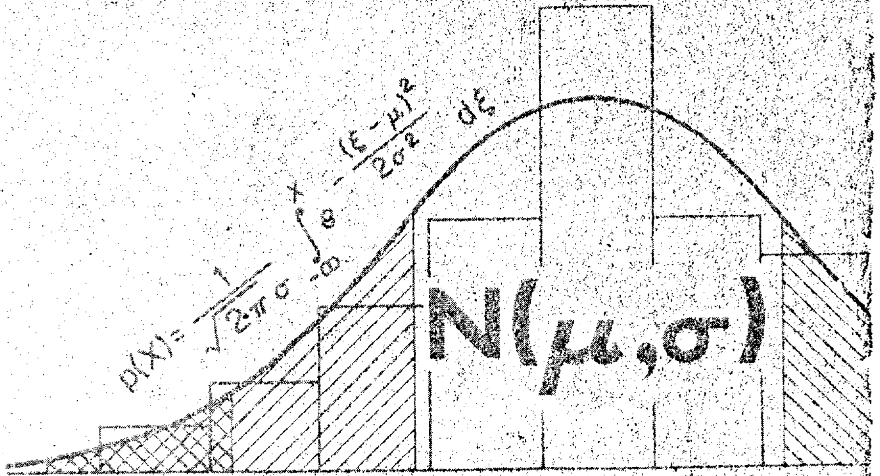
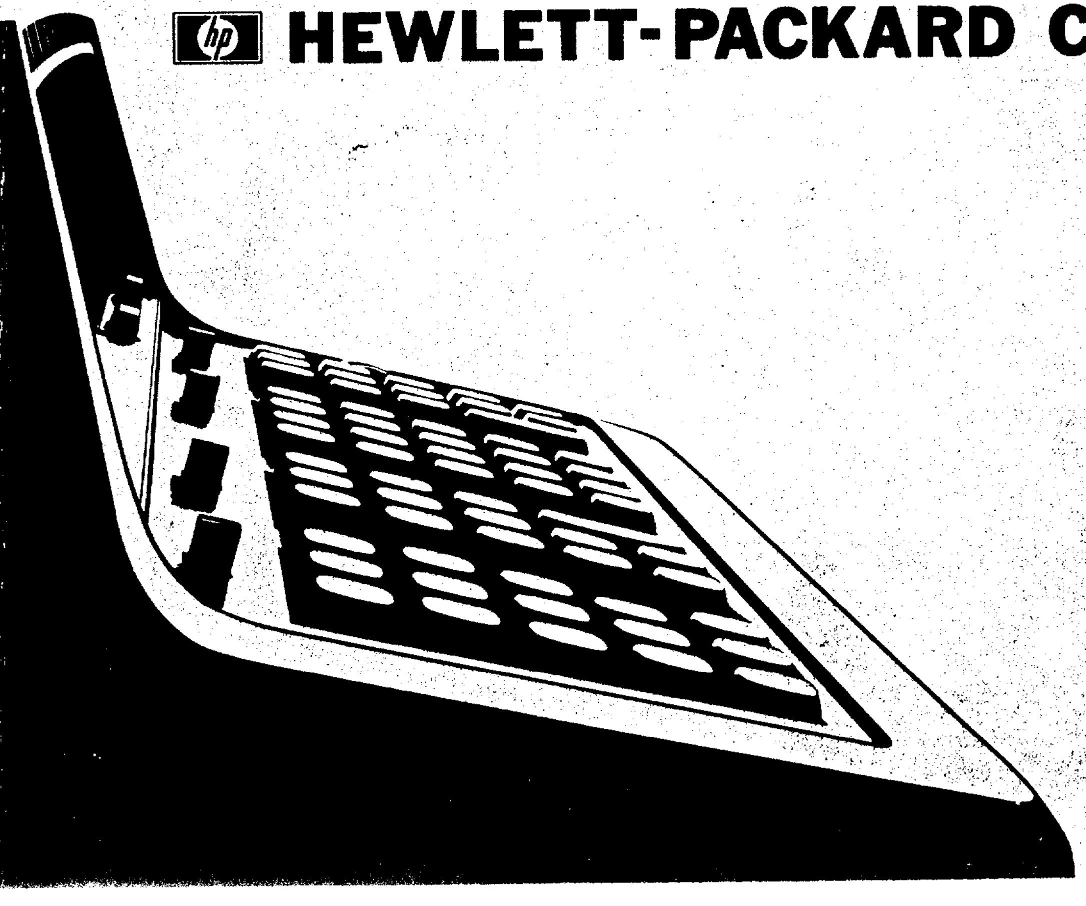
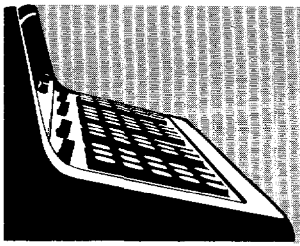


HEWLETT-PACKARD CALCULATOR

STAT-PAC

Vol. I





STAT-PAC VOLUME I

PART NO.
09100-70800

INTRODUCTION

This package of statistical programs for the Hewlett-Packard 9100A/B calculator has been designed to assist you in your statistical problem solving. Many of the programs contained in this Volume are actual customer supplied programs or customer suggested programs. A future package, Volume II will be generated from additional programs submitted by you, our customer.

We hope that you will find this package valuable in your everyday statistical computations. Your comments and suggestions for future statistically related calculator applications are welcome. If you develop a calculator program which you would like to have included in STAT-PAC VOLUME II, please forward it to us for review. Please write your programs in a format containing: Program Description, Reference, User Instructions, Program Steps, and a typical Example illustrating how you employ the program. All programs placed in Volume II will, of course, be included with credit to the author and company affiliation.

We wholeheartedly hope that this collection of statistical programs will significantly reduce your computational "chores" and increase the 9100 Calculator's usefulness to you.

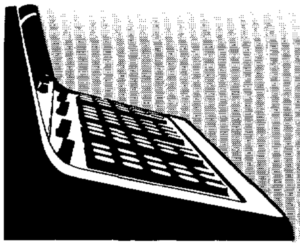
Sincerely,

HEWLETT-PACKARD COMPANY

Dave Cole
Applications Engineer
P.O. Box 301
Loveland, Colorado 80537

DC/bd

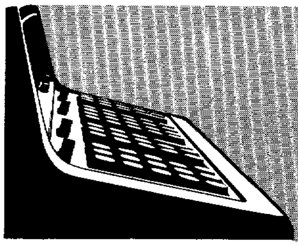
2



CONTENTS

This package is divided into ten sections, these being:

The programs included in each section are designed to be both functional and informational in that many are general enough to be applied directly to any statistical data reduction problem, whereas others are quite specific in application. The specialized programs such as Buffon Needle Experiment, Fourier Series - Sampled X(t), Gompertz Curve Fit, Weibull Distribution Via Failed and Suspended Sets, Control Ellipses Plot, and others demonstrate the variety and range of problems solvable on the Hewlett-Packard 9100 Calculator.

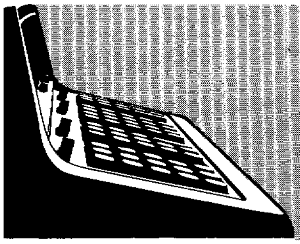


PROGRAM LISTING

Section I GENERAL STATISTICS

Section II DISTRIBUTION FUNCTIONS

Section X PLOTTER PROGRAMS (Con't)

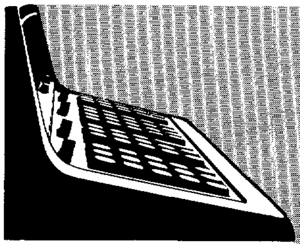


STAT-PAC -- PROGRAM LIBRARY DUPLICATION

The table below lists those programs which are found published in the 9100A/B Libraries or in the PLOTTER PACKET, 09100-76999. These programs, in most cases, have been modified for printer accommodation and are included in the STAT-PAC in order to arrange all your statistics programs into a single compact packet. These programs will be denoted by a ▲ following their STAT-PAC numbers.

TITLE	STAT-PAC No.	
Mean and Standard Deviation of Grouped Data	I-2	9100B Library
Histogram Generation (Without Plot	I-9	9100B Library
Random Number Generator (Uniform)	I-11	9100B Library
Normal Probability Integral	II-1	9100B Library
Poisson Distribution	II-4	9100B Library
χ^2 – Chi Square Distribution	II-5	9100B Library
F – Distribution	II-9	9100B Library
t Statistic for Means of Two Samples	III-1	9100A Library
χ^2 – Chi Square Evaluation Expected Values Equal ($E_i = E$)	III-5	9100B Library
χ^2 2 x 2 Contingency Table	III-6	9100A Library
χ^2 Chi Square Evaluation Expected Values Unequal ($E_i \neq E_j$)	III-7	9100B Library
Linear Regression and Correlation Coefficient	IV-1	9100B Library
Multiple Linear Regression 3 Variable	IV-3	9100B Library
Least Squares Fit – Power Curve	IV-6	9100B Library
Least Squares Fit – Exponential	IV-7	9100B Library
The Least Square Parabola	IV-8	9100B Library
Program For Simultaneous Solution of Four Equations in Four Unknowns With Printer	IV-13	9100B Library
One Way Analysis of Variance m x n	VII-1	9100B Library
Two Way Analysis of Variance	VII-2	9100B Library
Two Way Analysis of Variance With Replicates	VII-3	9100B Library
Weibull Distribution Parameter Calculation For Failure Data	IX-1	9100B Library
Histogram Generation (With Plot)	X-1	9100B Library
Normal Curve Overlay	X-2	9100B Library
Cumulative Probability Functions	X-4	9100B Library

TITLE	STAT-PAC No.	
Linear Regression And Plot	X-6	Plotter Packet
Power Curve Regression And Plot	X-8	Plotter Packet
Exponential Regression And Plot	X-10	Plotter Packet



9120A PRINTER

Each program has built-in printer commands eliminating your need to change the programs to accommodate a printer. These print commands have been incorporated both at input and output points in the programs. The USER INSTRUCTIONS do not assume that a printer is attached to the calculator, and contain PRESS: CONTINUE's between various output displays. If the printer is used, the USER will find that the calculator is automatically advanced through the output displays printing the results. However, if no printer is used, the operator advances through the output displays by use of the CONTINUE key as indicated in the instructions. Similarly, if no printer is used, the operator must press a final CONTINUE after the last display to place the calculator "in-ready" for another case.

To convert the STAT-PAC programs for use without a printer, the PRINT program commands should be replaced by STOP's and CONTINUE's. To assist in these changes, the program step pages have been annotated to locate the program steps requiring alteration. These annotations are:

s : change step to STOP

All other PRINT commands should be replaced by CONTINUE's

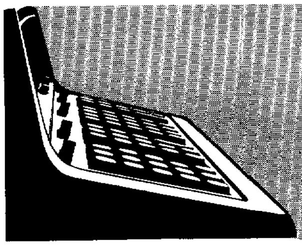
9125A PLOTTER PROGRAMS

Section X of the STAT-PAC contains 20 programs for the calculator/printer/plotter combination. These programs, such as Curve Fitting, Histogramming, Distribution Function Plotting, and Point Plotting, make optimum use of the plotter. The plotting section of the STAT-PAC contains general purpose plotting programs for:

a. Plotting data points prior to an analysis

b. Plotting general functions such as families of parametric distributions

It also contains very specific programs for plotting various distribution functions; normal, log normal, χ^2 (Chi-Squared), and others relating to regression analysis. Where possible, plot programs have been included for applications from the various ten sections.



STAT-PAC USAGE

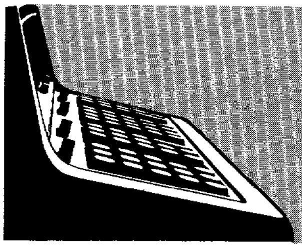
Each program in the STAT-PAC is documented with:

1. Program Description, Mathematical Formulas, References
2. USER INSTRUCTIONS
3. EXAMPLE Problem
4. Program STEPS

To ensure proper usage, the user should review the program description and mathematical formulation to verify its acceptance for his problem. Since many of the programs are customer programs, and since a variety of references were employed, a mathematical verification is justified.

Next, the program should be keyed into the calculator using the program steps. If the 9120A printer is not available, the PRINT commands (steps) should be replaced by the appropriate STOP or CONTINUE command.

Finally, the program EXAMPLE should be run to verify proper interpretation of the USER INSTRUCTIONS and error-free program step entry.

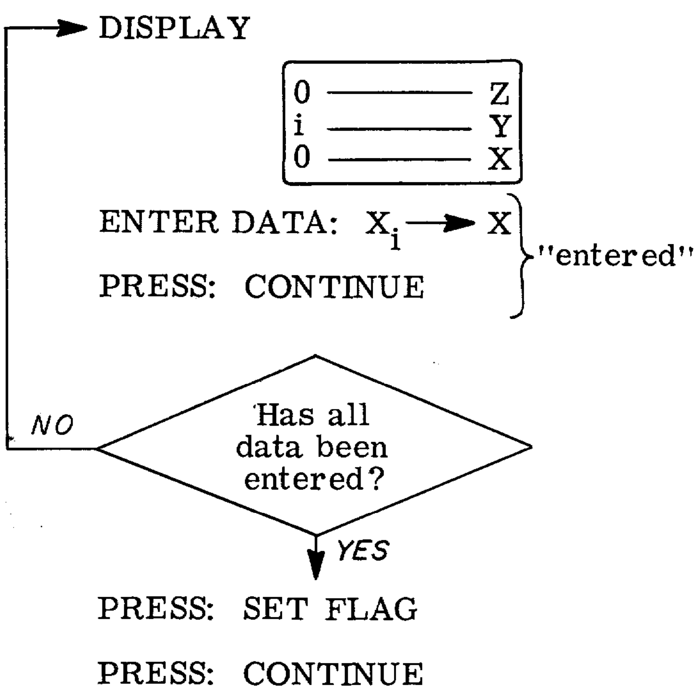


FLAG USAGE

Programs which generate statistics for sets of data $\{X_i\}$, $\{X_i, Y_i\}$, $\{X_i, Y_i, Z_i\}$, etc., use the FLAG to signal that the last entry $\{X_n\}$, $\{X_n, Y_n\}$, ..., has been entered. The USER INSTRUCTIONS for this operation are given below:

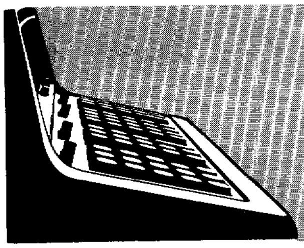
USER INSTRUCTIONS

USER RESPONSE



Here the user places the input data into the prescribed display register. The data is "entered" via the "CONTINUE".

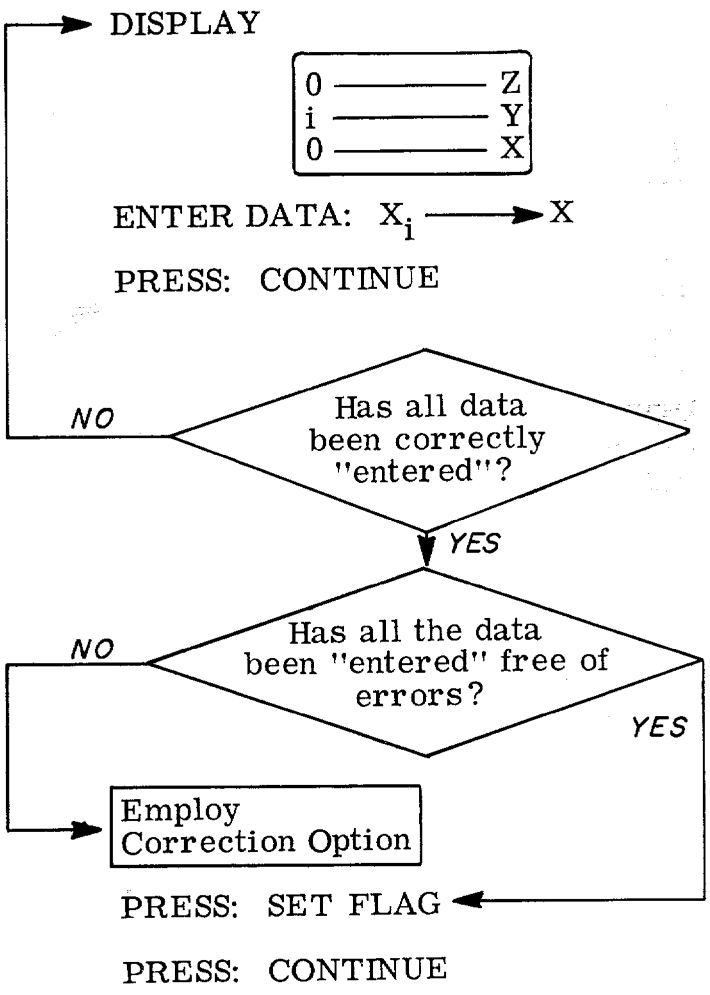
The FLAG is set after all data has been "entered".



