_2 \leftarrow a_1 + a_2$
A-016 R 1	
A-017 S+3	
A-018 R 3	
A-019 PRTX	$b_2 \leftarrow b_1 + b_2$
A-020 SPC	
A-021 RTN	- - - - - - - - - - - - - -
A-022 LBL2	Subtract (–)
A-023 R 0	
A-024 R 2	
A-025 -	
A-026 S 2	$a_2 \leftarrow a_1 - a_2$
A-027 PRTX	
A-028 R 1	$b_2 \leftarrow b_1 - b_2$
A-029 R 3	
A-030 -	
A-031 S 3	
A-032 PRTX	
A-033 SPC	
A-034 RTN	- - - - - - - - - - - - - -
A-035 LBL3	
A-036 R 1	Multiply (x)
A-037 R 0	
A-038 →P	
A-039 R 3	$r_1\ \theta_1$
A-040 R 2	
A-041 →P	$r_2\ \theta_2\ r_1\ \theta_1$
A-042 GTO0	- - - - - - - - - - - - - -
A-043 LBL4	Divide (÷)
A-044 R 1	
A-045 R 0	
A-046 →P	$r_1\ \theta_1$
A-047 R 3	
A-048 R 2	
A-049 →P	$r_2\ \theta_2\ r_1\ \theta_1$
A-050 X≷Y	
A-051 CHS	
A-052 X≷Y	
A-053 1/X	$1/r_2 - \theta_2\ r_1\ \theta_1$
A-054 LBL0	- - - - - - - - - - - - - -
A-055 X≷Y	
A-056 R↑	
A-057 +	
A-058 X≷Y	
A-059 R↑	
A-060 x	$r_1\ r_2\ (\theta_1 + \theta_2)$
A-061 LBL9	- - - - - - - - - - - - - -
A-062 →R	Output routine
A-063 S 2	
A-064 PRTX	
A-065 X≷Y	
A-066 S 3	
A-067 PRTX	
A-068 SPC	
A-069 RTN	- - - - - - - - - - - - - -
A-070 LBL5	1/z
A-071 R 3	
A-072 R 2	
A-073 →P	r θ
A-074 X≷Y	
A-075 CHS	
A-076 X≷Y	
A-077 1/X	$1/r - \theta$
A-078 GTO9	- - - - - - - - - - - - - -
A-079 LBL6	
A-080 S I	z^n
A-081 R 3	
A-082 R 2	
A-083 →P	r θ
A-084 I	
A-085 Y^x	
A-086 X≷Y	
A-087 I	
A-088 x	
A-089 X≷Y	$r^n\ n\theta$
A-090 GTO9	- - - - - - - - - - - - - -
A-091 LBL7	$z^{1/n}$
A-092 S I	
A-093 1	
A-094 CHS	
A-095 COS⁻¹	Compute 360/n (or 2π/n, or 400/n)
A-096 ENT↑	
A-097 +	
A-098 X≷Y	
A-099 ÷	
A-100 S 4	
A-101 R 3	
A-102 R 2	
A-103 →P	r θ
A-104 I	
A-105 1/X	
A-106 Y^x	
A-107 X≷Y	
A-108 I	
A-109 ÷	
A-110 X≷Y	$r^{1/n}\ \theta/n$
A-111 LBL8	- - - - - - - - - - - - - -
A-112 GSB9	Convert → R and print
A-113 DSZ	

A-114 GTOO A-115 RTN A-116 LBLO A-117 X≷Y A-118 →P	Loop n times Back → P ($r^{1/n}$, θ/n)	A-119 X≷Y A-120 R 4 A-121 + A-122 X≷Y A-123 GTO8	$\theta/n + \frac{360}{n} k$

REGISTERS

0 a_1	1 b_1	2 a_2	3 b_2
4 360/n	5	6	7
8	9	.0	.1
.2	.3	.4	.5	I n

STEP	INSTRUCTIONS	INPUTS DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Key in first complex number,			
	$a_1 + ib_1$.	a_1	ENTER↑	
		b_1	A	a_1
3	For arithmetic, go to step 4;			
	for a function, go to step 7.			
	A complex result is u + iv.			
	Arithmetic			
4	Key in second complex			
	number, $a_2 + ib_2$.	a_2	ENTER↑	
		b_2	A	a_2
5	Select one of four operations:			
	• Add (+)		GSB 1	u
				v
	• Subtract (−)		GSB 2	u
				v
	• Multiply (x)		GSB 3	u
				v
	• Divide (÷)		GSB 4	u
				v
6	The result of the operation has			
	been stored; go to step 7 for a			

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	function or to step 4 for further			
	arithmetic.			
	Functions			
7	Select one of three functions:			
	• Reciprocal (1/z)		GSB 5	u
				v
	• Raise z to an integer power			
	(z^n)	n	GSB 6	u
				v
	• Find the n^{th} root of z ($z^{1/n}$)	n	GSB 7	u
				v
8	The result has been stored; go			
	to step 4 for arithmetic or to			
	step 7 for another function.			

Example 1:

Evaluate the expression

$$\frac{z_1}{z_2 + z_3},$$

where $z_1 = 23 + 13i$, $z_2 = -2 + i$, $z_3 = 4 - 3i$. Since the program can remember only two numbers at a time, perform the calculation as

$$z_1 \times \left[1/(z_2 + z_3)\right].$$

Keystrokes:

2 A ENTER↑ 1 A 4 ENTER↑ 3

CHS A GSB 1 ⟶

GSB 5 ⟶

23 ENTER↑ 13 A GSB 3 ⟶

Outputs:

2.00 *** real ($z_2 + z_3$)
-2.00 *** imag ($z_2 + z_3$)

0.25 *** ($1/(z_2 + z_3)$)
0.25 ***

2.50 *** ($z_1/(z_2 + z_3)$)
9.00 ***

Example 2:

Find the 3 cube roots of 8.

Keystrokes:

8 ENTER↑ 0 A 3 GSB 7 ⟶

Outputs:

```
 2.00 ***
 0.00 ***

-1.00 ***
 1.73 ***

-1.00 ***
-1.73 ***
```

3 × 3 MATRIX OPERATIONS

This program performs some of the most common operations involving 3 × 3 matrices; namely, the calculation of the determinant and inverse of the matrix and the solution of a system of three simultaneous equations in three unknowns.

The method used in this program is Gaussian elimination. No pivoting is performed. There is one restriction on the matrix values, i.e., that the upper left-hand element (a_1) not be zero. If $a_1 = 0$, interchange the first row (or column) with another row (or column). This interchange will result in (1) a sign change in the determinant and (2) the interchange of the corresponding *columns* (or rows) in the inverse matrix.

Space does not permit a full treatment of the pertinent equations; however, the Comments section of the program listing shows the operations in detail. Basically, the input matrix A is transformed into an upper triangular matrix U, assuming A is nonsingular. The multipliers used to accomplish this transformation form a lower triangular matrix L, which has 1's along its diagonal. Disregarding one possible permutation matrix, the relationship among these matrices is U = LA. The matrix A is destroyed and the matrices U and L are used to compute the determinant, inverse, and solutions of simultaneous linear equations.

Equations:

$$\text{Let } A = \begin{matrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{matrix}, \; a_1 \neq 0.$$

The determinant of A, Det A, is found *after* its transformation to U by the product of the diagonal elements:

$$\text{Det A} = \pm a_1\, b_2\, c_3.$$

The sign is negative if a row interchange was performed by the program in the transformation of A to U.

A set of 3 simultaneous equations in 3 unknowns may be written as

$$a_1\, x + b_1\, y + c_1\, z = d_1$$

$$a_2\, x + b_2\, y + c_2\, z = d_2$$

$$a_3\, x + b_3\, y + c_3\, z = d_3$$

where x, y, z are unknowns and the $\{d_i\}$ constants.

In matrix notation this becomes $A\mathbf{x} = \mathbf{d}$, where $\mathbf{x}$ and $\mathbf{d}$ are the column vectors

$$\begin{matrix} x \\ y \\ z \end{matrix} \quad \text{and} \quad \begin{matrix} d_1 \\ d_2 \\ d_3 \end{matrix}, \text{ respectively.}$$

This problem is solved (neglecting row interchanges) as $U\mathbf{x} = L\mathbf{d}$.

Let T be the inverse of A; i.e., the 3 × 3 matrix is such that $AT = TA = I$, the 3 × 3 identity matrix.

$$T = \begin{matrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{matrix} \qquad I = \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

Let the column vectors of T be denoted $\mathbf{t}^{(j)}$ and let $\mathbf{I}^{(j)}$ be the column vectors of the identity matrix. Then the inverse matrix T is found by the solution of the equations

$$A\mathbf{t}^{(j)} = \mathbf{I}^{(j)},\ j = 1, 2, 3.$$

Remarks:

1. This program may also be used to solve 2 × 2 systems by representing the 2 × 2 matrix $\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}$ as a 3 × 3 matrix

$$\begin{matrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ 0 & 0 & 1 \end{matrix}$$

and a 2-dimensional column vector $\begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$ as a 3-dimensional column vector $\begin{pmatrix} d_1 \\ d_2 \\ 0 \end{pmatrix}$.

2. A determinant of 0 or an error halt in the exection of the program under LBL A with the message "See 5" indicates that the matrix input is singular.

A-000 LBLA		A-051 R 4	
A-001 S 2	Input	A-052 R 3	
A-002 R↓		A-053 S 4	
A-003 S 1		A-054 R↓	
A-004 R↓		A-055 S 3	- - - - - - - - - - - -
A-005 S.1		A-056 LBL0	
A-006 R/S		A-057 CHS	
A-007 S 4		A-058 S÷4	$m_{32} \leftarrow -\frac{b_3}{b_2}$
A-008 R↓		A-059 R 4	
A-009 S 3		A-060 R 5	
A-010 R↓		A-061 ×	
A-011 S.2		A-062 S+6	
A-012 R/S		A-063 R.1	$c_3 \leftarrow c_3 + m_{32}\ c_2$
A-013 S 6		A-064 R 3	
A-014 R↓		A-065 ×	
A-015 S 5		A-066 R 6	Determinant.
A-016 R↓		A-067 ×	
A-017 S.3	- - - - - - - - - - - -	A-068 R.0	
A-018 1		A-069 ×	
A-019 S.0		B-000 LBLB	- - - - - - - - - - - -
A-020 R.1		B-001 1	Inverse.
A-021 CHS	$m_{i1} \leftarrow -\frac{a_i}{a_1}$, i = 2, 3	B-002 ENT↑	
A-022 S÷1		B-003 0	
A-023 S÷2		B-004 ENT↑	
A-024 R 1		B-005 C	
A-025 R.2		B-006 0	
A-026 ×		B-007 ENT↑	
A-027 S+3	$b_2 \leftarrow b_2 + m_{21}\ b_1$	B-008 1	
A-028 R 1		B-009 ENT↑	
A-029 R.3		B-010 0	
A-030 ×		B-011 C	
A-031 S+5	$c_2 \leftarrow c_2 + m_{21}\ c_1$	B-012 0	
A-032 R 2		B-013 ENT↑	
A-033 R.2		B-014 ENT↑	
A-034 ×		B-015 1	
A-035 S+4	$b_3 \leftarrow b_3 + m_{31}\ b_1$	B-016 C	
A-036 R 2		C-000 LBLC	- - - - - - - - - - - -
A-037 R.3		C-001 S 9	Solve Ax = d.
A-038 ×		C-002 R↓	
A-039 S+6	$c_3 \leftarrow c_3 + m_{31}\ c_1$	C-003 S 8	
A-040 R 3		C-004 R↓	
A-041 X≠0	- - - - - - - - - - - -	C-005 S 7	- - - - - - - - - - - -
A-042 GTO0	If $b_2 = 0$, swap:	C-006 R 1	
A-043 1		C-007 R 7	Ld
A-044 CHS		C-008 ×	
A-045 S.0	$b_2 \rightleftharpoons b_3, c_2 \rightleftharpoons c_3$.	C-009 S+6	$d_2 \leftarrow d_2 + m_{21}\ d_1$
A-046 R 5		C-010 R 2	
A-047 R 6	Set pivot to −1.	C-011 R 7	
A-048 S 5		C-012 ×	
A-049 R↓		C-013 S+9	$d_3 \leftarrow d_3 + m_{31}\ d_1$
A-050 S 6		C-014 R.0	

C-015 X>0		C-034 S÷8	$y = (d_2 - c_2\ z)/b_2$
C-016 GTO0	If Pivot = −1, swap:	C-035 R 9	
C-017 R 8		C-036 R.3	
C-018 R 9	$d_2 \rightleftharpoons d_3$.	C-037 x	
C-019 S 8		C-038 R 8	$x = (d_1 - b_1\ y - c_1\ z)/a_1$.
C-020 R↓		C-039 R.2	
C-021 S 9		C-040 x	
C-022 LBL0	- - - - - - - - - - - - - - -	C-041 +	
C-023 R 4		C-042 S-7	
C-024 R 8	$d_3 \leftarrow d_3 + m_{32}\ d_2$.	C-043 R.1	
C-025 x		C-044 S÷7	
C-026 S+9		C-045 R 7	
C-027 R 6		C-046 PRTX	Output x, y, z.
C-028 S÷9	$z = d_3/c_3$.	C-047 R 8	
C-029 R 9		C-048 PRTX	
C-030 R 5		C-049 R 9	
C-031 x		C-050 PRTX	
C-032 S-8		C-051 SPC	
C-033 R 3			

REGISTERS

0	1 a_2, m_{21}	2 a_3, m_{31}	3 b_2	
4 b_3, m_{32}	5 c_2	6 c_3	7 d_1, x	
8 d_2, y	9 d_3, z	.0 ±1 – Pivot?	.1 a_1	
.2 b_1	.3 c_1	.4	.5	I

STEP	INSTRUCTIONS	INPUTS DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Key in matrix by columns and			
	find determinant.	a_1	ENTER↑	
		a_2	ENTER↑	
		a_3	A	a_1
		b_1	ENTER↑	
		b_2	ENTER↑	
		b_3	R/S	b_1
		c_1	ENTER↑	
		c_2	ENTER↑	
		c_3	R/S	Det A
3	Find inverse by columns.		B	t_{11}
				t_{21}
				t_{31}

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
				t_{12}
				t_{22}
				t_{32}
				t_{13}
				t_{23}
				t_{33}
4	To solve 3 simultaneous			
	equations, key in the			
	constants and find the			
	unknowns.	d_1	ENTER↑	
		d_2	ENTER↑	
		d_3	C	x
				y
				z

Example:

Solve the 3 × 3 system

$$\begin{aligned} x + 2y - z &= 4 \\ 3x - y + z &= 7 \\ -2x + 3z &= 8 \end{aligned}$$

Also find the determinant and inverse of the coefficient matrix.

Keystrokes:	**Outputs:**	
1 ENTER↑ 3 ENTER↑ 2 CHS A →	1.00	
2 ENTER↑ 1 CHS ENTER↑ 0 R/S →	2.00	
1 CHS ENTER↑ 1 ENTER↑ 3 R/S →	−23.00	(Det A)
B →	0.13 ***	(Inverse)
	0.48 ***	
	0.09 ***	
	0.26 ***	

```
                                  -0.04 ***
                                   0.17 ***
                                  -0.04 ***
                                   0.17 ***
                                   0.30 ***
4 ENTER↑ 7 ENTER↑ 8 C ——→          2.00 *** (x)
                                   3.00 *** (y)
                                   4.00 *** (z)
```

BASE CONVERSIONS

This program converts positive numbers to and from base 10 representations. The other base involved may be any integer from 2 to 99, inclusive.

Let x_b be the representation of the number in the original base b. Assume that it is to be converted to the representation x_B in base B. Either b or B must be 10. In general, the bases are stored manually (b in R_1, B in R_2) prior to keying in x_b and pressing **A**, which will cause the computation of x_B.

When converting numbers from base 10, b = 10. However, the number stored for b may be either 10 or 100. If the other base B < 10, then store b in R_1 as 10. If, however, B > 10, the value stored for b in R_1 should be 100.

Similarly, when numbers are converted to base 10 representations, B = 10. When b < 10, the value of B stored in R_2 should be 10; when b > 10, a value of 100 should be stored in R_2.

The table below shows examples of the four possible cases:

To convert	From Base	To Base	Store in R_1	Store in R_2
	10	2	10	2
	10	16	100	16
	2	10	2	10
	16	10	16	100

A number such as $4B6_{16}$ cannot be represented directly on the display because the display is strictly numeric. Therefore, some convention must be adopted to represent numbers R_a when a > 10. We use the convention of allocating two digit locations for each single character in R_a when a > 10.

For example, $4B6_{16}$ is represented as 041106_{16} by our convention (in hexadecimal system, A = 10, B = 11, C = 12, D = 13, E = 14, F = 15).

When displayed, this number may appear as 41106 or with an exponent

4.1106 04

which is interpreted as $4.B6 \times 16^2$.

The displayed exponent 4 is for base 10 and only serves to locate the decimal point (in the same manner as for decimal numbers).

When base $a > 10$ (as in the above example), divide the displayed exponent by 2 to get the true exponent of the number. When the displayed exponent is an odd integer, shift the decimal point of the displayed number one place (to the left or right) and adjust its exponent accordingly to make the true exponent an integer.

For example, the displayed number

$$1.112 \qquad -03$$

is interpreted as $B.C \times 16^{-2}$ or $0.BC \times 16^{-1}$.

Remarks:

1. When the magnitude of the number is very large or very small, this program will take a long time to execute.
2. The program will not give an error indication for invalid inputs for x_b. For example, 981_8 will be treated the same as 1201_8.
3. As the program now stands, the user is forced to make a decision at input time whether the number stored for base 10 is 10 or 100. An alternative approach would be to always store 10, never 100, and have the program decide whether to overwrite the 10 with 100 in some cases. Such an alteration of the program would require about 25 more program steps.

This program may generate roundoff error in the least significant digits for some problems.

A-000 LBL A	$x_b \rightarrow x_B$.	A-024 LBL 8	On entry R_3 contains
A-001 STO 3		A-025 RCL 6	normalized
A-002 RCL 1		A-026 RCL 3	
A-003 STO 5		A-027 x	
A-004 RCL 2		A-028 STO 3	x_b: $0 < x_b < 1$.
A-005 STO 6		A-029 GSB 7	
A-006 0		A-030 RCL 4	
A-007 STO 0		A-031 RCL 5	Build up x_B.
A-008 STO 4		A-032 x	
A-009 EEX		A-033 +	
A-010 1		A-034 STO 4	
A-011 2		A-035 RCL 3	
A-012 STO 8		A-036 GSB 7	
A-013 RCL 3	- - - - - - - - - - - - - - -	A-037 RCL 3	
A-014 LBL 9	Shift right until < 1.	A-038 -	
A-015 1		A-039 ABS	Do not build mantissa beyond
A-016 X>Y	R_0 keeps track of no. places	A-040 STO 3	10^{12}.
A-017 GTO 8	shifted (exponent)..	A-041 1	
A-018 STO+0		A-042 STO-0	
A-019 CL x		A-043 RCL 4	
A-020 RCL 6		A-044 RCL 8	
A-021 ÷		A-045 X≤Y	
A-022 STO 3		A-046 GTO 3	- - - - - - - - - - - - - - -
A-023 GTO 9	- - - - - - - - - - - - - - -	A-047 RCL 3	

A-048 X≠0 A-049 GTO8 A-050 LBL3 A-051 R 5 A-052 R 0 A-053 Y^x A-054 R 4 A-055 x A-056 S 4	x_B	A-057 RTN A-058 LBL7 A-059 EEX A-060 2 A-061 + A-062 EEX A-063 2 A-064 - A-065 INT	Eliminate round-off error

REGISTERS

0 Used	1 b	2 B	3 x_b
4 Used	5 b	6 B	7
8 10^{12}	9	.0	.1

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Store bases (one must be 10			
	or 100):			
	• Base to be converted from	b	STO 1	
	• Base to be converted to	B	STO 2	
3	Key in number in base b and			
	calculate number in base B.	x_b	A	x_B
4	For a new conversion between			
	the same bases, go to step 3;			
	to change either base, go to			
	step 2.			

Example 1:

Convert 0.2937_{10} to base 8 representation. (Since B = 8 < 10, b = 10.)

Keystrokes: **Outputs:**

10 STO 1 8 STO 2 f FIX 9

.2937 A ⟶ 0.226277543 (Base 8)

Example 2:

Convert $1.23_{10} \times 10^{-12}$ to base 16. (Since B = 16 > 10, b = 100.)

Keystrokes: **Outputs:**

100 STO 1 16 STO 2

1.23 EEX CHS 12 A ⟶ 1.051003061-20 (Base 16)

This is interpreted as $1.5A36A_{16} \times 16^{-10}$.

Example 3:

Convert $7.200067_8 \times 8^{-10}$ to base 10. (Since b = 8 < 10, B = 10.)

Keystrokes: **Outputs:**

8 STO 1 10 STO 2
f SCI 9 7.200067
EEX CHS 10 A ⟶ 6.752284066-09 (Base 10)

Example 4:

Convert $D.2EE4_{16} \times 16^{12}$ to base 10. (Since b = 16 > 10, B = 100.)

Keystrokes: **Outputs:**

16 STO 1 100 STO 2
13.02141404 EEX 24 A ⟶ 3.710731485 15 (Base 10)

PRIMES

This program will list all prime numbers beginning at a positive integer P (P ≥ 3) specified by the user.

The program tests all odd numbers as potential primes and potential divisors. The simplicty of this technique leads to a short program with relatively long execution time. A longer and faster program could be written using an algorithm which tests only a certain subset of odd numbers as potential primes and/or potential divisors. The algorithm might exclude from the test all numbers divisible by 3 or, with an even longer program, all numbers divisible by both 3 and 5.

A-000 LBL A A-001 2 A-002 ÷ A-003 INT A-004 2 A-005 × A-006 1 A-007 + A-008 S 0 A-009 LBL 8 A-010 3	Select 1st odd no. ≥ input.	A-011 S 1 A-012 LBL 9 A-013 R 0 A-014 R 1 A-015 ÷ A-016 LSTX A-017 X≷Y A-018 GTO 0 A-019 X≷Y A-020 INT A-021 LSTX	If divisor d > n/d, then d > $\sqrt{n}$; exit. If d/n, n is not prime; select new n.

A-022 X=Y A-023 GTO1 A-024 2 A-025 S+1 A-026 GTO9 A-027 LBL0	Else d ← d + 2; test new d.	A-028 R ↓ A-029 PRTX A-030 LBL1 A-031 2 A-032 S+0 A-033 GTO8	n is prime; print. Select next n.

REGISTERS			
0 n	1 d	2	3
4	5	6	7

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Key in lower bound of search for primes.	P	A	Primes

Example:

List primes greater than 100.

Keystrokes:

100 A ⟶

Outputs:

101.00 ***
103.00 ***
107.00 ***
109.00 ***
113.00 ***

R/S (Halt program.)

FACTORS

This program finds all prime factors of a positive integer.

As in the primes program, all odd numbers and 2 are tested as potential factors. The program could be made faster, though longer, by excluding from the test all potential factors divisible by 3 and 5.

A-000 LBL A A-001 S 0 A-002 1 A-003 S 1 A-004 2 A-005 S 2 A-006 LBL 7 A-007 R 0 A-008 R 2 A-009 ÷ A-010 LSTX A-011 X>Y A-012 GTO 0 A-013 X⇄Y A-014 INT A-015 LSTX A-016 X=Y	n is no. to be factored. d in R_2 is trial divisor. If d > n/d, then d > $\sqrt{n}$; n prime. If n/d integer, d is factor; print.	A-017 GTO 1 A-018 2 A-019 S+1 A-020 R 1 A-021 S 2 A-022 GTO 7 A-023 LBL 0 A-024 R 0 A-025 PRTX A-026 RTN A-027 LBL 1 A-028 R 2 A-029 PRTX A-030 R↓ A-031 S 0 A-032 GTO 7	Otherwise select next d. n is prime; print as factor. d is a factor. n ← n/d.

REGISTERS

0 n	1 1, 3, 5, ...	2 d	3
4	5	6	7

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Key in integer to be factored.	n	A	Factors

Example 1:

Find the prime factors of 924.

Keystrokes: **Outputs:**

924 A ⟶

2.00 ***
2.00 ***
3.00 ***
7.00 ***
11.00 ***

i.e., 924 = 2 × 2 × 3 × 7 × 11.

Example 2:

Find the prime factors of 3623.

Keystrokes: **Outputs:**

3623 A ⟶ 3623.00 ***

That is, 3623 is prime.

GREATEST COMMON DIVISOR, LEAST COMMON MULTIPLE

This program will find the greatest common divisor or the least common multiple of two positive integers.

Given two integers a and b, their greatest common divisor, GCD (a,b), is found as follows:

1. If b = 0, GCD (a,b) ← a and the program will halt.
2. If b ≠ 0, z ← a mod b, a ← b, and b ← z. Return to 1.

The least common multiple LCM (a,b) is found by

$$\text{LCM (a,b)} = \frac{ab}{\text{GCD (a,b)}}$$

A-000 LBL A		A-017 R 1	a ← b.
A-001 S 3	a ↑ b.	A-018 x	b ← z.
A-002 X⇄Y		A-019 -	
A-003 S 2		A-020 S 1	
A-004 RTN		A-021 X≠0	If b ≠ 0, loop again.
A-005 LBL 1	GCD (a, b)	A-022 GTO 5	
A-006 R 3		A-023 R 0	If b = 0, a = GCD.
A-007 S 1	b → R_1	A-024 RTN	
A-008 R 2		A-025 LBL 2	
A-009 S 0	a → R_0	A-026 R 2	
A-010 LBL 5		A-027 R 3	
A-011 R 0		A-028 x	
A-012 R 0		A-029 S 4	R_4 ← a * b.
A-013 R 1	z ← a mod b	A-030 GSB 1	
A-014 S 0		A-031 R 4	LCM = ab/GCD.
A-015 ÷	= a – b * INT (a/b).	A-032 X⇄Y	
A-016 INT		A-033 ÷	

REGISTERS

0 a	1 b	2 a	3 b
4 ab	5	6	7
8	9	.0	.1

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Key in integers.	a	ENTER↑	
		b	A	a
3	Find either or both:			
	• Greatest common divisor		GSB 1	GCD (a,b)
	• Least common multiple		GSB 2	LCM (a,b)

Example:

Find the GCD and LCM of 406 and 266.

Keystrokes:	**Outputs:**	
406 ENTER↑ 266 A GSB 1 ⟶	14.00	(GCD)
GSB 2 ⟶	7714.00	(LCM)

NOTES

FINANCE

ANNUITIES AND COMPOUND AMOUNTS

This program can be used to solve a variety of problems involving money, time and interest. The following variables can be inputs or outputs:

- n, which is the number of compounding periods. (For a 30 year loan with monthly payments, n = 12 × 30 = 360.)
- i, which is the periodic interest rate expressed as a percent. (For other than annual compounding, divide the annual percentage rate by the number of compounding periods in a year; i.e. 8% annual interest compounded monthly equals 8/12 or 0.667%.)
- PMT, which is the periodic payment.
- PV, which is the present value of the cash flows or compound amount.
- FV, which is the future value of a compounded amount or a series of cash flows.
- BAL, which is the balloon or remaining balance at the end of a series of payments.

Accumulated interest and remaining balance may also be computed with this program.

The program accommodates payments which are made at the end of compounding periods or at the beginning. Payments made at the end of compounding periods (ordinary annuity) are common in direct reduction loans and mortgages while payments at the beginning of compounding periods (annuity due) are common in leasing.

This program uses the convention that **cash outlays are input as negative,** and **cash incomes are input as positive.**

The initialization (A) performs two functions:

1. It sets PMT, PV, and BAL to zero (n and i are not affected).
2. It sets the ordinary annuity mode.

Pressing A provides a safe, convenient, easy to remember method of preparing the calculator for a new problem. It is not necessary to use A between problems containing the same combination of variables. For instance, any number of n, i, PMT, FV problems involving different numbers and/or different combinations of knowns could be done in succession without reinitializing. Only the values which change from problem to problem would have to be keyed in. To change the combination of variables without using A, simply input zero for any variable which is no longer applicable. To go from n, i, PMT, PV problems to n, i, PV, FV problems, a zero would be stored (0 STO 3) in place of PMT. Table I summarizes these procedures.

Table I

Possible Solutions Using *Annuities and Compound Amounts*

Allowable Combination of Variables	Applications		Initial Procedure
	Ordinary Annuity	Annuity Due	
n, i, PMT, PV (Input any three and calculate the fourth.)	Direct reduction loan Discounted notes Mortgages	Leases	Use A or set BAL to zero.
n, i, PMT, PV, BAL (Input any four and calculate the fifth.)	Direct reduction loan with balloon Discounted notes with balloon	Leases with residual values	None
n, i, PMT, FV (Input any three and calculate the fourth.)	Sinking fund	Periodic savings insurance	Use A or set PV to zero.
n, i, PV, FV (Input any three and calculate the fourth.)	Compound amount Savings (Annuity mode is not applicable and has no effect)		Use A or set PMT to zero.

Equations:

$$PV = \frac{PMT}{i} A\left[1 - (1 + i)^{-n}\right] + (BAL \text{ or } FV)(1 + i)^{-n}$$

where

$$A = \begin{cases} 1 & \text{ordinary annuity} \\ (1 + i) & \text{annuity due.} \end{cases}$$

Remarks:

The equation above is solved for i using Newton's method where:

$$i_n = i_{n-1} - \frac{f(i_{n-1})}{f'(i_{n-1})}$$

This is why solutions involving PMT and i take longer than other solutions. It is quite possible to define problems which cannot be solved by this technique. Such problems usually result in an error message but may simply continue to run indefinitely.