ERROR CORRECTING OPTIONS

Several of the STAT-PAC programs contain "error correcting options". These options allow the user to correct for data input errors without re-entering the entire data set. The USER INSTRUCTIONS are seen below:

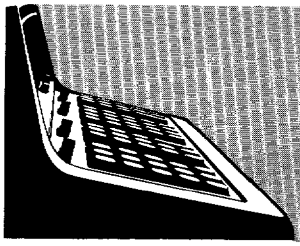
USER INSTRUCTIONS



USER RESPONSE

Enter the elements of the desired data set $\{X_n\}$.

Here the user reviews his input data to verify that each element of $\{X_n\}$ has been "entered". The set of data actually "entered" may contain errors, however, the "entered" set $\{X_n'\}$ contains the desired set $\{X_n\}$.



KEY MNEMONICS

The program step pages employ the following mnemonics:

KEY	MNEMONIC	CODE
0	0	00
1	1	01
2	2	02
3	3	03
4	4	04
5	5	05
6	6	06
7	7	07
8	8	10
9	9	11
e	e	12
a	a	13
b	b	14
f	f	15
c	c	16
d	d	17
CLEAR	CLR	20
•	•	21
ROLL ↑	RUP	22
$x\rightarrow( )$	XTO	23
$y\rightleftarrows( )$	YE	24
↓	DN	25
ENTER EXP	EEX	26

KEY	MNEMONIC	CODE
↑	UP	27
$x \rightleftarrows y$	XEY	30
ROLL ↓	RDN	31
CHG SIGN	CHS	32
+	+	33
−	-	34
÷	DIV	35
×	X	36
CLEAR x	CLX	37
$y \rightarrow ( )$	YTO	40
STOP	STP	41
FMT	FMT	42
IF FLAG	IFG	43
GO TO ( )	GTO	44
PRINT	PNT	45
END	END	46
CONT	CNT	47
IF $x=y$	X=Y	50
IF $x<y$	X<Y	52
IF $x>y$	X>Y	53
SET FLAG	SFL	54
$\|y\|$	|Y|	55
π	π	56
PAUSE	PSE	57
ACC +	AC+	60
RCL	RCL	61

KEY	MNEMONIC	CODE
TO POLAR	POL	62
ACC −	AC-	63
int x	INT	64
ln x	LN	65
TO RECT	RCT	66
$x \leftarrow (\ )$	XFR	67
hyper ▾	HYP	67
sin x	SIN	70
tan x	TAN	71
arc ▾	ARC	72
cos x	COS	73
e^x	EXP	74
log x	LOG	75
$\sqrt{x}$	$\sqrt{\ }$	76
▴SUB▾	SUB	77
RETURN	RTN	77

STAT-PAC NOMENCLATURE

The following list defines the mathematical notations used in the package.

NOTATION	DEFINITION
X_i, Y_i, x_i, y_i	subscripted sample points
$\bar{X}$, $\bar{Y}$, M	sample means
s_x, S, σ_x	sample standard deviations
s_{xy} , cov (x, y)	Covariance of X and Y
r_{xy}	Coefficient of Correlation between X and Y
pe_x	probable errors
n, N, n_x, n_y	number of data points
f_i	frequency of a data group
a_3	moment coefficient of skewness
a_4	moment coefficient of kurtosis
m_3	3rd moment about the mean
m_4	4th moment about the mean
${}_nP_r$, P^n_r	permutations of n things taken r at a time
${}_nC_r$, C^n_r , $\binom{n}{r}$	combinations of n things taken r at a time
A	arithmetic mean
G	geometric mean
H	harmonic mean
RN	random number
r.c.f.	relative cumulative frequency
p(X), f(X)	probability density function
P(X), f(X), P	probability distribution function
B(r, n, p)	binomial distribution

NOTATION	DEFINITION
γ, n, V, df	degrees of freedom
$\Gamma(\alpha)$	Gamma function
F(X)	cumulative distribution function
ln X	natural logarithm of X
log X	logarithm (base 10) of X
H_0	null hypothesis
H_A	alternative hypothesis
t	t statistic
α	level of significance
χ^2	chi-squared statistic
O_i	observed frequency
E	expected frequency
m, a, B	slope of linear regression line
b, c	intercept of linear regression line
X_{rms}, Y_{rms}	root mean square of X, Y
a_0, a_1, a_2, a_3	coefficients of polynomial regression
a_0, a_1, a_2, a_3	coefficients of multiple linear regression
$R_{x, yz}$, $R_{y, xz}$, $R_{z, xy}$	coefficients of linear multiple correlation
ω	radian frequency
Δt	time increment between sampled data
a_i	cosine coefficients in a Fourier Series
b_i	sine coefficients in a Fourier Series
F	F statistic
N!	N factorial
A	average (expected) value
R	range of a data set
$\hat{}$	estimate of some parameter (i.e., $\hat{s}_x$)
exp (X)	e^X
var (X)	variance of $\{X_i\}$

SECTION I GENERAL STATISTICS

MEAN, STANDARD DEVIATION AND PROBABLE ERROR

This program calculates the mean, $\overline{X}$; standard deviation, s_x; probable error, pe_x; the standard deviation from the mean, $s_{\overline{x}}$; and the probable error of the mean, $pe_{\overline{x}}$.

$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

$$s_x = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \overline{X})^2}{n - 1}} = \sqrt{\frac{\Sigma(X_i)^2 - n(\overline{X})^2}{n - 1}}$$

$$s_{\overline{x}} = \frac{s_x}{\sqrt{n}}$$

$$pe_x = .6745 s_x \qquad pe_{\overline{x}} = .6745 s_{\overline{x}}$$

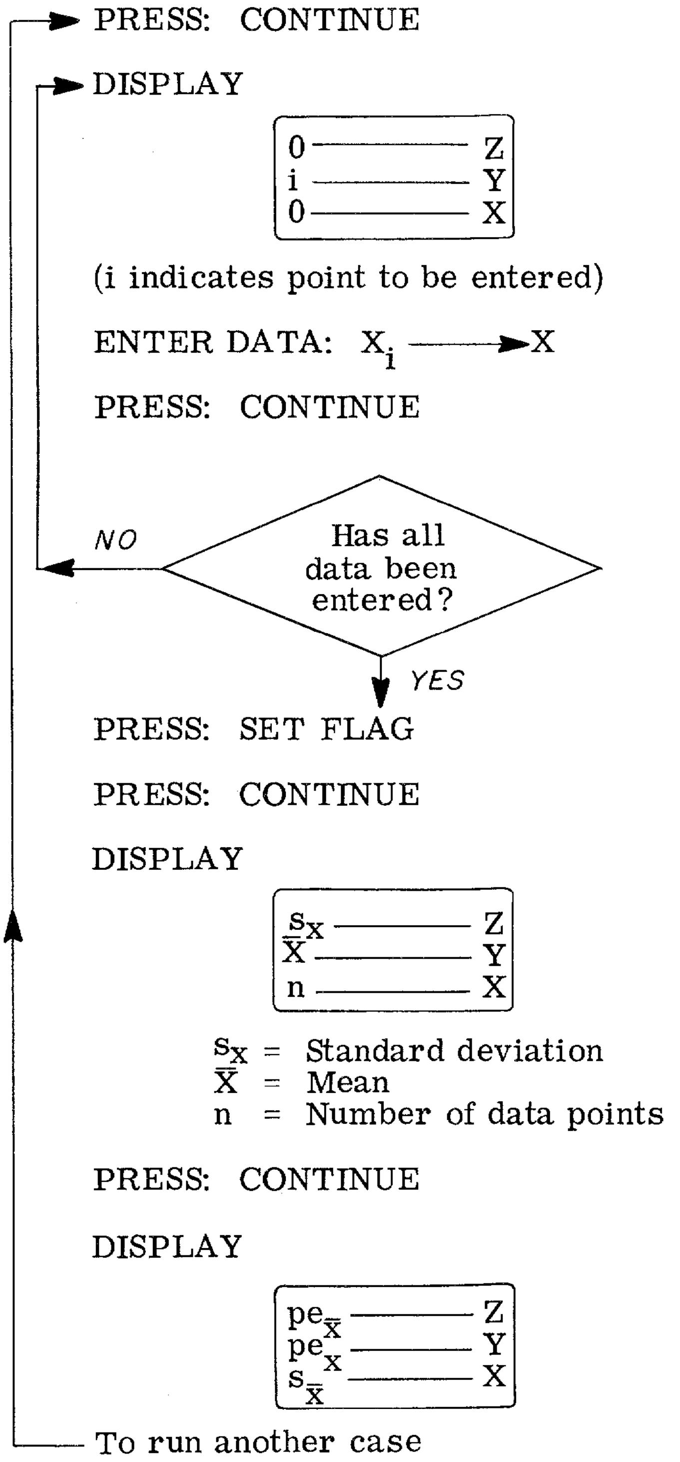
References:
1. Reference Data for Radio Engineers, International Telephone and Telegraph Corporation, 1956.
2. Treatment of Experimental Data, Worthing and Geffner, John Wiley And Sons, 1943.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM



PRESS: CONTINUE

DISPLAY

0 ———— Z
i ———— Y
0 ———— X

(i indicates point to be entered)

ENTER DATA: $X_i \longrightarrow X$

PRESS: CONTINUE

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

s_x ———— Z
$\bar{X}$ ———— Y
n ———— X

s_x = Standard deviation
$\bar{X}$ = Mean
n = Number of data points

PRESS: CONTINUE

DISPLAY

$pe_{\bar{x}}$ ———— Z
pe_x ———— Y
$s_{\bar{x}}$ ———— X

To run another case

EXAMPLE

Sample Data

1
2
3
4
5

Results:

1.581 ———— Z s_x
3.0 ———— Y $\bar{X}$
5 ———— X n

.477 ———— Z $pe_{\bar{x}}$
1.066 ———— Y pe_x
.707 ———— X $s_{\bar{x}}$

STAT-PAC I-1

```
00  CLR  20
01   1   01
02  XTO  23
03   d   17
04  XEY  30
05  STP  41     ENTRY
06  IFG  43
07   1   01
08   a   13
09  PNT  45
0a  PNT  45
0b  UP   27
0c   X   36
0d  AC+  60

10   0   00
11  UP   27
12   d   17
13  UP   27
14   1   01
15   +   33
16  YTO  40
17   d   17
18   0   00
19  GTO  44
1a   0   00
1b   5   05
1c   d   17
1d  UP   27

20   1   01
21   -   34
22  YTO  40
23   d   17
24  UP   27
25  RCL  61
26  RUP  22
27  DIV  35
28  YTO  40
29   f   15
2a   X   36
2b   f   15
2c   X   36
2d  DN   25

30   -   34
31   d   17
32  UP   27
33   1   01
34   -   34
35   f   15
36  RDN  31
37  DIV  35
38   d   17
39  RDN  31
3a   √   76
3b  XEY  30
3c  RUP  22
3d  PNT  45     s

40  PNT  45
41  XEY  30
42  DN   25
43   √   76
44  DIV  35
45  YTO  40
46   c   16
47   .   21
48   6   06
49   7   07
4a   4   04
4b   5   05
4c   X   36
4d  RUP  22

50   X   36
51   c   16
52  PNT  45     s
53  PNT  45
54  GTO  44
55   0   00
56   0   00
57  END  46
```

$s_{\overline{x}}$

n

Σx^2

Σx

MEAN AND STANDARD DEVIATION OF GROUPED DATA

This program calculates the mean, $\overline{X}$, and standard deviation, S, of a set of data points X_1, X_2, · · · , X_K with frequencies f_1, f_2, · · · , f_K respectively. The equations used are

$$\overline{X} = \frac{\sum_{i=1}^{K} f_i X_i}{\sum_{i=1}^{K} f_i}$$

$$S = \sqrt{\frac{\sum_{i=1}^{K} f_i (X_i - \overline{X})^2}{\sum_{i=1}^{K} f_i - 1}}$$

Reference: Introduction to the Theory of Statistics
by Mood and Graybill

McGraw - Hill 1963

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

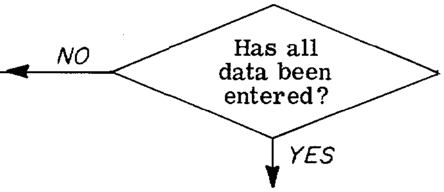
0 —— Z
i —— Y
0 —— X

(i indicates pair of points to be entered)

ENTER DATA:

$f_i \rightarrow Y$
$X_i \rightarrow X$

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

S —— Z
$\overline{X}$ —— Y
n —— X

S = Standard Deviation
$\overline{X}$ = Mean
n = Number of Data Points

EXAMPLES

SAMPLE DATA

X_i	f_i
1	3
2	3
3	1
4	2
5	1

s_x = 1.4337
$\overline{X}$ = 2.5
n = 10

SAMPLE DATA

X_i	f_i
41	3
38	5
37	2
39	18
40	22

s_x = .9010
$\overline{X}$ = 39.38
n = 50

00	CLR	20	
01	XTO	23	
02	c	16	
03	1	01	
04	XTO	23	
05	d	17	
06	XEY	30	
07	STP	41	ENTRY
08	IFG	43	
09	2	02	
0a	8	10	
0b	PNT	45	
0c	PNT	45	
0d	RUP	22	
10	c	16	
11	RUP	22	
12	+	33	
13	YTO	40	
14	c	16	
15	RUP	22	
16	X	36	
17	XEY	30	
18	X	36	
19	AC+	60	
1a	0	00	
1b	UP	27	
1c	d	17	
1d	UP	27	
20	1	01	
21	+	33	
22	YTO	40	
23	d	17	
24	0	00	
25	GTO	44	
26	0	00	
27	7	07	
28	YE	24	
29	f	15	
2a	UP	27	
2b	DN	25	
2c	c	16	
2d	DIV	35	
30	RDN	31	
31	X	36	
32	XTO	23	
33	f	15	
34	1	01	
35	RUP	22	
36	-	34	
37	DN	25	
38	DIV	35	
39	YE	24	
3a	e	12	
3b	CHS	32	
3c	DIV	35	
3d	e	12	
40	+	33	
41	DN	25	
42	√	76	
43	UP	27	
44	f	15	
45	UP	27	
46	c	16	
47	PNT	45	s
48	PNT	45	
49	GTO	44	
4a	0	00	
4b	0	00	
4c	END	46	

Σf_i

n

$\Sigma f_i X_i^2$

$\Sigma f_i X_i$

CUMULATIVE MEAN ($\overline{X}_c$) AND CUMULATIVE STANDARD DEVIATION (s_c)

This program calculates the cumulative mean ($\overline{X}_c$) and the cumulative standard deviation (s_c) from summary information, namely N, ΣX and $\Sigma(X)^2$, thus saving time where N_0 and N_1 are large, or where the individual values are not available from storage. The program employs the following formulas, where the zero (0) subscript refers to previous data and the one (1) subscript refers to current data.

$$\text{Cumulative Mean} = \overline{X}_c = \frac{\Sigma X_0 + \Sigma X_1}{N_0 + N_1} = \frac{N_0\overline{X}_0 + N_1\overline{X}_1}{N_0 + N_1}$$

Cumulative Standard Deviation = s_c

$$s_c = \left[\frac{(N_0 + N_1)\left(\Sigma(X_0^2) + \Sigma\,(X_1^2)\right) - (\Sigma X_0 + \Sigma X_1)^2}{(N_0 + N_1)(N_0 + N_1 - 1)} \right]^{\frac{1}{2}}$$

$$s_c^2 = (s_c)^2$$

$$N_c = N_1 + N_0$$

"This work was supported by the U.S. Atomic Energy Commission."

This program was written by Mr. L. E. Snodgrass of Sandia Corporation, Albuquerque, New Mexico.

USER INSTRUCTIONS

DEPRESS: X Y on the 9120A

PRESS: END

ENTER PROGRAM

→ PRESS: CONTINUE

DISPLAY

0 ———— Z
0 ———— Y
0 ———— X

ENTER DATA:

$\Sigma X_0^2 \longrightarrow Y$
$\Sigma X_1^2 \longrightarrow X$

PRESS: CONTINUE

ENTER DATA:

$\Sigma X_0 \longrightarrow Y$
$\Sigma X_1 \longrightarrow X$

PRESS: CONTINUE

ENTER DATA:

$N_0 \longrightarrow Y$
$N_1 \longrightarrow X$

PRESS: CONTINUE

DISPLAY

$\bar{X}_c$ ———— Z
s_c^2 ———— Y
s_c ———— X

PRINT

$\bar{X}_c$
s_c^2

PRESS: CONTINUE

DISPLAY

$\bar{X}_c$ ———— Z
s_c ———— Y
N_c ———— X

PRINT

s_c
N_c

To run another case

EXAMPLE

Sample Calculations

Original Experiment

Data	N_0	ΣX_0	ΣX_0^2	s_0	$\bar{X}_0$
2.5					
2.1					
2.5					
2.1					
2.1	5	11.3	25.73	.21909	2.260

Current Experiment

Data	N_1	ΣX_1	ΣX_1^2	s_1	$\bar{X}_1$
3.0					
3.1					
3.2					
4.0	4	13.3	44.85	.45735	3.325

Cumulative Results

N_c	s_c	s_c^2	$\bar{X}_c$
9	.64614	.41750	2.733

STAT-PAC I-3

```
00   CLR   20
01   STP   41     ENTRY
02   PNT   45
03   PNT   45
04    +    33
05   YTO   40
06    d    17
07   DN    25
08    1    01
09   STP   41     ENTRY
0a   PNT   45
0b   PNT   45
0c    +    33
0d   YTO   40

10    e    12
11   RDN   31
12   UP    27
13    X    36
14   YTO   40
15    c    16
16    0    00
17   UP    27
18   UP    27
19    2    02
1a   STP   41     ENTRY
1b   PNT   45
1c   PNT   45
1d    +    33

20   YTO   40
21    f    15
22    1    01
23    -    34
24   YTO   40
25    a    13
26    f    15
27    X    36
28   YTO   40
29    b    14
2a   RCL   61
2b   DIV   35
2c   UP    27
2d    d    17

30    X    36
31    c    16
32    -    34
33    b    14
34   DIV   35
35   DN    25
36   UP    27
37    √    76
38   CNT   47
39   RDN   31
3a   PNT   45   S
3b   PNT   45
3c    f    15
3d   XEY   30

40   RDN   31
41   PNT   45
42   PNT   45
43   END   46
```

$N_0 + N_1 - 1$

$(N_0 + N_1 - 1)(N_0 + N_1)$

$(\Sigma X_1 + \Sigma X_0)^2$

$\Sigma X_1^2 + \Sigma X_0^2$

$\Sigma X_1 + \Sigma X_0$

$N_1 + N_0$

TWO VARIABLE SUMS OF SQUARES
(WITH CORRECTION OPTION)

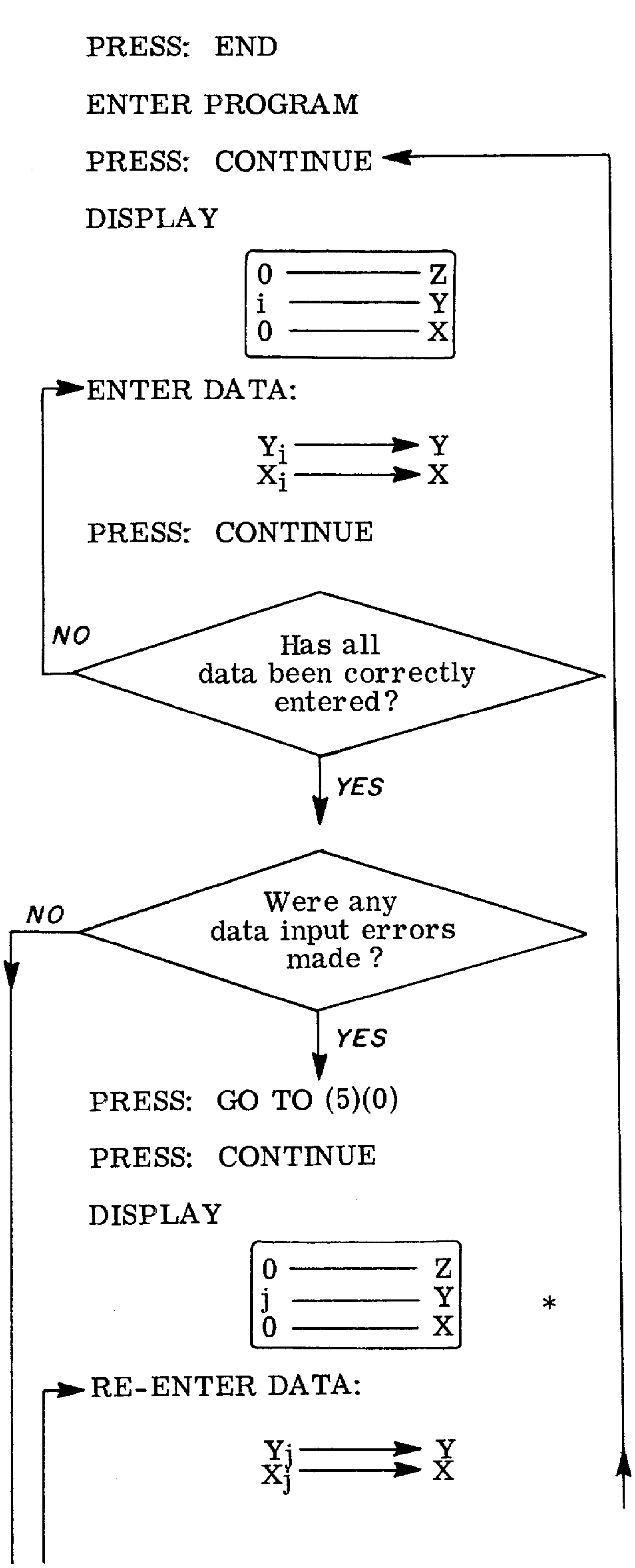
This program computes the sums of squares for a table of points (X_i, Y_i). The quantities computed are:

ΣX = Sum of X Values
ΣY = Sum of Y Values
ΣX^2 = Sum of Squared X Values
ΣY^2 = Sum of Squared Y Values
ΣXY = Sum of XY Products
n = Number of Data Pairs

This program has an option for correcting erroneously entered data. The program prints the computed quantities and also leaves them in the calculator memory for further use.

To correct for erroneously entered data the error data is re-entered (with the error) and the sums are corrected automatically.

USER INSTRUCTIONS



DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0 —— Z
i —— Y
0 —— X

ENTER DATA:

$Y_i \rightarrow Y$
$X_i \rightarrow X$

PRESS: CONTINUE

Has all data been correctly entered?

NO

YES

Were any data input errors made ?

NO

YES

PRESS: GO TO (5)(0)

PRESS: CONTINUE

DISPLAY

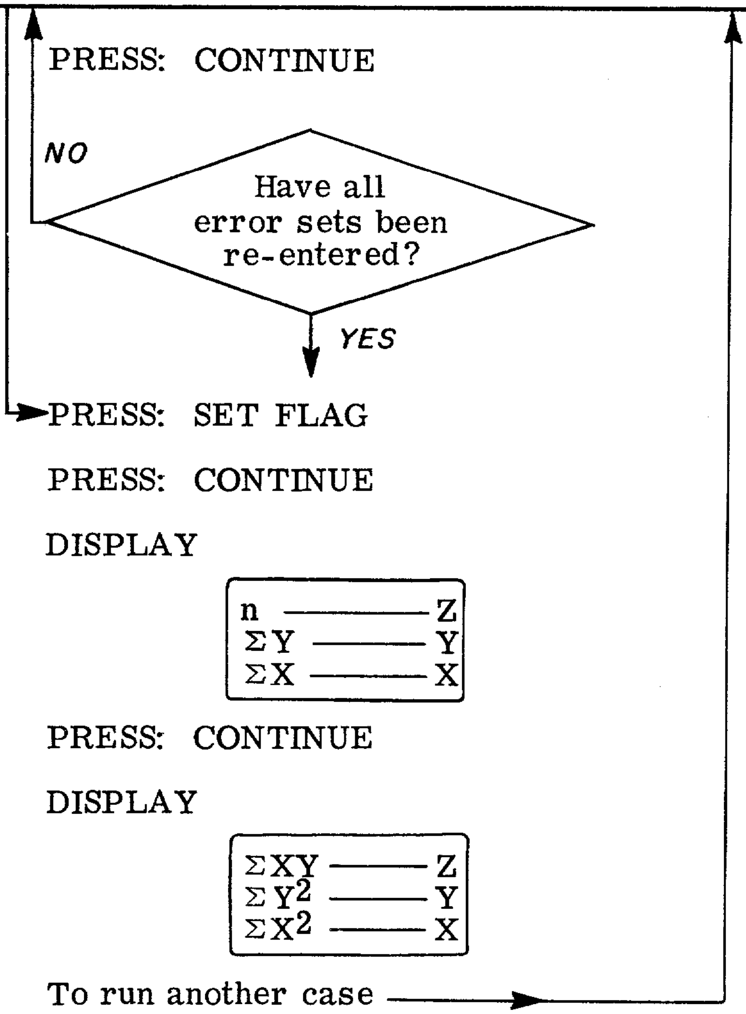
0 —— Z
j —— Y
0 —— X

*

RE-ENTER DATA:

$Y_j \rightarrow Y$
$X_j \rightarrow X$

USER INSTRUCTIONS (Con't)



PRESS: CONTINUE

Have all error sets been re-entered?

NO

YES

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

n —— Z
ΣY —— Y
ΣX —— X

PRESS: CONTINUE

DISPLAY

ΣXY —— Z
ΣY^2 —— Y
ΣX^2 —— X

To run another case →

*j represents an index on erroneously entered data sets, independent of i.

EXAMPLES

1)

i	X	Y
1	1	1
2	3	2
3	4	4
4	6	4
5	8	5
6	9	7
7	11	8
8	14	9

Results:

n = 8
ΣY = 40
ΣX = 56

ΣXY = 364
ΣY^2 = 256
ΣX^2 = 524

2)

Same as before but the first data set is entered erroneously as 1, 2

i	X	Y	
1	1	<2>	(error)
2	1	1	(entered
3	3	2	correct
4	4	4	set)
5	6	4	
6	8	5	
7	9	7	
8	11	8	
9	14	9	

After the entry of (14, 9)

PRESS: GO TO (5)(0)

PRESS: CONTINUE

DISPLAY

0 ———— Z
1 ———— Y
0 ———— X

RE-ENTER THE ERROR

$2 \longrightarrow Y$
$1 \longrightarrow X$

PRESS: CONTINUE

PRESS: SET FLAG

PRESS: CONTINUE

Results:

Same as Example 1

STAT-PAC I-4

Step	Key	Code	X	Y	Z
00	CLR	20			
01	XTO	23			
02	d	17			
03	XTO	23			
04	c	16			
05	XTO	23			
06	b	14			
07	XTO	23			
08	a	13			
09	1	01			
0a	XEY	30			
0b	STP	41		ENTRY	
0c	IFG	43			
0d	3	03			
10	a	13			
11	PNT	45			
12	PNT	45			
13	AC+	60			
14	UP	27			
15	X	36			
16	XEY	30			
17	YE	24			
18	d	17			
19	+	33			
1a	YEX	24			
1b	d	17			
1c	DN	25			
1d	X	36			
20	b	14			
21	+	33			
22	YTO	40			
23	b	14			
24	DN	25			
25	DN	25			
26	X	36			
27	c	16			
28	+	33			
29	YTO	40			
2a	c	16			
2b	a	13			
2c	UP	27			
2d	1	01			
30	+	33			
31	YTO	40			
32	a	13			
33	+	33			
34	0	00			
35	UP	27			
36	RDN	31			
37	GTO	44			
38	0	00			
39	b	14			
3a	a	13			
3b	UP	27			
3c	e	12			
3d	UP	27			

Step	Key	Code	X	Y	Z
40	f	15			
41	PNT	45	S		
42	PNT	45			
43	b	14			
44	UP	27			
45	c	16			
46	UP	27			
47	d	17			
48	PNT	45	S		
49	PNT	45			
4a	PNT	45			
4b	GTO	44			
4c	8	10			
4d	d	17			
50	0	00			
51	UP	27			
52	1	01			
53	UP	27			
54	0	00			
55	XTO	23			
56	9	11			
57	STP	41		ENTRY	
58	IFG	43			
59	3	03			
5a	a	13			
5b	PNT	45			
5c	PNT	45			
5d	AC-	63			
60	UP	27			
61	X	36			
62	XEY	30			
63	YE	24			
64	d	17			
65	-	34			
66	YE	24			
67	d	17			
68	DN	25			
69	X	36			
6a	b	14			
6b	XEY	30			
6c	-	34			
6d	YTO	40			
70	b	14			
71	DN	25			
72	DN	25			
73	X	36			
74	c	16			
75	XEY	30			
76	-	34			
77	YTO	40			
78	c	16			
79	a	13			
7a	UP	27			
7b	1	01			
7c	-	34			
7d	YTO	40			

Step	Key	Code	X	Y	Z
80	a	13			
81	YE	24			
82	9	11			
83	+	33			
84	YTO	40			
85	9	11			
86	+	33			
87	0	00			
88	UP	27			
89	RDN	31			
8a	GTO	44			
8b	5	05			
8c	7	07			
8d	END	46			

0		8	
1		9	n' (error number)
2		a	n
3		b	ΣXY
4		c	ΣY^2
5		d	ΣX^2
6		e	ΣY
7		f	ΣX

MEAN, VARIANCE, MOMENT COEFFICIENT OF SKEWNESS AND MOMENT COEFFICIENT OF KURTOSIS

This program computes the mean, variance, coefficients of skewness and kurtosis, and standard deviation of a table of data. The equations used are:

Mean = $\overline{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$

Variance = $S^2 = \frac{1}{n-1} \left[\sum_{i=1}^{n} X_i^2 - n (\overline{X})^2 \right] = \frac{SS}{n-1}$

m_3 = 3rd moment = $\frac{1}{n} \sum_{i=1}^{n} X_i^{\,3} - \frac{3}{n} \overline{X} \sum_{i=1}^{n} X_i^{\,2} + 2 (\overline{X})^3$

m_4 = 4th moment = $\frac{1}{n} \sum_{i=1}^{n} X_i^{\,4} - \frac{4}{n} \overline{X} \sum_{i=1}^{n} X_i^{\,3} + \frac{6}{n} (\overline{X})^2 \sum_{i=1}^{n} X_i^{\,2} - 3 (\overline{X})^4$

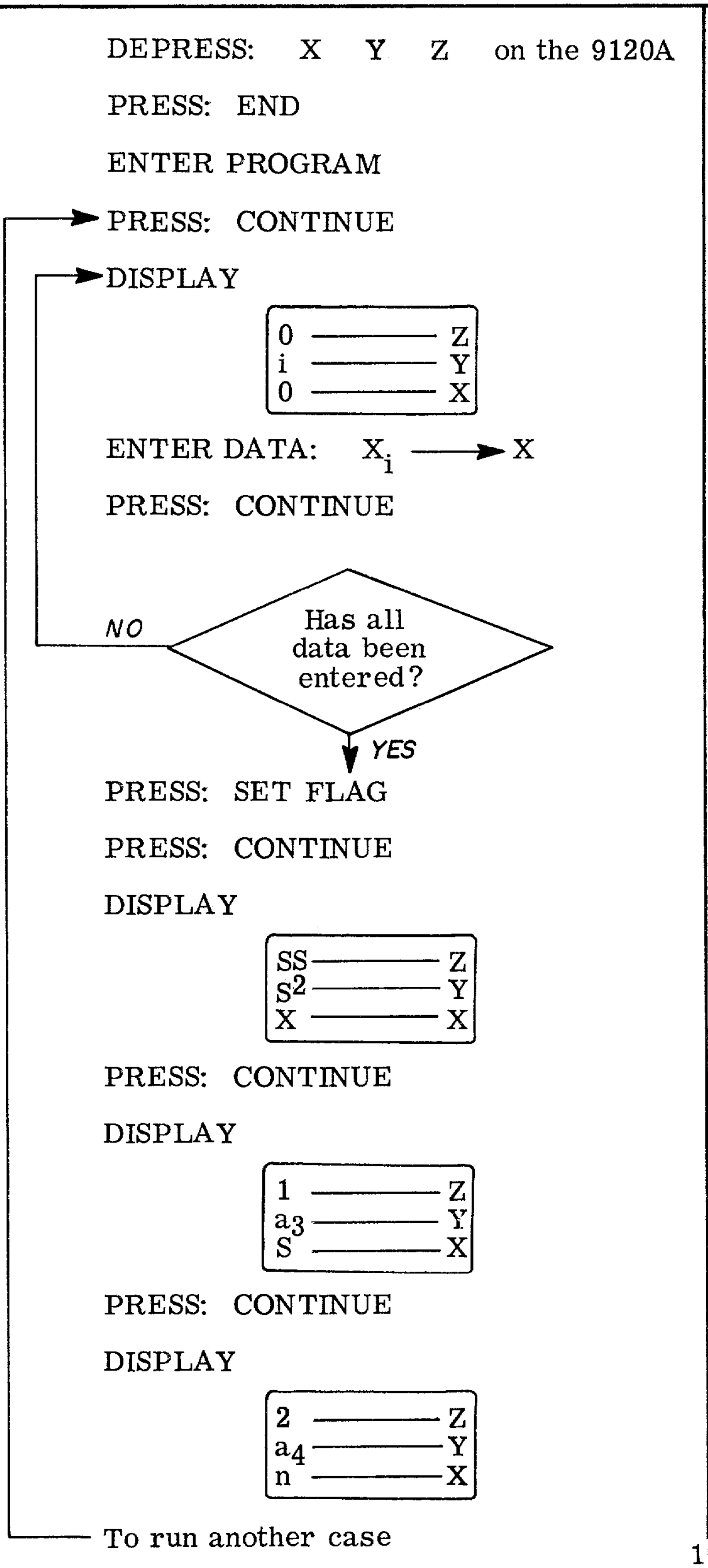
a_3 = moment coefficient of skewness = $\frac{m_3}{S^3}$

a_4 = moment coefficient of kurtosis = $\frac{m_4}{S^4}$

The user may wish to replace the (n - 1) divisor in the variance equation by n. If so, place a zero (0) in step (4)(3).