Interest problems with balloon payments of opposite sign to the periodic payments may have more than one mathematically correct answer (or no answer at all). While this program may find one of the answers, it has no way of finding or indicating other possibilities.

```
A-000 LBLA
A-001 CL X      Initialize by clearing PMT,
A-002 S 3       PV, FV (BAL) registers
A-003 S 4
A-004 S 5       ---------------
A-005 JPB0      Jump to ordinary annuity
B-000 LBLB      ---------------
B-001 R.2
B-002 X=0
B-003 GTO0
B-004    0      Annuity mode toggle
B-005 S.2       0 = annuity due
B-006 RTN
B-007 LBL0
B-008    1
B-009 S.2
B-010 RTN       ---------------
B-011 LBL1      Store dummy 0 for n.
B-012    0
B-013 S 1       ---------------
B-014 GSB0      Calculate subroutine.
B-015 R 5       ---------------
B-016 LSTX
B-017    -
B-018 R 4
B-019 LSTX      Solve for n and store it in R1.
B-020    +
B-021    ÷
B-022 CHS
B-023 LN
B-024 R 7
B-025 LN
B-026    ÷
B-027 S 1
B-028 RTN       ---------------
B-029 LBL2
B-030    0      Clear R2 for sum of i terms.
B-031 S 2       ---------------
B-032 R 5
B-033 R 1       ---------------
B-034 R 3       If PMT = 0, GTO
B-035 X=0       n, i, PV, FV solution.
B-036 GTO6      ---------------
B-037    x      Start guess of i.
B-038    +      n PMT + BAL
B-039 R 4       ---------------
B-040 X=0       If PV = 0, GTO FV guess.
B-041 GTO9      ---------------
B-042    +      PV guess for i.
B-043 R 1
B-044    ÷      n PMT + BAL + PV
B-045 R 4       ---------------
B-046 GTO7      ---------------
B-047 LBL9
B-048 R 5
B-049 LSTX
B-050    +
B-051 ENT↑
B-052    +      FV guess for i.
B-053 R 1
B-054    1
B-055    -
B-056 X²
B-057 R 3
B-058    x
B-059 R 5
B-060    -
B-061 LBL7      ---------------
B-062    ÷      Guess for i.
B-063 CHS       ---------------
B-064    .      If guess is less than -0.9
B-065    9      use -0.9 for guess.
B-066 CHS
B-067 X≤Y
B-068 X≠Y       ---------------
B-069 GSB9      Store guess as a %.
B-070 X=0       ---------------
B-071 RTN       If guess = 0 stop.
B-072 LBL8      ---------------
B-073 GSB0      Calculate f(i)
B-074    +
B-075 R 4
B-076    +      ---------------
B-077 R 8
B-078 R 1
B-079 R 7
B-080    ÷
B-081    x
B-082 S 6
B-083 R.0       Calculate f'(i)
B-084 R 9
B-085    ÷
B-086    -
B-087 R.1
B-088    x
B-089 R 0
B-090    x
B-091 R 6
B-092 R 5
B-093    x
B-094    -
B-095    ÷      ---------------
B-096 CHS       f(i)/f'(i)
B-097 GSB9      ---------------
B-098 R 2       Compute next i.
B-099    ÷      ---------------
B-100 ABS
B-101 EEX       Test increment to i for limit.
B-102 CHS
B-103    6
B-104 X≤Y
B-105 GTO8      ---------------
B-106 R 2       Stop and display.
B-107 RTN       ---------------
B-108 LBL6
B-109 R 5
B-110 R 4
B-111    ÷      Compute i for n, i, PV, FV
B-112 CHS       problem.
B-113 R 1
```

B-114 1/X		B-149 ÷	
B-115 Y^x		B-150 CHS	
B-116 1		B-151 S 5	
B-117 −		B-152 RTN	
B-118 LBL9	Convert i to % and add to R_2.	B-153 LBL0	$1 \rightarrow R_{.1}$ for ordinary annuity.
B-119 EEX		B-154 1	
B-120 2		B-155 S.1	
B-121 x		B-156 R 2	$i \rightarrow R_9$.
B-122 S+2		B-157 %	
B-123 RTN		B-158 S 9	$1 + i \rightarrow R_7$.
B-124 LBL3	Store dummy 1 for PMT.	B-159 +	
B-125 1		B-160 S 7	
B-126 S 3		B-161 R.2	If annuity due $1 + i \rightarrow R_{.1}$.
B-127 GSB0	Calculate subroutine.	B-162 X=0	
B-128 1/X		B-163 X≷Y	
B-129 R 4		B-164 S.1	
B-130 Rf	Solve for PMT and store it in R_3.	B-165 R 7	$(1 + i)^{-n} \rightarrow R_8$.
B-131 +		B-166 R 1	
B-132 x		B-167 CHS	
B-133 CHS		B-168 Y^x	
B-134 S 3		B-169 S 8	
B-135 RTN		B-170 R 5	
B-136 LBL4	Store dummy 1 for PV.	B-171 x	
B-137 1		B-172 1	
B-138 S 4		B-173 R 8	
B-139 GSB0	Calculate subroutine	B-174 −	$1 - (1+i)^{-n} \rightarrow R_{.0}$
B-140 +		B-175 S.0	
B-141 CHS	Solve for PV and store it in R_4.	B-176 R 3	
B-142 S 4		B-177 R 9	$PMT/i \rightarrow R_0$
B-143 RTN		B-178 ÷	
B-144 LBL5	Calculate subroutine	B-179 S 0	
B-145 GSB0		B-180 R.1	Calculate $\frac{PMT}{i}[1 - (1+i)^{-n}]R_{.1}$
B-146 R 4		B-181 x	
B-147 +	Solve for FV(BAL) and store it in R_5.	B-182 x	
B-148 R 8		B-183 RTN	

REGISTERS

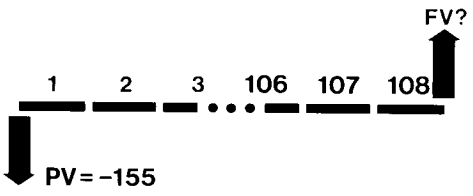
0 PMT/i	1 n	2 i	3 PMT
4 PV	5 FV(BAL)	6 $n(1+i)^{-n-1}$	7 1+i
8 $(1+i)^{-n}$	9 i/100	.0 $1 - (1+i)^{-n}$	.1 1 or 1 + i
.2 Annuity flag	.3	.4	.5

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Initialize. (1.00 = ordinary			
	annuity)		A	1.00
3	If payments occur at the			
	beginning of the period set			
	annuity due mode*.		B	0.00/1.00

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
4	Input the known values:			
	Number of periods	n	STO 1	n
	Periodic interest rate	i(%)	STO 2	i (%)
	Periodic payment	PMT	STO 3	PMT
	Present value	PV	STO 4	PV
	Future value, balloon or			
	balance	FV, (BAL)	STO 5	FV, (BAL)
5	Calculate the unknown value.			
	Number of periods		GSB 1	n
	Periodic interest rate		GSB 2	i (%)
	Periodic payment		GSB 3	PMT
	Present value		GSB 4	PV
	Future value, balloon or			
	balance		GSB 5	FV, (BAL)
6	For a new case, go to step 4			
	and change appropriate values.			
	Input zero for any value not			
	applicable in the new case.			
	*One or zero will be displayed			
	alternately after pressing B,			
	indicating that the ordinary			
	annuity mode is on or off.			

Example 1:

You place $155 in a savings account paying 5¾% compounded monthly. What sum of money may you withdraw at the end of 9 years?



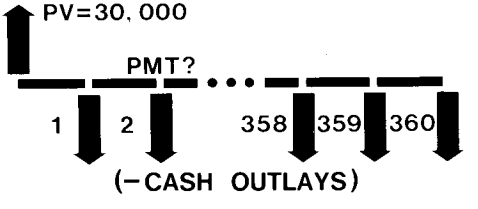
Keystrokes:	**Outputs:**	
A 155 CHS STO 4 ⟶	−155.00	(Cash outlay)
5.75 ENTER↑ 12 ÷ STO 2 ⟶	0.48	(.48% monthly interest rate)
9 ENTER↑ 12 × STO 1 ⟶	108.00	(# months of compounding)
GSB 5 ⟶	259.74	(FV)

If the interest is changed to 6%, what is the sum?

6 ENTER↑ 12 ÷ STO 2 ⟶	0.50	(.50% monthly interest rate)
GSB 5 ⟶	265.62	(FV)

Example 2:

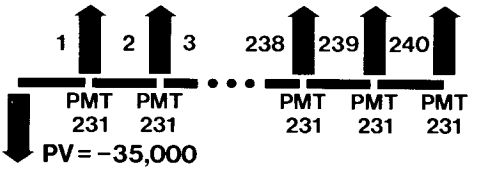
You receive $30000 from the bank as a 30 year, 9% mortgage. What monthly payment must you make to the bank to fully amortize the mortgage?



Keystrokes:	**Outputs:**	
A 30 ENTER↑ 12 × STO 1 ⟶	360.00	(# monthly payments)
30000 STO 4 ⟶	30000.00	(Loan amount)
9 ENTER↑ 12 ÷ STO 2 ⟶	0.75	(.75% monthly interest rate)
GSB 3 ⟶	−241.39	(PMT)

Example 3:

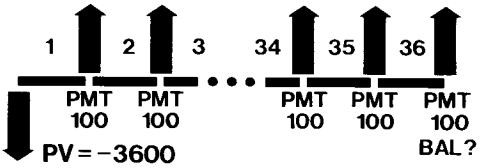
A fixed term annuity is available which requires a $35,000 initial deposit. In return the depositor will receive monthly payments of $231 for 20 years. What annual interest rate is being applied?



Keystrokes:	**Outputs:**	
[A] 35000 [CHS] [STO] [4] →	−35000.00	(Initial cash outlay)
231 [STO] [3] →	231.00	(Monthly income)
20 [ENTER↑] 12 [×] [STO] [1] →	240.00	(# monthly payments)
[GSB] [2] →	0.42	(0.42% monthly)
12 [×] →	5.00	(5% annual interest rate)

Example 4:

Two individuals are constructing a loan with a balloon payment. The loan amount is $3,600 and it is agreed that the annual interest rate will be 10% with 36 monthly paymens of $100. What balloon payment amount, to be paid coincident with the 36th payment, is required to fulfill the loan agreement? (Note the cashflow diagram below is with respect to the loaner. For the loanee, the appropriate diagram would be exactly the opposite.)



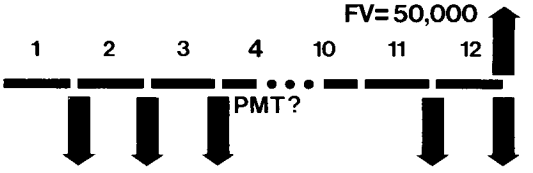
Keystrokes: **Outputs:**

[A] 3600 [CHS] [STO] [4] 10 [ENTER↑]
12 [÷] [STO] [2] 36 [STO] [1]
100 [STO] [3] [GSB] [5] → 675.27

(Note that the final payment is $675.27 + $100.00 = $775.27 since the final payment falls at the end of the last period.)

Example 5:

A corporation has determined that a certain piece of equipment costing $50,000 will be required in 3 years. Assuming a fund paying 7% compounded quarterly is available, what quarterly payment must be made in order to withdraw this cost from the fund if savings are to start at the end of this quarter?



Keystrokes: **Outputs:**

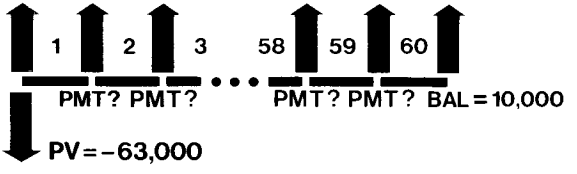
A 50000 STO 5 3 ENTER↑ 4 ×
STO 1 7 ENTER↑
4 ÷ STO 2 GSB 3 ⟶ −3780.69

What single amount, invested immediately, would provide the same effect?

0 STO 3 GSB 4 ⟶ −40602.89

Example 6:

A "third party" leasing firm is considering the purchase of a mini-computer priced at $63,000 and intends to achieve a 13% annual yield by leasing the computer to a customer for a 5-year period. Ownership is retained by the leasing firm, and at the end of the lease they expect to be able to sell the equipment for at least $10,000. What should they establish as the monthly payments in order to realize their desired yield? (Since lease payments occur at the start of the periods, this is an annuity due problem.)



Keystrokes: **Outputs:**

A B 63000 CHS STO 4 13 ENTER↑
12 ÷ STO 2 5 ENTER↑ 12 × STO 1
10000 STO 5 GSB 3 ⟶ 1300.16

If the price increased to \$70,000, what should the payments be?

70000 CHS STO 4 GSB 3 ⟶ 1457.73

If the payments were increased to \$1500 what would the yield be?

1500 STO 3 GSB 2 ⟶ 1.18 (% per month)

12 × ⟶ 14.12 (% per year)

Example 7:

This program may also be used to calculate accumulated interest/remaining balance for loans. The accumulated interest between two points in time, n_1 and n_2, is just the total payments made in that period less the principal reduction in that period. The principal reduction is the difference of the remaining balances for the two points in time. The following example demonstrates the concepts above.

For a 360 month, \$50000 loan at 9½% annual interest, find the remaining balance after the 24th payment and the accrued interest for payments 13–24 (between the 12th and 24th payments!).

First we must calculate the payment on the loan:

Keystrokes:	**Outputs:**	
A 360 STO 1 9.5 ENTER↑ 12 ÷ STO 2 50000 CHS STO 4 GSB 3 ⟶	420.43	payment

The remaining balance is found:

24 STO 1 GSB 5 ⟶	49352.76	Remaining balance at month 24

Store this remaining balance and calculate the remaining balance at period 12:

STO I 12 STO 1 GSB 5 ⟶ 49691.68

The principal reduction between payments 12 and 24 is:

I − ⟶ 338.92

The accrued interest is 12 payments less the principal reduction:

RCL 3 12 × ⟶	5045.13	Total paid out
x≷y − ⟶	4706.20	Accrued interest

NOTES

GAMES AND SUCH

MOON ROCKET LANDER

Imagine for a moment the difficulties involved in landing a rocket on the moon with a strictly limited fuel supply. You're coming down tail-first, free-falling toward a hard rock surface. You'll have to ignite your rockets to slow your descent; but if you burn too much too soon, you'll run out of fuel 100 feet up, and then you'll have nothing to look forward to but cold eternal moon dust coming faster every second. The object, clearly, is to space your burns just right so that you will alight on the moon's surface with no downward velocity.

The game starts off with the rocket descending at a velocity of 50 feet/sec from a height of 500 feet. The velocity and height are shown in a combined display as −50.0500, the height appearing to the right of the decimal point and the velocity to the left, with a negative sign on the velocity to indicate downward motion. If a velocity is ever displayed with no fractional part, for example, −15., it means that you have crashed at a speed of 15 feet/sec. In game terms, this means that you have lost; in real-life, it signifies an even less favorable outcome.

You will start the game with 120 units of fuel. You may burn as much or as little of your available fuel as you wish at each step of your descent; burns of zero are quite common. A burn of 5 units will just cancel gravity and hold your speed constant. Any burn over 5 will act to change your speed in an upward direction. You must take care, however, not to burn more fuel than you have; for if you do, no burn at all will take place, and you will free-fall to your doom! The final velocity shown will be your impact velocity (generally rather high). You may display your remaining fuel at any time by recalling R_7.

Equations:

We don't want to get too specific, because that would spoil the fun of the game; but rest assured that the program is solidly based on some old friends from Newtonian physics:

$$x = x_0 + v_0 t + \frac{1}{2} at^2 \qquad v = v_0 + at \qquad v^2 = v_0^2 + 2ax$$

where x, v, a, and t are distance, velocity, acceleration, and time.

Remarks:

1. If you crash before running out of fuel, the crash velocity shown will be the velocity before the burn, rather than the impact velocity.
2. Use only integer values for burns. Any decimal entry will cause an error in the display for V.X.

A-000 LBLA	Initialize	A-032 GTO3	If burn > fuel, prepare to crash.
A-001 5		A-033 S-7	
A-002 0		A-034 5	
A-003 0		A-035 -	Else update A, X, V.
A-004 S 9		A-036 S 6	
A-005 5		A-037 2	
A-006 0		A-038 ÷	
A-007 CHS		A-039 R 9	
A-008 S 8		A-040 +	X ← X + V + A/2.
A-009 1		A-041 R 8	
A-010 2		A-042 +	
A-011 0		A-043 S 9	
A-012 S 7		A-044 X<0	
A-013 LBL1	- - - - - - - - - - - - - -	A-045 GTO4	If X below ground, you've crashed.
A-014 FIX4		A-046 R 6	
A-015 R 9	Form display.	A-047 S+8	
A-016 EEX		A-048 GTO1	Else update V: V ← V + A.
A-017 4		A-049 LBL3	
A-018 ÷		A-050 R 8	
A-019 R 8		A-051 X²	All fuel gone, show crash velocity.
A-020 X<0		A-052 R 9	
A-021 GTO0		A-053 1	
A-022 +		A-054 0	
A-023 GTO2		A-055 x	
A-024 LBL0		A-056 +	
A-025 X≠Y		A-057 fX	
A-026 -	V. X.	A-058 CHS	
A-027 LBL2	- - - - - - - - - - - - - -	A-059 S 8	- - - - - - - - - - - - - -
A-028 RTN		A-060 LBL4	
A-029 R 7		A-061 R 8	Crash V.
A-030 X≠Y		A-062 FIX0	
A-031 X>Y			

REGISTERS

0	1	2	3
4	5	6 Accel.	7 Fuel
8 V	9 X	.0	.1
.2	.3	.4	.5

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Initialize.		A	-50.0500
3	Key in burn, display new			
	speed and altitude.	Burn	R/S	V.X

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
4	Perform step 3 until you land			
	or crash.			
5	To see remaining fuel at any			
	time.		RCL 7	Fuel
6	To display speed and altitude			
	at any time.		GSB 1	V.X
7	To start a new game, go to			
	step 2.			

Example:

Keystrokes:	**Outputs:**	
A →	-50.0500	
0 R/S →	-55.0448	
5 R/S →	-55.0393	(Note constant V when burn = 5)
30 R/S →	-30.0350	
0 R/S →	-35.0318	
0 R/S →	-40.0280	
0 R/S →	-45.0238	
0 R/S →	-50.0190	
RCL 7 →	85.0000	(Remaining fuel)
GSB 1 →	-50.0190	(Display V.X again)
10 R/S →	-45.0143	
0 R/S →	-50.0095	
RCL 7 →	75.0000	
10 R/S →	-45.0048	
25 R/S →	-25.0013	
20 R/S →	-25.	

Oops.

SUBMARINE HUNT

Using your destroyer, you try to locate the position of the enemy submarine in a 10 × 10 grid, and then to destroy it with your depth charge.

You input a seed (1 − 100) and the calculator will position the submarine in the center of one of the 100 squares (R, C), where R = row and C = column, and where R and C can each be 0, 1, 2, ..., 9.

You search for the submarine by taking sonar readings. If the submarine is in one of the 8 adjacent squares to your reading (or directly under your destroyer), the calculator will display "1." Otherwise a "0" will be shown.

When you think you've located the submarine, move your destroyer directly over it (move to the same square) and drop a depth charge. A blinking "SOS" signal from the sub indicates a hit, while a "0" shows a miss. If you miss, the submarine will move randomly to one of the 4 adjacent squares in the same row or column.

You can make the hunt easier or more difficult. For an easier game, increase the sensitivity of your sonar, allowing you to detect the submarine as far away as 2 squares in any direction (you cover a square region of the ocean 5 squares on a side.) You can switch between 1- and 2-square sensitivity as often as you like during the game. You are charged two sonar readings, however, when you use the 2-square sensitivity.

To make a more challenging game, you can allow the submarine to move after each sonar echo as well as after each depth charge miss. The submarine always moves randomly to an adjacent square in the same row or column.

A depth charge has a range of 0.9. When you position your destroyer for a depth charge drop, you may move anywhere on the board. For instance, a depth charge dropped from a (2.5, 6.5) location would destroy any submarine in the center of squares (2, 6), (2, 7), 3, 6) and (3, 7).

Try to destroy the submarine using no more than 10 sonar readings and 1 depth charge, playing a regular game with 1 square sensitivity. You can check your status any time after sonar readings or depth charges.

Status format is dd.ss

Where dd = Number of depth charges fired
ss = Number of sonar readings

Reference: This program is based on an HP-65 Users' Library program written by Moshe Breiner.

```
A-000 LBLA
A-001 CL R
A-002   .
A-003   5
A-004   2
A-005   6
A-006   4
A-007   1
A-008   6
A-009   x
A-010 S 0
A-011 GSB7
A-012 S 1
A-013 GSB7
A-014 S 2
A-015   1
A-016 S.1
A-017   0
A-018 R/S
A-019 LBL1
A-020 R.0
A-021   1
A-022 X=Y
A-023   -
A-024 S.0
A-025 R/S
A-026 LBL7
A-027 JPC7
B-000 LBLB
B-001 R.1
B-002   1
B-003 X=Y
B-004   +
B-005 S.1
C-000 LBLC
C-001 R.1
C-002 S+8
C-003 S 6
C-004 R↓
C-005 GSB6
C-006 R.0
C-007 S 5
C-008 GSB9
C-009 R 3
C-010 R/S
C-011 JPD6
C-012 LBL9
C-013 GSB7
C-014   4
C-015 X≠Y
C-016 X>Y
C-017 GTO0
C-018 R 5
C-019 CHS
C-020 S 5
C-021 LBL0
C-022 GSB7
C-023   5
C-024 X>Y
C-025 GTO0
```

Initialize

Generate random submarine position.

Toggle for submarine movement after sonar.

Link to random # gen.

Toggle for sonar sensitivity.

Sonar reading.

0. or 1.

Link to status display.

Submarine movement routine.

Movement of ±1 unit

```
C-026   1
C-027 GTO1
C-028 LBL0
C-029   2
C-030 LBL1
C-031 S I
C-032 R i
C-033 R 5
C-034   +
C-035 X<0
C-036 GTO0
C-037   9
C-038 X≠Y
C-039 X≤Y
C-040 GTO1
C-041 LBL0
C-042 R 5
C-043   2
C-044   x
C-045   -
C-046 LBL1
C-047 S i
C-048 R 3
C-049 RTN
C-050 LBL8
C-051 R 2
C-052   -
C-053 X≠Y
C-054 R 1
C-055   -
C-056 →P
C-057 R 6
C-058   -
C-059   .
C-060   9
C-061   -
C-062 X<0
C-063 GTO0
C-064   0
C-065 GTO1
C-066 LBL0
C-067   1
C-068 LBL1
C-069 S 3
C-070 RTN
C-071 LBL7
C-072 FIX0
C-073 R 0
C-074   9
C-075   9
C-076   7
C-077   x
C-078 FRAC
C-079 S 0
C-080   1
C-081   0
C-082   x
C-083 INT
C-084 RTN
D-000 LBLD
```

Select movement in row or column

Is move on playing board?

Store next position

Response to sonar or depth charge.

Random # generator.

D-001 1	Depth charge.	D-020 5	
D-002 S+7		D-021 LBL1	
D-003 0		D-022 PSE	
D-004 S 6		D-023 DSZ	
D-005 R↓		D-024 GTO1	
D-006 R↓		D-025 R/S	
D-007 GSB8	Compute response	D-026 LBL6	- - - - - - - - - - - -
D-008 X≠0	Hit?	D-027 FIX2	
D-009 GTO0	- - - - - - - - - - - -	D-028 R 7	Display status
D-010 1		D-029 R 8	
D-011 S 5	Move submarine after depth	D-030 EEX	
D-012 GSB9	charge.	D-031 2	
D-013 R/S		D-032 ÷	
D-014 GTO6	- - - - - - - - - - - -	D-033 +	
D-015 LBL0		D-034 RTN	- - - - - - - - - - - -
D-016 5		D-035 LBL8	Link to movement and
D-017 S I	Submarine hit!	D-036 JPC8	response routines.
D-018 5		D-037 LBL9	
D-019 0	Send "SOS"	D-038 JPC9	

REGISTERS

0 Seed	1 R	2 C	3 Response
4	5 ±1, 0	6 Used	7 dd
8 ss	9	.0 0,1 Movement after sonar.	.1 1,2 sonar sensitivity.
.2	.3	.4	.5 I Used

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Key in SEED (any number			
	between 1 and 100).	SEED	A	0.
3	Select difficult game if desired			
	(submarine always moving).		R/S	1.
4	To change sonar sensitivity:		B	2. or 1.
	"2" means sensitivity dis-			
	tance is 2 squares.			
	"1" means sensitivity dis-			
	tance is 1 square.			
5	**Sonar**	R	ENTER↑	
	"0" means no echo	C	C	0. or 1.
	"1" means echo received			
	or			

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	Depth Charge	R	ENTER↑	
	"0" means miss	C	D	0. or
	Blinking "SOS" means HIT!			"SOS"
	To review status		R/S	dd.ss
5	For a new game, go to step 2.			

Example 1:

Keystrokes:	**Outputs:**	
58 A ⟶	0.	
2 ENTER↑ 1 C ⟶	1.	Echo

You now know your enemy is in one of the "X" squares below.

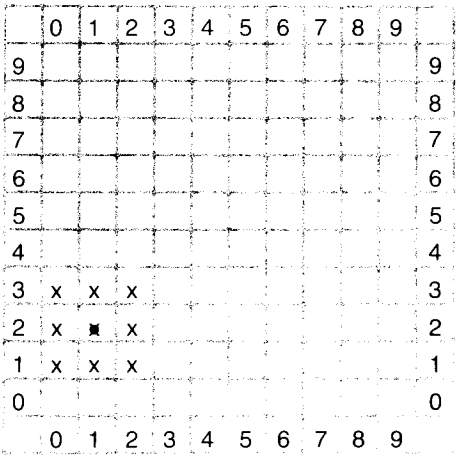


Diagram of 1st move

The submarine cannot be in the X squares below.

1 ENTER↑ 2 C ——→ 0. No echo

	0	1	2	3	4	5	6	7	8	9	
9											9
8											8
7											7
6											6
5											5
4											4
3	x	x	x								3
2	x	⊗	⊗								2
1	x	⊗	▣								1
0											0
	0	1	2	3	4	5	6	7	8	9	

Diagram of 2nd move

3 ENTER↑ 0 C ——→ 0. No echo

Check your status:

R/S ——→ 0.03

You've narrowed down the submarine's location to just 2 squares, those containing an "X" with no circle.

	0	1	2	3	4	5	6	7	8	9	
9											9
8											8
7											7
6											6
5											5
4											4
3	▣	⊗	x								3
2	⊗	⊗	⊗								2
1	x	⊗	⊗								1
0											0
	0	1	2	3	4	5	6	7	8	9	

Diagram of 3rd move

1 ENTER↑ 0 C ——→ 1. Echo

This eliminates (3, 2) as a submarine location, so you've found it!

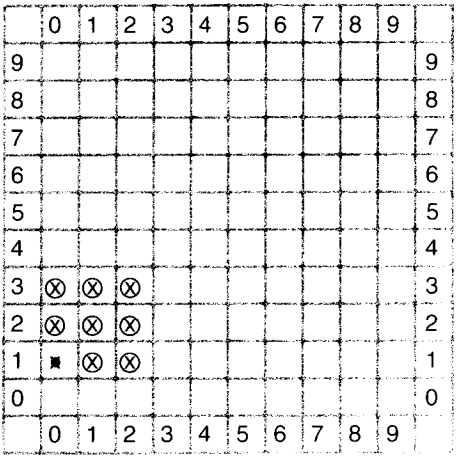


Diagram of 4th move

1 ENTER↑ 0 D ——→ "SOS" A hit!

R/S ——→ 1.04 Four sonar readings and one depth charge

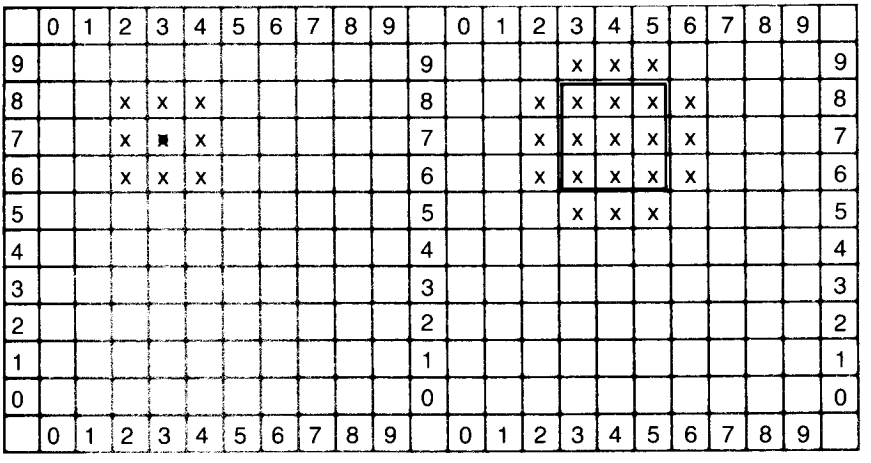
Example 2:

Keystrokes:	**Outputs:**
60 A ——→	0.
R/S ——→	1.

Submarine will now move on sonar echos as well as on depth charge misses.

7 ENTER↑ 4 C ——→ 1. Echo

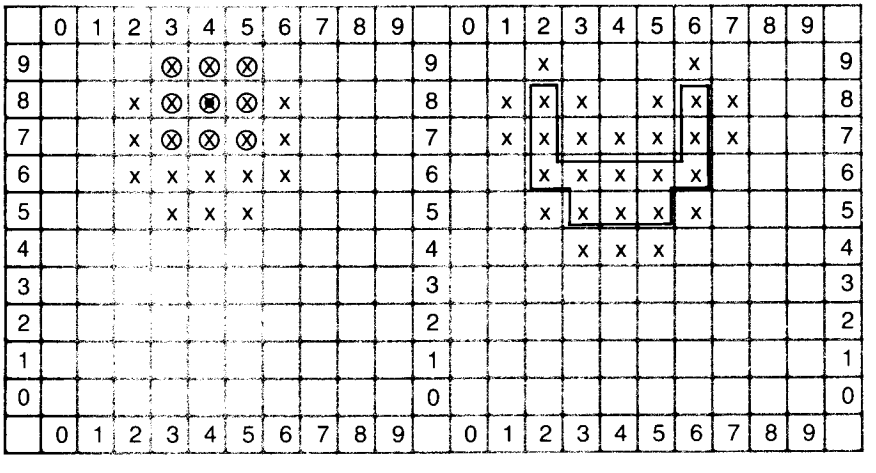
The submarine is in one of the "X" squares in the left diagram below. But the submarine moves, so now it could be in any of the "X" squares in the right diagram below.