Reference: Theory and Problems of Statistics: M.R. Spiegel McGraw-Hill, 1961.

USER INSTRUCTIONS



DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

ENTER DATA: $X_i \longrightarrow X$

PRESS: CONTINUE

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

PRESS: CONTINUE

DISPLAY

PRESS: CONTINUE

DISPLAY

To run another case

EXAMPLE

i	X_i
1	2.1
2	3.5
3	4.2
4	6.5
5	4.1
6	3.6
7	5.3
8	3.7
9	4.9

Results:

SS = 12.509
S^2 = 1.564
$\overline{X}$ = 4.211

a_3 = .198
S = 1.250

a_4 = .068
n = 9.000

STAT-PAC I-5

00	CLR	20	
01	XTO	23	
02	c	16	
03	XTO	23	
04	d	17	
05	XTO	23	
06	b	14	
07	1	01	
08	XEY	30	
09	STP	41	ENTRY
0a	IFG	43	
0b	3	03	
0c	2	02	
0d	PNT	45	
10	PNT	45	
11	UP	27	
12	X	36	
13	AC+	60	
14	X	36	
15	UP	27	
16	DN	25	
17	X	36	
18	b	14	
19	+	33	
1a	YTO	40	
1b	b	14	
1c	DN	25	
1d	c	16	
20	+	33	
21	YTO	40	
22	c	16	
23	d	17	
24	UP	27	
25	1	01	
26	+	33	
27	YTO	40	
28	d	17	
29	+	33	
2a	0	00	
2b	UP	27	
2c	RDN	31	
2d	GTO	44	
30	0	00	
31	9	11	
32	RCL	61	
33	UP	27	
34	d	17	
35	DIV	35	
36	YTO	40	
37	f	15	
38	DN	25	
39	UP	27	
3a	X	36	
3b	d	17	
3c	X	36	
3d	RDN	31	
40	-	34	
41	RUP	22	
42	XEY	30	
43	1	01	
44	-	34	
45	DN	25	
46	DIV	35	
47	f	15	
48	PNT	45	s
49	PNT	45	
4a	YTO	40	
4b	a	13	
4c	c	16	
4d	UP	27	
50	f	15	
51	UP	27	
52	3	03	
53	X	36	
54	e	12	
55	X	36	
56	DN	25	
57	-	34	
58	d	17	
59	DIV	35	
5a	f	15	
5b	UP	27	
5c	X	36	
5d	X	36	
60	2	02	
61	X	36	
62	DN	25	
63	+	33	
64	a	13	
65	DIV	35	
66	√	76	
67	DIV	35	
68	UP	27	
69	1	01	
6a	RDN	31	
6b	PNT	45	s
6c	PNT	45	
6d	b	14	
70	UP	27	
71	f	15	
72	UP	27	
73	4	04	
74	X	36	
75	c	16	
76	X	36	
77	DN	25	
78	-	34	
79	f	15	
7a	UP	27	
7b	X	36	
7c	6	06	
7d	X	36	
80	e	12	
81	X	36	
82	DN	25	
83	+	33	
84	d	17	
85	DIV	35	
86	f	15	
87	UP	27	
88	X	36	
89	X	36	
8a	X	36	
8b	3	03	
8c	X	36	
8d	DN	25	
90	-	34	
91	d	17	
92	DIV	35	
93	DIV	35	
94	d	17	
95	UP	27	
96	2	02	
97	RDN	31	
98	PNT	45	
99	PNT	45	
9a	END	46	

S^2 (m_2)

ΣX^4

ΣX^3

n

ΣX^2

ΣX_i $\bar{X}$

COVARIANCE AND COEFFICIENT OF CORRELATION

This program determines the covariance and coefficient of correlation derived from a table of data containing X and Y observations as given below:

X	Y
X_1	Y_1
X_2	Y_2
.	.
.	.
.	.
X_n	Y_n

$$\text{Covariance} = S_{XY} = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y})$$

$$= \frac{1}{n-1} \left\{ \Sigma X_i Y_i - \frac{1}{n} \Sigma X_i \Sigma Y_i \right\}$$

$$\text{Coefficient of Correlation} = r_{XY} = \frac{S_{XY}}{S_X S_Y}$$

This program computes

$$\overline{X} = \frac{1}{n} \Sigma X_i$$

$$\overline{Y} = \frac{1}{n} \Sigma Y_i$$

$$S_X = \frac{1}{n-1} \left\{ \Sigma X_i^2 - \frac{(\Sigma X_i)^2}{n} \right\}$$

$$S_Y = \frac{1}{n-1} \left\{ \Sigma Y_i^2 - \frac{(\Sigma Y_i)^2}{n} \right\}$$

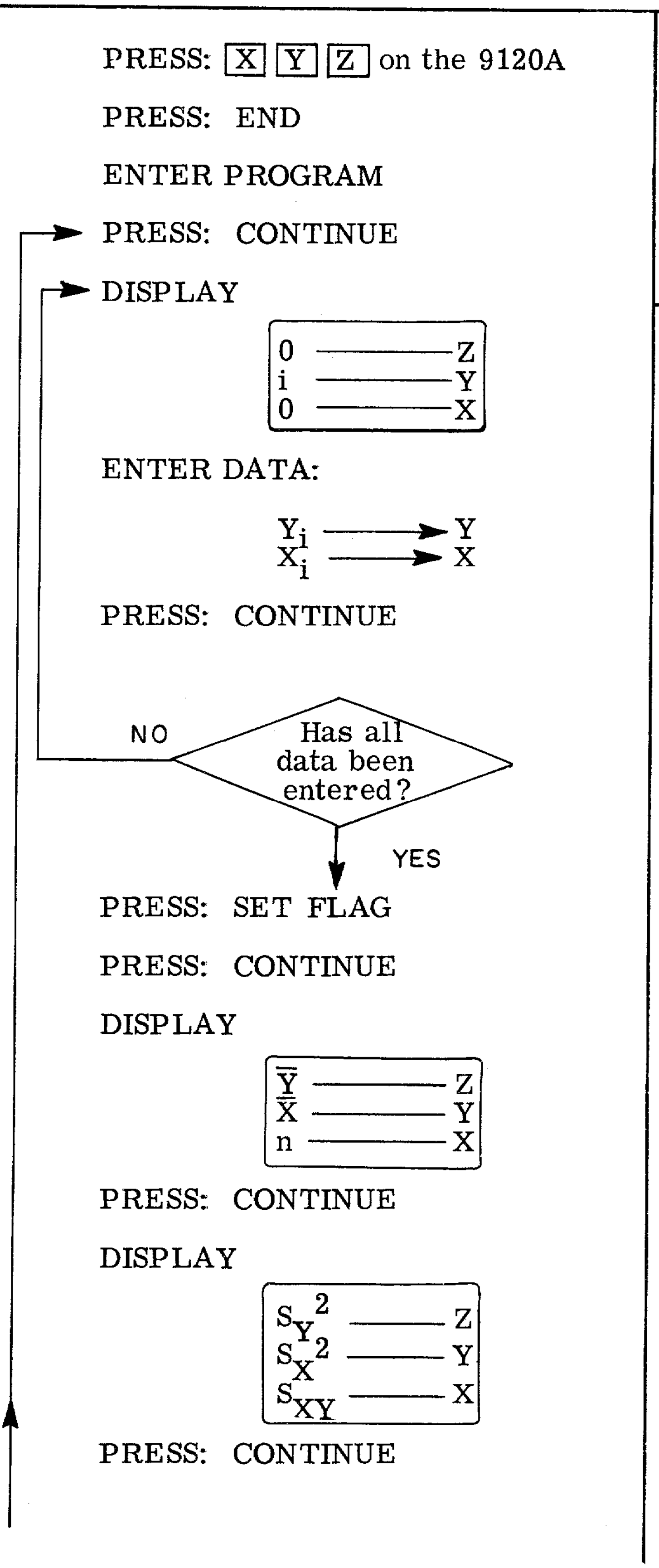
$$S_{XY}$$

and

$$r_{XY}$$

Reference: Statistical Theory with Engineering Applications, A. Hald, John Wiley & Sons, 1960.

USER INSTRUCTIONS



PRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0 ——— Z
i ——— Y
0 ——— X

ENTER DATA:

$Y_i \longrightarrow Y$
$X_i \longrightarrow X$

PRESS: CONTINUE

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

$\overline{Y}$ ——— Z
$\overline{X}$ ——— Y
n ——— X

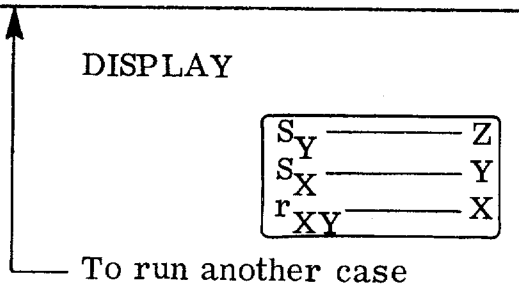
PRESS: CONTINUE

DISPLAY

$S_Y{}^2$ ——— Z
$S_X{}^2$ ——— Y
S_{XY} ——— X

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)



DISPLAY

S_Y ——— Z
S_X ——— Y
r_{XY} ——— X

To run another case

EXAMPLE

X	Y
1	1
3	2
4	4
6	4
8	5
9	7
11	8
14	9

Results:

5.	$\overline{Y}$
7.	$\overline{X}$
8.	n
8.00	$S_Y{}^2$
18.86	$S_X{}^2$
12.00	S_{XY}
2.83	S_Y
4.34	S_X
.977	r_{XY}

STAT-PAC I-6

Step	Key	Code	X	Y	Z
00	CLR	20			
01	XTO	23			
02	d	17			
03	XTO	23			
04	c	16			
05	XTO	23			
06	b	14			
07	XTO	23			
08	a	13			
09	1	01			
0a	XEY	30			
0b	STP	41		ENTRY	
0c	IFG	43			
0d	3	03			
10	c	16			
11	PNT	45			
12	PNT	45			
13	AC+	60			
14	ARC	72			
15	UP	27			
16	X	36			
17	XEY	30			
18	YE	24			
19	d	17			
1a	+	33			
1b	YE	24			
1c	d	17			
1d	DN	25			
20	X	36			
21	b	14			
22	+	33			
23	YTO	40			
24	b	14			
25	RUP	22			
26	UP	27			
27	X	36			
28	c	16			
29	+	33			
2a	YTO	40			
2b	c	16			
2c	a	13			
2d	UP	27			
30	1	01			
31	+	33			
32	YTO	40			
33	a	13			
34	+	33			
35	0	00			
36	XEY	30			
37	UP	27			
38	0	00			
39	GTO	44			
3a	0	00			
3b	b	14			
3c	d	17			
3d	UP	27			

Step	Key	Code	X	Y	Z
40	f	15			
41	UP	27			
42	X	36			
43	a	13			
44	DIV	35			
45	DN	25			
46	-	34			
47	a	13			
48	UP	27			
49	1	01			
4a	-	34			
4b	DN	25			
4c	DIV	35			
4d	YTO	40			
50	d	17			
51	c	16			
52	UP	27			
53	e	12			
54	UP	27			
55	X	36			
56	a	13			
57	DIV	35			
58	DN	25			
59	-	34			
5a	a	13			
5b	UP	27			
5c	1	01			
5d	-	34			
60	DN	25			
61	DIV	35			
62	YTO	40			
63	c	16			
64	b	14			
65	UP	27			
66	f	15			
67	UP	27			
68	e	12			
69	X	36			
6a	CNT	47			
6b	a	13			
6c	DIV	35			
6d	DN	25			
70	-	34			
71	a	13			
72	UP	27			
73	1	01			
74	-	34			
75	DN	25			
76	DIV	35			
77	YTO	40			
78	b	14			
79	e	12			
7a	UP	27			
7b	a	13			
7c	DIV	35			
7d	f	15			

Step	Key	Code	X	Y	Z
80	UP	27			
81	a	13			
82	DIV	35			
83	PNT	45	s		
84	PNT	45			
85	c	16			
86	UP	27			
87	d	17			
88	UP	27			
89	b	14			
8a	PNT	45	s		
8b	PNT	45			
8c	RUP	22			
8d	√	76			
90	RUP	22			
91	√	76			
92	X	36			
93	DN	25			
94	DIV	35			
95	c	16			
96	√	76			
97	UP	27			
98	d	17			
99	√	76			
9a	RUP	22			
9b	PNT	45			
9c	PNT	45			
9d	END	46			

n

ΣXY/SXY

$\Sigma Y^2/S_Y{}^2$

$\Sigma X^2/S_X{}^2$

ΣY

ΣX

This program evaluates the formula for the permutations of n objects taken r at a time:

$$ {}_nP_r = \frac{n!}{(n-r)!} = P_r^n $$

$$ = n(n-1)(n-2)\ .\ .\ .\ (n-r+1) $$

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

ENTER PROGRAM

→ PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA:

n ——→ Y
r ——→ X

PRESS: CONTINUE

DISPLAY

P^n_r ——— Z
n ——— Y
r ——— X

To run another case

EXAMPLE

1. $_5P_5 = 5! = 120$

2. $_8P_4 = 1680$

3. How many different numbers can be formed by using six out of the nine digits, 1, 2, 3, ... 9 ?

$$_9P_6 = 60480.$$

STAT-PAC I-7

```
00  CLR  20    ENTRY
01  STP  41
02  UP   27
03  DN   25
04  XTO  23
05   c   16
06   -   34
07  CLX  37
08  X>Y  53
09  DN   25
0a  STP  41
0b  RDN  31
0c  AC+  60
0d  UP   27

10   1   01
11  RDN  31
12  X=Y  50
13   2   02
14   0   00
15  DN   25
16   X   36
17  UP   27
18   1   01
19   -   34
1a   f   15
1b  GTO  44
1c   1   01
1d   2   02

20  RCL  61
21   c   16
22  PNT  45
23  PNT  45
24  END  46
```

r

n

n - r

COMBINATIONS

This program evaluates the formula for the combination of n objects taken r at a time:

$$ {}_{n}C_{r} = \frac{n!}{(n-r)!\,r!} = C_{r}^{n} $$

$$ = \frac{n(n-1)(n-2)\ \ldots\ (n-r+1)}{r!} $$

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

→ PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA:

n ——→ Y
r ——→ X

PRESS: CONTINUE

DISPLAY

C^n_r ——— Z
n ——— Y
r ——— X

To run another case

EXAMPLE

1. $_8C_4 = 70.$

2. In how many ways can a group of 600 people be selected from 602 people? Use the identity.

$$_{602}C_{600} = {}_{602}C_2 = 180901.$$

$$_nC_r = {}_nC_{n-r}$$

ENTRY

```
00  CLR   20
01  STP   41
02  XTO   23
03   d    17
04   1    01
05  AC+   60
06  RDN   31
07   f    15
08  XEY   30
09   d    17
0a  X<Y   52
0b   1    01
0c   a    13
0d  RCL   61

10  DIV   35
11  DN    25
12   X    36
13   1    01
14  UP    27
15  CHS   32
16  AC-   63
17  GTO   44
18   0    00
19   7    07
1a   e    12
1b  YE    24
1c   d    17
1d  XEY   30

20   +    33
21  PNT   45
22  PNT   45
23  END   46
```

r

n, n - 1, . . . , n - r

1, 2, 3

HISTOGRAM GENERATION (WITHOUT PLOT)

This program generates a histogram table of ten windows given a data set of positive numbers. In addition, it determines the mean (M_x) and the variance ($\sigma_x 2$) of the raw data, and the mean (M_h) and variance ($\sigma_h 2$) of the normalized histogram data. Since the raw data is normalized by the program to values $0 \leq h \leq 10$, the new mean and variance are given by

$$M_h = \frac{M_x}{W}$$

$$\sigma_h{}^2 = \frac{\sigma_x{}^2}{W^2}$$

where W is the histogram window width (normalization factor). The window width W is chosen such that all normalized data entries X/W will lie between 0 and 10. Thus, if the data ranges from 0 ➝ 200, a W of 20 would be proper.

This program uses Indirect Addressing. The (+) registers are used for storage whereas the (—) registers are used for program steps.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

ENTER PROGRAM: (Starting Address is (-0)(0))

PRESS: GO TO (-) (0) (0)

PRESS: CONTINUE

DISPLAY

0 ———— Z
0 ———— Y
0 ———— X

ENTER DATA: W → X

PRESS: CONTINUE

→ DISPLAY

N ———— Z
N ———— Y
1 ———— X

ENTER DATA: X → X

PRESS: CONTINUE

Has all data been entered? NO → return to DISPLAY; YES ↓

PRESS: SET FLAG

PRESS: CONTINUE

→ DISPLAY

% ———— Z (% of data points in Group)
K_A ———— Y (Number of data points in Group)
Group ———— X (Group #)

PRESS: CONTINUE

Is Group less than 10? YES → PRESS: CONTINUE, return to DISPLAY; NO ↓

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

σ_x^2 ———— Z
M_x ———— Y
0 ———— X

PRESS: CONTINUE

DISPLAY

σ_h^2 ———— Z
M_h ———— Y
1 ———— X

PRESS: GO TO
PRESS: —
PRESS: 0
PRESS: 0

} To consider another set of data.

EXAMPLE

The data set is:

104, 92, 83, 78, 58, 135, 146, 24, 74, 85, 81, 128, 140, 113, 79, 78, 53, 42, 34, 85, 96, 110, 133, 158, 171, 108, 84, 90, 73, 11, 51, 118, 68, 139, 92, 109, 89, 124, 91, 116.

The data varies between 0 and 200 so W is chosen to be 20.

Result

Group	K_A	%
1	1	2.5
2	2	5
3	4	10
4	6	15
5	11	27.5
6	7	17.5
7	5	12.5
8	3	7.5
9	1	2.5
10	0	0

N = 40

σ_x^2 = 1252.644 σ_h^2 = 3.132

M_x = 93.575 M_h = 4.679

STAT-PAC I-9

Step	Key	Code	X	Y	Z
00	CLR	20		Minus Page	
01	XTO	23			
02	0	00			
03	XTO	23			
04	1	01			
05	XTO	23			
06	2	02			
07	XTO	23			
08	3	03			
09	XTO	23			
0a	4	04			
0b	XTO	23			
0c	5	05			
0d	XTO	23			
10	6	06			
11	XTO	23			
12	7	07			
13	XTO	23			
14	8	10			
15	XTO	23			
16	9	11			
17	STP	41		ENTRY	
18	XTO	23			
19	a	13			
1a	1	01			
1b	CHS	32			
1c	RDN	31			
1d	DN	25			
20	1	01			
21	+	33			
22	UP	27			
23	DN	25			
24	STP	41		ENTRY	
25	IFG	43			
26	5	05			
27	a	13			
28	PNT	45			
29	PNT	45			
2a	UP	27			
2b	X	36			
2c	AC+	60			
2d	XEY	30			
30	a	13			
31	DIV	35			
32	DN	25			
33	INT	64			
34	UP	27			
35	EEX	26			
36	9	11			
37	+	33			
38	YTO	40			
39	-	34			
3a	4	04			
3b	CLX	37			
3c	CLX	37			
3d	YE	24			

Step	Key	Code	X	Y	Z
40	6	06			
41	0	00			
42	0	00			
43	0	00			
44	0	00			
45	0	00			
46	0	00			
47	0	00			
48	0	00			
49	1	01			
4a	9	11			
4b	0	00			
4c	0	00			
4d	0	00			
50	IFG	43			
51	1	01			
52	d	17			
53	CLX	37			
54	1	01			
55	+	33			
56	SFL	54			
57	GTO	44			
58	3	03			
59	d	17			
5a	YTO	40			
5b	b	14			
5c	CLX	37			
5d	UP	27			
60	EEX	26			
61	9	11			
62	+	33			
63	YTO	40			
64	-	34			
65	7	07			
66	-	34			
67	0	00			
68	UP	27			
69	0	00			
6a	0	00			
6b	0	00			
6c	0	00			
6d	YE	24			
70	0	00			
71	0	00			
72	0	00			
73	0	00			
74	0	00			
75	0	00			
76	0	00			
77	0	00			
78	0	00			
79	1	01			
7a	9	11			
7b	0	00			
7c	0	00			
7d	0	00			

Step	Key	Code	X	Y	Z
80	YTO	40			
81	c	16			
82	b	14			
83	DIV	35			
84	EEX	26			
85	2	02			
86	X	36			
87	1	01			
88	RUP	22			
89	+	33			
8a	XFR	67			
8b	c	16			
8c	XEY	30			
8d	PNT	45	S		
90	PNT	45			
91	UP	27			
92	1	01			
93	0	00			
94	X>Y	53			
95	6	06			
96	0	00			
97	RCL	61			
98	UP	27			
99	b	14			
9a	DIV	35			
9b	YTO	40			
9c	f	15			
9d	X	36			
a0	f	15			
a1	X	36			
a2	DN	25			
a3	-	34			
a4	b	14			
a5	DIV	35			
a6	f	15			
a7	UP	27			
a8	0	00			
a9	PNT	45			
aa	PNT	45	S		
ab	a	13			
ac	DIV	35			
ad	XEY	30			

Storage

0	K_0	8	K_8
1	K_1	9	K_9
2	K_2	a	W
3	K_3	b	N
4	K_4	c	K_A
5	K_5	d	
6	K_6	e	ΣX^2
7	K_7	f	

STAT-PAC I-9

Step	Key	Code	X	Y	Z
b0	RDN	31		Minus	
b1	DIV	35		Page	
b2	DIV	35			
b3	DN	25			
b4	XEY	30			
b5	UP	27			
b6	1	01			
b7	PNT	45	s		
b8	PNT	45			
b9	GTO	44			
ba	-	34			
bb	0	00			
bc	0	00			

ARITHMETIC, GEOMETRIC, HARMONIC MEANS

This program calculates the arithmetic, geometric, and harmonic means of a set of n data values. The program evaluates the following formulas:

$$A = \frac{1}{n} \sum_{i=1}^{n} X_i \quad \text{(arithmetic mean)}$$

$$G = \sqrt[n]{X_1 \cdot X_2 \ldots X_n} \quad \text{(geometric mean)}$$

$$H = \frac{n}{\sum_{i=1}^{n} \frac{1}{X_i}}$$

Reference: Theory and Problems of Statistics, Murray R. Spiegel, McGraw-Hill, 1961.

USER INSTRUCTIONS

DEPRESS: X on the 9120A

PRESS: END

ENTER PROGRAM

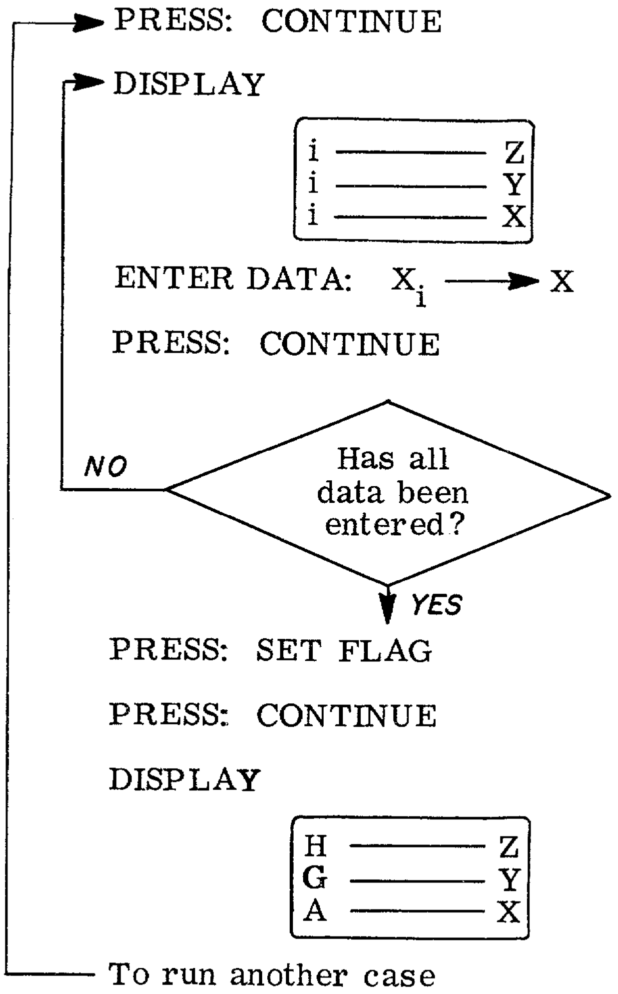
PRESS: CONTINUE

DISPLAY

i —— Z
i —— Y
i —— X

ENTER DATA: $X_i \longrightarrow X$

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

H —— Z
G —— Y
A —— X

To run another case

EXAMPLE

Data: 2, 4, 8

Results:

3.42857 —— Z Harmonic mean
4.00000 —— Y Geometric mean
4.66667 —— X Arithmetic mean

STAT-PAC I-10

00	CLR	20	
01	1	01	
02	XTO	23	
03	d	17	
04	YTO	40	
05	c	16	
06	UP	27	
07	UP	27	
08	STP	41	ENTRY
09	IFG	43	
0a	2	02	
0b	a	13	
0c	PNT	45	
0d	PNT	45	
10	UP	27	
11	1	01	
12	AC+	60	
13	d	17	
14	XEY	30	
15	X	36	
16	YTO	40	
17	d	17	
18	UP	27	
19	1	01	
1a	XEY	30	
1b	DIV	35	
1c	c	16	
1d	+	33	
20	YTO	40	
21	c	16	
22	f	15	
23	UP	27	
24	1	01	
25	+	33	
26	DN	25	
27	GTO	44	
28	0	00	
29	6	06	
2a	d	17	
2b	LN	65	
2c	UP	27	
2d	f	15	
30	DIV	35	
31	DN	25	
32	EXP	74	
33	XTO	23	
34	b	14	
35	e	12	
36	UP	27	
37	f	15	
38	DIV	35	
39	YTO	40	
3a	a	13	
3b	UP	27	
3c	c	16	
3d	DIV	35	
40	b	14	
41	UP	27	
42	a	13	
43	RUP	22	
44	PNT	45	
45	RUP	22	
46	PNT	45	
47	RUP	22	
48	PNT	45	
49	PNT	45	
4a	PNT	45	s
4b	GTO	44	
4c	0	00	
4d	0	00	
50	END	46	

A

G

$\Sigma\frac{1}{X_i}$

πX_i

ΣX_i

n

RANDOM NUMBER GENERATOR
(UNIFORM)

This program calculates uniformly distributed random numbers (RN) in the range $0 \leq RN_i \leq 1$ using the formula given below:

$$RN_i = \left[\pi + RN_{(i-1)}\right]^8 - \text{Int.}\left\{\left[\pi + RN_{(i-1)}\right]^8\right\}$$

RN_i is the current random number and $RN_{(i-1)}$ is the last calculated random number. More than 10,000 random numbers may be generated before values are repeated.

USER INSTRUCTIONS

DEPRESS: X on the 9120A

PRESS: END

ENTER PROGRAM:

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
RN_i ——— X

TO RESET PROBLEM

PRESS: END

EXAMPLE

$RN_1 = .5310$

$RN_2 = .6918$

$RN_3 = .3712$

$RN_4 = .6890$

STAT-PAC I-11

```
00  CLR   20
01  UP    27
02   π    56
03   +    33
04  DN    25
05  UP    27
06   X    36
07  DN    25
08  UP    27
09   X    36
0a  DN    25
0b  UP    27
0c   X    36
0d  DN    25

10  UP    27
11  INT   64
12   -    34
13  DN    25
14  PNT   45    S
15  GTO   44
16   0    00
17   1    01
18  END   46
```

RANDOM NUMBER GENERATION (NORMAL)

This program generates a sequence of Normally distributed random numbers with mean and variance $\bar{n}$ and σ_n^2 . The generation technique involves transforming Uniform random variables to normal variables by the formulas:

$$N_i = (-2 \ln \mu_i)^{\frac{1}{2}} \cos (2\pi \cdot \mu_{i+1})$$

$$N_{i+1} = (-2 \ln \mu_i)^{\frac{1}{2}} \sin (2\pi \cdot \mu_{i+1})$$

where (μ_i, μ_{i+1}) are independent uniform random variables, $0 \leq \mu_i \leq 1$.

The N_i thus generated are normally distributed with mean <u>zero</u> and <u>unity</u> variance.

The set $\{N_i\}$ are modified to generate a more general set of mean ($\bar{n}$) and variance (σ_n^2) by

$$n_i = \sigma_n N_i + \bar{n}$$

$$n_{i+1} = \sigma_n N_{i+1} + \bar{n}$$

The set $\{n_i\}$ is now distributed normally with mean $\bar{n}$ and variance σ_n^2 .

Reference: Handbook of Mathematical Functions, U. S. Dept of Commerce, Applied Mathematics Series, 1964, page 953.

USER INSTRUCTIONS

DEPRESS: X Y on the 9120A

SET: ☐ RADIANS

SET: Decimal Wheel as required

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
1 ——— X

ENTER DATA:

$\sigma_n^2 \longrightarrow Y$
$\bar{n} \longrightarrow X$

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
2 ——— X

ENTER DATA:

$\mu_2 \longrightarrow Y$ *
$\mu_1 \longrightarrow X$

→ PRESS: CONTINUE

DISPLAY

n_{i+1} ——— Z
n_{i+1} ——— Y
n_i ——— X

The program will automatically continue to generate and print a series of $\{n_i\}$ until the operator presses STOP.