Diagrams of 1st move

8 ENTER 4 C ⟶ 0. No echo

You've eliminated some positions (left diagram below: X), but new possible positions have been created by the enemy's random move (right diagram).



Diagrams of 2nd move

7 ENTER↑ 5 C ⟶ 1. Echo

This eliminates many possible positions (left diagram), but again, new ones are created (right diagram).

	0	1	2	3	4	5	6	7	8	9		0	1	2	3	4	5	6	7	8	9	
9			⊗				⊗				9						x	x				9
8		⊗	⊗	⊗		x	x	⊗			8					x	x	x	x			8
7		⊗	⊗	⊗	x	■	x	⊗			7				x	x	x	x	x			7
6		⊗	⊗	⊗	x	x	x	⊗			6				x	x	x	x	x			6
5			⊗	⊗	⊗	⊗	⊗				5					x	x	x				5
4				⊗	⊗	⊗					4											4
3											3											3
2											2											2
1											1											1
0											0											0
	0	1	2	3	4	5	6	7	8	9		0	1	2	3	4	5	6	7	8	9	

Diagrams of 3rd move

Fourth move. You try a depth charge.

7.5 ENTER↑ 4.5 D ⟶ "SOS" A hit!

R/S ⟶ 1.03 dd.ss

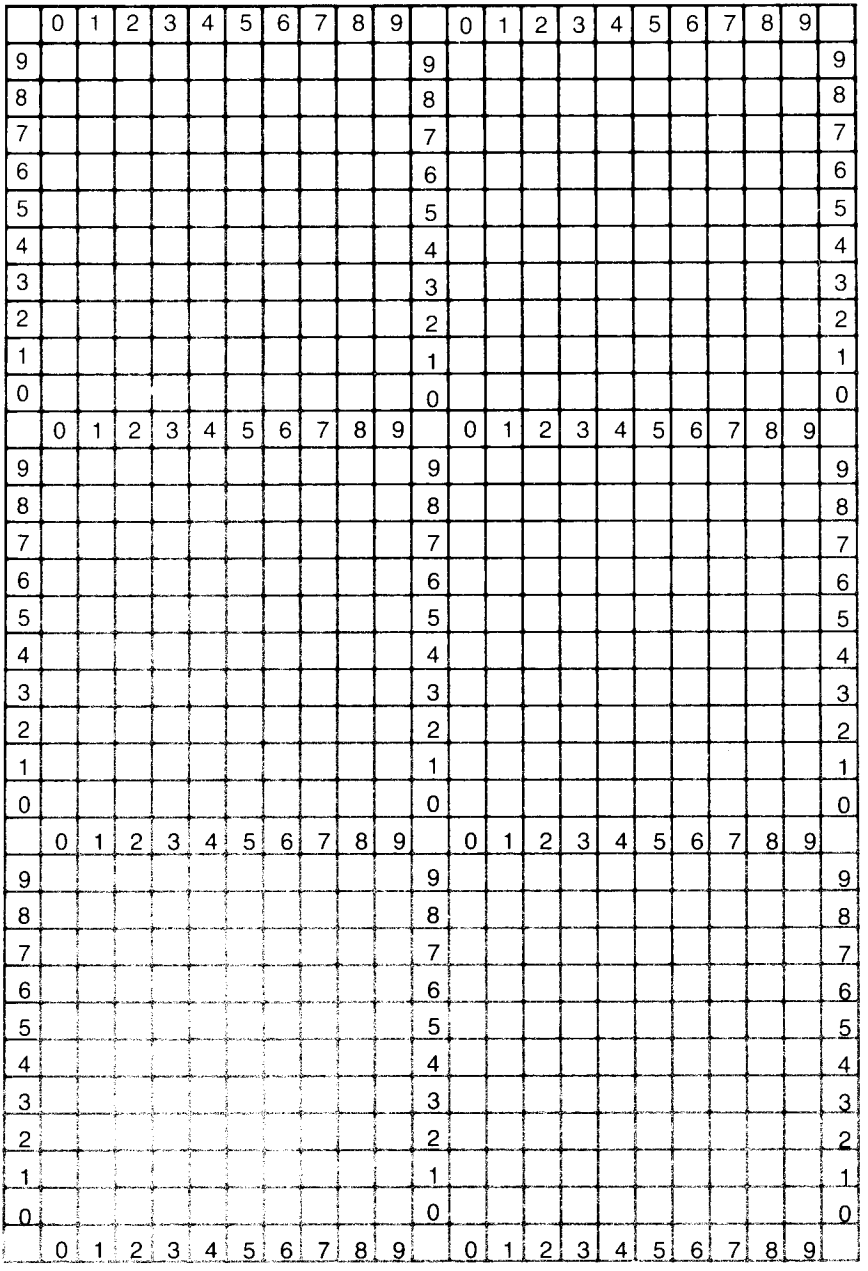
The submarine used to be in one of these 4 squares:

	0	1	2	3	4	5	6	7	8	9	
9						x	x				9
8					x	x	x	x			8
7				x	x	x	x	x			7
6				x	x	x	x	x			6
5					x	x	x				5
4											4
3											3
2											2
1											1
0											0
	0	1	2	3	4	5	6	7	8	9	

RCL 1 ⟶ 8.00

RCL 2 ⟶ 4.00

This shows the submarine was sending its "SOS" from square (8,4).



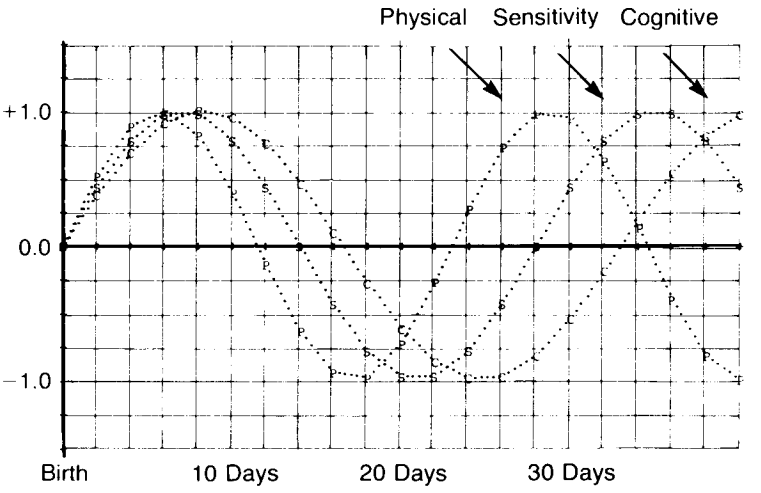
Playing boards for Submarine Hunt. You might wish to use copies of this page for your games.

BIORHYTHMS

From the ancient days, philosophers and sages have taught that human happiness lies in the harmonious integration of body, mind, and heart. Now a twentieth-century theory claims to be able to quantitatively gauge the functioning of these three aspects of our selves: the physical, sensitive, and cognitive.

The biorhythm theory is based on the assumption that the human body has inner clocks or metabolic rhythms with constant cycle times. Currently, three cycles starting at birth in a positive direction are postulated. The 23-day or physical cycle relates with physical vitality, endurance and energy. The 28-day or sensitivity cycle relates with sensitivity, intuition and cheerfulness. The 33-day or cognitive cycle relates with mental alertness and judgement.

For each cycle, a day is considered either high, low, or critical. Let x be the output value for a given cycle. The high ($0 < x \leq 1$) times are regarded as energetic times, you are your most dynamic in the cycle. The low ($-1 \leq x\ 0$) times are regarded as the recuperative periods. The critical days ($x = 0$) are regarded as your accident prone days especially for the physical and sensitivity cycles.



Remarks:

1. The birthdate and biodate must be between January 1, 1901, and December 31, 2099.
2. The format for input of dates is MM.DDYYYY. For example, June 3, 1976, is keyed in as 6.031976. The program does not check input data. Thus, if an improper format or an invalid date (e.g., February 30) is keyed in, erroneous answers may result.
3. This program sets the angular mode to radians (RAD).

A-000 LBLA		A-044 1	Next day
A-001 RAD	Birthdate	A-045 S+8	
A-002 GSB9	Store N	A-046 GSB8	- - - - - - - - - - - - -
A-003 S 9		A-047 GTO6	
A-004 RTN	- - - - - - - - - - - - -	A-048 LBL9	Compute N(M, D, Y)
A-005 LBL1		A-049 ENT↑	
A-006 GSB9	Biodate	A-050 INT	
A-007 R 9		A-051 S 6	M
A-008 -		A-052 -	
A-009 S 8	Store $N_2 - N_1$.	A-053 EEX	
A-010 LBL8		A-054 2	
A-011 1		A-055 x	
A-012 8		A-056 ENT↑	D
A-013 S 7		A-057 INT	
A-014 SPC		A-058 S 5	
A-015 GSB7	23-day cycle	A-059 -	
A-016 GSB7	28-day cycle	A-060 EEX	
A-017 LBL7	- - - - - - - - - - - - -	A-061 4	
A-018 5		A-062 x	Y
A-019 S+7	# days	A-063 S 4	
A-020 R 8		A-064 2	
A-021 R 7		A-065 R 6	
A-022 ÷		A-066 X>Y	
A-023 FRAC		A-067 GTO2	
A-024 2		A-068 1	
A-025 x		A-069 S-4	
A-026 ↑		A-070 1	
A-027 x		A-071 2	
A-028 SIN		A-072 S+6	
A-029 ENT↑		A-073 LBL2	
A-030 ABS		A-074 1	
A-031 X≠0		A-075 S+6	
A-032 ÷		A-076 R 6	
A-033 LSTX		A-077 3	
A-034 EEX		A-078 0	
A-035 7		A-079 .	
A-036 +		A-080 6	
A-037 EEX		A-081 x	
A-038 7		A-082 INT	
A-039 -		A-083 R 4	
A-040 x		A-084 3	
A-041 PRTX	Bio value	A-085 6	
A-042 RTN	- - - - - - - - - - - - -	A-086 5	
A-043 LBL6		A-087 .	

A-088 2 A-089 5 A-090 × A-091 INT		A-092 + A-093 R 5 A-094 +	N - - - - - - - - - - - -

REGISTERS			
0	1	2	3
4 Y	5 D	6 M	7 23, 28, 33
8 $N_2 - N_1$	9 N_1	.0	.1
.2	.3	.4	.5	I

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Key in birthdate.	MM.DDYYYY	A	day#*
3	Key in biodate and find bio values.	MM.DDYYYY	GSB 1	P S C
4	To find bio values for succeeding days.		R/S	P S C
5	For a new birthdate, go to step 2; for a new biodate, go to step 3.			
	*See Calendar Functions for an explanation of this number.			

Example:

Calculate the Bio values for June 29, 1976, for a person born March 27, 1948. Find the values for the two days following also.

Keystrokes: **Outputs:**

3.271948 A

6.291976 GSB 1 ⟶

−1.00 *** (P)
−0.62 *** (S)
−1.00 *** (C)

R/S ⟶

−0.98 *** (June 30)
−0.78 ***
−0.97 ***

−0.89 *** (July 1)
−0.90 ***
−0.91 ***

R/S halts output.

COUNT-DOWN TIMER

This program provides a count-down timer and a calibration routine for measuring elapsed time. When using this program, you should remember that the clock circuits of the HP-95C are designed for calculator use, not for accurate time keeping. Although the routine may be calibrated quite accurately, highly stable performance should not be expected due to variable conditions about the calculator.

Equations:

$$C_{a_{new}} = C_{a_{old}} \frac{\text{HP-95C time}}{\text{actual time}}$$

A-000 LBLA A-001 FIX4 A-002 S 0 A-003 LBL2 A-004 0 A-005 PRTX A-006 R/S A-007 S 1 A-008 +H A-009 R 0 A-010 x A-011 S I A-012 R 1 A-013 R/S A-014 LBL1	Store constant. --- "Alarm" --- Store time. Store counter. ---	A-015 DSZ A-016 GTO1 A-017 GTO2 B-000 LBLB B-001 HMS- B-002 +H B-003 R 1 B-004 +H B-005 ÷ B-006 1/X B-007 R 0 B-008 x B-009 PRTX B-010 GTOA	Loop on counter. --- Go to "alarm". --- Calibrate constant. ---

REGISTERS

0 Ca	1 Time	2	3
4	5	6	7

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Key in timer constant (try 17000)	C_a	A	0.0000
3	Key in desired time.	t H.MS	R/S	t
4	Start timer.		R/S	0.0000
5	Timer loops for time t. When 0.0000 is displayed, time has elapsed. For new time t, execute steps 3 and 4. To calibrate, proceed to step 6.			
6	Key in ending time and starting time and compute new constant.	t_e	ENTER↑	
		t_s	B	C_a
	For new time t, execute steps 3 and 4.			
	The program executes a PRINTx to provide an audible signal for its completion of the timing. This may be omitted from the program (A-005) if desired.			

Example:

Measure elapsed times of 35 seconds and 1 minute 8 seconds.

First, key in the timer constant:

Keystrokes:	**Outputs:**
17000 A →	0.0000
0.0035 R/S →	0.0035

Start timer:

R/S ⟶ 0.0000 ***

Timer runs for approximately 35 seconds.

For second desired time:

0.0108 R/S ⟶ 0.0108

Start timer:

R/S ⟶ 0.0000 ***

Supposing you had noticed the *actual* ending and starting times of the second example were 9:58:16 and 9:57:01, respectively. Calibrate the timer with this information:

9.5816 ENTER↑ 9.5701 B ⟶ 15413.3336

Now try the calibrated timer for 2 minutes 5 seconds:

0.0205 R/S ⟶ 0.0205

Start timer:

R/S ⟶ 0.0000 ***

Under the same conditions, the new timer constant 15413 should be keyed in to A for subsequent use of this program. Your HP-95C will have its own "best" constant for calibration.

CALENDAR FUNCTIONS

For the period March 1, 1900 through February 28, 2100, these programs solve for dates and days.

Given a date, the first program computes an associated day number*. By using this program on two dates, the number of days between those dates may be found.

The second program takes a day number* and finds the corresponding date.

The third program computes the day of the week from a given day number*.

By using the first two programs together, a second date may be computed from a date and a specified number of days (see example).

A date must be input in mm.ddyyyy format. For instance, June 3, 1975, is keyed in as 6.031975. It is important that the zero between the decimal point and the day of the month be included when the day of the month is less than 10. The day of the week is represented by the digits 0 through 6 where zero is Sunday.

*The Julian Day number is an astronomical convention representing the number of days since January 1, 4713 B.C.

Equations:

To compute the day number from the date:

Julian Day number = INT (365.25 y′) + INT (30.6001 m′) + d + 1,720,982

where

$$y' = \begin{cases} \text{year} - 1 & \text{if } m = 1 \text{ or } 2 \\ \text{year} & \text{if } m > 2 \end{cases}$$

$$m' = \begin{cases} \text{month} + 13 & \text{if } m = 1 \text{ or } 2 \\ \text{month} + 1 & \text{if } m > 2 \end{cases}$$

Then days between dates is found by

$$\text{Days} = \text{Day number}_2 - \text{Day number}_1$$

To compute the date from a day number:

$$\text{Day \#} = \text{Julian Day Number} - 1{,}720{,}982$$

$$y' = \text{INT}\left[\frac{\text{Day \#} - 122.1}{365.25}\right]$$

$$m' = \text{INT}\left[\frac{\text{Day \#} - \text{INT}(365.25\ y')}{30.6001}\right]$$

$$\text{Day of the month} = \text{Day \#} - \text{INT}\left[365.25\ y'\right] - \text{INT}\left[30.6001\ m'\right]$$

$$\text{Month} = m = \begin{cases} m' - 13 \text{ if } m' = 14 \text{ or } 15 \\ m' - 1 \text{ if } m' < 14 \end{cases}$$

$$\text{Year} = \begin{cases} y' & \text{if } m > 2 \\ y' + 1 & \text{if } m = 1 \text{ or } 2 \end{cases}$$

To compute the day of the week:

$$\text{Day of the week} = 7 \times \text{FRAC}\left[(\text{Day \#} + 5)/7\right]$$

Remarks:

No checking is done to determine if input data represents valid dates.

Although the programs have been written to function independently, some programming savings can be made by the user if the programs are used concurrently.

Storage registers R_0–R_3 are available to the user.

```
A-000 LBLA
A-001 ENT1
A-002 INT
A-003 S 7
A-004   -
A-005 EEX
A-006   2
A-007   x
A-008 ENT1
A-009 INT
A-010 S 8
A-011   -
A-012 EEX
A-013   4
A-014   x
A-015 S 9
A-016 R 7
A-017   1
A-018   +
A-019 ENT1
A-020 1/X
A-021   .
A-022   7
A-023   +
A-024 CHS
A-025 INT
A-026 S+9
A-027   1
A-028   2
A-029   x
A-030   -
A-031   3
A-032   0
A-033   .
A-034   6
A-035   0
A-036   0
A-037   1
A-038 S 6
A-039   x
A-040 INT
A-041 R 9
A-042   3
A-043   6
A-044   5
A-045   .
A-046   2
A-047   5
A-048 S 5
A-049   x
A-050 INT
A-051   +
A-052 R 8
A-053   +
A-054 FIX0
A-055 RTN
B-000 LBLB
B-001 S 4
B-002   1
```

Break date input into the individual components of mm,dd,yyyy.

---

m + 1

---

m + 1 → m′
y → y′

If input to this routine has absolute value 1 or greater:
y = y ± 1
(+ for plus input)
m = m ± 12

Compute day number.

```
B-003   2
B-004   2
B-005   .
B-006   1
B-007   -
B-008   3
B-009   6
B-010   5
B-011   .
B-012   2
B-013   5
B-014 S 5
B-015   ÷
B-016 INT
B-017 S 9
B-018 R 5
B-019   x
B-020 INT
B-021 R 4
B-022   -
B-023 CHS
B-024 S 4
B-025   3
B-026   0
B-027   .
B-028   6
B-029   0
B-030   0
B-031   1
B-032 S 6
B-033   ÷
B-034 INT
B-035 S 7
B-036 R 4
B-037 X≠Y
B-038 R 6
B-039   x
B-040 INT
B-041   -
B-042 S 6
B-043 R 7
B-044   1
B-045 R 8
B-046   %
B-047   -
B-048   -
B-049 R 7
B-050   1
B-051   4
B-052   ÷
B-053 INT
B-054 S+9
B-055   1
B-056   2
B-057   x
B-058   -
B-059 R 9
B-060 EEX
B-061   6
```

Calculate y′

Calculate m′

Calculate day of month.

Build (m′ − 1) .dd part of display.

Correct m′ − 1 and y′ to m and y.

Finish building mm.ddyyyy result and display answer.

B-062 ÷ B-063 + B-064 FIX6 B-065 RTN C-000 LBLC C-001 5 C-002 +		C-003 7 C-004 ÷ C-005 FRAC C-006 7 C-007 x C-008 RTN	Calculate day of the week from day #.

REGISTERS			
0	1	2	3
4 Day #	5 365.25	6 30.6001	7 m
8 d	9 y	.0	.1
.2	.3	.4	.5

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	For day #'s from dates.			
1	Key in program A.			
2	Key in date and compute day #.	date	A	day #
3	Repeat step 2 for any desired date from March 1, 1900 to February 28, 2100.			
4	For # days between dates, compute day #'s for each and find difference.	$date_1$	A	day #1
			STO 1	day #1
		$date_2$	A	day #2
			RCL 1	day #1
			−	day #2 − day #1
	For dates from day #'s			
1	Key in program B.			
2	Key in day # and compute date.	day #	B	date
	For day of the week from day #.			
1	Key in program C.			

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
2	Key in day # and compute			
	day of the week.	day #	C	0, ..., 6

Example 1:

Senior Lieutenant Yuri Gagarin flew Vostok I into space on April 12, 1961. On July 21, 1969, Neil Armstrong set foot on the moon. How many days had passed between the first manned space flight and the moon landing? On what day of the week did each event take place.

Keystrokes:	**Outputs:**	
4.121961 A STO 1 →	716420.	(Day #1)
7.211969 A STO 2 →	719442.	(Day #2)
RCL 1 − →	3022.	(Days)
RCL 1 C →	3.	(Wednesday)
RCL 2 C →	1.	(Monday)

Example 2:

A short term note is due in 200 days. If the issue date is June 11, 1976, what is the maturity date?*

Keystrokes:	**Outputs:**	
6.111976 A →	721959.	
200 + →	722159.	
B →	12.281976	(December 28, 1976)

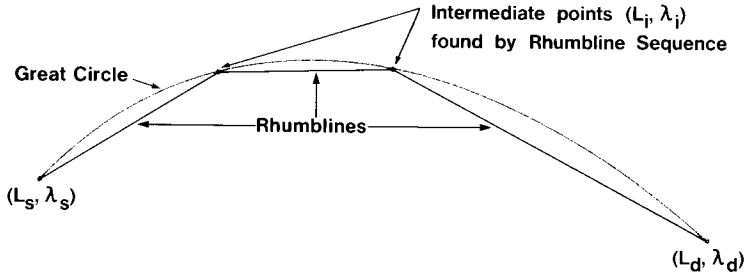
*Note this calculation requires both the A and B programs be entered in the calculator. First a day number is computed for the known date, the number of days (200) is added to it, and this new day number is converted to a date.

Some securities use a 30/360 day calendar while this program performs all calculations using the actual number of days. Do not use the program for financial purposes unless you are sure that actual calendar days are correct.

NAVIGATION

RHUMBLINE SEQUENCE

This program is designed to assist in the activity of course planning. You must supply the source and destination latitude and longitude. Then the rhumbline course and distance from source to destination may be found.



If a longitude interval ($\Delta\lambda$) is then keyed in, the program will compute a sequence of rhumblines which approximate the great circle route from source to destination, where the rhumblines join points separated in longitude by $\Delta\lambda$. The output will be as follows: latitude (L) and longitude (λ) of source, course (C) and distance (D) from source to first intermediate point, L and λ of first intermediate point, C and D from first to second intermediate point, L and λ of second intermediate point, and so on, the final outputs being L and λ of the destination. At the end of all calculations, the sum of all the rhumbline distances is printed out.

The next intermediate point is always found on the great circle from the previous point to the destination. Thus, the points do not all lie on the same great circle from source to destination.

The longitude interval $\Delta\lambda$ should be keyed in as a positive number if the course is westward or as a negative number if the course is eastward.

Latitudes and longitudes are input and output in degrees-minutes-seconds format; course and $\Delta\lambda$ are in decimal degrees. Southern latitudes and eastern longitudes are input and output as negative numbers.

Equations:

$$L_i = \tan^{-1}\left[\frac{\tan(L_d)\sin(\lambda_i - \lambda_{i-1}) - \tan(L_{i-1})\sin(\lambda_i - \lambda_d)}{\sin(\lambda_d - \lambda_{i-1})}\right]$$

$$C = \tan^{-1} \frac{\pi(\lambda_{i-1} - \lambda_i)}{180\left[\ln \tan(45 + \tfrac{1}{2}L_i) - \ln \tan(45 + \tfrac{1}{2}L_{i-1})\right]}$$

$$D = \begin{cases} 60(\lambda_i - \lambda_{i-1})\cos L_i; & \cos C = 0 \\ 60\,\dfrac{(L_i - L_{i-1})}{\cos C}\ ; \text{otherwise} \end{cases}$$

where

$L_0, \lambda_0 = L_s, \lambda_s$ = latitude, longitude of source

L_d, λ_d = latitude, longitude of destination

L_i, λ_i = latitude, longitude of i^{th} point.

Remarks:

This program assumes the calculator is set in DEG mode.

A-000 LBLA		B-020 S I	Clear R_5 for Σ.
A-001 →H		B-021 0	
A-002 S 0	L_1	B-022 S 5	
A-003 S 6		B-023 SPC	
A-004 RTN		B-024 R 0	
A-005 →H		B-025 S 6	Print L_1, λ_1.
A-006 S 1	λ_1	B-026 →HMS	
A-007 S 7		B-027 PRTX	
A-008 RTN		B-028 R 1	
A-009 →H		B-029 S 7	
A-010 S 2	L_2	B-030 →HMS	
A-011 S 8		B-031 PRTX	
A-012 RTN		B-032 SPC	
A-013 →H		B-033 R 1	
A-014 S 3	λ_2	B-034 LBL1	Loop to compute
A-015 S 9		B-035 R 4	successive rhumblines.
A-016 RTN		B-036 +	
A-017 JPB9	Compute rhumbline C, D.	B-037 1	
B-000 LBLB	- - - - - - - - - - - - - -	B-038 →R	
B-001 FIX4	$\Delta\lambda$	B-039 →P	
B-002 S 4		B-040 RI	
B-003 R 1		B-041 S 9	λ_i
B-004 R 3		B-042 GSB7	
B-005 -		B-043 S 8	L_i
B-006 2		B-044 GSB9	Rhumb line from
B-007 ÷		B-045 S+5	(L_{i-1}, λ_{i-1}) to (L_i, λ_i)
B-008 SIN		B-046 SPC	
B-009 SIN⁻¹		B-047 R 8	
B-010 2		B-048 S 6	
B-011 ×		B-049 →HMS	$L_i \rightarrow L_{i-1}$
B-012 R 4	$n = \lvert\Delta\theta/\Delta\lambda\rvert$	B-050 PRTX	$\lambda_i \rightarrow \lambda_{i-1}$
B-013 ÷	where $\Delta\theta =$	B-051 R 9	
B-014 ABS	$2\sin^{-1}\ \sin\ \dfrac{\lambda_1 - \lambda_2}{2}$	B-052 S 7	
B-015 EEX		B-053 →HMS	
B-016 CHS		B-054 PRTX	
B-017 7		B-055 SPC	
B-018 -		B-056 R 9	
B-019 INT		B-057 DSZ	- - - - - - - - - - - - - -

B-058 GTO1		B-111 ÷	
B-059 R 2	$L_i \leftarrow L_2$	B-112 ↑	
B-060 S 8		B-113 x	
B-061 R 3	$\lambda_i \leftarrow \lambda_2$	B-114 R 8	
B-062 S 9		B-115 GSB4	
B-063 GSB9	ΣD	B-116 R 6	
B-064 S+5		B-117 GSB4	
B-065 R 2		B-118 −	
B-066 →HMS	L_2	B-119 →P	
B-067 PRTX		B-120 R↓	C
B-068 R 3	λ_2	B-121 S.1	
B-069 →HMS		B-122 R.0	
B-070 PRTX		B-123 SIN	
B-071 SPC		B-124 SIN⁻¹	x < 0 means East to West.
B-072 SPC	Final ΣD	B-125 X<0	W to E, C is answer.
B-073 R 5		B-126 GTO0	
B-074 FIX1		B-127 R.1	
B-075 PRTX	– – – – – – – – – – – – –	B-128 GTO2	
B-076 RTN	$\lambda_i \rightarrow L_i$	B-129 LBL0	E to W, 360 − C.
B-077 LBL7		B-130 3	
B-078 R 7	Computes L_i for λ_i on great	B-131 6	
B-079 −	circle from (L_{i-1}, λ_{i-1}) to	B-132 0	
B-080 SIN	(L_2, λ_2).	B-133 R.1	
B-081 R 2		B-134 ABS	
B-082 TAN		B-135 −	
B-083 x		B-136 LBL2	
B-084 R 9		B-137 ABS	
B-085 R 3		B-138 PRTX	Course.
B-086 −		B-139 R.0	
B-087 SIN		B-140 R 8	
B-088 R 6		B-141 COS	
B-089 TAN		B-142 x	
B-090 x		B-143 ENT↑	
B-091 −		B-144 R 8	
B-092 R 3		B-145 R 6	
B-093 R 7		B-146 −	
B-094 −		B-147 R.1	
B-095 SIN		B-148 COS	
B-096 ÷		B-149 X≠0	
B-097 TAN⁻¹		B-150 ÷	
B-098 RTN	– – – – – – – – – – – – –	B-151 ENT↑	
B-099 LBL9	Rhumb line C, D	B-152 X=0	
B-100 FIX1	from (L_{i-1}, λ_{i-1})	B-153 R↑	
B-101 R 7	to (L_i, λ_i).	B-154 6	
B-102 R 9		B-155 0	
B-103 −		B-156 x	
B-104 S.0		B-157 ABS	Distance.
B-105 2		B-158 PRTX	
B-106 ÷	Make	B-159 SPC	
B-107 SIN	$-180 \leqslant \lambda_{i-1} - \lambda_i \leqslant 180$.	B-160 FIX4	– – – – – – – – – – – – –
B-108 SIN⁻¹		B-161 RTN	
B-109 9		B-162 LBL4	
B-110 0		B-163 2	

B-164 ÷ B-165 4 B-166 5		B-167 + B-168 TAN B-169 LN	

REGISTERS

0 L_1	1 λ_1	2 L_2	3 λ_2
4 $\Delta\lambda$	5 ΣD	6 L_{i-1}	7 λ_{i-1}
8 L_i	9 λ_i	.0 $\lambda_i - \lambda_{i-1}$	.1 Used
.2	.3	.4	.5	1 Used

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Key in latitude and longitude			
	of source.	L_s	A	
		λ_s	R/S	
3	Key in latitude and longitude			
	of destination.	L_d	R/S	
		λ_d	R/S	
4	(optional) Compute rhumbline			
	course and distance from			
	source to destination.		R/S	C (dec. deg.)
				D (n. m.)
5	Key in longitude interval and			
	compute rhumbline sequence.	$\Delta\lambda$	B	L_s
				λ_s
				C
				D
				L_i
				λ_i
				C

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
				D
				L_d
				λ_d

Example:

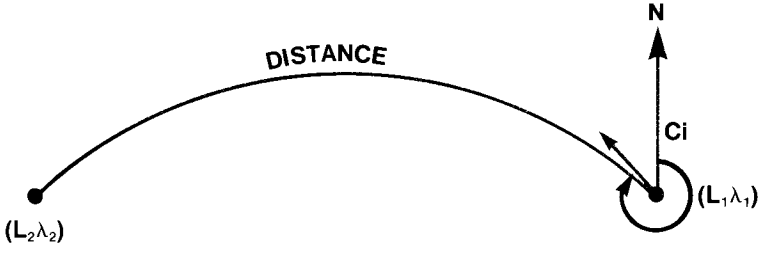
Find the intended track for a voyage from Tokyo to San Francisco which follows rhumblines approximating the great circle track. Use intermediate points separated by 20° in longitude. Since the voyage is eastward, use $\Delta\lambda = -20$. Tokyo is at (L35°40′N, λ139°45′E), San Francisco at (L37°49′N, λ122°25′W).