*μ_1 and μ_2 are two initializing uniform random variables such that:

USER INSTRUCTIONS (Con't)

$$\mu_1 \neq \mu_2$$
$$0 \le \mu_1 \le 1$$
$$0 \le \mu_2 \le 1$$

EXAMPLE

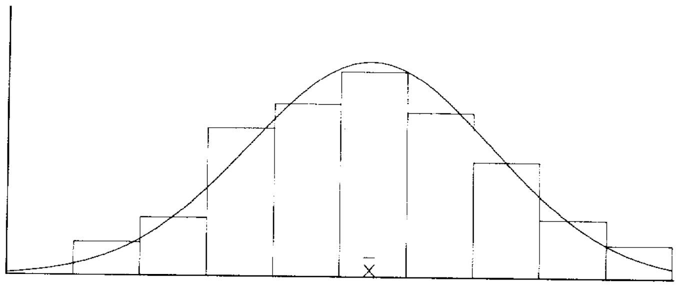
$\sigma_n^2 = 1$
$\bar{n} = 3$

$\mu_2 = .75$
$\mu_1 = .25$

Results:

The histogram seen below represents the first 120 samples generated by this program. The histogram was generated by program STAT-PAC X-1. The superimposed normal curve was generated by program STAT-PAC X-2. The computed mean and variance are:

$\bar{n}_{\text{computed}} = 2.99$
$\sigma_n^2 \text{ computed} = 1.01$
$w = .55$



STAT-PAC I-12

00	CLR	20	
01	1	01	
02	STP	41	ENTRY
03	PNT	45	
04	PNT	45	
05	XTO	23	
06	d	17	
07	DN	25	
08	√	76	
09	XTO	23	
0a	c	16	
0b	2	02	
0c	STP	41	ENTRY
0d	PNT	45	
10	PNT	45	
11	AC+	60	
12	LN	65	
13	UP	27	
14	2	02	
15	X	36	
16	DN	25	
17	CHS	32	
18	√	76	
19	RUP	22	
1a	UP	27	
1b	π	56	
1c	X	36	
1d	2	02	
20	X	36	
21	DN	25	
22	UP	27	
23	COS	73	
24	RUP	22	
25	X	36	
26	RUP	22	
27	SIN	70	
28	X	36	
29	c	16	
2a	X	36	
2b	RUP	22	
2c	X	36	
2d	d	17	
30	+	33	
31	RUP	22	
32	+	33	
33	DN	25	
34	PNT	45	S
35	SFL	54	
36	f	15	
37	UP	27	
38	π	56	
39	+	33	
3a	DN	25	
3b	UP	27	
3c	X	36	
3d	DN	25	
40	UP	27	
41	X	36	
42	DN	25	
43	UP	27	
44	X	36	
45	DN	25	
46	UP	27	
47	INT	64	
48	-	34	
49	YE	24	
4a	e	12	
4b	YE	24	
4c	f	15	
4d	IFG	43	
50	3	03	
51	6	06	
52	e	12	
53	UP	27	
54	f	15	
55	GTO	44	
56	1	01	
57	2	02	
58	END	46	

NORMAL PROBABILITY PAPER PLOT POINTS

This program determines the Relative Cumulative Frequencies, in percent, from a set of grouped data. The percentages can be plotted on Normal Probability graph paper for determining the normalcy of the data.

Formulas used:

$$r.c.f.(i) = \frac{100}{N} \sum_{j=1}^{i} f_j$$

$$N = \sum_{i=1}^{n} f_i$$

n = Number of Groups

f_i = Frequency of the i^{th} group

The input data consists of a Cumulative Frequency table of grouped data as illustrated below.

Group	Group Description	Frequency
1	Below Boundary 1	f_1
2	Boundary 1 to Boundary 2	f_2
.	.	.
.	.	.
.	.	.
n	Above Boundary (n-1)	f_n

Reference: Introduction to Statistical Inference, J.C.R. Li, Edward Brothers, Inc., 1957.

USER INSTRUCTIONS

DEPRESS: X on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

```
0 ———— Z
0 ———— Y
0 ———— X
```

ENTER DATA: N ⟶ X

PRESS: CONTINUE

DISPLAY

```
0 ———— Z
i ———— Y
0 ———— X
```

ENTER DATA: Upper Boundary and Frequency

$UB_i \longrightarrow Y$
$f_i \longrightarrow X$

PRESS: CONTINUE

DISPLAY

```
i ———— Z
r.c.f.% — Y
r.c.f.% — X
```

*

After the n^{th} group is entered, N is printed again.

To run another case

*If printer is used, the print blocks will be:

$$\left\{\begin{array}{l} i \\ UB_i \\ f_i \\ r.c.f._i\ \% \end{array}\right.$$

EXAMPLE

Group	Grouped description	Frequency
1	Below 20	6
2	20 to 30	12
3	30 to 40	25
4	40 to 50	39
5	50 to 60	48
6	60 to 70	47
7	70 to 80	36
8	80 to 90	22
9	90 to 100	10
10	100 to 110	5

N = 250

Results:

i = 1
UB_i = 20
f_i = 6
$r.c.f._i\%$ = 2.40

= 2
= 30
= 12
= 7.20

= 3
= 40
= 25
= 17.20

= 4
= 50
= 39
= 32.80

= 5
= 60
= 48
= 52.00

EXAMPLE (Con't)

i = 6
UB_i = 70
f_i = 47
$r.c.f._i\%$ = 70.80

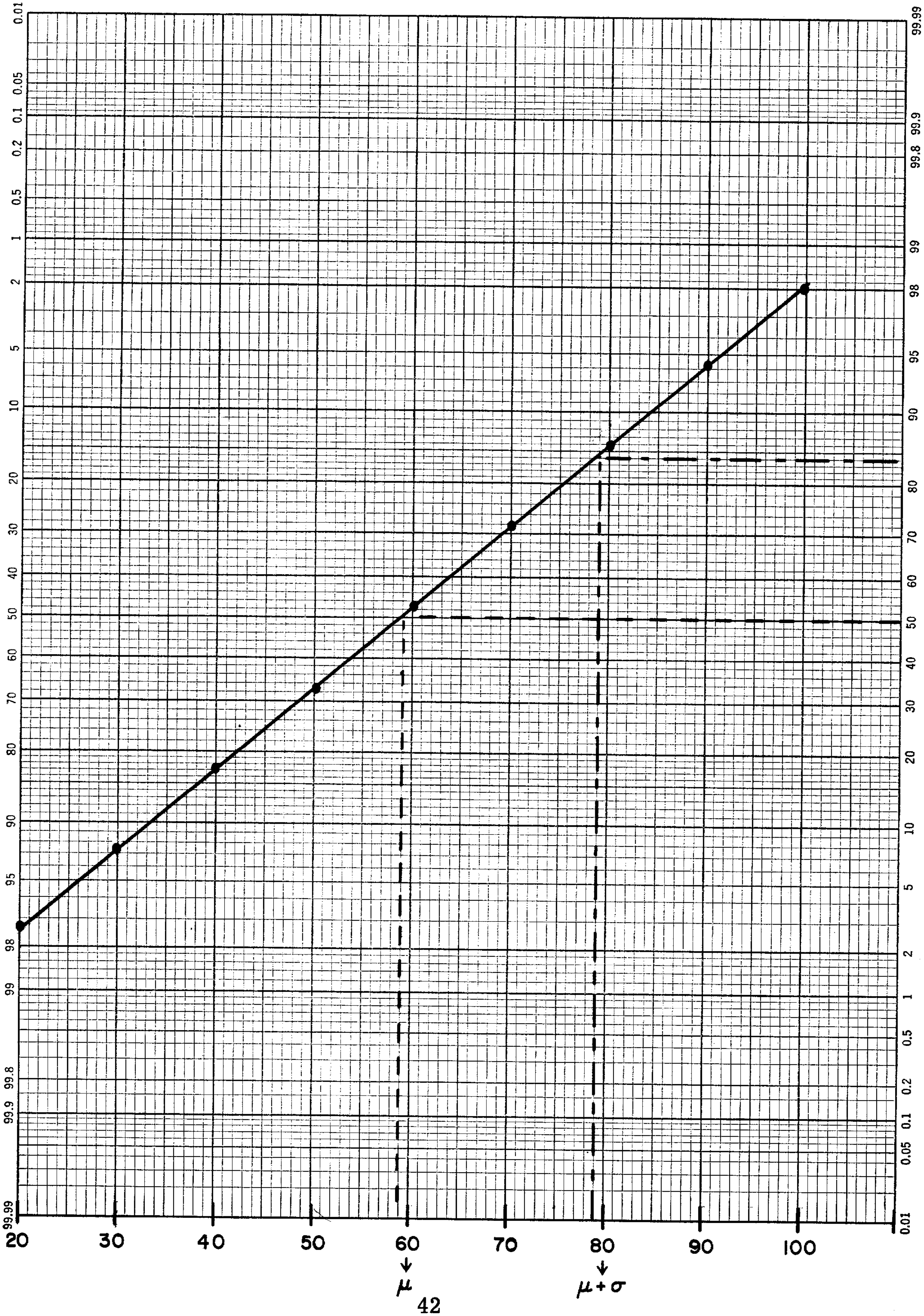
= 7
= 80
= 36
= 85.20

= 8
= 90
= 22
= 94.00

= 9
=100
= 10
= 98.00

= 10
=110
= 5
=100.00

The $r.c.f._i\%$ are plotted on the following graph.


0.01
0.05
0.1
0.2
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
99.8
99.9
99.99
99.99
99.9
99.8
99
98
95
90
80
70
60
50
40
30
20
10
5
2
1
0.5
0.2
0.1
0.05
0.01
20
30
40
50
60
70
80
90
100
μ
μ+σ

Step	Key	Code	X	Y	Z
00	CLR	20			
01	XTO	23			
02	d	17			
03	STP	41		ENTRY	
04	XTO	23			
05	e	12			
06	PNT	45			
07	PNT	45			
08	PNT	45			
09	1	01			
0a	XEY	30			
0b	STP	41		ENTRY	
0c	RUP	22			
0d	PNT	45			
10	RUP	22			
11	PNT	45			
12	RUP	22			
13	PNT	45			
14	UP	27			
15	f	15			
16	+	33			
17	YTO	40			
18	f	15			
19	e	12			
1a	DIV	35			
1b	EEX	26			
1c	2	02			
1d	X	36			
20	d	17			
21	UP	27			
22	1	01			
23	+	33			
24	DN	25			
25	XTO	23			
26	d	17			
27	RDN	31			
28	PNT	45	S		
29	PNT	45			
2a	PNT	45			
2b	DN	25			
2c	f	15			
2d	UP	27			
30	e	12			
31	X=Y	50			
32	3	03			
33	c	16			
34	0	00			
35	RDN	31			
36	1	01			
37	+	33			
38	0	00			
39	GTO	44			
3a	0	00			
3b	b	14			
3c	e	12			
3d	UP	27			
40	UP	27			
41	PNT	45			
42	PNT	45			
43	PNT	45			
44	PNT	45			
45	PNT	45	S		
46	GTO	44			
47	0	00			
48	0	00			
49	END	46			

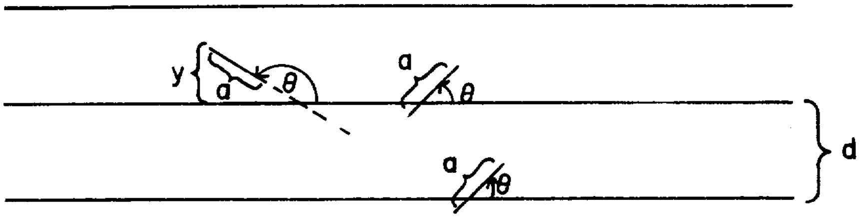
Storage

0		8	
1		9	
2		a	
3		b	
4		c	
5		d	group #
6		e	N
7		f	c.f.

BUFFON NEEDLE EXPERIMENT

STAT-PAC
I-14

This program simulates the well known Buffon Needle Experiment described in the reference. The problem is to determine the probability that a needle of length a will intersect a line on a grid of lines separated by distance d. The geometry of the experiment is seen below:



The needle of length $a \leq d$ is randomly tossed on the grid. By considering the angle θ between the needle and a line of the grid, and the distance Y between the upper tip of the needle and the line it intersects or the nearest line below it, the possible relations can be derived:

$$0 \leq \theta \leq \pi$$

$$0 \leq Y \leq d$$

The needle intersects a line when:

$$0 \leq Y \leq a \sin \theta .$$

It can be shown that the probability of an intersection is theoretically given by:

$$P_T = \frac{2a}{\pi d} .$$

This formula involving π has intrigued mathematicians since it allows one to experimentally determine π .

This program randomly chooses a θ from a uniform distribution such that:

$$0 \leq \theta \leq \pi$$

then independently choose a Y from a uniform distribution such that:

$$0 \leq Y \leq 1$$

The grid distance d is defined to be 1.

Thus $P_T = \frac{2a}{\pi}$.

The value of Y is then compared against

$$Y \leq a \sin \theta.$$

If Y is less than or equal to a sin θ , an intersection has occurred. The number of failing throws is then used to evaluate the experimental probability:

$$P_E = \frac{\#\ \text{successful}}{\text{successful} + \text{failed}}$$

The program displays:

\# trials
P_E
P_T

and proceeds to toss another needle.

Reference: Theory of Probability, M. E. Munroe, McGraw-Hill, 1951.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

SET: ☐ RADIANS

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA:

$R_Y \longrightarrow Z$ *
$R_\theta \longrightarrow Y$
$a \longrightarrow X$

PRESS: CONTINUE

DISPLAY

Trials ——— Z
P_E ——— Y
P_T ——— X

The program continues indefinitely.

*R_Y is an initializing value for the routine which generates the random values of Y. R_Y should be $0 \le R_Y \le 1$.

R_θ is a similar initializer for generating R_θ. R_θ should not equal R_Y. R_θ should be $0 \le R_\theta \le \pi$.

EXAMPLE

R_Y = .75
R_θ = .5
a = .5

Results after 2100 trials,

P_E = .312
P_T = .318

```
00  CLR   20
01  STP   41      ENTRY
02  PNT   45
03  PNT   45
04  XTO   23
05   d    17
06  RUP   22
07  XTO   23
08   a    13
09   2    02
0a   X    36
0b   π    56
0c  DIV   35
0d  YTO   40

10   c    16
11   π    56
12  RUP   22
13   +    33
14  DN    25
15  UP    27
16   X    36
17  DN    25
18  UP    27
19   X    36
1a  DN    25
1b  UP    27
1c   X    36
1d  DN    25

20  UP    27
21  INT   64
22   -    34
23  YTO   40
24   b    14
25   π    56
26   X    36
27  DN    25
28  SIN   70
29  UP    27
2a   d    17
2b   X    36
2c   a    13
2d  UP    27

30   π    56
31   +    33
32  DN    25
33  UP    27
34   X    36
35  DN    25
36  UP    27
37   X    36
38  DN    25
39  UP    27
3a   X    36
3b  DN    25
3c  UP    27
3d  INT   64

40   -    34
41  YTO   40
42   a    13
43  DN    25
44  X<Y   52
45   5    05
46   6    06
47  X=Y   50
48   5    05
49   6    06
4a   e    12
4b  UP    27
4c   1    01
4d   +    33

50  YTO   40
51   e    12
52   f    15
53  GTO   44
54   6    06
55   0    00
56   f    15
57  UP    27
58   1    01
59   +    33
5a  YTO   40
5b   f    15
5c   e    12
5d  XEY   30

60   +    33
61  UP    27
62  DN    25
63  XEY   30
64  DIV   35
65   c    16
66  PSE   57
67   b    14
68  RDN   31
69  GTO   44
6a   1    01
6b   1    01
6c  END   46
```

MEDIAN FINDER

This program determines the <u>median</u> of a set of n numbers (X_i). The median is defined as:

M = The middle value, or the arithmetic mean of the two middle values, of a set of numbers which have been arranged in order of magnitude.

Thus the median of the set { 8, 5, 4, 8, 10, 4, 3, 6, 8 } is 6 because when arranged in order { 3, 4, 4, 5, <u>6</u>, 8, 8, 8, 10 } 6 is the middle value.

This program determines the median M of a set of data <u>unordered</u> by magnitude.

The "median finding" portion of this program was written by Mr. E. J. Schmidt, ITT Federal Electric Corporation, Paramus, N. J. Various input-output modifications were made for user convenience.

Reference: Theory and Problems of Statistics, Murray Spiegel, McGraw-Hill Book Company, 1961, page 47.

USER INSTRUCTIONS

DEPRESS: X Y on the 9120A

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: GO TO (-)(0)(0)

PRESS: CONTINUE

DISPLAY

```
0 ———— Z
0 ———— Y
0 ———— X
```

ENTER DATA: $M_{guess} \longrightarrow X$

(M_{guess} can be any member of $\{X_i\}$ except $X_{guess} \geq X_{max}$, or $X_{guess} \leq X_{min}$)

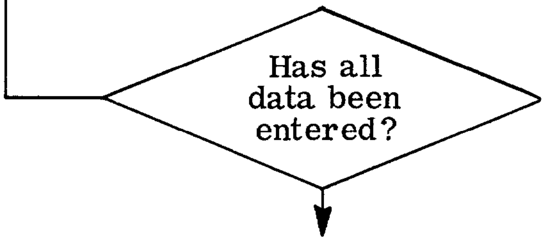
PRESS: CONTINUE

DISPLAY

```
0 ———— Z
i ———— Y
0 ———— X
```

ENTER DATA: $X_i \longrightarrow X$

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

```
M ———— Z
M ———— Y
M ———— X
```

USER INSTRUCTIONS (Con't)

or

DISPLAY

```
0 ———— Z
1 ———— Y      *
0 ———— X
```

To run another case

*If this display appears the value of M_{guess} was too close to X_{min} or X_{max}. A new value of M_{guess} was calculated by the program and the program is ready for re-entry of $\{X_i\}$. Return to the point of X_i entry, and repeat USER INSTRUCTIONS.