Keystrokes:	**Outputs:**
35.40 A 139.45 CHS R/S →	−139.75
37.49 R/S 122.25 R/S →	122.42
20 CHS B →	35.4000 *** (L_s)
	−139.4500 *** (λ_s)
	60.5 *** (C)
	1053.7 *** (D)
	44.1829 *** (L_1)
	−159.4500 *** (λ_1)
	74.4 *** (C)
	861.1 *** (D)
	48.0934 *** (L_2)
	−179.4500 *** (λ_2)
	89.3 *** (C)
	799.3 *** (D)
	48.1843 *** (L_3)
	160.1500 *** (λ_3)
	104.3 *** (C)
	851.0 *** (D)
	44.4839 *** (L_4)
	140.1500 *** (λ_4)

```
117.6 *** (C)
905.5 *** (D)
37.4900 *** (Ld)
122.2500 *** (λd)
4470.7 *** (Total D)
```

GREAT CIRCLE NAVIGATION

This program computes the great circle distance between two points and the initial course from the first point. Coordinates are input in degrees-minutes-seconds format. Outputs are distance in nautical miles and initial course in decimal degrees.



Equations:

$$D = 60 \cos^{-1} \left[\sin L_1 \sin L_2 + \cos L_1 \cos L_2 \cos (\lambda_2 - \lambda_1)\right]$$

$$C = \cos^{-1} \left[\frac{\sin L_2 - \sin L_1 \cos (D/60)}{\sin (D/60) \cos L_1}\right]$$

$$C_i = \begin{cases} C & ; \sin (\lambda_2 - \lambda_1) < 0 \\ 360\text{-}C & ; \sin (\lambda_2 - \lambda_1) \geq 0 \end{cases}$$

where

L_1, λ_1 = coordinates of initial point

L_2, λ_2 = coordinates of final point

D = distance from initial to final point

C_i = initial course from initial to final point

Remarks:

Southern latitudes and eastern longitudes must be entered as negative numbers.

Truncation and round off errors occur when the source and destination are very close together (1 mile or less).

Do not use coordinates located at diametrically opposite sides of the earth.
Do not use latitudes of +90° or −90°.
Do not try to compute initial heading along a line of longitude ($L_1 = L_2$).
This program assumes the calculator is set in DEG mode.

A-000 LBLA		B-020 S 6	
A-001 →H	L_1	B-021 6	
A-002 S 0		B-022 0	
A-003 RTN	- - - - - - - - - - - - - -	B-023 ×	
A-004 →H		B-024 PRTX	D.
A-005 S 1	λ_1	B-025 R 2	
A-006 R/S		B-026 SIN	
A-007 →H	- - - - - - - - - - - - - -	B-027 R 0	
A-008 S 2	L_2	B-028 SIN	
A-009 R/S	- - - - - - - - - - - - - -	B-029 R 5	
A-010 →H		B-030 ×	
A-011 S 3	λ_2	B-031 -	
A-012 R/S	- - - - - - - - - - - - - -	B-032 R 0	
B-000 LBLB		B-033 COS	
B-001 R 0		B-034 ÷	
B-002 SIN		B-035 R 6	
B-003 R 2		B-036 SIN	
B-004 SIN		B-037 ÷	
B-005 ×		B-038 COS⁻¹	C.
B-006 R 0		B-039 R 4	
B-007 COS		B-040 SIN	
B-008 R 2		B-041 X<0	
B-009 COS		B-042 GTO0	
B-010 ×		B-043 R↓	
B-011 R 3		B-044 3	
B-012 R 1		B-045 6	
B-013 -		B-046 0	
B-014 S 4		B-047 X≷Y	
B-015 COS		B-048 -	C_i
B-016 ×		B-049 RTN	
B-017 +		B-050 LBL0	C_i
B-018 S 5		B-051 R↓	
B-019 COS⁻¹		B-052 PRTX	

REGISTERS

0 L_1	1 λ_1	2 L_2	3 λ_2
4 $\lambda_2 - \lambda_1$	5 cos D/60	6 D/60	7
8	9	.0	.1
.2	.3	.4	.5	I

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Key in latitude and longitude			
	of source.	L_1 (D.MS)	A	L_1 (dec. deg.)

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
		λ_1 (D.MS)	R/S	λ_1 (dec. deg.)
3	Key in latitude and longitude			
	of destination.	L_2 (D.MS)	R/S	L_2 (dec. deg.)
		λ_2 (D.MS)	R/S	λ_2 (dec. deg.)
4	Compute distance and			
	initial course.		B	D (n.m.)
				C_i (dec. deg.)

Example:

Find the distance and initial course for the great circle from Tokyo (L35°40′N, λ139°45′E) to San Francisco (L37°49′N, λ122°25′E).

Keystrokes:	**Outputs:**
35.40 A 139.45 CHS R/S →	-139.75
37.49 R/S 122.25 R/S →	122.42
B →	4460.04 *** (D)
	54.37 *** (C_i)

SIGHT REDUCTION TABLE

This program calculates the computed altitude Hc and azimuth Zn of a celestial body given the observer's latitude L and the local hour angle LHA and declination (d) of the body. It thus becomes a replacement for the nine volumes of HO 214. However, the user need not bother with the distinctions of same name and contrary name; the program itself resolves all ambiguities of this type.

Equations:

$$Hc = \sin^{-1}\left[\sin d \sin L + \cos d \cos L \cos LHA\right]$$

$$Zn = \begin{cases} Z & ; \sin LHA < 0 \\ 360\text{-}Z & ; \sin LHA \geq 0 \end{cases} \qquad Z = \cos^{-1}\left[\frac{\sin d - \sin L \sin Hc}{\cos L \cos Hc}\right]$$

Remarks:

Southern latitudes and southern declinations must be entered as negative numbers.

The meridian angle t may be input in place of LHA, but if so, eastern meridian angles must be input as negative numbers.

This program assumes the calculator is set in DEG mode.

A-000 LBLA		A-030 R 1	
A-001 →H	L.	A-031 SIN	
A-002 S 0		A-032 R 3	
A-003 RTN	- - - - - - - - - - - - - -	A-033 R 0	
A-004 →H		A-034 SIN	
A-005 S 1	d.	A-035 ×	
A-006 R/S		A-036 −	
A-007 →H	- - - - - - - - - - - - - -	A-037 R 0	
A-008 S 2	LHA.	A-038 COS	
A-009 R 0		A-039 ÷	
A-010 SIN		A-040 R 4	
A-011 R 1		A-041 COS	
A-012 SIN		A-042 ÷	
A-013 ×		A-043 COS⁻¹	Z
A-014 R 0		A-044 R 2	
A-015 COS		A-045 SIN	
A-016 R 1		A-046 X<0	
A-017 COS		A-047 GTO0	
A-018 ×		A-048 R↓	
A-019 R 2		A-049 3	
A-020 COS		A-050 6	
A-021 ×		A-051 0	
A-022 +		A-052 X≠Y	
A-023 S 3		A-053 −	
A-024 SIN⁻¹		A-054 PRTX	Zn.
A-025 S 4	Hc, dec. deg.	A-055 RTN	
A-026 →HMS		A-056 LBL0	
A-027 FIX4		A-057 R↓	
A-028 PRTX	Hc, D.MS.	A-058 PRTX	Zn.
A-029 FIX1			

REGISTERS

0 L	1 d	2 LHA	3 sin Hc
4 Hc	5	6	7
8	9	.0	.1
.2	.3	.4	.5	I

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Key in the following:			
	Observer's latitude	L, D.MS	A	L, dec. deg.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	Declination	d, D.MS	R/S	d, dec. deg.
	Local hour angle	LHA, D.MS	R/S	Hc, D.MS
				Zn, dec. deg.

Example:

Compute the altitude and azimuth of the moon if its LHA is 2°39′54″W and its declination 13°51′06″S. The assumed latitude is 33°20′N.

Keystrokes:	**Outputs:**
33.20 A →	33.33
13.5106 CHS R/S →	−13.85
2.3954 R/S →	42.4447 *** (Hc, D.MS)
	183.5 *** (Zn, dec. deg.)

TIME OF SUNRISE AND SUNSET

The following program computes the time of sunrise and sunset given the month and day and the observer's latitude and longitude. All times given are Greenwich Mean Time.

Equations:

Sunrise: $R = [\lambda - \cos^{-1}(-\tan \phi_s \tan L)]/15 - E + 12$

Sunset: $S = [\lambda + \cos^{-1}(-\tan \phi_s \tan L)]/15 - E + 12$

where

L, λ = latitude, longitude of observer

ϕ_s = subsolar latitude (declination of sun)

E = equation of time

ϕ_s and E are approximated by

$\phi_s = -23.5 \cos(t + 10)$

$E = 0.123 \cos(t + 87) - 1/6 \sin(2t + 20)$

$t = 0.988 [D - 1 + 30.3(m - 1)]$

where D and m are day and month, respectively.

Remarks:

These equations compute the time at which the middle of the sun is on the horizon. Atmospheric refraction, which would cause the sun to rise earlier and set later, is not accounted for.

Southern latitudes and eastern longitudes are expressed as negative values.

The approximate values of ϕ_s and E cause s to exhibit a maximum error of +4.7 minutes and −0.6 minutes at 45° north latitude, based on 1973 ephemeris data. Refraction and secular changes in the ephemeris can result in errors as large as +8 minutes from observed data at 45° north. Errors decrease as latitudes approach 0°. Large errors exist above 65°.

This program assumes the calculator is set in DEG mode.

A-000 LBLA		A-040 2	
A-001 S 0	D.	A-041 0	
A-002 RTN		A-042 +	
A-003 S 1	- - - - - - - - - - - - -	A-043 SIN	
A-004 R/S	m.	A-044 6	
A-005 →H	- - - - - - - - - - - - -	A-045 ÷	E
A-006 S 2		A-046 -	
A-007 R/S		A-047 1	
A-008 →H		A-048 2	
A-009 S 3	- - - - - - - - - - - - -	A-049 -	
A-010 R 1	λ.	A-050 S 5	
A-011 1		A-051 R 4	
A-012 -		A-052 1	
A-013 3		A-053 0	
A-014 0		A-054 +	
A-015 .		A-055 COS	
A-016 3		A-056 2	
A-017 x		A-057 3	
A-018 R 0		A-058 .	
A-019 +		A-059 5	
A-020 1		A-060 x	ϕ_s.
A-021 -		A-061 TAN	
A-022 .		A-062 R 2	
A-023 9		A-063 TAN	
A-024 6		A-064 x	
A-025 6		A-065 COS⁻¹	
A-026 x		A-066 S 6	
A-027 S 4	t	A-067 R 3	
A-028 8		A-068 R/S	
A-029 7		A-069 LBL1	- - - - - - - - - - - - -
A-030 +		A-070 R 3	Sunrise.
A-031 COS		A-071 R 6	
A-032 .		A-072 -	
A-033 1		A-073 1	
A-034 2		A-074 5	
A-035 3		A-075 ÷	
A-036 x		A-076 R 5	
A-037 R 4		A-077 -	
A-038 ENT↑		A-078 X<0	
A-039 +		A-079 GSB9	

A-080 →HMS A-081 RTN A-082 LBL9 A-083 2 A-084 4 A-085 + A-086 RTN A-087 LBL2 A-088 R 3 A-089 R 6 A-090 + A-091 1 A-092 5	Sunset.	A-093 ÷ A-094 R 5 A-095 - A-096 2 A-097 4 A-098 X≤Y A-099 GSB8 A-100 X≥Y A-101 →HMS A-102 RTN A-103 LBL8 A-104 - A-105 ENT↑	

REGISTERS

0 D	1 m	2 L, dec. deg.	3 λ, dec. deg.
4 t	5 E - 12	6 $\Delta\theta$	7
8	9	.0	.1

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Key in date:			
	Day	D	A	D
	Month	m	R/S	m
3	Key in position:			
	Latitude	L, D.MS	R/S	L, dec. deg.
	Longitude	λ, D.MS	R/S	λ, dec. deg.
4	Compute either or both:			
	Time of sunrise		GSB 1	H.MS
	Time of sunset		GSB 2	H.MS

Example:

What time does the sun rise in San Francisco (L37°49′N, λ122°25′W) on Christmas Day? What time does it set?

Keystrokes:	**Outputs:**
25 A 12 R/S →	12.00
37.49 R/S 122.25 R/S →	122.42
GSB 1 →	15.28
GSB 2 →	0.50

The time of sunrise is thus 15:28 GMT or 7:28 AM Pacific Standard Time; sunset is at 00:50 GMT or 4:50 PM PST.

NUMERICAL METHODS

SOLUTION TO f(x) = 0

This program is designed to find one real root of the equation f(x) = 0, where f(x) is a function specified by the user. The user must also specify two initial guesses, a and b, which the program will refine by a modified secant method until it finds either an exact root x_0 ($f(x_0) = 0$) or until two successive approximations are nearly indistinguishable. At this point it will stop, displaying the root found.

If the initial guesses a and b bracket the root (i.e., f(a) x f(b) < 0), then convergence is guaranteed if the function is continuous over the interval [a,b]. There may also be cases in which the program will iterate forever without converging to a root.

The user-defined function f(x) should be keyed in under LBL B. The value of x will be in the display (X-register) when the function is entered. The stack and register R_0–R_9 are available for use in defining f(x).

Remarks:

Of the initial guesses a and b, your better guess should be used for a.

As a and b are keyed in, their function values will be computed and displayed. This can be of assistance in finding values to bracket a root (f(a) × f(b) < 0).

After a root has been found, key B may be pressed to see if the function value is close enough to zero.

A-000 LBL A		A-022 S.5	New a = old c.
A-001 S.1		A-023 R.5	
A-002 B	a → f(a)	A-024 R.3	
A-003 S.2		A-025 -	
A-004 R/S	- - - - - - - - - - - - - - -	A-026 R.4	
A-005 S.3		A-027 R.2	
A-006 B	b → f(b)	A-028 ÷	
A-007 S.4		A-029 1	
A-008 R/S	- - - - - - - - - - - - - - -	A-030 -	
A-009 2		A-031 EEX	
A-010 GTO 1		A-032 CHS	
A-011 LBL 0	If AC = 0, exit.	A-033 9	
A-012 X=Y		A-034 -	
A-013 GTO 9		A-035 ÷	c = a +
A-014 LST X		A-036 +	(a–b)/(B/A – 1.000000001)
A-015 S.4	New B = old A.	A-037 X≠Y	If c ≠ a, GTO 2.
A-016 R.5		A-038 GTO 2	
A-017 S.3	New b = old a.	A-039 R.3	
A-018 2	- - - - - - - - - - - - - - -	A-040 -	
A-019 LBL 1		A-041 9	
A-020 S.0		A-042 ÷	
A-021 R.1	New p = 2 or 2 x (old p).	A-043 -	c = a – (a–b)/9.

A-044 X≠Y		A-061 GTO0	
A-045 GTO2	If c = a, halt.	A-062 R↓	
A-046 S.1		A-063 X≤Y	Else if AB ≥ 0, GTO 0.
A-047 R/S	- - - - - - - - - - - - - - -	A-064 GTO0	
A-048 LBL2	If c ≠ a, then save c.	A-065 R.0	If AC > 0 and
A-049 S.1		A-066 R.4	AB < 0, then
A-050 6		A-067 X≠Y	new B = old B/p.
A-051 R.4	c = f(c)	A-068 ÷	
A-052 R.2		A-069 S.4	
A-053 x		A-070 CL X	
A-054 0		A-071 LSTX	
A-055 LSTX		A-072 ENT↑	New p = 2 x (old p).
A-056 R↑		A-073 +	
A-057 S.2	New A = old C.	A-074 GTO1	- - - - - - - - - - - - - - -
A-058 X≠Y		A-075 LBL9	
A-059 x		A-076 R.1	Root.
A-060 X≤Y	If AC ≤ 0, then GTO 0.		- - - - - - - - - - - - - - -

REGISTERS

0	1	2	3	
4	5	6	7	
8	9	.0 $p = 2^n$	.1 c	
.2 A	.3 b	.4 B	.5 a	I

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Switch to PRGM.		g B	(CLEAR B)
	Key in function f(x).			
	Switch to RUN.			
3	Key in initial guesses.	a	A	f(a)
		b	R/S	f(b)
4	Compute root.		R/S	Root
5	To find f(x) for any x.	x	B	f(x)

Example:

Find a root of the equation $\ln x + 3x - 10.8074$ in the interval $[1, 5]$. Store the constant 10.8074 in R_1. Find the actual function value of the reported root.

Keystrokes: **Outputs:**

Switch to PRGM.
g CLEAR B
f LN f LAST x 3 ×
+ RCL 1 −

Switch to RUN.

10.8074 STO 1 1 A ⟶	−7.81	(f(1))
5 R/S ⟶	5.80	(f(5)
R/S ⟶	3.21	(Root)
B ⟶	0.00	(f(3.21))

NUMERICAL INTEGRATION, DISCRETE CASE

Let $x_0, x_1, \ldots, x_n$ be equally spaced points $x_i = x_0 + ih$ for $i = 1, 2, \ldots, n$, at which corresponding values $f(x_0), f(x_1), \ldots, f(x_n)$ of a function $f(x)$ are known. Using only this information, with no explicit expression for $f(x)$ itself,

$$\int_{x_0}^{x_n} f(x)\,dx$$

may be approximated using

(1) The trapezoidal rule:

$$\int_{x_0}^{x_n} f(x)\,dx \cong \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n)\right] = I_1$$

(2) The Simpson's rule:

$$\int_{x_0}^{x_n} f(x)\,dx \cong \frac{h}{3}\Big[f(x_0) + 4f(x_1) + 2f(x_2) + \ldots + 4f(x_{n-3}) + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\Big] = I_2$$

In order to apply Simpson's rule, n must be even.

Remarks:

Attempting to compute by Simpson's rule with n odd will cause the error display, SEE 5.

A-000 LBLA		A-022 GTO9	
A-001 S S	h	A-023 LBL1	
A-002 R/S		A-024 2	Trapezoidal area.
A-003 CL Σ		A-025 R.1	
A-004 S.1		A-026 GTO0	
A-005 S.3	$f(x_0)$	A-027 LBL2	
A-006 CL X		A-028 R.0	Simpson area.
A-007 LBL9		A-029 2	
A-008 R/S		A-030 ÷	
A-009 S 8	$f(x_j)$, j odd.	A-031 FRAC	
A-010 ENT↑		A-032 CHS	Test n even.
A-011 +		A-033 X≠0	
A-012 ENT↑		A-034 JX	
A-013 +		A-035 3	
A-014 LSTX	$2\,f(x_j)$, $4\,f(x_j)$.	A-036 R.3	
A-015 Σ+		A-037 LBL0	
A-016 R/S		A-038 R 8	$2\,f(x_n)$ were added, so subtract $f(x_n)$.
A-017 S 8		A-039 −	
A-018 ENT↑	$f(x_j)$, j even.	A-040 R 9	
A-019 +		A-041 x	
A-020 ENT↑		A-042 X≷Y	Area.
A-021 Σ+	$2\,f(x_j)$, $2\,f(x_j)$.	A-043 ÷	

REGISTERS

0	1	2	3
4	5	6	7
8 $f(x_j)$	9 h	.0 n	.1 Σ TRAP
.2 Used	.3 Σ SIMP	.4 Used	.5 Used

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Key in spacing.	h	A	
3	Key in $f(x_0)$.	$f(x_0)$	R/S	0
4	Repeat step 4 for all function			
	values.	$f(x_j)$	R/S	j
5	After all function values have			
	been keyed in, calculate either			
	or both:			
	Trapezoidal approx.		GSB 1	TRAP
	Simpson's rule.		GSB 2	SIMP

Example:

Given the values below for $f(x_j)$, $j = 0, 1, \ldots, 8$, compute the approximations to the integral

$$\int_0^2 f(x)\,dx$$

by the trapezoidal rule and by Simpson's rule.

The value for h is 0.25.

i	0	1	2	3	4	5	6	7	8
x_i	0	.25	.5	.75	1	1.25	1.5	1.75	2
$f(x_i)$	2	2.8	3.8	5.2	7	9.2	12.1	15.6	20

Keystrokes:	**Outputs:**	
.25 A 2 R/S 2.8 R/S 3.8 R/S 5.2 R/S 7 R/S 9.2 R/S 12.1 R/S 15.6 R/S 20 R/S →	8.00	(n)
GSB 1 →	16.68	(Trapezoidal)
GSB 2 →	16.58	(Simpson's)

NUMERICAL INTEGRATION, SIMPSON'S RULE

This program approximates the definite integral $\int_a^b f(x)\,dx$ by Simpson's rule when a formula for f(x) is known explicitly. The user must specify the endpoints a and b of the interval over which integration is to be performed and the number of subintervals n into which the interval (a, b) is to be divided. This n must be even; if it is not, the error message "See 5" will be displayed. The program will go on to compute $x_0 = a$, $x_j = x_0 + jh$ for $j = 1, 2, \ldots, n-1$, and $x_n = b$ where

$$h = \frac{b - a}{n}$$

The integral $\int_a^b f(x)\,dx$ is approximated by Simpson's rule:

$$\int_{x_0}^{x_n} f(x)\,dx \cong \frac{h}{3}\Big[f(x_0) + 4f(x_1) + 2f(x_2) + \ldots + 4f(x_{n-3}) + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\Big]$$

The program assumes that f(x) will be keyed into program memory under LBL B. Registers R_0–R_9 and the stack are available for use in defining f(x). The routine f(x) will find the value of x in the X-register upon entry. Two levels of subroutines are allowed in defining f(x).

A-000 LBL A		A-020 R.3	
A-001 S.3	b	A-021 R.2	
A-002 X≷Y		A-022 S.5	$x_0 \leftarrow a$.
A-003 S.2	a	A-023 -	
A-004 R/S		A-024 I	
A-005 S I	n	A-025 ÷	
A-006 2		A-026 S.4	$h \leftarrow (b - a)/n$.
A-007 ÷		A-027 2	
A-008 FRAC		A-028 S.0	R_0 will be 2 or 4.
A-009 CHS	Display "See 5" if n is not even.	A-029 DSZ	
A-010 X≠0		A-030 LBL 9	
A-011 fX		A-031 R.4	
A-012 R.2		A-032 R.5	
A-013 B		A-033 +	$x \leftarrow x + h$
A-014 S.1		A-034 S.5	f(x)
A-015 R.3		A-035 B	
A-016 B	$R_1 \leftarrow f(a) + f(b)$.	A-036 6	
A-017 R.1		A-037 R.0	
A-018 +		A-038 -	
A-019 S.1		A-039 S.0	

A-040 x A-041 R.1 A-042 + A-043 S.1 A-044 DSZ A-045 GT09	$R_{.1} \leftarrow R_{.1} + 4$ (or 2) f(x).	A-046 R.1 A-047 R.4 A-048 x A-049 3 A-050 ÷	Area.

REGISTERS			
0	1	2	3
4	5	6	7
8	9	.0 4, 2	.1 Σ Simp

.2	.3	.4	.5	I
a	b	h	x	n

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Switch to PRGM.		g B	(CLEAR B)
	Key in f(x).			
	Switch to RUN.			
3	Key in endpoints.	a	ENTER↑	
		b	A	a
4	Key in number of sub-intervals.	n	R/S	Area

Example:

Find the value of $\int_0^{2\pi} \frac{dx}{1 - \cos x + 0.25}$ for u = 16. Note that x is assumed to be in radians. For safety, if you work mostly in degrees, it is good programming practice to set the angular mode to radians at the beginning of the routine, then back to degrees at the end.

Keystrokes:

Switch to PRGM.
g CLEAR B
g RAD COS 1
x≷y − .25 +
1/x g DEG

Outputs:

Switch to RUN.

0 ENTER↑ 2 g π × A ⟶	0.00	
16 R/S ⟶	8.36	(Area)

The exact solution is $\frac{8\pi}{3} = 8.38$.

DIFFERENTIAL EQUATIONS

This program solves first-order differential equations by the fourth-order Runge-Kutta method. A first-order equation is of the form $y' = f(x,y)$ with initial values (x_0, y_0). The solution is a numerical solution which calculates y_i for $x_i = x_0 + ih$, $i = 1, 2, 3, \ldots$, where h is an increment specified by the user.

The user must specify the function f(x,y) by keying it into program memory under LBL B. The stack and registers R_0–R_9 and I are available for defining f(x,y). The function should assume that x and y will be in the X- and Y-registers, respectively, upon entry.

Equations:

$$y_{i+1} = y_i + \frac{1}{6}(c_1 + 2c_2 + 2c_3 + c_4)$$

where

$$c_1 = hf(x, y_i)$$

$$c_2 = hf\left(x_i + \frac{h}{2}, y_i + \frac{c_1}{2}\right)$$

$$c_3 = hf\left(x_i + \frac{h}{2}, y_1 + \frac{c_2}{2}\right)$$

$$c_4 = hf(x_i + h, y_i + c_3)$$

A-000 LBL A		A-029 LSTX	
A-001 2		A-030 ENT↑	
A-002 ÷		A-031 +	$y_i + c_3$
A-003 S.4	h/2	A-032 R.3	
A-004 R/S		A-033 +	
A-005 S.3	y_0	A-034 R.2	
A-006 X≷Y		A-035 R.4	
A-007 S.2	x_0	A-036 +	
A-008 SPC	- - - - - - - - - - - - - - -	A-037 S.2	$x_i + h \rightarrow x_{i+1}$
A-009 LBL 9		A-038 B	
A-010 B		A-039 R.1	
A-011 S.1	$c_1/2$	A-040 +	
A-012 R.3		A-041 R.0	
A-013 +	$y_i + (c_1/2)$	A-042 ENT↑	
A-014 R.2		A-043 +	
A-015 R.4		A-044 +	
A-016 +		A-045 3	$\Delta = \frac{1}{6}(c_1 + 2c_2 + 2c_3 + c_4)$
A-017 S.2	$x_i + (h/2)$	A-046 ÷	
A-018 B		A-047 R.3	
A-019 S.0	$c_2/2$	A-048 +	$y_{i+1} = y_i + \Delta$
A-020 R.3		A-049 S.3	
A-021 +	$y_i + (c_2/2)$	A-050 R.2	
A-022 R.2	$x_i + (h/2)$	A-051 PRTX	
A-023 B		A-052 R.3	x_{i+1}
A-024 R.0		A-053 PRTX	
A-025 X≷Y		A-054 SPC	y_{i+1}
A-026 +		A-055 X≷Y	
A-027 S.0	$c_3/2$	A-056 GTO 9	
A-028 CL X			

REGISTERS

0	1	2	3
4	5	6	7
8	9	.0 $c_2 + c_3$	.1 c_1
.2 x_i	.3 y_i	.4 h/2	.5

I

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	**To key in f(x):**			
	Switch to PRGM.		g B	(CLEAR B)
	Key in function and f(x), end with		RCL .	
			4 ×	
	Switch to RUN.			
3	Key in spacing.	h	A	h/2
4	Key in initial x and y values			
	and output succeeding values.	x_0	ENTER↑	

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
		y_0	R/S	x_i
				y_i

Example:

Solve numerically the first-order differential equation

$$y' = \frac{\sin x + \tan^{-1}(y/x)}{y - \ln(\sqrt{x^2 + y^2})}$$

where $x_0 = y_0 = 1$. Let $h = 0.5$. The angular mode must be set to radians.

Keystrokes: **Outputs:**

Switch to PRGM.

g CLEAR B
g RAD STO 1 x≷y
STO 2 x≷y g →P
f LN STO 3 R↓
RCL 1 f SIN +
RCL 2 RCL 3 − ÷
g DEG RCL • 4 ×

Switch to RUN.

.5 A ⟶ 0.25 (h/2)

1 ENTER↑ 1 R/S ⟶ 1.50 *** (x_1)
2.06 *** (y_1)
2.00 *** (x_2)
2.78 *** (y_2)
2.50 *** (x_3)
3.28 *** (y_3)

R/S (halts program)

STATISTICS

CURVE FITTING

This program performs curve fits by three different methods: exponential, logarithmic, and power. The linear curve fit (linear regression) is already a preprogrammed function so is not included here. For each curve fit, the data points are entered by keying in the y-value, pressing ENTER↓, and keying in the x-value. After all (x,y) data pairs have been keyed in, the following calculations may be made: regression constants, coefficient of determination, estimate of y ($\hat{y}$), and estimate of x ($\hat{x}$).

Equations:

Exponential

The exponential equation $y = a\,e^{bx}$ is fit by applying linear regression to the equation

$$\ln y = bx + \ln a.$$

All y-values must be positive.

$$b = \frac{\Sigma x_i \ln y_i - \frac{1}{n}(\Sigma x_i)(\Sigma \ln y_i)}{\Sigma x_i^2 - \frac{1}{n}(\Sigma x_i)^2}$$

$$a = \exp\left[\frac{\Sigma \ln y_i}{n} - b\frac{\Sigma x_i}{n}\right]$$

$$\hat{y} = ae^{bx}$$

$$\hat{x} = \frac{1}{b}(\ln y - \ln a)$$

Logarithmic

The logarithmic equation is $y = a + b \ln x$. In this curve fit, all x-values must be positive.

$$b = \frac{\Sigma y_i \ln x_i - \frac{1}{n}\Sigma \ln x_i \,\Sigma y_i}{\Sigma (\ln x_i)^2 - \frac{1}{n}(\Sigma \ln x_i)^2}$$

$$a = \frac{1}{n}(\Sigma y_i - b \Sigma \ln x_i)$$

$$\hat{y} = a + b (\ln x)$$

$$\hat{x} = e^{[(y - a)/b]}$$

Power

A power curve is of the form $y = ax^b$. This is linearized to the form

$$\ln y = \ln a + b (\ln x).$$

Here both x- and y-values must be positive.

$$b = \frac{\Sigma (\ln x_i)(\ln y_i) - \frac{(\Sigma \ln x_i)(\Sigma \ln y_i)}{n}}{\Sigma (\ln x_i)^2 - \frac{(\Sigma \ln x_i)^2}{n}}$$

$$a = \exp\left[\frac{\Sigma \ln y_i}{n} - b \frac{\Sigma \ln x_i}{n}\right]$$

$$\hat{y} = ax^b$$

$$\hat{x} = e^{[(\ln y - \ln a)/b]}$$

Coefficient of determination

For all cases, the value of r^2 is computed as shown below:

$$r^2 = \left(b \frac{s_x}{s_y}\right)^2$$

where s_x, s_y = standard deviations of x and y.

Remarks:

Each curve fit routine (under LBL's B, C, and D) is completely self-contained and, if desired, may be keyed into program memory without any of the other routines.

A-000 LBLA	
A-001 CL Σ	Clear.
B-000 LBLB	- - - - - - - - - - - - -
B-001 X≷Y	Exponential.
B-002 LN	
B-003 X≷Y	
B-004 Σ+	
B-005 RTN	
B-006 LBL1	- - - - - - - - - - - - -
B-007 LR	
B-008 e^x	
B-009 PRTX	
B-010 X≷Y	a
B-011 PRTX	
B-012 RTN	b
B-013 LBL2	- - - - - - - - - - - - -
B-014 LR	
B-015 X≷Y	
B-016 S	
B-017 LSTX	
B-018 x	
B-019 X≷Y	
B-020 ÷	
B-021 X^2	r^2
B-022 RTN	- - - - - - - - - - - - -
B-023 LBL3	
B-024 ŷ	x → y
B-025 e^x	
B-026 RTN	- - - - - - - - - - - - -
B-027 LBL4	y → x
B-028 LR	
B-029 LSTX	
B-030 LN	
B-031 X≷Y	
B-032 -	
B-033 X≷Y	
B-034 ÷	
B-035 RTN	- - - - - - - - - - - - -
C-000 LBLC	Logarithmic.
C-001 LN	
C-002 Σ+	
C-003 RTN	- - - - - - - - - - - - -
C-004 LBL1	
C-005 LR	a
C-006 PRTX	
C-007 X≷Y	b
C-008 PRTX	- - - - - - - - - - - - -
C-009 RTN	
C-010 LBL2	
C-011 LR	
C-012 X≷Y	
C-013 S	
C-014 LSTX	
C-015 x	
C-016 X≷Y	
C-017 ÷	
C-018 X^2	r^2
C-019 RTN	- - - - - - - - - - - - -
C-020 LBL3	
C-021 LN	x → y
C-022 ŷ	
C-023 RTN	- - - - - - - - - - - - -
C-024 LBL4	
C-025 LR	y → x
C-026 LSTX	
C-027 X≷Y	
C-028 -	
C-029 X≷Y	
C-030 ÷	
C-031 e^x	
D-000 LBLD	- - - - - - - - - - - - -
D-001 LN	Power.
D-002 X≷Y	
D-003 LN	
D-004 X≷Y	
D-005 Σ+	
D-006 RTN	- - - - - - - - - - - - -
D-007 LBL1	
D-008 LR	
D-009 e^x	a
D-010 PRTX	
D-011 X≷Y	b
D-012 PRTX	- - - - - - - - - - - - -
D-013 RTN	
D-014 LBL2	
D-015 LR	
D-016 X≷Y	
D-017 S	
D-018 LSTX	
D-019 x	
D-020 X≷Y	
D-021 ÷	
D-022 X^2	r^2
D-023 RTN	- - - - - - - - - - - - -
D-024 LBL3	x → y
D-025 LN	
D-026 ŷ	
D-027 e^x	
D-028 RTN	- - - - - - - - - - - - -
D-029 LBL4	y → x
D-030 LN	
D-031 R↓	
D-032 LR	

D-033 R↑ D-034 X≷Y D-035 −		D-036 X≷Y D-037 ÷ D-038 e^x	
REGISTERS			
0	1	2	3
4	5	6	7
8	9	.0 n	.1 Σx
.2 Σx^2	.3 Σy	.4 Σy^2	.5 Σxy	I

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Initialize.		A	
3	Key in y-value and x-value.	y_i	ENTER↑	
		x_i		
4	Select one of curve fits below:			
	Exponential		B	i
	Logarithmic		C	i
	Power		D	i
5	Repeat steps 3 and 4 for all			
	data pairs.			
6	Find regression constants.		GSB 1	a
				b
7	Find coefficient of deter-			
	mination.		GSB 2	r^2
8	For a given x-value, find an			
	estimated y.	x	GSB 3	$\hat{y}$
9	For a given y-value, find an			
	estimated x.	y	GSB 4	$\hat{x}$
10	For a new curve fit, go to			
	step 2.			

Example 1:

Determine whether a power curve or an exponential curve is the better fit to the data below:

x	16	27	33	41	56
y	23	30	35	38	42

Keystrokes:	**Outputs:**	
A		(Start power curve)
23 ENTER↑ 16 D →	1.00	
30 ENTER↑ 27 D →	2.00	
35 ENTER↑ 33 D →	3.00	
38 ENTER↑ 41 D →	4.00	
42 ENTER↑ 56 D →	5.00	
GSB 1 →	5.93 ***	(a)
	0.50 ***	(b)

Thus the power curve is $y = 5.93\ x^{0.50}$.

GSB 2 →	0.98	(r^2)
A		(Start exponential)
23 ENTER↑ 16 B →	1.00	
30 ENTER↑ 27 B →	2.00	
35 ENTER↑ 33 B →	3.00	
38 ENTER↑ 41 B →	4.00	
42 ENTER↑ 56 B →	5.00	
GSB 1 →	19.71 ***	(a)
	0.01 ***	(b)

Thus the exponential curve is $y = 19.71\ e^{0.01x}$.

GSB 2 →	0.90	(r^2)

Since r^2 for the power curve $0.98 > 0.90$, we conclude that the power curve fits the data better than the exponential curve.

Example 2:

A manufacturer observes declining sales of a soon-to-be-obsoleted product, of which there were originally 10,000 units in inventory. The cumulative sales

figures for a number of months are shown below. If these are fit by a logarithmic curve, find the projected cumulative sales after 7 months. Find also the month in which the last unit (number 10,000) is projected to be sold.

Month (x)	1	2	3	4	5	6
Cumulative Sales (y)	1431	3506	5177	6658	7810	8592

Keystrokes:	**Outputs:**	
A		(Start logarithmic)
1431 ENTER↑ 1 C ⟶	1.00	
3506 ENTER↑ 2 C ⟶	2.00	
5177 ENTER↑ 3 C ⟶	3.00	
6658 ENTER↑ 4 C ⟶	4.00	
7810 ENTER↑ 5 C ⟶	5.00	
8592 ENTER↑ 6 C ⟶	6.00	
GSB 1 ⟶	1066.15 ***	(a)
	4069.93 ***	(b)
GSB 2 ⟶	0.99	(r^2)
7 GSB 3 ⟶	8985.87	($x \rightarrow \hat{y}$)

At the end of 7 months, the cumulative sales are expected to total 8986 units.

10000 GSB 4 ⟶	8.98	($y \rightarrow \hat{x}$)

The final unit is projected to be sold near the end of the ninth month.

COVARIANCE AND CORRELATION COEFFICIENT

For a set of given data points $\{(x_i, y_i), i = 1, 2, \ldots, n\}$, the covariance and correlation coefficient are found by

$$\text{covariance } s_{xy} = \frac{1}{n-1}\left(\Sigma x_i y_i - \frac{1}{n}\Sigma x_i \Sigma y_i\right)$$

$$\text{or } s_{xy}' = \frac{1}{n}\left(\Sigma x_i y_i - \frac{1}{n}\Sigma x_i \Sigma y_i\right)$$

$$\text{correlation coefficient } r = \frac{B\, s_x}{s_y}$$

where B = slope from linear regression

s_x, s_y = standard deviations of x and y

Remarks:

The correlation coefficient r always lies in the range $-1 \leq r \leq 1$. It may also be found by

$$r = \frac{s_{xy}}{s_x\, s_y}$$

A-000 LBLA A-001 CL Σ A-002 RTN A-003 LBL1 A-004 R.5 A-005 R.1 A-006 R.3 A-007 x A-008 R.0 A-009 ÷ A-010 - A-011 R.0 A-012 1 A-013 - A-014 S 0	Clear.	A-015 ÷ A-016 RTN A-017 R 0 A-018 x A-019 R.0 A-020 ÷ A-021 RTN A-022 LBL2 A-023 LR A-024 X≷Y A-025 S A-026 LSTX A-027 x A-028 X≷Y A-029 ÷	s_{xy}. s_{xy}'. r.

REGISTERS

0 n − 1	1	2	3
4	5	6	7
8	9	.0 n	.1 Σx
.2 Σx^2	.3 Σy	.4 Σy^2	.5 Σxy

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Initialize.		A	
3	Key in y-value and x-value.	y_i	ENTER↑	
		x_i	Σ+	i
4	Repeat step 3 for all data			
	pairs.			
5	Find covariance.		GSB 1	s_{xy}
			R/S	s_{xy}
6	Find correlation coefficient.		GSB 2	r
7	For a new case go to step 2.			

Example:

Find s_{xy}, s_{xy}', and r for the data below:

x	26	30	44	50	62
y	92	85	78	81	54

Keystrokes:	**Outputs:**	
A		
92 ENTER↑ 26 Σ+ →	1.00	
85 ENTER↑ 30 Σ+ →	2.00	
78 ENTER↑ 44 Σ+ →	3.00	
81 ENTER↑ 50 Σ+ →	4.00	
54 ENTER↑ 62 Σ+ →	5.00	
GSB 1 →	−191.00	(s_{xy})
R/S →	−152.80	(s_{xy}')
GSB 2 →	−0.90	(r)

MOMENTS AND SKEWNESS

This program computes the first three moments and the moment coefficient of skewness for a set of data $\{x_1, x_2, \ldots, x_n\}$.

Equations:

$$1^{st} \text{ moment} \quad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$2^{nd} \text{ moment} \quad m_2 = \frac{1}{n}\Sigma x_i^2 - \bar{x}^2$$

$$3^{rd} \text{ moment} \quad m_3 = \frac{1}{n}\Sigma x_i^3 - \frac{3}{n}\bar{x}\,\Sigma x_i^2 + 2\bar{x}^3$$

moment coefficient of skewness

$$\gamma_1 = \frac{m_3}{m_2^{3/2}}$$

A-000 LBLA		B-020 ÷	
A-001 CL Σ	Clear.	B-021 R.1	
B-000 LBLB		B-022 R 0	
B-001 ENT↑	Sum x, x^2.	B-023 ×	
B-002 X²		B-024 R.0	
B-003 Σ+		B-025 ÷	
B-004 RTN		B-026 3	
B-005 LBL1		B-027 ×	
B-006 $\bar{x}$		B-028 −	
B-007 X≷Y		B-029 R 0	
B-008 S 0		B-030 ENT↑	
B-009 RTN	$\bar{x}$	B-031 X²	
B-010 R.1		B-032 ×	
B-011 R.0		B-033 2	
B-012 ÷		B-034 ×	
B-013 R 0		B-035 +	
B-014 X²		B-036 S 2	m_3
B-015 −		B-037 RTN	
B-016 S 1	m_2	B-038 R 2	
B-017 RTN		B-039 R 1	
B-018 R.5		B-040 1	
B-019 R.0		B-041 .	

B-042 5 B-043 y^x		B-044 ÷	γ_1
REGISTERS			
0 $\bar{x}$	1 m_2	2 m_3	3
4	5	6	7
8	9	.0 n	.1 Σx^2
.2 Σx^4	.3 Σx	.4 Σx^2	.5 Σx^3	1

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Initialize.		A	
3	Key in x-value.	x_i	B	i
4	Repeat step 3 for all x-values.			
5	Compute the moments and			
	skewness.		GSB 1	$\bar{x}$
			R/S	m_2
			R/S	m_3
			R/S	γ_1

Example:

Compute the statistics of moment and skewness for the x-values below:

i	1	2	3	4	5	6
x_i	2.1	3.5	4.2	6.5	4.1	3.6

Keystrokes:	**Outputs:**	
A 2.1 B →	1.00	
3.5 B 4.2 B →	3.00	
6.5 B 4.1 B 3.6 B →	6.00	
GSB 1 →	4.00	($\bar{x}$)
R/S →	1.72	(m_2)
R/S →	1.43	(m_3)
R/S →	0.63	(γ_1)

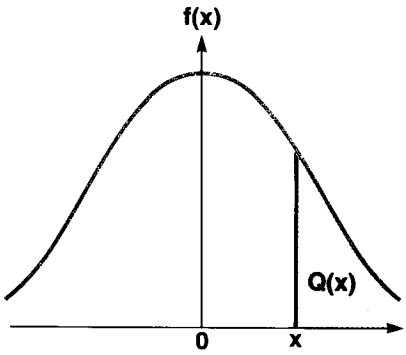
NORMAL DISTRIBUTION

The density function for a standard normal variable is

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}.$$

The upper tail area is

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt.$$



For $x \geq 0$, polynomial approximation is used to compute Q(x):

$$Q(x) = f(x)(b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5) + \epsilon(x)$$

where $|\epsilon(x)| < 7.5 \times 10^{-8}$

$$t = \frac{1}{1 + rx}, \quad r = 0.2316419$$

$$b_1 = .31938153, \quad b_2 = -.356563782$$

$$b_3 = 1.781477937, \quad b_4 = -1.821255978$$

$$b_5 = 1.330274429$$

Remarks:

The program only works for $x \geq 0$. Equations $f(-x) = f(x)$, $Q(-x) = 1 - Q(x)$, where $x \geq 0$, can be used to find f and Q for negative numbers.

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, 1968.

A-000 LBL A A-001 STO 6 A-002 ENT↑ A-003 x A-004 2 A-005 ÷ A-006 CHS A-007 e^x A-008 1 A-009 2 A-010 x A-011 1/X A-012 ÷ A-013 STO 7 A-014 R/S A-015 RCL 0 A-016 RCL 6 A-017 x A-018 1 A-019 +	x f(x)	A-020 1/X A-021 ENT↑ A-022 ENT↑ A-023 ENT↑ A-024 RCL 5 A-025 x A-026 RCL 4 A-027 + A-028 x A-029 RCL 3 A-030 + A-031 x A-032 RCL 2 A-033 + A-034 x A-035 RCL 1 A-036 + A-037 x A-038 RCL 7 A-039 x	t Q(x)

REGISTERS

0 r	1 b_1	2 b_2	3 b_3
4 b_4	5 b_5	6 x	7 f(x)
8	9	.0	.1
.2	.3	.4	.5	I

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Store constants.	r	STO 0	
		b_1	STO 1	
		b_2	STO 2	
		b_3	STO 3	
		b_4	STO 4	
		b_5	STO 5	
3	Key in x and compute f(x) and			
	Q(x).	x	A	f(x)
			R/S	Q(x)
4	For a new x, go to step 3.			

Example:

Find f(x) and Q(x) for x = 1.18 and x = 2.28.

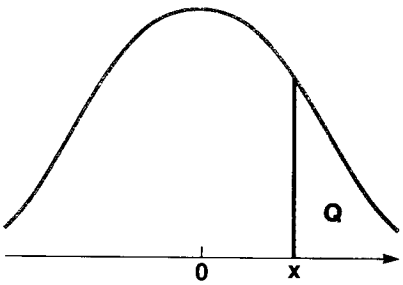
Keystrokes:	**Outputs:**	
.2316419 STO 0 →	0.23	(r)
.31938153 STO 0 →	0.32	(b_1)
.356563782 CHS STO 2 →	−0.36	(b_2)
1.781477937 STO 3 →	1.78	(b_3)
1.821255978 CHS STO 4 →	−1.82	(b_4)
1.330274429 STO 5 →	1.33	(b_5)
1.18 A →	0.20	(f(x))
R/S →	0.12	(Q(x))
2.28 A →	0.03	(f(x))
R/S →	0.01	(Q(x))

INVERSE NORMAL INTEGRAL

This program determines the value of x such that

$$Q = \int_x^{\infty} \frac{e^{-\frac{t^2}{2}}}{\sqrt{2\pi}}\, dt$$

where Q is given and $0 < Q \leq 0.5$.



The following rational approximation is used:

$$x = t - \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3} + \epsilon(Q)$$

where $|\epsilon(Q)| < 4.5 \times 10^{-4}$

$$t = \sqrt{\ln \frac{1}{Q^2}}$$

$c_0 = 2.515517$ $d_1 = 1.432788$

$c_1 = 0.802853$ $d_2 = 0.189269$

$c_2 = 0.010328$ $d_3 = 0.001308$

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, 1968.

A-000 LBLA	Q(x)	A-016 +	
A-001 ENT↑		A-017 ×	
A-002 x		A-018 1	
A-003 1/X		A-019 +	
A-004 LN		A-020 S 7	
A-005 √X		A-021 CL X	
A-006 S 6	t	A-022 R 2	
A-007 ENT↑		A-023 x	
A-008 ENT↑		A-024 R 1	
A-009 ENT↑		A-025 +	
A-010 R 5		A-026 ×	
A-011 x		A-027 R 0	
A-012 R 4		A-028 +	
A-013 +		A-029 R 7	
A-014 ×		A-030 ÷	x
A-015 R 3		A-031 −	

REGISTERS

0 c_0	1 c_1	2 c_2	3 d_1
4 d_2	5 d_3	6 t	7 Used
8	9	.0	.1

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Store constants.	c_0	STO 0	
		c_1	STO 1	
		c_2	STO 2	
		d_1	STO 3	

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
		d_2	STO 4	
		d_3	STO 5	
3	Key in Q and find x.	Q	A	x

Example:

Find the x-values corresponding to Q = 0.12 and Q = 0.05.

Keystrokes:	**Outputs:**	
2.515517 STO 0 →	2.52	(c_0)
0.802853 STO 1 →	0.80	(c_1)
0.010328 STO 2 →	0.01	(c_2)
1.432788 STO 3 →	1.43	(d_1)
0.189269 STO 4 →	0.19	(d_2)
0.001308 STO 5 →	1.308000000–03	(d_3)
.12 A →	1.18	(x)
.05 A →	1.65	(x)

PERMUTATION AND COMBINATION

A permutation is an ordered subset of a set of distinct objects. The number of possible permutations, each containing n objects, that can be formed from a collection of m distinct objects is given by

$$ {}_mP_n = \frac{m!}{(m-n)!} = m(m-1)\dots(m-n+1) $$

where m, n are integers and $0 \leq n \leq m$.

A combination is a selection of one or more of a set of distinct objects without regard to order. The number of possible combinations, each containing n objects, that can be formed from a collection of m distinct objects is given by

$$ {}_mC_n = \frac{m!}{(m-n)!\,n!} = \frac{m(m-1)\dots(m-n+1)}{1 \cdot 2 \cdot \dots \cdot n} $$

where m, n are integers and $0 \leq n \leq m$.

This program computes ${}_mC_n$ using the following algorithm:

1. If $n \leq m - n$

$$ {}_mC_n = \frac{m-n+1}{1} \cdot \frac{m-n+2}{2} \cdot \ldots \cdot \frac{m}{n} . $$

2. If $n > m - n$, program computes ${}_mC_{m-n}$.

Remarks:

${}_mP_n$ can also be denoted by P_n^m, P(m, n) or $(m)_n$.

${}_mP_0 = 1$, ${}_mP_1 = m$, ${}_mP_m = m!$

${}_mC_n$, which is also called the binomial coefficient, can be denoted by C_n^m, C(m, n), or $\binom{m}{n}$.

${}_mC_n = {}_mC_{m-n}$

${}_mC_0 = {}_mC_m = 1$

${}_mC_1 = {}_mC_{m-1} = m$

A-000 LBLA	m ↑ n → ${}_mP_n$	A-031 R↓	
A-001 X=Y		A-032 RTN	- - - - - - - - - - - -
A-002 GTO5		A-033 LBL5	${}_mP_m = m!$
A-003 X>Y		A-034 N!	- - - - - - - - - - - -
A-004 GTO2		B-000 LBLB	m ↑ n → ${}_mC_n$
A-005 X=0		B-001 X>Y	
A-006 GTO3		B-002 GTO2	
A-007 1		B-003 -	
A-008 X=Y		B-004 LSTX	
A-009 GTO4		B-005 X≤Y	
A-010 -		B-006 X≠Y	Store max (n, m−n)
A-011 S I		B-007 S 9	
A-012 R↓		B-008 1	Let m − n > n
A-013 S 9	- - - - - - - - - - - -	B-009 S 8	
A-014 LBL1	m(m−1) ... (m − n + 1)	B-010 +	m − n + 1 → R_7
A-015 R 9		B-011 S 7	
A-016 1		B-012 R↓	
A-017 -		B-013 X=0	
A-018 S 9		B-014 GTO3	
A-019 X		B-015 LBL5	
A-020 DSZ		B-016 1	
A-021 GTO1		B-017 R 8	
A-022 RTN	- - - - - - - - - - - -	B-018 +	
A-023 LBL2	n > m: error	B-019 S 8	
A-024 0		B-020 X>Y	If j > n, exit
A-025 ÷	- - - - - - - - - - - -	B-021 GTO6	
A-026 LBL3	${}_mP_0 = 1$	B-022 R 9	
A-027 1		B-023 +	
A-028 RTN	- - - - - - - - - - - -	B-024 R 8	$\frac{m-n+j}{j}$
A-029 LBL4	${}_mP_1 = m$	B-025 ÷	
A-030 R↓		B-026 SX7	

B-027 R↓ B-028 GTO5 B-029 LBL6 B-030 R 7 B-031 RTN	$_mC_n$	B-032 LBL2 B-033 0 B-034 ÷ B-035 LBL3 B-036 1	n > m: error $_mC_0$

REGISTERS			
0	1	2	3
4	5	6	7 Π
8 j	9 m − n	.0	.1
.2	.3	.4	.5 I Used

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Key in m and n.	m	ENTER↑	
		n		
3	Find either:			
	Permutations		A	$_mP_n$
	Combinations		B	$_mC_n$

Examples:

Find $_{43}P_3$ and $_{43}C_3$.

Keystrokes:	**Outputs:**	
43 ENTER↑ 3 A ⟶	74046.00	$(_{43}P_3)$
43 ENTER↑ 3 B ⟶	12341.00	$(_{43}C_3)$

RANDOM NUMBER GENERATOR

This program calculates uniformly distributed pseudo random numbers u_i in the range

$$0 \leq u_i \leq 1$$

using the multiplicative linear congruential method:

$$u_{i+1} = \text{Fractional part of } (997\ u_i)$$

where i = 0, 1, 2, ...

The period of this generator is 500,000 numbers and the generator passes the frequency test (chi square) for uniformity, the serial test and the run test. The most significant digits (the left hand digits) are the most random; the rightmost digits are significantly less random.

The seed used in the example is a good one. Other seeds may be selected but the quotient of (seed $\times$ 10^7) divided by two or five must not be an integer. Also, it would be wise to statistically test other seeds before using them.

A-000 LBL A A-001 S G A-002 R/S A-003 R D A-004 9	u_0	A-005 9 A-006 7 A-007 x A-008 FRAC A-009 GTO A	u_{i+1} = Frac (997 u_i)
REGISTERS			
0 Used	1	2	3
4	5	6	7

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Input seed.	u_0	A	u_0
3	Generate random number.		R/S	u_i
4	Repeat step 3 as often as			
	desired.			

Example:

Generate random numbers from the seed 0.5284163.

Keystrokes:	**Outputs:**	
.5284163 A →	0.53	(u_0)
R/S →	0.83	(u_1)
R/S →	0.56	(u_2)
R/S →	0.27	(u_3)
R/S →	0.04	(u_4)

CHI-SQUARE EVALUATION

This program calculates the value of the χ^2 statistic for the goodness of fit test by the equation

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

where O_i = observed frequency

E_i = expected frequency

If the expected values are equal

$$\left(E = E_i = \frac{\Sigma O_i}{n} \text{ for all } i\right)$$

then

$$\chi^2 = \frac{n\Sigma O_i^2}{\Sigma O_i} - \Sigma O_i$$

Remarks:

In order to apply the goodness of fit test to a set of given data, combining some classes may be necessary to make sure that each expected frequency is not too small (say, not less than 5).

A-000 LBLA		A-018 RTN	χ^2
A-001 0		A-019 LBL2	- - - - - - - - - - - -
A-002 S 7	Initialize.	A-020 S+8	Equal E's.
A-003 S 8		A-021 X²	
A-004 S 9		A-022 S+9	
A-005 RTN	- - - - - - - - - - - -	A-023 1	
A-006 LBL1		A-024 S+7	
A-007 S 8	Unequal E_i's.	A-025 R 7	i
A-008 -		A-026 RTN	- - - - - - - - - - - -
A-009 X²	$\frac{(O_i - E_i)^2}{E_i}$	A-027 R 7	
A-010 R 8		A-028 R 9	
A-011 ÷		A-029 ×	
A-012 S+9		A-030 R 8	
A-013 1		A-031 ÷	
A-014 S+7		A-032 R 8	
A-015 R 7	i	A-033 -	χ^2
A-016 RTN	- - - - - - - - - - - -	A-034 RTN	- - - - - - - - - - - -
A-017 R 9		A-035 R 8	

A-036 R 7	E	A-037 ÷	
REGISTERS			
0	1	2	3
4	5	6	7 n
8 Used	9 Used	.0	.1
.2	.3	.4	.5	I

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Initialize.		A	0.00
3	For expected values unequal,			
	go to step 4; for expected			
	values equal, go to step 8.			
	Unequal			
4	Key in observed and expected			
	value.	O_i	ENTER↑	
		E_i	GSB 1	i
5	Repeat step 4 until all values			
	have been keyed in.			
6	Compute chi-square statistic.		R/S	χ^2
7	For a new case go to step 2.			
	Equal			
8	Key in observed value.	O_i	GSB 2	i
9	Repeat step 8 until all ob-			
	served values have been			
	keyed in.			
10	Compute chi-square statistic.		R/S	χ^2
11	Compute E.		R/S	E
12	For a new case, go to			
	step 2.			

Example 1:

Find the χ^2 statistic for the data below (expected values unequal):

O_i	8	50	47	56	5	14
E_i	9.6	46.7	51.8	54.4	8.25	9.15

Keystrokes:	**Outputs:**	
A →	0.00	
8 ENTER↑ 9.6 GSB 1 →	1.00	
50 ENTER↑ 46.7 GSB 1 →	2.00	
47 ENTER↑ 51.8 GSB 1 →	3.00	
56 ENTER↑ 54.4 GSB 1 →	4.00	
5 ENTER↑ 8.25 GSB 1 →	5.00	
14 ENTER↑ 9.15 GSB 1 →	6.00	
R/S →	4.84	(χ^2)

Example 2:

The following table shows the observed frequencies in tossing a die 120 times. χ^2 can be used to test if the die is fair.

Number	1	2	3	4	5	6
Frequency	25	17	15	23	24	16

Keystrokes:	**Outputs:**	
A →	0.00	
25 GSB 2 →	1.00	
17 GSB 2 →	2.00	
15 GSB 2 →	3.00	
23 GSB 2 →	4.00	
24 GSB 2 →	5.00	
16 GSB 2 →	6.00	
R/S →	5.00	(χ^2)
R/S →	20.00	(E)

PAIRED t STATISTIC

Given a set of paired observations from two normal populations with means μ_1, μ_2 (unknown)

x_i	x_1	x_2	...	x_n
y_i	y_1	y_2	...	y_n

let

$$D_i = x_i - y_i$$

$$\overline{D} = \frac{1}{n}\sum_{i=1}^{n} D_i$$

$$s_D = \sqrt{\frac{\Sigma D_i^2 - \frac{1}{n}(\Sigma D_i)^2}{n-1}}$$

$$s_{\overline{D}} = \frac{s_D}{\sqrt{n}}$$

The test statistic

$$t = \frac{\overline{D}}{s_{\overline{D}}},$$

which has $n - 1$ degrees of freedom (df), can be used to test the null hypothesis

$$H_0: \mu_1 = \mu_2.$$

A-000 LBLA A-001 CL Σ B-000 LBLB B-001 − B-002 Σ+ B-003 RTN B-004 LBL1 B-005 S B-006 R.0	Clear. Sum $x_i - y_i$.	B-007 ΣX B-008 ÷ B-009 $\bar{x}$ B-010 LSTX B-011 ÷ B-012 RTN B-013 R.0 B-014 1 B-015 −	t df

REGISTERS

0	1	2	3
4	5	6	7
8	9	.0 n	.1 ΣD
.2 ΣD^2	.3 Used	.4 Used	.5 Used

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Initialize.		A	
3	Key in x- and y-values.	x_i	ENTER↑	
		y_i	B	i
4	Repeat step 3 until all values			
	have been keyed in.			
5	Compute t statistic and			
	degrees of freedom.		GSB 1	t
			R/S	df

Example:

Find the paired t-statistic for the data below:

x	14	17.5	17	17.5	15.4
y	17	20.7	21.6	20.9	17.2

Keystrokes:	**Outputs:**	
A 14 ENTER↑ 17 B →	1.00	
17.5 ENTER↑ 20.7 B →	2.00	
17 ENTER↑ 21.6 B →	3.00	
17.5 ENTER↑ 20.9 B →	4.00	
15.4 ENTER↑ 17.2 B →	5.00	(n)
GSB 1 →	−7.16	(t)
R/S →	4.00	(df)

t STATISTIC FOR TWO MEANS

Suppose $\{x_1, x_2, \ldots, x_{n_1}\}$ and $\{y_1, y_2, \ldots, y_{n_2}\}$ are independent random samples from two normal populations having means μ_1, μ_2 (unknown) and the same unknown variance σ^2.