EXAMPLE

$\{X_i\} = \{ 8, 5, 4, 8, 10, 4, 3, 6, 8 \}$

Let $M_{guess} = 7$

Result: M = 6

Let $M_{guess} = 9$

Note that the DISPLAY:

```
0 ———— Z
1 ———— Y
0 ———— X
```

appears indicating that M_{guess} was too near X_{max} or X_{min}. But, by re-entering the data set:

Result: M = 6

STAT-PAC I-15

Step	Key	Code	X	Y	Z	Step	Key	Code	X	Y	Z	Step	Key	Code	X	Y	Z
00	CHS	32		Plus Page		40	DN	25				80	CNT	47			
01	X=Y	50				41	GTO	44				81	CNT	47			
02	5	05				42	0	00				82	CNT	47			
03	4	04				43	a	13				83	CNT	47			
04	4	04				44	c	16				84	CNT	47			
05	CHS	32				45	GTO	44				85	CNT	47			
06	X=Y	50				46	0	00				86	CNT	47			
07	5	05				47	a	13				87	CNT	47			
08	8	10				48	c	16				88	CNT	47			
09	a	13				49	UP	27				89	CNT	47			
0a	UP	27				4a	b	14				8a	CNT	47			
0b	UP	27				4b	+	33				8b	CNT	47			
0c	PNT	45	s			4c	2	02				8c	CNT	47			
0d	PNT	45				4d	DIV	35				8d	CNT	47			
10	GTO	44				50	DN	25				90	CNT	47			
11	-	34				51	GTO	44				91	CNT	47			
12	0	00				52	0	00				92	CNT	47			
13	0	00				53	a	13				93	CNT	47			
14	f	15				54	b	14				94	CNT	47			
15	GTO	44				55	GTO	44				95	CNT	47			
16	0	00				56	0	00				96	CNT	47			
17	a	13				57	a	13				97	CNT	47			
18	f	15				58	b	14				98	CNT	47			
19	UP	27				59	UP	27				99	CNT	47			
1a	e	12				5a	a	13				9a	CNT	47			
1b	+	33				5b	+	33				9b	CNT	47			
1c	2	02				5c	2	02				9c	CNT	47			
1d	DIV	35				5d	DIV	35				9d	CNT	47			
20	DN	25				60	DN	25				a0	CNT	47			
21	GTO	44				61	GTO	44				a1	CNT	47			
22	0	00				62	0	00				a2	CNT	47			
23	a	13				63	a	13				a3	CNT	47			
24	e	12				64	CNT	47				a4	CNT	47			
25	GTO	44				65	CNT	47				a5	CNT	47			
26	0	00				66	CNT	47				a6	CNT	47			
27	a	13				67	CNT	47				a7	CNT	47			
28	e	12				68	CNT	47				a8	CNT	47			
29	UP	27				69	CNT	47				a9	CNT	47			
2a	d	17				6a	CNT	47				aa	CNT	47			
2b	+	33				6b	CNT	47				ab	CNT	47			
2c	2	02				6c	CNT	47				ac	CNT	47			
2d	DIV	35				6d	CNT	47				ad	CNT	47			
30	DN	25				70	CNT	47									
31	GTO	44				71	CNT	47									
32	0	00				72	CNT	47									
33	a	13				73	CNT	47									
34	d	17				74	CNT	47									
35	GTO	44				75	CNT	47									
36	0	00				76	CNT	47									
37	a	13				77	CNT	47									
38	d	17				78	CNT	47									
39	UP	27				79	CNT	47									
3a	c	16				7a	CNT	47									
3b	+	33				7b	CNT	47									
3c	2	02				7c	CNT	47									
3d	DIV	35				7d	CNT	47									

```
b0  CNT  47
b1  CNT  47
b2  CNT  47
b3  CNT  47
b4  CNT  47
b5  CNT  47
b6  CNT  47
b7  CNT  47
b8  CNT  47
b9  CNT  47
ba  CNT  47
bb  CNT  47
bc  CNT  47
bd  CNT  47

c0  CNT  47
c1  CNT  47
c2  CNT  47
c3  CNT  47
c4  CNT  47
c5  CNT  47
c6  CNT  47
c7  CNT  47
c8  CNT  47
c9  CNT  47
ca  CNT  47
cb  CNT  47
cc  CNT  47
cd  CNT  47

d0  CNT  47
d1  CNT  47
d2  CNT  47
d3  CNT  47
d4  CNT  47
d5  CNT  47
d6  CNT  47
d7  CNT  47
d8  CNT  47
d9  CNT  47
da  CNT  47
db  CNT  47
dc  CNT  47
dd  CNT  47

00  CLR  20   Minus
              Page
01  STP  41
02  PNT  45   ENTRY
03  PNT  45
04  XTO  23
05   9   11
06   1   01
07  XEY  30
08  XTO  23
09   c   16
0a  XTO  23
0b   7   07
0c  XTO  23
0d   8   10

10  XTO  23
11   -   34
12   f   15
13  EEX  26
14   9   11
15  XTO  23
16   d   17
17   0   00
18  STP  41   ENTRY
19  IFG  43
1a   8   10
1b   1   01
1c  PNT  45
1d  PNT  45

20  YTO  40
21   -   34
22   f   15
23  YE   24
24   9   11
25  YTO  40
26   9   11
27  X>Y  53
28   5   05
29   1   01
2a  RDN  31
2b  YE   24
2c   7   07
2d   1   01

30   +   33
31  YTO  40
32   7   07
33   c   16
34  RUP  22
35  XEY  30
36  X<Y  52
37   6   06
38   b   14
39   b   14
3a  X<Y  52
3b   6   06
3c   d   17
3d   a   13

40  X<Y  52
41   7   07
42   1   01
43   0   00
44  UP   27
45   1   01
46  UP   27
47  XFR  67
48   -   34
49   f   15
4a   +   33
4b   0   00
4c  GTO  44
4d   1   01

50   8   10
51  RDN  31
52  YE   24
53   8   10
54   1   01
55   +   33
56  YTO  40
57   8   10
58   d   17
59  RUP  22
5a  XEY  30
5b  X>Y  53
5c   7   07
5d   6   06

60   e   12
61  X>Y  53
62   7   07
63   8   10
64   f   15
65  X>Y  53
66   7   07
67   a   13
68  GTO  44
69   4   04
6a   3   03
6b  YE   24
6c   c   16
6d  YE   24

70   b   14
71  YEX  24
72   a   13
73  GTO  44
74   4   04
75   3   03
76  YE   24
77   d   17
78  YE   24
79   e   12
7a  YE   24
7b   f   15
7c  GTO  44
7d   4   04
```

Step	Key	Code	Step	Key	Code
80	3	03	c0	ARC	72
81	XFR	67	c1	1	01
82	7	07	c2	X=Y	50
83	YE	24	c3	+	33
84	8	10	c4	3	03
85	-	34	c5	4	04
86	5	05	c6	ARC	72
87	CHS	32	c7	0	00
88	X<Y	52	c8	X=Y	50
89	9	11	c9	+	33
8a	9	11	ca	3	03
8b	YEX	24	cb	8	10
8c	a	13	cc	ARC	72
8d	1	01	cd	1	01
90	-	34	d0	CHS	32
91	YTO	40	d1	X=Y	50
92	9	11	d2	+	33
93	0	00	d3	4	04
94	UP	27	d4	4	04
95	RUP	22	d5	ARC	72
96	GTO	44	d6	2	02
97	0	00	d7	CHS	32
98	4	04	d8	X=Y	50
99	CHS	32	d9	+	33
9a	X>Y	53	da	4	04
9b	a	13	db	8	10
9c	6	06	dc	ARC	72
9d	YE	24	dd	3	03
a0	f	15			
a1	1	01			
a2	+	33			
a3	GTO	44			
a4	9	11			
a5	1	01			
a6	X=Y	50			
a7	+	33			
a8	1	01			
a9	4	04			
aa	ARC	72			
ab	4	04			
ac	X=Y	50			
ad	+	33			
b0	1	01			
b1	8	10			
b2	ARC	72			
b3	3	03			
b4	X=Y	50			
b5	+	33			
b6	2	02			
b7	4	04			
b8	ARC	72			
b9	2	02			
ba	X=Y	50			
bb	+	33			
bc	2	02			
bd	8	10			

UNIVERSAL FOUR OPERATIONS PROGRAM

This program will perform a given arithmetic operation on a list of numbers. It will add, subtract, multiply or divide each number in the list by a given constant designated by the operator. The user enters the constant and the type of operation in a single entry. The program saves time in operations such as unit conversions, and it is not necessary to write a special program for each type of operation.

Although printout of each result is included, the program may be used without a printer by changing the PRINT commands in steps (2)(4), (3)(6), (4)(6) and (5)(6) to CONTINUE commands.

This program was written by Monsieur Claude Cardot, of Laboratoires De Marcoussis, Centre De Recherches De La Compangnie Général D'Electricité, Marcoussis, France.

USER INSTRUCTIONS	EXAMPLES

PRESS: Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: END

PRESS: CONTINUE

DISPLAY

0 ——— Z
1 ——— Y
0 ——— X

ENTER DATA: K (operating number) → X

PRESS: +, -, x, or ÷ to designate desired operation

PRESS: CONTINUE

DISPLAY

* ——— Z
K ——— Y
1 ——— X

*Denotes Insignificant Display

ENTER DATA: $X_i \rightarrow X$

PRESS: CONTINUE

DISPLAY

Result ——— Z
K ——— Y
X_i ——— X

ENTER DATA: Next $X_i \rightarrow X$

PRESS: CONTINUE

For new operating quantity K and/or new type of operation.

1. $K = \pi \rightarrow X$

Operation: ÷

X_i	RESULT
5.770	1.837
3.438	1.094
7.123	2.267
4.999	1.591
6.283	2.000

2. $K = \sin 65^\circ \rightarrow X$

Operation: x (Multiply)

X_i	RESULT
231.4	209.72
579.6	525.30
788.9	714.99
187.3	169.75
422.7	383.10

STAT-PAC I-16

```
00  CLR   20
01  1     01
02  RUP   22
03  STP   41
04  AC+   60
05  -     34
06  1     01
07  X=Y   50
08  2     02
09  c     16
0a  RCL   61
0b  +     33
0c  1     01
0d  X=Y   50

10  3     03
11  b     14
12  RCL   61
13  UP    27
14  1     01
15  XEY   30
16  DIV   35
17  DN    25
18  X=Y   50
19  4     04
1a  b     14
1b  RCL   61
1c  RUP   22
1d  1     01

20  STP   41
21  X     36
22  XEY   30
23  RDN   31
24  PNT   45
25  STP   41
26  RUP   22
27  f     15
28  RDN   31
29  GTO   44
2a  2     02
2b  1     01
2c  RCL   61
2d  RUP   22

30  1     01
31  STP   41
32  +     33
33  RUP   22
34  f     15
35  XEY   30
36  PNT   45
37  STP   41
38  GTO   44
39  3     03
3a  2     02
3b  RCL   61
3c  RUP   22
3d  1     01

40  STP   41
41  XEY   30
42  UP    27
43  DN    25
44  -     34
45  RUP   22
46  PNT   45
47  STP   41
48  GTO   44
49  4     04
4a  1     01
4b  RCL   61
4c  RUP   22
4d  1     01

50  STP   41
51  XEY   30
52  UP    27
53  DN    25
54  DIV   35
55  RUP   22
56  PNT   45
57  STP   41
58  GTO   44
59  5     05
5a  1     01
5b  END   46
```

SECTION II DISTRIBUTION FUNCTIONS

NORMAL PROBABILITY INTEGRAL

This program computes the integral of the standardized normal distribution

$$P(X) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{X} e^{-\frac{1}{2}z^{2}} dz \qquad (X \leq 7)$$

The following equations are used

$$P(X) = \frac{\mathrm{erf}\left(\frac{X}{\sqrt{2}}\right)}{2} + \frac{1}{2}$$

where

$$\mathrm{erf}(X) = \frac{2}{\sqrt{\pi}} e^{-X^{2}} \sum_{n=0}^{\infty} \frac{2^{n}}{1 \cdot 3 \cdots (2n+1)} X^{2n+1}$$

Reference: Handbook of Mathematical Functions
by Abramowitz and Stegun

National Bureau of Standards 1964

USER INSTRUCTIONS

DEPRESS: X Y on the 9120A
PRESS: END
ENTER PROGRAM
PRESS: CONTINUE

→ DISPLAY

0 —— Z
0 —— Y
1 —— X

ENTER DATA: X → X

PRESS: CONTINUE

DISPLAY

0 —— Z
P(X) —— Y
X —— X

To run another case

EXAMPLES

X = .3	P(X) = .618
X = -.3	P(X) = .382
X = 0	P(X) = .5
X = 3	P(X) = .999

STAT-PAC II-1

```
00   CLR   20
01   1     01
02   STP   41    ENTRY
03   CNT   47
04   X=Y   50
05   0     00
06   a     13
07   GTO   44
08   1     01
09   2     02
0a   .     21
0b   5     05
0c   XEY   30
0d   GTO   44

10   6     06
11   0     00
12   XTO   23
13   c     16
14   X<Y   52
15   SFL   54
16   CHS   32
17   UP    27
18   2     02
19   √     76
1a   DIV   35
1b   DN    25
1c   UP    27
1d   X     36

20   UP    27
21   DN    25
22   RUP   22
23   EXP   74
24   DIV   35
25   2     02
26   X     36
27   π     56
28   √     76
29   DIV   35
2a   1     01
2b   YTO   40
2c   a     13
2d   RUP   22

30   DIV   35
31   YTO   40
32   b     14
33   2     02
34   DIV   35
35   EEX   26
36   1     01
37   0     00
38   CHS   32
39   RUP   22
3a   YE    24
3b   b     14
3c   RDN   31
3d   +     33

40   RDN   31
41   DIV   35
42   RDN   31
43   YE    24
44   a     13
45   +     33
46   YE    24
47   a     13
48   YE    24
49   b     14
4a   X>Y   53
4b   3     03
4c   a     13
4d   a     13

50   UP    27
51   2     02
52   DIV   35
53   .     21
54   5     05
55   +     33
56   1     01
57   IFG   43
58   XEY   30
59   -     34
5a   CLX   37
5b   RDN   31
5c   XEY   30
5d   c     16

60   PNT   45    S
61   PNT   45
62   GTO   44
63   0     00
64   0     00
65   END   46
```

INVERSE NORMAL INTEGRAL

This program determines the value of X such that

$$\int_{-\infty}^{X} \frac{e^{-\frac{\lambda^2}{2}}}{\sqrt{2}\,\pi} \, d\lambda = P$$

where P is given. This program performs the inverse of Program STAT-PAC II-1 Normal Probability Integral. P represents an area under the normal curve where X is the upper limit of integration.

The following equations are used:

$$P(X) = \frac{\operatorname{erf}\left(\frac{X}{\sqrt{2}}\right)}{2} + \frac{1}{2}$$

$$\operatorname{erf}(X) = \frac{2}{\sqrt{\pi}} e^{-X^2} \sum_{n=0}^{\infty} \frac{2^n}{1 \cdot 3 \ldots \cdot (2n+1)} X^{2n+1}$$

Reference: Handbook of Mathematical Functions, Abramowitz and Stegun,
National Bureau of Standards, 1964.

USER INSTRUCTIONS

PRESS: X on the 9120A

PRESS: END

ENTER PROGRAM

→ PRESS: CONTINUE

DISPLAY

0 → Z
0 → Y
0 → X

ENTER DATA: P → X

PRESS: CONTINUE

DISPLAY

X — Z
X — Y
X — X

To run another case

To observe the program converging on the proper value of X, place a PAUSE in step (6)(7).