We want to test the null hypothesis

$$H_0: \mu_1 - \mu_2 = D$$

where D is a given number.

Define

$$\bar{x} = \frac{1}{n_1} \sum_{i=1}^{n_1} x_i$$

$$\bar{y} = \frac{1}{n_2} \sum_{i=1}^{n_2} y_i$$

$$t = \frac{\bar{x} - \bar{y} - D}{\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \quad \sqrt{\frac{\Sigma x_i^2 - n_1 \bar{x}^2 + \Sigma y_i^2 - n_2 \bar{y}^2}{n_1 + n_2 - 2}}}$$

We can use this t statistic, which has the t distribution with $n_1 + n_2 - 2$ degrees of freedom, to test the null hypothesis H_0.

A-000 LBLA	Clear.	B-020 +	
A-001 CL Σ		B-021 1/X	
B-000 LBLB	Save parameters of x-values.	B-022 ÷	
B-001 R.0		B-023 R 1	
B-002 S 0		B-024 R 2	
B-003 R.2		B-025 X²	
B-004 S 1		B-026 R 0	
B-005 x̄		B-027 x	
B-006 S 2		B-028 −	
B-007 CL Σ	Clear Σ for y-values.	B-029 R.2	
B-008 RTN		B-030 +	
B-009 x̄		B-031 R 3	
B-010 S 3		B-032 X²	
B-011 LSTX		B-033 R.0	
B-012 +		B-034 x	
B-013 CHS		B-035 −	
B-014 R 2		B-036 R 0	
B-015 +	$\bar{x} - \bar{y} - D$	B-037 R.0	
B-016 R 0		B-038 +	
B-017 1/X		B-039 2	
B-018 R.0		B-040 −	
B-019 1/X		B-041 ÷	

B-042 √x		B-043 ÷	t
REGISTERS			
0 n_1	1 Σx^2	2 $\bar{x}$	3 $\bar{y}$
4	5	6	7
8	9	.0 n_2	.1 Σy
.2 Σy^2	.3 Used	.4 Used	.5 Used	I

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Initialize.		A	
3	Key in x-value.	x_i	Σ+	i
4	Repeat step 4 for all x-values.			
5	Compute mean of x-values.		B	$\bar{x}$
6	Key in y-value.	y_i	Σ+	i
7	Repeat step 6 for all y-values.			
8	Input D and compute t.	D	R/S	t
9	To find the mean of y-values.		$\bar{x}$	$\bar{y}$

Example:

Test the hypothesis $\mu_1 = \mu_2$ (i.e., D = 0) for the data below:

x: 79, 84, 108, 114, 120, 103, 122, 120

y: 91, 103, 90, 113, 108, 87, 100, 80, 99, 54.

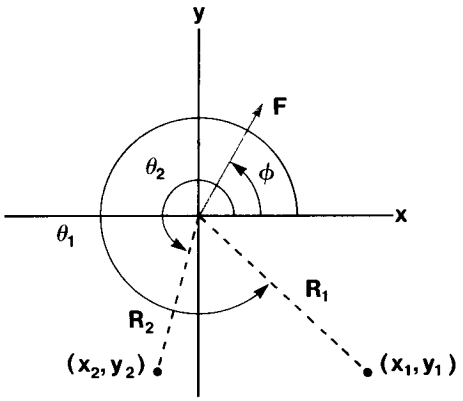
Keystrokes:	**Outputs:**	
A 79 Σ+ 84 Σ+ ⟶	2.00	
108 Σ+ 114 Σ+ 120 Σ+ ⟶	5.00	
103 Σ+ 122 Σ+ 120 Σ+ ⟶	8.00	(n_1)
B ⟶	106.25	($\bar{x}$)
91 Σ+ 103 Σ+ 90 Σ+ ⟶	3.00	
113 Σ+ 108 Σ+ 87 Σ+ ⟶	6.00	
100 Σ+ 80 Σ+ 99 Σ+		
54 Σ+ ⟶	10.00	(n_2)

0 R/S →	1.73	(t)
x̄ →	92.50	($\bar{y}$)

STRUCTURES

STATIC EQUILIBRIUM OF A POINT

This program calculates the two reaction forces necessary to balance a given two-dimensional force vector. The direction of the reaction forces may be specified as a vector of arbitrary length or by Cartesian coordinates using the point of force application as the origin.



Equations:

$$R_1 \cos \theta_1 + R_2 \cos \theta_2 = F \cos \phi$$

$$R_1 \sin \theta_1 + R_2 \sin \theta_2 = F \sin \phi$$

where:

F is the known force;

ϕ is the direction of the known force;

R_1 is one reaction force;

θ_1 is the direction of R_1;

R_2 is the second reaction force;

θ_2 is the direction of R_2;

The coordinates x_1 and y_1 are referenced from the point where F is applied to the end of the member along which R_1 acts; x_2 and y_2 are the coordinates referenced from the point where F is applied to the end of the member along which R_2 acts.

Remarks:

This program assumes the calculator is set in DEG mode. This program may be used in conjunction with the Vector Operations program and leaves registers R_6–R_9 available for combining the programs.

A-000 LBL A		A-027 R 3	
A-001 LBL 1	Y_1, X_1 input	A-028 ×	
A-002 →P		A-029 R 5	
A-003 X⇄Y		A-030 R 2	
A-004 LBL 2	Input θ_1	A-031 ×	
A-005 1		A-032 −	
A-006 →R	and store	A-033 R 1	
A-007 S 0		A-034 R 2	
A-008 X⇄Y	$\sin\theta_1$, $\cos\theta_1$	A-035 ×	
A-009 S 1		A-036 R 0	
A-010 R/S		A-037 R 3	
A-011 LBL 3	Y_2, X_2 input	A-038 ×	
A-012 →P		A-039 −	
A-013 X⇄Y		A-040 ÷	
A-014 LBL 4		A-041 SPC	
A-015 1	Input θ_2	A-042 PRTX	Output R_1
A-016 →R		A-043 LSTX	
A-017 S 2	and store	A-044 R 5	
A-018 X⇄Y		A-045 R 0	
A-019 S 3	$\sin\theta_2$, $\cos\theta_2$	A-046 ×	
A-020 R/S		A-047 R 4	
A-021 LBL 5	Input ϕ and F	A-048 R 1	
A-022 →R		A-049 ×	
A-023 S 4	and compute	A-050 −	
A-024 X⇄Y		A-051 R↑	
A-025 S 5	reaction forces,	A-052 ÷	
A-026 R 4		A-053 PRTX	Output R_2

REGISTERS

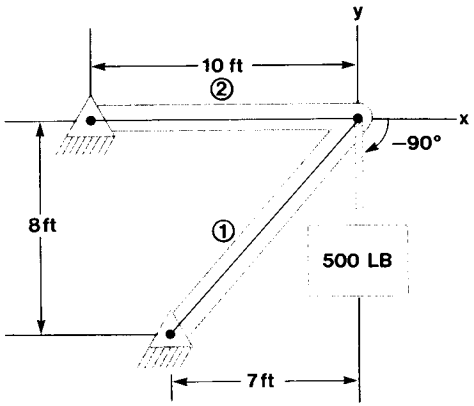
0 $\cos\theta_1$	1 $\sin\theta_1$	2 $\cos\theta_2$	3 $\sin\theta_2$
4 F $\cos\phi$	5 F $\sin\phi$	6	7
8	9	.0	.1

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Define reaction directions as			
	Cartesian coordinates or as			
	vectors of arbitrary magnitude.			
	(Use the point of force appli-			
	cations as the origin):			
	define direction one in rec-			

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	tangular form	y_1	ENTER↑	y_1
		x_1	GSB 1	sin θ_1
	or in polar form	θ_1	GSB 2	sin θ_1
	and			
	define direction two in rec-			
	tangular form	y_2	ENTER↑	y_2
	or in polar form	x_2	GSB 3	sin θ_2
3	Key in known force			
	direction	ϕ	ENTER↑	
	then magnitude *and* compute			
	reactions	F	GSB 5	R_1, R_2
4	To change force, go to step 3.			
	To change either or both reac-			
	tion directions, go to step 2.			

Example 1:

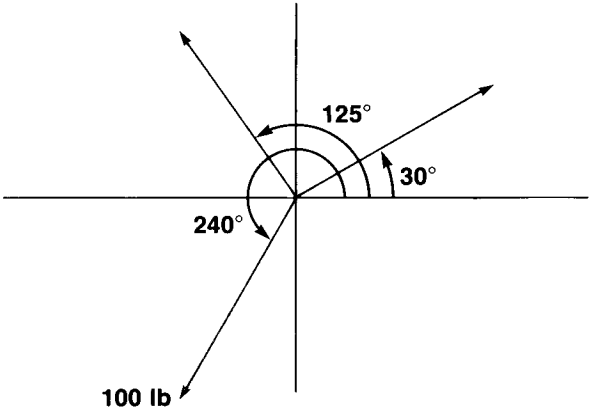
Find the reaction forces in the pin-jointed structure shown below.



Keystrokes:	**Outputs:**
8 CHS ENTER↑ 7 CHS GSB 1 ⟶	−0.75
0 ENTER↑ 10 CHS GSB 3 ⟶	0.06
90 CHS ENTER↑ 500 GSB 5 ⟶	−664.38 *** (R_1) 437.50 *** (R_2)

Example 2:

Find the reaction forces for the diagram below.



Keystrokes:	**Outputs:**
30 GSB 2 ⟶	0.50
125 GSB 4 ⟶	0.82
240 ENTER↑ 100 GSB 5 ⟶	90.98 *** (R_1) 50.19 *** (R_2)

COMPOSITE SECTION PROPERTIES

The properties of arbitrarily shaped sections can be evaluated using this program. Exact solutions are obtained when the section is broken into a finite number of rectangles. Approximate solutions can be achieved by assuming that finite areas are concentrated at their centers.

The program calculates the area of the section, the centroid of the area, the moments of inertia about any specified set of axes, the polar moment of inertia about the specified axis, the moments of inertia about an axis translated to the centroid, the moments of inertia of the principal axis, the rotation angle between the translated axis and the principal axis, and the polar moment of inertia about the principal axis.

Equations:

$$A_{si} = \Delta x_i \, \Delta y_i$$

$$A = A_{s1} + A_{s2} + A_{s3} + \ldots + A_{sn}$$

$$\overline{x} = \frac{\sum_{i=1}^{n} x_{0i} \, A_{si}}{A}$$

$$\overline{y} = \frac{\sum_{i=1}^{n} y_{0i} \, A_{si}}{A}$$

$$I_x = \sum_{i=1}^{n} \left(y_{0i}^2 + \frac{\Delta y_i^2}{12} \right) A_{si}$$

$$I_y = \sum_{i=1}^{n} \left(x_{0i}^2 + \frac{\Delta x_i^2}{12} \right) A_{si}$$

$$J = I_x + I_y$$

$$I_{xy} = \sum_{i=1}^{n} x_{0i} \, y_{0i} \, A_{si}$$

$$I_{\overline{x}} = I_x - A\overline{y}^2$$

$$I_{\bar{y}} = I_y - A\bar{x}^2$$

$$I_{\bar{x}\bar{y}} = I_{xy} - A\bar{x}\bar{y}$$

$$\phi = \frac{1}{2}\tan^{-1}\left(\frac{-2I_{\bar{x}\bar{y}}}{I_{\bar{x}} - I_{\bar{y}}}\right)$$

$$I_x' = I_{\bar{x}}\cos^2\theta + I_{\bar{y}}\sin^2\theta - I_{\bar{x}\bar{y}}\sin 2\theta$$

$$I_y' = I_{\bar{y}}\cos^2\theta + I_{\bar{x}}\sin^2\theta + I_{\bar{x}\bar{y}}\sin 2\theta$$

$$J' = I_x' + I_y'$$

$$I_{xy}' = \frac{(I_{\bar{x}} - I_{\bar{y}})}{2}\sin 2\theta + I_{\bar{x}\bar{y}}\cos 2\theta$$

where:

Δx_i is the width of a rectangular element;

Δy_i is the height of a rectangular element;

A_{si} is the area of an element;

A is the total area of the section;

$\bar{x}$ is the x coordinate of the centroid;

$\bar{y}$ is the y coordinate of the centroid;

x_{0i} is the x coordinate of the centroid of an element;

y_{0i} is the y coordinate of the centroid of an element;

I_x is the moment of inertia about the x-axis;

I_y is the moment of inertia about the y-axis;

J is the moment of inertia about the origin;

I_{xy} is the product of inertia;

$I_{\bar{x}}$ is the moment of inertia about the x-axis translated to the centroid;

$I_{\bar{y}}$ is the moment of inertia about the y-axis translated to the centroid;

$I_{\bar{x}\bar{y}}$ is the product of inertia about the translated axis;

ϕ is the angle between the translated axis and the principal axis;

θ is the angle between the original axis and an arbitrary axis;

I_x' is the x moment of inertia about the arbitrary axis;

I_y' is the y moment of inertia about the arbitrary axis;

J' is the polar moment of inertia about the arbitrary axis;

I_{xy}' is the product of inertia about the arbitrary axis.

Remarks:

This program assumes the calculator is set in DEG mode.

Reference:

Wojciechowski, Felix; "Properties of Plane Cross Sections"; *Machine Design;* P. 105, Jan 22, 1976.

```
A-000 LBLA
A-001 S 6
A-002 R↓
A-003 S 7
A-004 R/S
A-005 S 8
A-006 X≷Y
A-007   x
A-008 S.5
A-009 LSTX
A-010 LBL2
A-011 X²
A-012    1
A-013    2
A-014   ÷
A-015 R 7
A-016 X²
A-017   +
A-018   x
A-019 S+3
A-020 R 8
A-021 X²
A-022    1
A-023    2
A-024   ÷
A-025 R 6
A-026 X²
A-027   +
A-028 R.5
A-029   x
A-030 S+4
A-031 R.5
A-032 R 6
A-033   x
A-034 S+1
A-035 R 7
A-036 R.5
A-037   x
A-038 S+2
A-039   x
A-040 R.5
A-041   ÷
A-042 S+5
A-043 R.5
A-044 S+0
A-045 R/S
A-046 LBL1
A-047 S.5
A-048    0
A-049 S 8
A-050 GTO2
B-000 LBL6
B-001 GSB1
```

Store x_{0i} and y_{0i}.

Input Δx and Δy and accumulate moments and area.

Input area for approximate solution.

Compute $\bar{x}$ and $\bar{y}$.

```
B-002 R 5
B-003 R 0
B-004 R.0
B-005 R.1
B-006   x
B-007   x
B-008   -
B-009 ENT↑
B-010   +
B-011 R 4
B-012 R.1
B-013 X²
B-014 R 0
B-015   x
B-016   -
B-017 R.0
B-018 X²
B-019 R 0
B-020   x
B-021 CHS
B-022 R 3
B-023   +
B-024   -
B-025 X≠0
B-026   ÷
B-027 TAN⁻¹
B-028    2
B-029   ÷
B-030 R 0
B-031 PRTX
B-032 R.0
B-033 PRTX
B-034 R.1
B-035 PRTX
B-036 R↑
B-037 PRTX
B-038 R/S
B-039 LBL1
B-040 R 1
B-041 R 0
B-042   ÷
B-043 S.1
B-044 R 2
B-045 R 0
B-046   ÷
B-047 S.0
C-000 LBLC
C-001 ENT↑
C-002 S 1
C-003   +
C-004 S 9
C-005 R↓
C-006 S.3
```

Compute ϕ.

Output A, $\bar{y}$, $\bar{x}$, ϕ.

Compute $\bar{x}$ and $\bar{y}$ subroutine.

Store y, x, and θ.

C-007 R↓		C-054 1	
C-008 S.2	- - - - - - - - - - - - -	C-055 →R	
C-009 GSB1		C-056 X²	
C-010 ×		C-057 S 6	
C-011 R 0		C-058 R.2	
C-012 ×		C-059 ×	
C-013 CHS		C-060 X≷Y	
C-014 R 5		C-061 X²	
C-015 +		C-062 S 7	
C-016 R.3		C-063 R.3	
C-017 R.1		C-064 ×	
C-018 -		C-065 +	
C-019 R.2		C-066 R 9	
C-020 R.0		C-067 SIN	
C-021 -		C-068 R.4	
C-022 ×		C-069 ×	
C-023 R 0		C-070 -	
C-024 ×		C-071 PRTX	Output I_x'
C-025 +		C-072 LSTX	
C-026 S.4		C-073 R 6	
C-027 R.2		C-074 R.3	
C-028 X²		C-075 ×	
C-029 R.2		C-076 +	
C-030 R.0	Calculate I_x', I_y', J', and I_{xy}'.	C-077 R 7	
C-031 ×		C-078 R.2	
C-032 2		C-079 ×	
C-033 ×		C-080 +	
C-034 -		C-081 PRTX	Output I_y'
C-035 R 0		C-082 +	
C-036 ×		C-083 PRTX	Output J'
C-037 R 3		C-084 R.2	
C-038 +		C-085 R.3	
C-039 S.2		C-086 -	
C-040 R.3		C-087 2	
C-041 X²		C-088 ÷	
C-042 R.3		C-089 R 9	
C-043 R.1		C-090 SIN	
C-044 ×		C-091 ×	
C-045 2		C-092 R.4	
C-046 ×		C-093 R 9	
C-047 -		C-094 COS	
C-048 R 0		C-095 ×	
C-049 ×		C-096 +	
C-050 R 4		C-097 PRTX	Output I_{xy}'
C-051 +		C-098 R/S	
C-052 S.3		C-099 LBL1	Subroutine to B1.
C-053 I		C-100 JPB1	

REGISTERS

0 ΣA	1 $\Sigma x_{0i}A_{si}$	2 $\Sigma y_{0i}A$	3 ΣI_x
4 ΣI_y	5 ΣI_{xy}	6 x_{0i}, $\cos^2\theta$	7 y_{0i}, $\sin^2\theta$
8 Δx_{0i}	9 2θ	.0 $\bar{y}$	.1 $\bar{x}$

.2 y, Used	.3 x, Used	.4 Used	.5 A_{si}	I θ

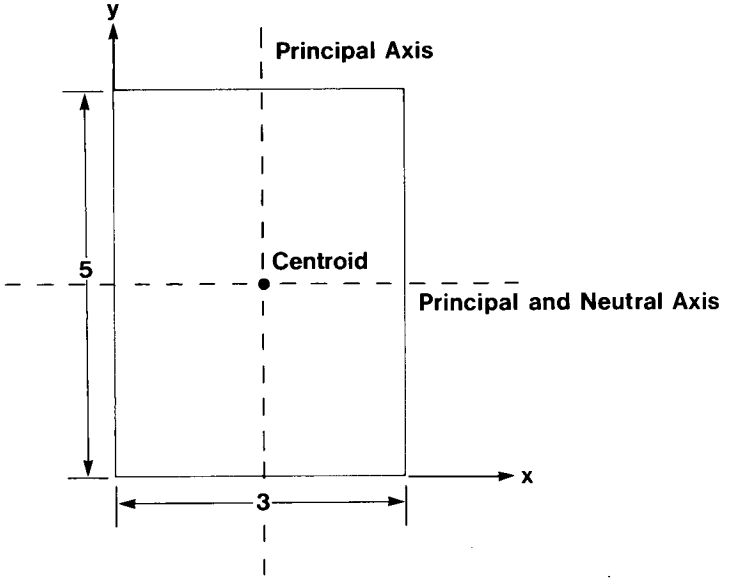
STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Initialize.		f REG	
3	Key in coordinates of centroid			
	of element.	y_{oi}	ENTER↑	y_{oi}
		x_{oi}	A	y_{oi}
4	Key in height and width of rec-			
	tangular element.	Δ_{yi}	ENTER↑	Δ_{yi}
		Δ_{xi}	R/S	A_{si}
	or key in area of element			
	for approximate solution.			
		A_{si}	GSB 1	A_{si}
5	Repeat steps 3-4 for each			
	element.			
6	Calculate area, centroid,			
	and ϕ.		B	$A, \bar{y}, \bar{x}, \phi$
7	Specify arbitrary origin and			
	rotation and calculate			
	properties*	y	ENTER↑	
		x	ENTER↑	
		θ	C	Ix', Iy', J', Ixy'
	*Note C may be pressed			
	immediately after B for $I_{\bar{x}\phi}$,			
	$I_{\bar{y}\phi}$, $J\phi$, $I_{\bar{x}\bar{y}\phi}$ since B leaves			
	$\bar{y}, \bar{x}, \phi$ in position in the			
	stack.			
	$I_{\bar{x}}$, $I_{\bar{y}}$, J, $I_{\bar{x}\bar{y}}$ may be obtained by			
	B CL x C, or by recalling y			
	and x from storage (RCL • 0			
	RCL • 1 0 C).			

Example 1:

What is the moment of inertia about the neutral axis through the centroid of the section ($I_{\bar{x}\phi}$) for the rectangular section shown? What is the moment of inertia about the X-axis (I_x)?

Table of Inputs

Section	y_0	x_0	Δy	Δx
1	2.5	1.5	5	3



Keystrokes:

Outputs:

g REG 2.5 ENTER↑ 1.5 A
5 ENTER↑ 3 R/S ⟶ 15.00 (A_{si})

B ⟶ 15.00 *** (A)
2.50 *** ($\bar{y}$)
1.50 *** ($\bar{x}$)
0.00 *** (ϕ)

C ⟶ 31.25 *** ($I_{\bar{x}\phi}$)
11.25 *** ($I_{\bar{y}\phi}$)
42.50 *** (J_ϕ)
0.00 *** ($I_{\bar{x}\bar{y}\phi}$)

(Note the centroid and ϕ are already properly positioned for calculation of properties about the neutral axis through the centroid.)

0 ENTER↑ 0 ENTER↑ 0 C ⟶ (Key in zeroes for properties about the origin and axes shown.)

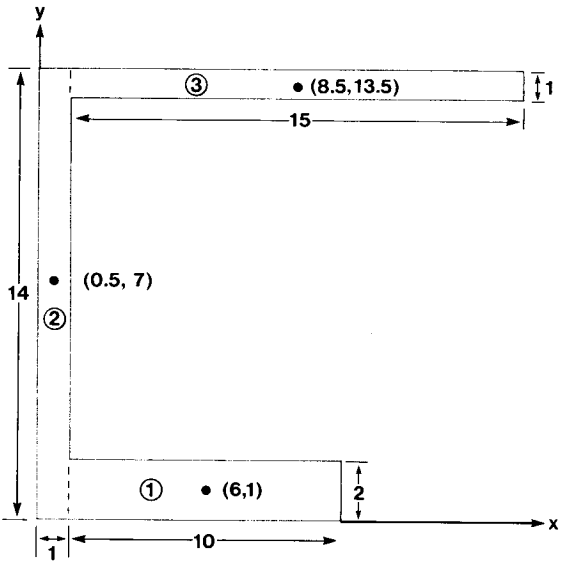
125.00 *** (I_x)
45.00 *** (I_y)
170.00 *** (J)
56.25 *** (I_{xy})

Example 2:

Calculate the section properties for the beam shown below.

Table of Inputs

Section	y_{0i}	x_{0i}	Δy	Δx
1	1	6	2	10
2	7	0.5	14	1
3	13.5	8.5	1	15



Keystrokes:

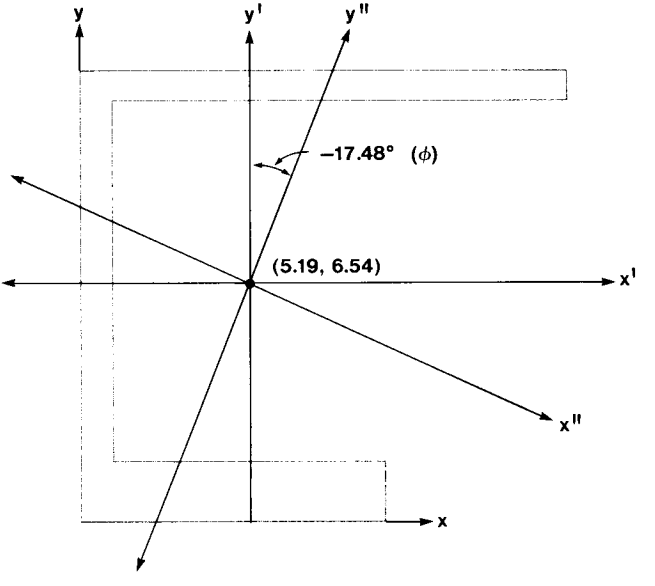
g REG 1 ENTER↑ 6 A 2 ENTER↑
10 R/S 7 ENTER↑ .5 A 14 ENTER↑
1 R/S 13.5 ENTER↑ 8.5 A
1 ENTER↑ 15 R/S B ⟶

Outputs:

49.00 *** (A)
6.54 *** ($\bar{y}$)

	5.19 *** $(\bar{x})$
	−17.48 *** (ϕ)
C →	1651.04 *** $(I_{\bar{x}\phi})$
	863.46 *** $(I_{\bar{y}\phi})$
	2514.49 *** (J_{ϕ})
	0.00 *** $(I_{\bar{x}\bar{y}\phi})$
RCL • 0 →	6.54 $(\bar{y})$
RCL • 1 →	5.19 $(\bar{x})$
0	0. (θ)
C →	1580.00 *** $(I_{\bar{x}})$
	934.49 *** $(I_{\bar{y}})$
	2514.49 *** (J)
	225.61 *** $(I_{\bar{x}\bar{y}})$
0 ENTER↑ 0 ENTER↑ 0 C →	3676.33 *** (I_x)
	2256.33 *** (I_y)
	5932.67 *** (J)
	1890.25 *** (I_{xy})

Below is a figure showing the translated axis and the rotated, principal axis of example 2.

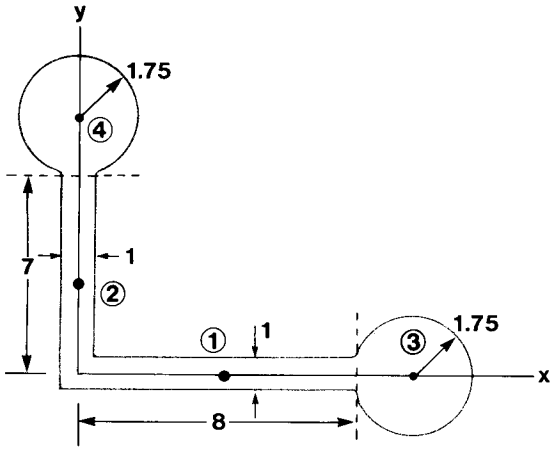


Example 3:

Find the approximate section properties of the member below by assuming that the circular parts are concentrated at their centers. Also assume that the boundaries between the circular and rectangular parts are straight lines.

Table of Inputs

Section	y_{0i}	x_{0i}	Δy	Δx	A
1	0.0	4.5	1	8	
2	3.0	0.0	7	1	
3	0.0	10.25			9.62
4	8.25	0.0			9.62



Keystrokes:

9 REG 0 ENTER↑ 4.5 A 1 ENTER↑
8 R/S 3 ENTER↑ 0 A 7 ENTER↑
1 R/S 0 ENTER↑ 10.25 A 9.62
GSB 1 8.25 ENTER↑ 0 A
9.62 GSB 1 B ⟶

Outputs:

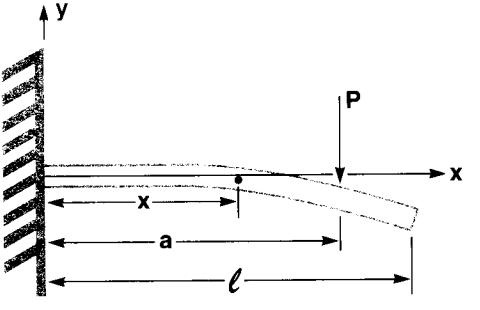
34.24 *** (A)
2.93 *** ($\overline{y}$)
3.93 *** ($\overline{x}$)
−36.74 *** (ϕ)

C →	158.27 ***	($I_{\bar{x}\phi}$)
	981.34 ***	($I_{\bar{y}\phi}$)
	1139.61 ***	(J_ϕ)
	0.00 ***	($I_{\bar{x}\bar{y}\phi}$)
RCL • 0 →	2.93	($\bar{y}$)
RCL • 1 →	3.93	($\bar{x}$)
0 C →	452.82 ***	($I_{\bar{x}}$)
	686.79 ***	($I_{\bar{y}}$)
	1139.61 ***	(J)
	-394.56 ***	($I_{\bar{x}\bar{y}}$)
0 ENTER↑ 0 ENTER↑ 0 C →	747.01 ***	(I_x)
	1215.95 ***	(I_y)
	1962.96 ***	(J)
	0.00 ***	(I_{xy})

CANTILEVER BEAMS—POINT LOAD

This program calculates deflection, slope, moment and shear at any specified point along a rigidly fixed, cantilever beam of uniform cross section for point loads. By using the principle of superposition, complicated beams with multiple point loads may be analyzed.

Equations:



$$y = \frac{PX_1^2}{6EI}(X_1-3a) - \frac{Pa^2}{2EI}(x-a)(x>a)^* \qquad \text{(deflection due to point load)}$$

$$\theta = \frac{PX_1}{2EI}(X_1-2a) \qquad \text{(slope due to point load)}$$

$$M = P(X_1-a) \qquad \text{(moment due to point load)}$$

$$V = P(x \leq a) \qquad \text{(shear due to point load)}$$

where

y is the deflection at a distance x from the wall;

θ is the slope (change in y per change in x) at x;

M is the moment at x;

V is the shear at x;

I is the moment of inertia of the beam;

E is the modulus of elasticity of the beam;

ℓ is the length of the beam;

P is a concentrated load;

a is the distance from the foundation to the point load.

*The notation $(x > a)$ is interpreted as 1.00 if x is greater than a and as 0.00 if x is less than or equal to a. $x_1 = x$ for x less than a; $x_1 = a$ for x greater than a.

Remarks:

Deflections must not significantly alter the geometry of the problem. Beams must be of constant cross section for deflection and slope equations to be valid. Stresses must be in the elastic region.

Registers $R_{.0}$–$R_{.5}$ are available for user storage.

Sums of y, θ, M and V may be stored in $R_{.1}$, $R_{.2}$, $R_{.3}$ and $R_{.4}$, respectively.

Refer to page 128 for sign conventions for beams.

A-000 LBL A A-001 S 0 A-002 S 8 A-003 R 4 A-004 X≥Y A-005 S 6 A-006 R 8 A-007 R 4 A-008 3 A-009 x A-010 − A-011 S 6 A-012 R 5 A-013 R 2 A-014 ÷ A-015 R 1 A-016 ÷ A-017 2 A-018 ÷ A-019 S 7 A-020 5 A-021 ÷ A-022 x A-023 R 8 A-024 X^2 A-025 x A-026 R 4 A-027 X^2 A-028 R 7 A-029 x A-030 R 0	Store specified point Compare x and a Compute y.	A-031 R 4 A-032 − A-033 X<0 A-034 CL X A-035 x A-036 − A-037 R/S A-038 R 6 A-039 R 4 A-040 + A-041 S 6 A-042 R 7 A-043 x A-044 R 8 A-045 x A-046 R/S A-047 R 5 A-048 R 6 A-049 R 4 A-050 + A-051 x A-052 R/S A-053 R 5 A-054 R 4 A-055 R 0 A-056 − A-057 X<0 A-058 CL X A-059 X≠0 A-060 X≠Y	Compute θ. Compute M. Compute V.

REGISTERS

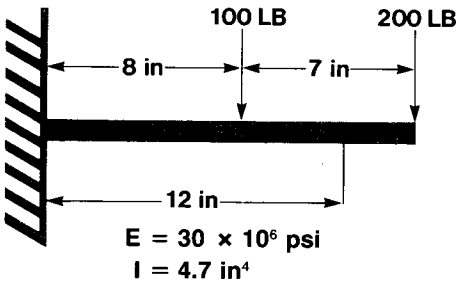
0 x	1 I	2 E	3 ℓ
4 a	5 P	6 Used	7 $\frac{P}{2EI}$
8 x, a	9	.0	.1
.2	.3	.4	.5	I

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Store beam parameters:			
	Moment of inertia of beam	I	STO 1	I
	Modulus of elasticity of beam	E	STO 2	E
	Length of beam	ℓ	STO 3	ℓ
	Distance from foundation			
	to point load	a	STO 4	a
	Concentrated load	P	STO 5	P

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
3	Key in x to specify point of			
	interest and calculate deflection;	x	A	y
	calculate slope at x;		R/S	θ
	calculate moment at x;		R/S	M
	calculate shear at x.		R/S	V
4	For a new location with the			
	same loading, go to step 3.			
	For new beam properties or			
	loading, go to step 2 and			
	change the appropriate			
	values.			

Example 1:

What is the deflection at x = 12? Neglect the weight of the beam.



Keystrokes: **Outputs:**

4.7 STO 1 30 EEX 6 STO 2
15 STO 3 f ENG 3

Compute deflection at 12 inches due to 100 lb. weight:

8 STO 4 100 STO 5 12 A ⟶ −211.8 −06

Store deflection due to 100 lb. load for addition to deflection due to 200 lb. load:

STO • 1 ⟶ −211.8 −06

Compute deflection at 12 inches due to 200 lb. load:

15 STO 4 200 STO 5 12 A → -1.123 -03

Compute total deflection.

RCL • 1 + → -1.335 -03

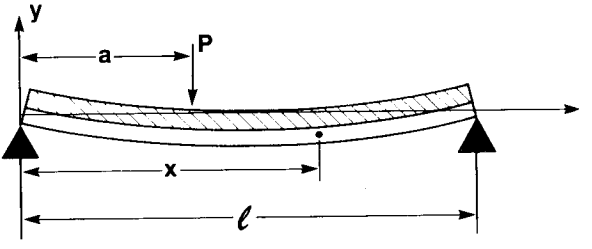
What are the deflection, slope, moment and shear at 15 inches due to the 200 lb. load?

15 A →	-1.596	-03
R/S →	-159.6	-06
R/S →	0.000	00
R/S →	0.000	00

SIMPLY SUPPORTED BEAMS—POINT LOAD

This program calculates deflection, slope, moment and shear at any specified point along a simply supported beam of uniform cross section for point loads. By using the principle of superposition, complicated beams with multiple point loads can be analyzed.

Equations:



$$y = \frac{P(\ell-a)x}{6EI}\left[x^2 + (\ell-a)^2 - \ell^2\right]^* \qquad \text{(deflection due to point load)}$$

$$\theta = \frac{P(\ell-a)}{6EI}\left[3x^2 + (\ell-a)^2 - \ell^2\right]^* \qquad \text{(slope due to point load)}$$

$$M = \frac{P(\ell-a)x}{\ell}^* \qquad \text{(moment due to point load)}$$

$$V = \frac{P(\ell-a)}{\ell}^* \qquad \text{(shear due to point load)}$$

*If x is greater than a, (ℓ − a) is replaced by −a and x is replaced by x − ℓ.

where

y is the deflection at a distance x from the left support;

θ is the slope (change in y per change in x) at x;

M is the moment at x;

V is the shear at x;

I is the moment of inertia of the beam;

E is the modulus of elasticity of the beam;

ℓ is the length of the beam;

P is the length of the beam;

a is the distance from the left support to the point load.

Remarks:

Deflections must not significantly alter the geometry of the problem. Beams must be of constant cross section for deflection and slope equations to be valid. Stresses must be in the elastic region.

Registers $R._0$–$R._5$ are available for user storage.

Sums of y, θ, M and V may be stored in $R._1$, $R._2$, $R._3$, and $R._4$, respectively.

Refer to page 128 for sign conventions for beams.

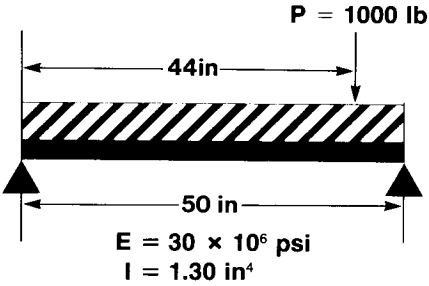
B-000 LBL6		B-026 R/S	
B-001 S 0	Store specified point.	B-027 R 7	
B-002 R 4		B-028 R 0	Compute M.
B-003 X>Y	Compare x and a.	B-029 x	
B-004 GTO0		B-030 R/S	
B-005 CHS		B-031 R 7	Recall V.
B-006 S 6		B-032 R/S	
B-007 CL X	Store –a and x–ℓ	B-033 LBL2	
B-008 R 3		B-034 R 0	
B-009 -		B-035 x	
B-010 S 0		B-036 R 6	
B-011 GTO1		B-037 →P	
B-012 LBL0		B-038 X²	
B-013 R 3		B-039 R 3	Calculate subroutine.
B-014 -	Store ℓ–a	B-040 X²	
B-015 CHS		B-041 -	
B-016 S 6		B-042 R 5	
B-017 LBL1		B-043 R 6	
B-018 1		B-044 x	
B-019 GSB2	Compute y.	B-045 R 3	
B-020 R 0		B-046 ÷	
B-021 x		B-047 S 7	
B-022 R/S		B-048 R 2	
B-023 3		B-049 ÷	
B-024 √X	Compute θ.	B-050 R 1	
B-025 GSB2		B-051 ÷	

B-052 6 B-053 ÷		B-054 x B-055 RTN	
REGISTERS			
0 x, x−ℓ	1 I	2 E	3 ℓ
4 a	5 P	6 ℓ−a, −a	7 $\frac{P(\ell-a)}{\ell}$
8	9	.0	.1

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Store beam parameters:			
	Moment of inertia of beam	I	STO 1	I
	Modulus of elasticity of beam	E	STO 2	E
	Length of beam	ℓ	STO 3	ℓ
	Distance from foundation			
	to point load	a	STO 4	a
	Concentrated load	P	STO 5	P
3	Key in x to specify point of			
	interest and calculate deflection;	x	B	y
	calculate slope at x;		R/S	θ
	calculate moment at x;		R/S	M
	calculate shear at x.		R/S	V
4	For a new location with the			
	same loading, go to step 3.			
	For new beam properties or			
	loading, go to step 2 and			
	change the appropriate			
	values.			

Example 1:

What are the deflection, slope, moment, and shear of the beam below at x = 38 inches?

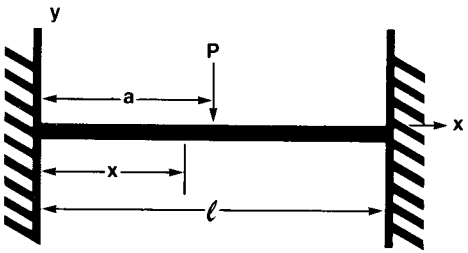


Keystrokes:	**Outputs:**	
1.3 STO 1 30 EEX 6 STO 2 50 STO 3 44 STO 4 1000 STO 5 f ENG 3		
38 B →	-19.88	-03
R/S →	957.9	-06
R/S →	4.560	03
R/S →	120.0	00

FIXED-FIXED BEAM-POINT LOAD

This program calculates deflection, slope, moment and shear at any specified point along a beam of uniform cross section fixed at both ends for point loads. By using the principle of superposition, complicated beams with multiple point loads can be analyzed.

Equations:



$$y = \frac{P(\ell-a)^2x^2}{6EI\ell^3}\left[x(\ell + 2a) - 3a\ell\right]^* \qquad \text{(deflection due to point load)}$$

$$\theta = \frac{P(\ell-a)^2x}{2EI\ell^3}\left[x(\ell + 2a) - 2a\ell\right]^* \qquad \text{(slope due to point load)}$$

$$M = \frac{P(\ell-a)^2}{\ell^3}\left[x(\ell + 2a) - a\ell\right]^* \qquad \text{(moment due to point load)}$$

$$V = \frac{P(\ell-a)^2}{\ell^3}(\ell + 2a)^* \qquad \text{(shear due to point load)}$$

*If x is greater than a, a is replaced by (ℓ − a) and x is replaced by (ℓ − x). The signs of θ and V are also changed.

where

y is the deflection at a distance x from the left support;

θ is the slope (change in y per change in x) at x;

M is the moment at x;

V is the shear at x;

I is the moment of inertia of the beam;

E is the modulus of elasticity of the beam;

ℓ is the length of the beam;

P is a concentrated load;

a is the distance from the left support to the point load.

Remarks:

Deflections must not significantly alter the geometry of the problem. Beams must be of constant cross section for deflection and slope equations to be valid. Stresses must be in the elastic region.

Registers $R._1$–$R._5$ are available for user storage.

Sums of y, θ, M and V may be stored in $R._1$, $R._2$, $R._3$, and $R._4$, respectively.

Sign Conventions for Beams

NAME	VARIABLE	SENSE	SIGN
DEFLECTION	y	↑	+
SLOPE	θ	↑	+
INTERNAL MOMENT	M		+
SHEAR	V		+
EXTERNAL FORCE OR LOAD	P	↓	+

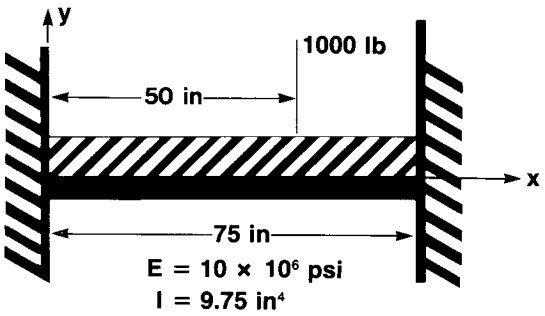
C-000 LBL C	Store specified point.	C-040 ÷	
C-001 S 0		C-041 6	
C-002 R 4		C-042 ÷	
C-003 X>Y	Compare x and a.	C-043 3	
C-004 GTO0		C-044 GSB2	Compute y.
C-005 R 3		C-045 R/S	
C-006 -		C-046 R 7	
C-007 CHS		C-047 R 0	
C-008 S 6	Store ℓ-a	C-048 x	
C-009 R 3	and ℓ-x	C-049 R.0	
C-010 R 0		C-050 ÷	
C-011 -		C-051 2	
C-012 S 0		C-052 ÷	
C-013 1		C-053 2	
C-014 CHS		C-054 GSB2	Compute θ.
C-015 S 9		C-055 R 9	
C-016 GTO1		C-056 x	
C-017 LBL0		C-057 R/S	
C-018 S 6	Store a.	C-058 R 7	
C-019 1		C-059 1	Compute M.
C-020 S 9		C-060 GSB2	
C-021 LBL1		C-061 R/S	
C-022 R 3		C-062 R 7	
C-023 R 6		C-063 R 8	
C-024 -		C-064 x	Compute V.
C-025 X²		C-065 R 9	
C-026 R 5		C-066 x	
C-027 x	$\frac{P(\ell-a)^2}{\ell^3}$	C-067 R/S	
C-028 R 3		C-068 LBL2	
C-029 3		C-069 CHS	
C-030 Y^x		C-070 R 6	
C-031 ÷		C-071 R 3	
C-032 S 7		C-072 x	
C-033 R 0		C-073 x	
C-034 X²		C-074 R 6	
C-035 x		C-075 2	Compute subroutine.
C-036 R 1		C-076 x	
C-037 R 2		C-077 R 3	
C-038 x		C-078 +	
C-039 S.0	$EI \rightarrow R_{.0}$	C-079 S 8	

C-080 R 0 C-081 x C-082 +		C-083 x C-084 RTN	
REGISTERS			
0 x, ℓ−x	1 I	2 E	3 ℓ
4 a	5 P	6 a, ℓ−a	7 $P(\ell-a)^2/\ell^3$
8 ℓ+2a	9 ±1	.0 EI	.1
.2	.3	.4	.5	I

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Store beam parameters:			
	Moment of inertia of beam	I	STO 1	I
	Modulus of elasticity of beam	E	STO 2	E
	Length of beam	ℓ	STO 3	ℓ
	Distance from foundation			
	to point load	a	STO 4	a
	Concentrated load	P	STO 5	P
3	Key in x to specify point of			
	interest and calculate deflection;	x	C	y
	calculate slope at x;		R/S	θ
	calculate moment at x;		R/S	M
	calculate shear at x.		R/S	V
4	For a new location with the			
	same loading, go to step 3.			
	For new beam properties or			
	loading, go to step 2 and			
	change the appropriate			
	values.			

Example 1:

Find the deflection, slope, moment and shear at x = 0 and at x = 40 inches for the configuration below.



Keystrokes: **Outputs:**

9.75 STO 1 10 EEX 6 STO 2
75 STO 3 50 STO 4 1000 STO 5
f ENG 3

0 C →	0.000	00
R/S →	0.000	00
R/S →	-5.556	03
R/S →	259.3	00
40 C →	-17.22	-03
R/S →	-151.9	-06
R/S →	4.815	03
R/S →	259.3	00

NOTES

SURVEYING

FIELD ANGLE TRAVERSE

This program uses horizontal distances and angles or deflections turned from a reference azimuth to compute the coordinates of successive points in a traverse. For a closed traverse, the area enclosed and closure distance and azimuth are computed.

Equations:

$$N_{i+1} = N_i + HD \cos Az$$

$$E_{i+1} = E_i + HD \sin Az$$

$$\text{Area} = \sum_{k=1}^{n} LAT_k \left(\tfrac{1}{2} DEP_k + \sum_{j=1}^{k-1} DEP_j \right)$$

where $DEP_k = E_{k+1} - E_k$ and $LAT_k = N_{k+1} - N_k$

Remarks:

Some programming savings may be made by the user if any of programs Field Angle Traverse, Bearing Traverse, and Inverse are combined.

This program assumes the calculator is set in DEG mode.

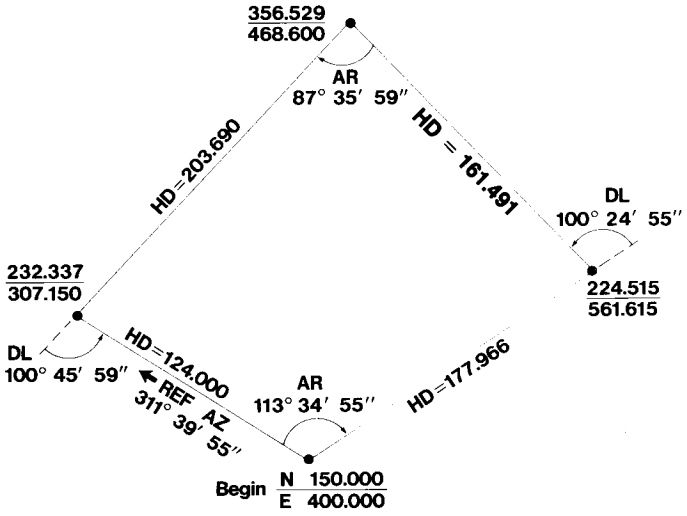
A-000 LBLA		A-025 →P	
A-001 FIX4		A-026 LBL5	
A-002 CL R		A-027 X⇄Y	Compute azimuth.
A-003 S 0	Store starting point	A-028 X>0	
A-004 X⇄Y	coordinates and 180.	A-029 GTO3	
A-005 S 1		A-030 3	
A-006 1		A-031 6	
A-007 6		A-032 0	
A-008 0		A-033 +	
A-009 S 2		A-034 LBL3	
A-010 R/S	- - - - - - - - - - - - -	A-035 S 3	
A-011 R 2		A-036 →HMS	
A-012 HMS+	Reference azimuth.	A-037 R/S	- - - - - - - - - - - - -
A-013 →H		A-038 S+4	Input horizontal distance.
A-014 GTO4	- - - - - - - - - - - - -	A-039 R 3	
A-015 LBL1		A-040 X⇄Y	
A-016 R 2	Angle input.	A-041 →R	
A-017 HMS+	- - - - - - - - - - - - -	A-042 S+5	
A-018 LBL2	Deflection input.	A-043 X⇄Y	
A-019 →H	- - - - - - - - - - - - -	A-044 S+6	
A-020 R 3		A-045 2	
A-021 +		A-046 ÷	
A-022 LBL4		A-047 R 6	
A-023 1		A-048 -	Compute next coordinates.
A-024 →R		A-049 x	and accumulate area.

A-050 S+7 A-051 R 5 A-052 R 1 A-053 + A-054 SPC A-055 PRTX A-056 R 6 A-057 R 0 A-058 + A-059 PRTX		D-000 LBL0 D-001 R 7 D-002 ABS D-003 R/S D-004 R 6 D-005 R 5 D-006 +P D-007 R/S D-008 JPA5	Area Set-up for closure.

REGISTERS			
0 Beg E	1 Beg N	2 180	3 Az
4 ΣHD	5 LAT	6 DEP	7 Area
8	9	.0	.1

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Key in beginning coordinates.	BEG N	ENTER↑	
		BEG E	A	180.00
3	Key in reference azimuth			
	away from beginning point.	REF AZ(D.MS)	R/S	Az (D.MS)
4	Key in field angle:			
	angle right		GSB 1	Az (D.MS)
	or angle left		CHS GSB 1	Az (D.MS)
	or deflection right		GSB 2	Az (D.MS)
	or deflection left		CHS GSB 2	Az (D.MS)
5	Key in horizontal distance and			
	compute coordinates.	HD	R/S	N, E
6	Repeat steps 4 and 5 for suc-			
	cessive courses.			
7	For closed figure,			
	compute area,		D	Area
	error distance, and		R/S	Error Dist.
	error azimuth		R/S	Error Az (D.MS)

Example: Traverse the figure below starting at $\frac{\text{N } 150}{\text{E } 400}$.



Keystrokes:	**Outputs:**	
150 ENTER↑ 400 A →	180.0000	
311.3955 R/S →	131.3955	
113.3455 GSB 1 →	65.1450	
177.966 R/S →	224.5150 ***	N
	561.6150 ***	E
100.2455 CHS GSB 2 →	324.4955	
161.491 R/S →	356.5285 ***	N
	468.6000 ***	E
87.3559 GSB 1 →	232.2554	
203.690 R/S →	232.3372 ***	N
	307.1498 ***	E
100.4559 CHS GSB 2 →	131.3955	
124 R/S →	149.9048 ***	N
	399.7829 ***	E
D →	26558.8204	Area
R/S →	0.2371	Error distance
R/S →	246.1844	Error azimuth

BEARING TRAVERSE

This program uses quadrant bearings and horizontal distances to compute the coordinates of successive points in a traverse. For a closed traverse, the area enclosed and error distance and azimuth are computed.

Equations:

$$N_{i+1} = N_i + HD \cos Az$$

$$E_{i+1} = E_i + HD \sin Az$$

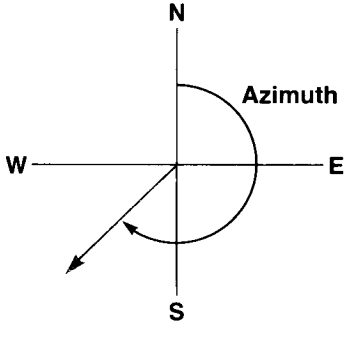
$$\text{Area} = \sum_{k=1}^{n} LAT_k \left(\tfrac{1}{2} DEP_k + \sum_{j=1}^{k-1} DEP_j \right)$$

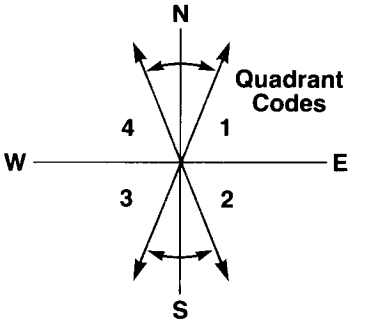
where $DEP_k = E_{k+1} - E_k$ and $LAT_k = N_{k+1} - N_k$

Remarks:

Some programming savings may be made by the user if any of programs Field Angle Traverse, Bearing Traverse, and Inverse are combined.

This program assumes the calculator is set in DEG mode.





B-000 LBLE B-001 FIX4 B-002 CL R B-003 S 0 B-004 X≷Y B-005 S 1 B-006 1 B-007 8 B-008 0	Store starting coordinates and 180.	B-009 S 2 B-010 R/S B-011 LBL1 B-012 ENT↑ B-013 ENT↑ B-014 2 B-015 ÷ B-016 INT B-017 R 2	Convert bearing and quadrant code to azimuth.

B-018 x		B-046 →R	
B-019 X≷Y		B-047 S+5	
B-020 R 2		B-048 X≷Y	
B-021 x		B-049 S+6	
B-022 COS		B-050 2	
B-023 R↑		B-051 ÷	Accumulate area and compute next coordinates.
B-024 →H		B-052 R 6	
B-025 x		B-053 -	
B-026 -	- - - - - - - - - - - - - -	B-054 x	
B-027 LBL4		B-055 S+7	
B-028 1		B-056 R 5	
B-029 →R		B-057 R 1	
B-030 →P		B-058 +	
B-031 LBL2		B-059 SPC	
B-032 X≷Y		B-060 PRTX	
B-033 X>0		B-061 R 6	
B-034 GTO3	0 < Az ⩽ 360.	B-062 R 0	
B-035 3		B-063 +	
B-036 6		B-064 PRTX	- - - - - - - - - - - - - -
B-037 0		D-000 LBL0	
B-038 +		D-001 R 7	
B-039 LBL3		D-002 ABS	Area
B-040 S 3		D-003 R/S	
B-041 →HMS		D-004 R 6	- - - - - - - - - - - - - -
B-042 R/S	- - - - - - - - - - - - - -	D-005 R 5	
B-043 S+4	Input horizontal distance.	D-006 →P	Set-up for closure.
B-044 R 3		D-007 R/S	
B-045 X≷Y		D-008 JPBZ	- - - - - - - - - - - - - -

REGISTERS

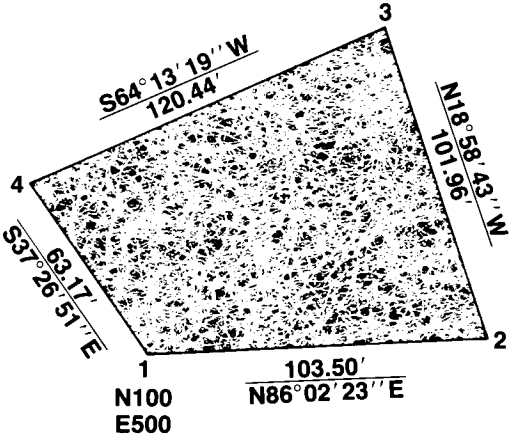
0 Beg E	1 Beg N	2 180	3 Az
4 ΣHD	5 LAT	6 DEP	7 Area
8	9	.0	.1

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Key in beginning coordinates.	BEG N	ENTER↑	
		BEG E	B	180.00
3	Key in bearing and quadrant	BRG (D.MS)	ENTER↑	
	code.	QD	GSB 1	Az (D.MS)
4	Key in horizontal distance and			
	compute coordinates.	HD	R/S	N, E
5	Repeat steps 3 and 4 for suc-			
	cessive courses.			
6	For closed figure,			
	compute area,		D	Area

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	error distance, and		R/S	Error Dist.
	error azimuth		R/S	Error Az (D.MS)

Example:

Traverse the figure below starting at $\frac{\text{N } 100}{\text{E } 500}$.



Keystrokes: **Outputs:**

100 ENTER↑ 500 B →	180.0000
86.0223 ENTER↑ 1 GSB 1 →	86.0223
103.50 R/S →	107.1482 *** N
	603.2529 *** E
18.5843 ENTER↑ 4 GSB 1 →	341.0117
101.96 R/S →	203.5657 *** N
	570.0939 *** E
64.1319 ENTER↑ 3 GSB 1 →	244.1319
120.44 R/S →	151.1880 *** N
	461.6395 *** E
37.2651 ENTER↑ 2 GSB 1 →	142.3309

63.17 R/S →	101.0366 ***	N
	500.0490 ***	E
D →	8855.4922	Area
R/S →	1.0378	Error distance
R/S →	2.4219	Error azimuth

INVERSE

This program calculates the distance and azimuth of the line joining two points, given the coordinates of the points. For a closed inverse, the area enclosed and closure distance and azimuth are computed.

Equations:

$$HD = \sqrt{(N_i - N_{i-1})^2 + (E_i - E_{i-1})^2}$$

$$Az = \tan^{-1} \frac{E_i - E_{i-1}}{N_i - N_{i-1}}$$

$$Area = \sum_{k=1}^{n} LAT_k \left(\tfrac{1}{2} DEP_k + \sum_{j=1}^{k-1} DEP_j \right)$$

where $DEP_k = E_{k+1} - E_k$ and $LAT_k = N_{k+1} - N_k$

Remarks:

Some programming savings may be made by the user if any of programs Field Angle Traverse, Bearing Traverse, and Inverse are combined.

This program assumes the calculator is set in DEG mode.

C-000 LBL C C-001 FIX4 C-002 CL R C-003 S 0 C-004 X≷Y C-005 S 1 C-006 1 C-007 8 C-008 0 C-009 S 2 C-010 R/S C-011 LBL 1 C-012 R 6 C-013 - C-014 R 0 C-015 -	Store starting coordinates and 180. - - - - - - - - - - - - - - -	C-016 S+6 C-017 S 8 C-018 X≷Y C-019 R 5 C-020 - C-021 R 1 C-022 - C-023 S+5 C-024 S 9 C-025 X≷Y C-026 2 C-027 ÷ C-028 R 6 C-029 - C-030 x C-031 S+7	Accumulate Area. - - - - - - - - - - - - - - -

C-032 R 8 C-033 R 9 C-034 LBL2 C-035 →P C-036 SPC C-037 PRTX C-038 X≷Y C-039 X>0 C-040 GTO3 C-041 3 C-042 6 C-043 0	Compute distance and azimuth.	C-044 + C-045 LBL3 C-046 →HMS C-047 PRTX D-000 LBL0 D-001 R 7 D-002 ABS D-003 SPC D-004 PRTX D-005 R 6 D-006 R 5 D-007 JPC2	Area Set-up for closure.

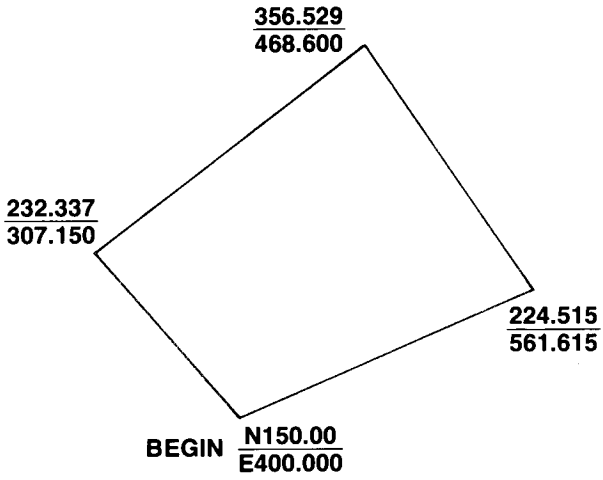
REGISTERS

0 Beg E	1 Beg N	2 180	3
4	5 LAT	6 DEP	7 Area
8 ΔE	9 ΔN	.0	.1
.2	.3	.4	.5	I

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Key in beginning coordinates.	BEG N	ENTER↑	
		BEG E	C	180.00
3	Key in coordinates of next			
	point	N	ENTER↑	
	and compute horizontal	E	GSB 1	HD
	distance and azimuth.			Az (D.MS)
4	Repeat step 3 for suc-			
	cessive courses.			
6	For closed figure,			
	compute area,		D	Area
	error distance, and			Error Dist.
	error azimuth			Error Az (D.MS)

Example:

Inverse the figure below starting at $\frac{\text{N } 150.00}{\text{E } 400.00}$.



Keystrokes:	**Outputs:**
150 ENTER↑ 400 C →	180.0000
224.515 ENTER↑ 561.615	
GSB 1 →	177.9660 *** HD
	65.1450 *** Az
356.529 ENTER↑ 468.600	
GSB 1 →	161.4914 *** HD
	324.4955 *** Az
232.337 ENTER↑ 307.150	
GSB 1 →	203.6903 *** HD
	232.2553 *** Az
150 ENTER↑ 400 GSB 1 →	124.0988 *** HD
	131.3357 *** Az
D →	26545.4956 *** Area
	0.0000 *** Error distance
	360.0000 *** Error azimuth

Note the area differs slightly from the Field Angle Traverse example area since the area closed exactly for the Inverse.

SIDESHOTS

This program is used to make sideshots or radials from a point. Two methods may be used for a sideshot, 1) input a bearing and distance and calculate the point coordinates, or 2) input the point coordinates and calculate the azimuth and distance to the point.

Equations:

$$N = N_0 + HD \cos Az$$

$$E = E_0 + HD \sin Az$$

Remarks:

This program assumes the calculator is set in DEG mode.

0-000 LBL0	Store hub coordinates and 180.	0-033 X>0	AZ positive?
0-001 FIX4		0-034 GT03	
0-002 CL R		0-035 3	
0-003 S 0		0-036 6	
0-004 X⇄Y		0-037 0	
0-005 S 1		0-038 +	360 + AZ
0-006 1		0-039 LBL3	
0-007 8		0-040 S 3	
0-008 0		0-041 →HMS	Display AZ
0-009 S 2		0-042 RTN	
0-010 R/S		0-043 R 3	
0-011 LBL1		0-044 X⇄Y	
0-012 ENT↑		0-045 →R	
0-013 ENT↑		0-046 R 1	
0-014 2		0-047 +	
0-015 ÷		0-048 SPC	
0-016 INT		0-049 PRTX	Print Northing
0-017 R 2		0-050 X⇄Y	
0-018 ×	180 INT(QD/2)	0-051 R 0	
0-019 X⇄Y		0-052 +	Print Easting
0-020 R 2		0-053 PRTX	
0-021 ×		0-054 R/S	
0-022 COS	cos (180 QD)	0-055 LBL2	
0-023 R↑		0-056 R 0	
0-024 →H		0-057 −	ΔE
0-025 ×		0-058 X⇄Y	
0-026 −		0-059 R 1	
0-027 LBL4	AZ	0-060 −	ΔN
0-028 1		0-061 →P	
0-029 →R		0-062 SPC	
0-030 →P		0-063 PRTX	Print distance
0-031 LBL5		0-064 GSB5	Print AZ
0-032 X⇄Y		0-065 PRTX	

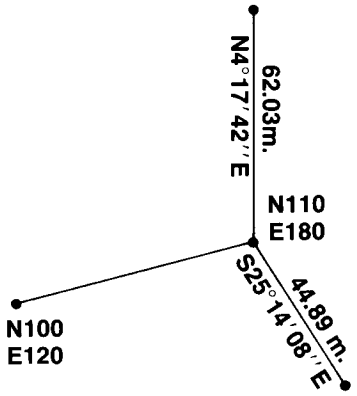
REGISTERS

0 E_0	1 N_0	2 180	3
4	5	6	7

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Key in hub	N	ENTER↓	
	coordinates	E	D	180.00
3a	For bearing sideshot:			
	Key in bearing	BRG (D.MS)	ENTER↓	
	and quadrant.	QD	GSB 1	Az (D.MS)
	Key in horizontal distance and			
	compute point coordinates.	HD	R/S	N, E
3b	For inverse sideshot:			
	Key in coordinates of sideshot			
	point and compute horizontal	N	ENTER↓	
	distance and azimuth.	E	GSB 2	HD,
				Az (D.MS)
4	Repeat step 3 or all desired			
	sideshots.			

Example:

Calculate the sideshots for the figure shown below:

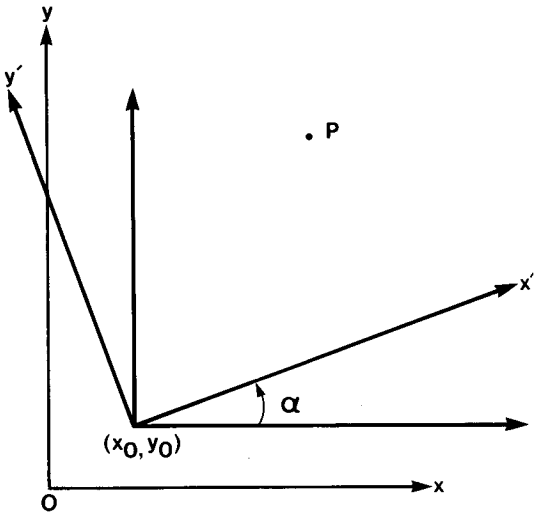


Keystrokes:	**Outputs:**
110 ENTER↑ 180 D →	180.0000
4.1742 ENTER↑ 1 GSB 1 →	4.1742 Az
62.03 R/S →	171.8558 *** N
	184.6455 *** E
25.1408 ENTER↑ 2 GSB 1 →	154.4552 Az
44.89 R/S →	69.3942 *** N
	199.1384 *** E
100 ENTER↑ 120 GSB 2 →	60.8276 *** HD
	260.3216 *** Az

TRIGONOMETRY AND ANALYTICAL GEOMETRY

COORDINATE TRANSLATION AND ROTATION

This program allows for two-dimensional translation and rotation of coordinate axes. Suppose the origin of a coordinate system is translated to a new point, (x_0, y_0), and the x and y axes are rotated through an angle α to give new axes, x′ and y′. Suppose that a point P has coordinates (x, y) with respect to the old system of x and y axes, and coordinates (x′, y′) with respect to the new axes. Then given α and one pair of coordinates, the program allows you to find the other pair of coordinates.



Equations:

Let Rect (r, θ) denote the operation [→R] when r is in the X-register and θ is in the Y-register. Let Pol (x, y) denote the operation [→P] when x is in X and y is in Y.

Then $(x', y') = \text{Rect}\ (r, \theta - \alpha)$
where $(r, \theta) = \text{Pol}\ (x - x_0, y - y_0)$

and $(x, y) = (x_0, y_0) + \text{Rect}\ (r', \theta' + \alpha)$
where $(r', \theta') = \text{Pol}\ (x', y')$

Remarks:

The program may be used to solve a problem of translation only, or of rotation only, or of combined translation and rotation. If the problem involves translation alone, a value of $\alpha = 0$ must be input. For rotation alone, the values $x_0 = y_0 = 0$ must be input.

The program assumes the following sign convention: α should be input as a positive number if the rotation is counterclockwise, and negative if clockwise.

This program assumes the calculator is set in DEG mode.

A-000 LBLA A-001 S 7 A-002 R↓ A-003 S 9 A-004 R↓ A-005 S 8 A-006 RTN A-007 LBL1 A-008 R 9 A-009 - A-010 X≷Y A-011 R 8 A-012 - A-013 →P A-014 X≷Y A-015 R 7 A-016 - A-017 X≷Y A-018 →R A-019 PRTX	$x_0 \uparrow y_0 \uparrow \theta$ --- $x \uparrow y \rightarrow x', y'$	A-020 X≷Y A-021 PRTX A-022 SPC A-023 RTN A-024 LBL2 A-025 X≷Y A-026 →P A-027 X≷Y A-028 R 7 A-029 + A-030 X≷Y A-031 →R A-032 R 8 A-033 + A-034 PRTX A-035 X≷Y A-036 R 9 A-037 + A-038 PRTX A-039 SPC	--- $x' \uparrow y' \rightarrow x, y$ ---

REGISTERS

0	1	2	3
4	5	6	7 θ
8 x_0	9 y_0	.0	.1
.2	.3	.4	.5

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Key in new origin and angle			
	of rotation.	x_0	ENTER↑	
		y_0	ENTER↑	
		α	A	x_0
3	Key in coordinates in old			
	system and compute co-			

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	ordinates in the new.	x	ENTER↑	
		y	GSB 1	x′
				y′
4	Key in coordinates in new			
	system and compute co-			
	ordinates in the old.	x′	ENTER↑	
		y′	GSB 2	x
				y

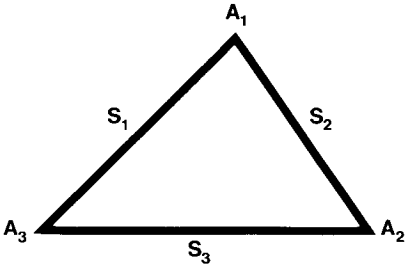
Example:

The origin of a coordinate system is translated to the point $(-1, 4)$ and rotated 30° in a positive (counterclockwise) direction. Find the new coordinates of the point whose coordinates in the old system are (1, 3). If the coordinates of a point in the new system are (5, 7), what are its coordinates in the old system?

Keystrokes:	**Outputs:**
1 CHS ENTER↑ 4 ENTER↑ 30 A ▸	−1.00
1 ENTER↑ 3 GSB 1 ⟶	1.23 *** (x′)
	−1.87 *** (y′)
5 ENTER↑ 7 GSB 2 ⟶	−0.17 *** (x)
	12.56 *** (y)

TRIANGLE SOLUTIONS

This program may be used to find the sides, the angles, and the area of a plane triangle.



In general the specifications of any three of the six parameters of a triangle (3 sides, 3 angles) is sufficient to define the triangle. (The exception is that the three angles will not define a triangle.) There are thus five possible cases that this program will handle: two sides and the included angle (SAS), two angles and the included side (ASA), two sides and the adjacent angle (SSA—the ambiguous case), two angles and the adjacent side (AAS), and three sides (SSS).

The calculator will successively print the values of the sides, the angles, and the area. The order of output is determined by the order of input: if input values are selected in a clockwise order, output values will be printed in a clockwise order. The order of outputs is as follows:

First side input (S_1)

Adjacent angle (A_1)

Adjacent side (S_2)

Adjacent angle (A_2)

Adjacent side (S_3)

Adjacent angle (A_3)

Area

Equations:

SAS (S_1, A_1, S_2):

$$S_3 = \sqrt{S_1^2 + S_2^2 - 2\,S_1 S_2 \cos A_1}$$

$$A_2 = \tan^{-1}\frac{S_1 \sin A_1}{S_2 - S_1 \cos A_1}$$

$$A_3 = \cos^{-1}\left[-\cos\left(A_1 + A_2\right)\right]$$

ASA (A_3, S_1, A_1):

$$S_2 = S_1\frac{\sin A_3}{\sin A_2} = S_1\frac{\sin A_3}{\sin\left(A_1 + A_3\right)}$$

Now go to SAS.

SSA (S_1, S_2, A_2):

$$A_3 = \sin^{-1}\left(\frac{S_2 \sin A_2}{S_1}\right)$$

$$A_1 = \cos^{-1}\left[-\cos\left(A_2 + A_3\right)\right]$$

Now go to SAS.

AAS (A_2, A_1, S_1):

$$S_2 = S_1\frac{\sin A_3}{\sin A_2} = S_1\frac{\sin\left(A_1 + A_2\right)}{\sin A_2}$$

Now go to SAS.

SSS (S_1, S_2, S_3):

$$A^1 = \cos^{-1}\left(\frac{S_1^2 + S_2^2 - S_3^2}{2\,S_1\,S_2}\right)$$

Now go to SAS.

Remarks:

Any angular mode may be used.

Note that the triangle described by the program does not conform to standard triangle notation; i.e., A_1 is not opposite S_1.

Angles must be entered as decimals. The [H.MS→H] conversion can be used to convert degrees, minutes, and seconds to decimal degrees.

Accuracy of solution may degenerate for triangles containing extremely small angles.

A-000 LBLA		A-060 R 3	
A-001 S 3	SAS	A-061 ×	
A-002 R↓		A-062 2	
A-003 S 2		A-063 ×	
A-004 R↓		A-064 ÷	
A-005 S 1	----------	A-065 COS⁻¹	
A-006 LBL9		A-066 S 2	
A-007 R 2	Compute A_2, S_3 by law of	A-067 GTO9	
A-008 R 1	cosines.	B-000 LBLB	
A-009 →R		B-001 S 2	----------
A-010 R 3		B-002 R↓	ASA
A-011 X≷Y		B-003 S 1	
A-012 −		B-004 X≷Y	
A-013 →P		B-005 S 6	
A-014 S 5		B-006 SIN	
A-015 X≷Y		B-007 R 2	Find S_2, go to SAS.
A-016 S 4		B-008 R 6	
A-017 R 2		B-009 +	
A-018 +		B-010 SIN	
A-019 COS	cos A_3 = −cos (A_1 + A_2)	B-011 ÷	
A-020 CHS		B-012 ×	
A-021 COS⁻¹		B-013 S 3	
A-022 S 6		B-014 JPA9	----------
A-023 SIN		C-000 LBLC	
A-024 ×		C-001 S 4	SSA
A-025 R 1		C-002 R↓	
A-026 ×		C-003 S 3	
A-027 2	Area	C-004 X≷Y	
A-028 ÷		C-005 S 1	Find A_1, go to SAS.
A-029 S 0		C-006 R 4	
A-030 R 1	----------	C-007 SIN	
A-031 PRTX	Output solutions.	C-008 X≷Y	
A-032 R 2		C-009 ÷	
A-033 PRTX		C-010 ×	
A-034 R 3		C-011 SIN⁻¹	
A-035 PRTX		C-012 R 4	
A-036 R 4		C-013 +	
A-037 PRTX		C-014 COS	
A-038 R 5		C-015 CHS	
A-039 PRTX		C-016 COS⁻¹	
A-040 R 6		C-017 S 2	
A-041 PRTX		C-018 GSB1	
A-042 SPC		C-019 R 1	
A-043 R 0		C-020 R 3	Two solutions exist
A-044 PRTX		C-021 X≤Y	if $S_2 > S_1$.
A-045 SPC		C-022 RTN	
A-046 RTN	----------	C-023 R 6	
A-047 LBL1	SSS	C-024 COS	
A-048 S 5		C-025 CHS	
A-049 R↓		C-026 COS⁻¹	
A-050 S 3		C-027 S 6	
A-051 R↓		C-028 R 4	$A_3 \leftarrow 180 - A_3$.
A-052 S 1	Find A_1 by law of	C-029 +	
A-053 R 3	cosines, go to SAS.	C-030 COS	
A-054 →P		C-031 CHS	Find A_1, go to SAS.
A-055 X²		C-032 COS⁻¹	
A-056 R 5		C-033 S 2	
A-057 X²		C-034 LBL1	
A-058 −		C-035 JPA9	----------
A-059 R 1		D-000 LBLD	

D-001 S 1 D-002 R↓ D-003 S 2 D-004 R↓ D-005 S 4 D-006 R↑ D-007 + D-008 SIN	AAS Find S_2, go to SAS.	D-009 R 4 D-010 SIN D-011 ÷ D-012 R 1 D-013 x D-014 S 3 D-015 JPA9	

REGISTERS

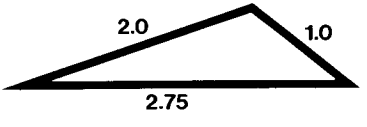
0 Area	1 S_1	2 A_1	3 S_2
4 A_2	5 S_3	6 A_3	7
8	9	.0	.1

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Select case.			
	SAS			
3	Key in side, angle, and side.	S	ENTER↑	
		A	ENTER↑	
		S	A	Output
	ASA			
4	Key in angle, side, and angle.	A	ENTER↑	
		S	ENTER↑	
		A	B	Output
	SSA			
5	Key in side, side, and angle.	S	ENTER↑	
		S	ENTER↑	
		A	C	Output*
	AAS			
6	Key in angle, angle, and side.	A	ENTER↑	
		A	ENTER↑	
		S	D	Output
	SSS			
7	Key in three sides.	S	ENTER↑	
		S	ENTER↑	

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
		S	[9] [JUMP]	
			[A] [1] [R/S]	Output
	Output in each case consists			
	of the following variables in			
	this order:			
	First side input			
	Adjacent angle			
	Adjacent side			
	Adjacent angle			
	Adjacent side			
	Adjacent angle			
	Area			
	*If two solutions exist, both will			
	be printed.			

Example 1:

Find the angles and the area for the following triangle.



Keystrokes:

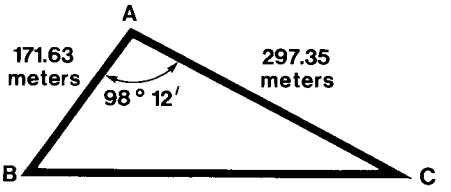
2 [ENTER↑] 1 [ENTER↑] 2.75 [9] [JUMP]
[A] [1] [R/S] ⟶

Outputs:

2.00 ***
129.84 *** (A_1)
1.00 ***
33.95 *** (A_2)
2.75 ***
16.21 *** (A_3)
0.77 *** (Area)

Example 2:

A surveyor is to find the area and dimensions of a triangular land parcel. From point A, the distances to B and C are measured with an electronic distance meter. The angle between AB and AC is also measured. Find the area and other dimensions of the triangle.



This is a side-angle-side problem where:

$$S_1 = 171.63,\ A_1 = 98°12'\ \text{and}\ S_2 = 297.35.$$

Keystrokes:

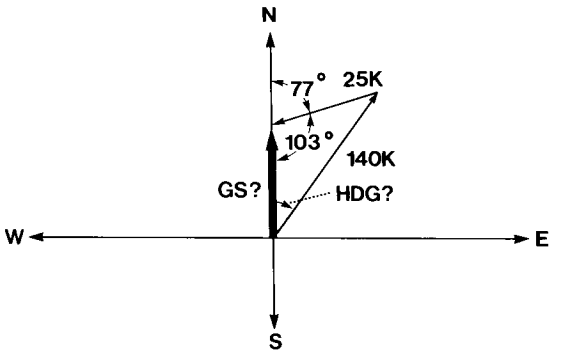
171.63 ENTER↑ 98.12 f H.MS→H
297.35 A →

Outputs:

171.63 *** (AB)
98.20 *** (A)
297.35 *** (AC)
27.83 *** (C)
363.91 *** (CB)
53.97 *** (B)
25256.21 *** (Area)

Example 3:

A pilot wishes to fly due north. The wind is reported as 25 knots at 77°. Because winds are reported opposite to the direction they blow, this is interpreted as 77 + 180 or 257°. The true airspeed of the aircraft is 140 knots. What heading (HDG) should be flown? What is the ground speed (GS)?



By subtracting the wind direction from 180 (yielding an angle of 103°), the problem reduces to a S_1, S_2, SA_2 triangle.

Keystrokes:

140 ENTER↑ 25 ENTER↑ 103

C →

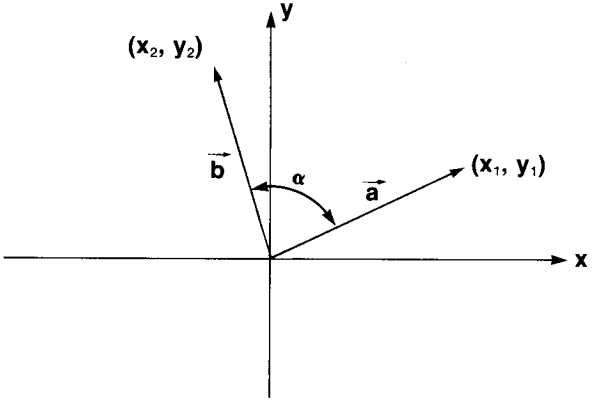
Outputs:

```
140.00 *** (TAS)
 66.98 ***
 25.00 *** (Wind velocity)
103.00 ***
132.24 *** (GS)
 10.02 *** (HDG)
1610.64 ***
```

Thus, the pilot should fly a heading 10.02° east of due north. His ground speed equals 132.24 knots.

VECTOR OPERATIONS

This program performs some of the most common operations on two-dimensional vectors, including addition, subtraction, dot product, and angle between the vectors.



Vectors may be input in either polar or rectangular form. The two vectors input need not be in the same form. Output vectors are printed in rectangular coordinates; pressing R/S will cause them to be printed in polar coordinates as well.

Equations:

Let the vectors a and b be (x_1, y_1) and (x_2, y_2) in rectangular coordinates or (r_1, θ_1) and (r_2, θ_2) in polar coordinates. Let α be the angle between $\vec{a}$ and $\vec{b}$.

Then

$$\vec{a} + \vec{b} = (x_1 + x_2, y_1 + y_2)$$

$$\vec{a} - \vec{b} = (x_1 - x_2, y_1 - y_2)$$

$$\vec{a} \cdot \vec{b} = x_1x_2 + y_1y_2$$

$$\alpha = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{r_1 r_2}\right)$$

Remarks:

This program assumes the calculator is set in DEG mode.

A-000 LBLA		A-037 X≷Y	
A-001 R 7	Input (x, y).	A-038 PRTX	
A-002 S 9		A-039 SPC	
A-003 R↓		A-040 RTN	
A-004 S 7		A-041 LBL3	
A-005 R↓		A-042 GSB8	
A-006 R 6		A-043 PRTX	Dot product.
A-007 S 8		A-044 SPC	
A-008 R↓		A-045 RTN	
A-009 S 6		A-046 LBL4	
A-010 RTN		A-047 GSB8	Angle between.
A-011 LBL1		A-048 R 9	
A-012 R 6	Addition.	A-049 R 8	
A-013 R 8		A-050 →P	
A-014 +		A-051 X≷Y	
A-015 PRTX		A-052 R↓	
A-016 R 7		A-053 R 7	
A-017 R 9		A-054 R 6	
A-018 +		A-055 →P	
A-019 PRTX		A-056 X≷Y	
A-020 SPC		A-057 R↓	
A-021 GTO9		A-058 ×	
A-022 LBL2		A-059 ÷	
A-023 R 8	Subtraction.	A-060 COS⁻¹	
A-024 R 6		A-061 PRTX	
A-025 −		A-062 SPC	
A-026 PRTX		A-063 RTN	
A-027 R 9		A-064 LBL8	
A-028 R 7		A-065 R 6	Subroutine for dot product called from LBL 3 and LBL 4.
A-029 −		A-066 R 8	
A-030 PRTX		A-067 ×	
A-031 SPC		A-068 R 7	
A-032 LBL9		A-069 R 9	
A-033 R/S	Convert from rectangular to polar.	A-070 ×	
A-034 X≷Y		A-071 +	
A-035 →P		B-000 LBLB	
A-036 PRTX		B-001 X≷Y	

B-002 →R B-003 X≷Y	Input (r, θ), convert to (x, y).	B-004 GTOA	

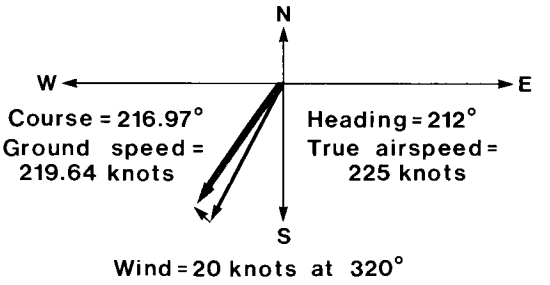
REGISTERS

0	1	2	3
4	5	6 x_2	7 y_2
8 x_1	9 y_1	.0	.1
.2	.3	.4	.5	I

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Key in vector $\vec{a}$:			
	• Rectangular	x_1	ENTER↑	
		y_1	A	x_1
	• Polar	r_1	ENTER↑	
		θ_1	B	x_1
3	Key in vector $\vec{b}$:			
	• Rectangular	x_2	ENTER↑	
		y_2	A	x_2
	• Polar	r_2	ENTER↑	
		θ_2	B	x_2
4	Compute:			
	• Sum $(\vec{a} + \vec{b})$		GSB 1	x
				y
	• Difference $(\vec{a} - b)$		GSB 2	x
				y
	• Dot product $(\vec{a} \cdot b)$		GSB 3	$\vec{a} \cdot \vec{b}$
	• Angle between		GSB 4	α

Example 1:

An aircraft flies a heading of 212 degrees at 225 knots. The wind blows at 20 knots from 140 degrees, i.e., towards 320 degrees. What is the course and ground speed of the aircraft?



Keystrokes:	**Outputs:**
225 ENTER↑ 212 B →	−190.81
20 ENTER↑ 320 B →	15.32
GSB 1 →	−175.49 *** (x)
	−132.09 *** (y)
R/S →	219.64 *** (Knots)
	−143.03 *** (Degrees)

Example 2:

Find the dot product of and the angle between the vectors (2, −5) and (4, 1).

Keystrokes:	**Outputs:**
2 ENTER↑ 5 CHS A →	2.00
4 ENTER↑ 1 A →	4.00
GSB 3 →	3.00 *** (Dot product)
GSB 4 →	82.23 *** (Angle)

HYPERBOLICS

This program computes the hyperbolic functions and their inverses. Note in the equations below the restrictions on the values of the argument in each case.

Equations:

Hyperbolic functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\operatorname{csch} x = \frac{1}{\sinh x} \qquad (x \neq 0)$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\coth x = \frac{1}{\tanh x} \qquad (x \neq 0)$$

Inverse Hyperbolic Functions

$$\sinh^{-1} x = \ln \left[x + (x^2 + 1)^{1/2}\right]$$

$$\cosh^{-1} x = \ln \left[x + (x^2 - 1)^{1/2}\right] \qquad x \geqslant 1$$

$$\tanh^{-1} x = \tfrac{1}{2} \ln \left[\frac{1 + x}{1 - x}\right] \qquad x^2 < 1$$

$$\operatorname{csch}^{-1} x = \sinh^{-1} \left[\frac{1}{x}\right] \qquad x \neq 0$$

$$\operatorname{sech}^{-1} x = \cosh^{-1} \left[\frac{1}{x}\right] \qquad 0 < x \leqslant 1$$

$$\coth^{-1} x = \tanh^{-1} \left[\frac{1}{x}\right] \qquad x^2 > 1$$

Remarks:

This program assumes the calculator is set in RAD mode.

A-000 LBLA A-001 LBL1 A-002 e^x A-003 ENT↑ A-004 1/X A-005 − A-006 2 A-007 ÷ A-008 RTN A-009 LBL2 A-010 e^x A-011 ENT↑ A-012 1/X A-013 + A-014 2 A-015 ÷ A-016 RTN A-017 LBL3 A-018 S 9 A-019 GSB1 A-020 R 9 A-021 GSB2 A-022 ÷ A-023 RTN A-024 LBL4 A-025 ENT↑ A-026 X^2	Sinh. Cosh. Tanh. $Sinh^{-1}$	A-027 1 A-028 + A-029 √X A-030 + A-031 LN A-032 RTN A-033 LBL5 A-034 ENT↑ A-035 X^2 A-036 1 A-037 − A-038 √X A-039 + A-040 LN A-041 RTN A-042 LBL6 A-043 1 A-044 X⇄Y A-045 + A-046 1 A-047 LSTX A-048 − A-049 ÷ A-050 LN A-051 2 A-052 ÷	 $Cosh^{-1}$ $Tanh^{-1}$

REGISTERS

0	1	2	3
4	5	6	7
8	9 Used	.0	.1
.2	.3	.4	.5

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in program.			
2	Initialize.		GTO A	
3	For hyperbolics, go to step 4;			
	for inverse hyperbolics, go to			
	step 5.			
	HYPERBOLIC FUNCTIONS			
4	Key in argument and compute			
	• hyperbolic sine	x	GSB 1	sinh x
	• hyperbolic cosine	x	GSB 2	cosh x

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	• hyperbolic tangent	x	GSB 3	tanh x
	• hyperbolic cotangent	x	GSB 3 1/x	coth x
	• hyperbolic secant	x	GSB 2 1/x	sech x
	• hyperbolic cosecant	x	GSB 1 1/x	csch x
	INVERSE HYPERBOLIC			
	FUNCTIONS			
5	• inverse hyperbolic sine	x	GSB 4	$\sinh^{-1}$ x
	• inverse hyperbolic cosine	x	GSB 5	$\cosh^{-1}$ x
	• inverse hyperbolic tangent	x	GSB 6	$\tanh^{-1}$ x
	• inverse hyperbolic cotangent	x	1/x GSB 6	$\coth^{-1}$ x
	• inverse hyperbolic secant	x	1/x GSB 5	sech^{-1} x
	• inverse hyperbolic cosecant	x	1/x GSB 4	csch^{-1} x

Example 1:

Evaluate the following hyperbolic functions:

sinh 2.5; cosh 3.2; tanh 1.9; coth −2.01; sech −0.25; csch 4.6.

Keystrokes:	**Outputs**	
GTO A		
2.5 GSB 1 →	6.05	(sinh 2.5)
3.2 GSB 2 →	12.29	(cosh 3.2)
1.9 GSB 3 →	0.96	(tanh 1.9)
2.01 CHS GSB 3 1/x →	-1.04	(coth -2.01)
.25 CHS GSB 2 1/x →	0.97	(sech -0.25)
4.6 GSB 1 1/x →	0.02	(csch 4.6)

Example 2:

Evaluate the following inverse hyperbolic functions: $\sinh^{-1}$ 2.4; $\cosh^{-1}$ 90; $\tanh^{-1}$ −0.65; $\coth^{-1}$ 3.4; sech^{-1} 0.4; csch^{-1} 2.

Keystrokes:	**Outputs:**	
GTO A		
2.4 GSB 4 →	1.61	($\sinh^{-1}$ 2.4)

90 GSB 5 →	5.19	($\cosh^{-1}$ 90)
.65 C GSB 6 →	−0.78	($\tanh^{-1}$ −0.65)
3.4 1/x GSB 6 →	0.30	($\coth^{-1}$ 3.4)
.4 1/x GSB 5 →	1.57	(sech^{-1} 0.4)
2 1/x GSB 4 →	0.48	(csch^{-1} 2)

1000 N.E. Circle Blvd., Corvallis, OR 97330

00095-90003