EXAMPLES

1. P = .5000, result, X = 0.0
2. P = .841344746, result, X = 1.0
3. P = .999999713, result, X = 5.0
4. P = .022750132, result, X = -2.0

STAT-PAC II-2

```
00  CLR  20
01  STP  41      ENTRY
02  PNT  45
03  XTO  23
04  b    14
05  UP   27
06  1    01
07  XTO  23
08  c    16
09  CHS  32
0a  6    06
0b  .    21
0c  5    05
0d  YTO  40
10  d    17
11  UP   27
12  c    16
13  +    33
14  YTO  40
15  d    17
16  CLX  37
17  XEY  30
18  X<Y  52
19  SFL  54
1a  CHS  32
1b  UP   27
1c  2    02
1d  √    76
20  DIV  35
21  DN   25
22  UP   27
23  X    36
24  UP   27
25  DN   25
26  RUP  22
27  EXP  74
28  DIV  35
29  2    02
2a  X    36
2b  π    56
2c  √    76
2d  DIV  35
30  1    01
31  YTO  40
32  f    15
33  RUP  22
34  DIV  35
35  YTO  40
36  e    12
37  2    02
38  DIV  35
39  EEX  26
3a  1    01
3b  0    00
3c  CHS  32
3d  RUP  22
40  YE   24
41  e    12
42  RDN  31
43  +    33
44  RDN  31
45  DIV  35
46  RDN  31
47  YE   24
48  f    15
49  +    33
4a  YE   24
4b  f    15
4c  YE   24
4d  e    12
50  X>Y  53
51  4    04
52  0    00
53  f    15
54  UP   27
55  2    02
56  DIV  35
57  .    21
58  5    05
59  +    33
5a  1    01
5b  IFG  43
5c  XEY  30
5d  -    34
60  CLX  37
61  RDN  31
62  XEY  30
63  b    14
64  UP   27
65  DN   25
66  -    34
67  CNT  47
68  UP   27
69  DN   25
6a  |Y|  55
6b  EEX  26
6c  CHS  32
6d  9    11
70  X>Y  53
71  9    11
72  6    06
73  DN   25
74  0    00
75  X>Y  53
76  8    10
77  c    16
78  UP   27
79  c    16
7a  X>Y  53
7b  8    10
7c  2    02
7d  GTO  44
80  1    01
81  0    00
82  UP   27
83  CHS  32
84  .    21
85  1    01
86  X    36
87  YTO  40
88  c    16
89  GTO  44
8a  1    01
8b  0    00
8c  UP   27
8d  c    16
90  X<Y  52
91  8    10
92  2    02
93  GTO  44
94  1    01
95  0    00
96  d    17
97  UP   27
98  UP   27
99  PNT  45
9a  PNT  45
9b  END  46

P(X)
```

BINOMIAL DISTRIBUTION

This program evaluates the Binomial (Bernoulli) distribution

$$B(r, n, p) = {}_nC_r\ (p)^r\ (1-p)^{n-r}$$

where:

p = probability of a success

r = number of successes

n = number of trials

Reference: Probabilistic Reliability, Martin Shooman, McGraw-Hill, 1968.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

→ PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA: Probability

p → Z
n → Y
r → X

PRESS: CONTINUE

DISPLAY

B(n, r, p) — Z
n ——— Y
r ——— X

To run another case

EXAMPLE

What is the probability of twice obtaining obtaining a 3 on six rolls of a die?

$p = \frac{1}{6}$
$n = 6$
$r = 2$

$B(2,\ 6,\ \frac{1}{6}) = .2009388$

```
00  CLR  20
01  STP  41     ENTRY
02  XTO  23
03   d   17
04  YTO  40
05   c   16
06   1   01
07  AC+  60
08  RDN  31
09  YTO  40
0a   b   14
0b   f   15
0c  XEY  30
0d   d   17

10  X<Y  52
11   1   01
12   d   17
13  RCL  61
14  DIV  35
15  DN   25
16   X   36
17   1   01
18  UP   27
19  CHS  32
1a  AC-  63
1b  GTO  44
1c   0   00
1d   b   14

20  LN   65
21  XEY  30
22   d   17
23   X   36
24  DN   25
25  EXP  74
26   X   36
27   1   01
28  UP   27
29   b   14
2a   -   34
2b   e   12
2c  XEY  30
2d  LN   65

30   X   36
31  DN   25
32  EXP  74
33   X   36
34   c   16
35  UP   27
36   d   17
37  PNT  45
38  PNT  45
39  END  46
```

p

n

r

n

1

POISSON DISTRIBUTION

The Poisson density function is defined by $f(n; \lambda) = \frac{\lambda^n e^{-\lambda}}{n!}$ where λ may be estimated by

$\hat{\lambda}$ = the expected value of X_i (n = 0, 1, 2, ...)

$$\sum_{n=0}^{\infty} \frac{\lambda^n e^{-\lambda}}{n!} = e^{-\lambda} \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} = e^{-\lambda} e^{\lambda} = 1$$

The Poisson density function is a discrete density which is used to evaluate such things as component failure probabilities. It is also used to approximate the Binomial Distribution when the number of events (N) is large and the probability that an event will happen (p) is small i.e., a general rule of thumb is for $p \leq .1$ and $\hat{\lambda} = Np \leq 5$.

Reference: Mathematical Statistics
by John E. Freund

1962 Prentice-Hall

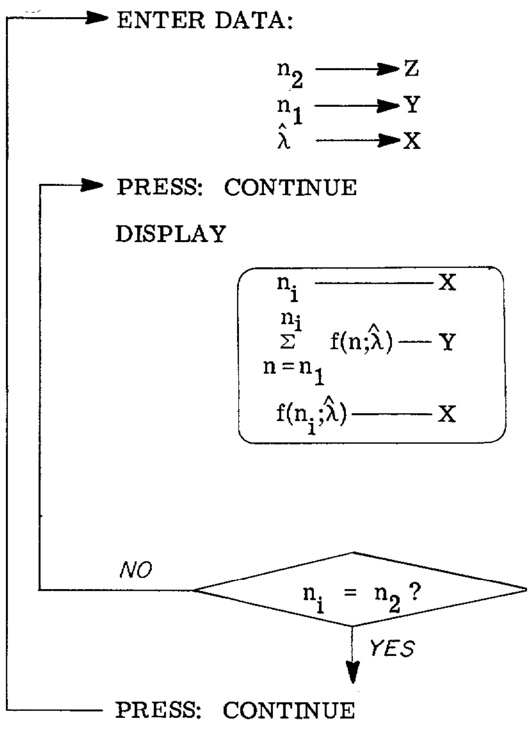
USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE



EXAMPLE

A Poisson distribution is given by

$$f(n;.72) = \frac{(.72)^n e^{-.72}}{n!}$$

Find: a) $f(0;.72)$

$\hat{\lambda} = .72,\ n_1 = 0,\ n_2 = 0$

$f(0;.72) = .48675226$

b) $f(3;.72)$

$\hat{\lambda} = .72,\ n_1 = 3,\ n_2 = 3$

$f(3;.72) = .03027988$

If 3% of the electric bulbs manufactured by a company are defective find the probability that in a sample of 100 bulbs

a) Between 1 and 3 bulbs will be defective.

$$\left.\begin{array}{l}\text{Expected no.}\\ \text{of bulbs that}\\ \text{are defective}\end{array}\right\} = \hat{\lambda} = (.03)(100) = 3$$

$$\sum_{n=1}^{3} f(n;3) = .59744482$$

where $\hat{\lambda} = 3,\ n_1 = 1,\ n_2 = 3$

b) Less than or equal to 2 bulbs are defective

$$\sum_{n=0}^{2} f(n;3) = .42319008$$

where $\hat{\lambda} = 3,\ n_1 = 0,\ n_2 = 2$

STAT-PAC II-4

```
00   CLR   20
01   STP   41        ENTRY
02   PNT   45
03   XTO   23
04    c    16
05   YTO   40
06    b    14
07   RUP   22
08   XTO   23
09    d    17
0a   XEY   30
0b   CHS   32
0c   EXP   74
0d   UP    27

10    c    16
11   LN    65
12   UP    27
13    b    14
14    X    36
15   XEY   30
16   EXP   74
17   RUP   22
18    X    36
19   YTO   40
1a    a    13
1b    b    14
1c   UP    27
1d   UP    27

20    1    01
21    -    34
22   X>Y   53
23    2    02
24    b    14
25   RDN   31
26    X    36
27   RUP   22
28   GTO   44
29    2    02
2a    1    01
2b    a    13
2c   RUP   22
2d   UP    27

30    0    00
31   XEY   30
32   X=Y   50
33   ARC   72
34    1    01
35   XEY   30
36   DN    25
37   DIV   35
38   XEY   30
39   UP    27
3a   AC+   60
3b    b    14
3c   RDN   31
3d   RCL   61

40   STP   41
41   PNT   45
42   PNT   45
43   CLX   37
44   XTO   23
45    f    15
46    d    17
47   RUP   22
48   X=Y   50
49    0    00
4a    0    00
4b   UP    27
4c    1    01
4d    +    33

50   YTO   40
51    b    14
52    c    16
53   GTO   44
54    0    00
55    b    14
56   END   46
```

χ^2 CHI SQUARE DISTRIBUTION

This program evaluates the Chi Square Distribution Integral for a given value of χ^2 and ν degrees of freedom; i.e., the program evaluates

$$P(\chi^2,\nu) = \frac{1}{2^{\nu/2}\Gamma(\frac{\nu}{2})} \int_0^{\chi^2} T^{\nu/2-1} e^{-T/2}\, dT \qquad 0 \leq \chi^2$$

The series approximation used to evaluate the integral is

$$P(\chi^2,\nu) = (\tfrac{1}{2}\chi^2)^{\nu/2} \frac{e^{-\chi^2/2}}{\Gamma(\frac{\nu+2}{2})} \left\{1 + \sum_{r=1}^{\infty} \frac{\chi^{2r}}{(\nu+2)(\nu+4)\cdots(\nu+2r)}\right\}$$

Reference: Handbook of Mathematical Functions
by Abramowitz and Stegun

National Bureau of Standards 1964

ER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

ENTER DATA:

$\chi^2 \longrightarrow Y$
$\nu \longrightarrow X$

PRESS: CONTINUE

DISPLAY

$P(\chi^2, \nu)$ — Z
χ^2 — Y
ν — X

EXAMPLES

$P(7.88, 1) = .995$

$P(10.6, 2) = .995$

$P(12.8, 17) = .251$

GENERAL FORM

$P(X^2, \nu)$

```
00  CLR   20
01  STP   41   ENTRY
02  CNT   47
03  XTO   23
04   d    17
05  YTO   40
06   c    16
07   2    02
08  DIV   35
09  YTO   40
0a   f    15
0b  DN    25
0c  LN    65
0d  UP    27

10   d    17
11  UP    27
12   1    01
13   +    33
14   2    02
15  DIV   35
16  YTO   40
17   e    12
18  DN    25
19   X    36
1a  DN    25
1b  EXP   74
1c  UP    27
1d   f    15

20  CHS   32
21  EXP   74
22   X    36
23   e    12
24  UP    27
25  INT   64
26  X=Y   50
27   4    04
28   4    04
29   2    02
2a  LN    65
2b   X    36
2c  DN    25
2d  EXP   74

30   X    36
31   c    16
32   √    76
33  DIV   35
34   d    17
35  DIV   35
36  UP    27
37   2    02
38   -    34
39   0    00
3a  X=Y   50
3b   6    06
3c   a    13
3d  DN    25

40  DIV   35
41  GTO   44
42   3    03
43   6    06
44   4    04
45  LN    65
46   X    36
47  DN    25
48  EXP   74
49   X    36
4a   d    17
4b  UP    27
4c   2    02
4d   X    36

50  DN    25
51  DIV   35
52  UP    27
53   4    04
54   -    34
55  X<Y   52
56   5    05
57   0    00
58   2    02
59  X>Y   53
5a   6    06
5b   1    01
5c  DN    25
5d  DIV   35

60  UP    27
61   c    16
62  |Y|   55
63  DIV   35
64   π    56
65  DIV   35
66  DN    25
67   √    76
68   X    36
69  UP    27
6a   d    17
6b  XTO   23
6c   f    15
6d   0    00

70  XTO   23
71   e    12
72   1    01
73  XEY   30
74   2    02
75  AC+   60
76   c    16
77   X    36
78   f    15
79  DIV   35
7a  EEX   26
7b   9    11
7c  CHS   32
7d  X<Y   52

80   7    07
81   4    04
82   e    12
83   +    33
84  DN    25
85   X    36
86   c    16
87  UP    27
88   d    17
89  PNT   45
8a  PNT   45
8b  END   46
```

HYPERGEOMETRIC DISTRIBUTION

This program computes various functions associated with the hypergeometric distribution which is given by:

$$f(X) = \frac{\binom{k}{X}\binom{N-k}{n-X}}{\binom{N}{n}} \qquad X = 0,\ 1,\ 2,\ \ldots,\ \ [\,n,\ k\,]$$

where [n, k] is the smaller of n and k.

N - is the number of items (successes and failures) in a finite population.

n - is the number of items drawn in sample without replacement from the population.

k - is the number of failures in the finite population.

X - is the number of failures in the sample.

f(X) represents the probability of exactly X successes and n - X failures in the sample of n items.

The function

$$F(X) = \sum_{r=0}^{X} f(r)$$

yields the probability of X or fewer failures in the sample of n items.

The mean and variance of the distribution are given by

$$\mu = \frac{nk}{N}$$

$$\sigma^2 = \frac{k(N-k)n(N-n)}{N^2(N-1)}$$

This program determines f(X), F(X), μ, σ, and σ^2. It uses program STAT-PAC I-8, COMBINATIONS, as a subroutine.

Reference: Handbook of Tables for Probability and Statistics, Chemical Rubber Co. 1966.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM: Side A followed by Side B.

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
1 ——— X

ENTER DATA:

N ——→ Z
n ——→ Y
k ——→ X

PRESS: CONTINUE

DISPLAY

F(X) ——— Z
f(X) ——— Y
X ——— X

(X = 0, 1, 2, . . .)

When X exceeds [n, k] the final display appears:

DISPLAY

σ^2 ——— Z
σ ——— Y
μ ——— X

To run another case

EXAMPLE

N = 5
n = 3
k = 2

.1000	Z	
.1000	Y	X = 0
0.	X	
.7000	Z	
.6000	Y	X = 1
1.	X	
1.0000	Z	
.3000	Y	X = 2
2.	X	
.3600	Z	
.6000	Y	(μ, σ, σ^2)
1.2000	X	

STAT-PAC II-6 9100B ONLY

00	CLR	20	Plus Page	40	UP	27		80	CNT	47
01	XTO	23		41	a	13		81	CNT	47
02	c	16		42	GTO	44		82	CNT	47
03	XTO	23		43	SUB	77		83	CNT	47
04	7	07		44	-	34		84	CNT	47
05	1	01		45	0	00		85	CNT	47
06	STP	41	ENTRY	46	0	00		86	CNT	47
07	PNT	45		47	XFR	67		87	CNT	47
08	PNT	45		48	8	10		88	CNT	47
09	XTO	23		49	XEY	30		89	CNT	47
0a	b	14		4a	DIV	35		8a	CNT	47
0b	YTO	40		4b	XFR	67		8b	CNT	47
0c	a	13		4c	7	07		8c	CNT	47
0d	RDN	31		4d	XEY	30		8d	CNT	47
10	YTO	40		50	+	33		90	CNT	47
11	9	11		51	YTO	40		91	CNT	47
12	UP	27		52	7	07		92	CNT	47
13	c	16		53	UP	27		93	CNT	47
14	X>Y	53		54	c	16		94	CNT	47
15	6	06		55	PNT	45	S	95	CNT	47
16	2	02		56	PNT	45		96	CNT	47
17	b	14		57	UP	27		97	CNT	47
18	UP	27		58	1	01		98	CNT	47
19	c	16		59	+	33		99	CNT	47
1a	X>Y	53		5a	YTO	40		9a	CNT	47
1b	6	06		5b	c	16		9b	CNT	47
1c	2	02		5c	a	13		9c	CNT	47
1d	GTO	44		5d	GTO	44		9d	CNT	47
20	SUB	77		60	1	01		a0	CNT	47
21	-	34		61	2	02		a1	CNT	47
22	0	00		62	XFR	67		a2	CNT	47
23	0	00		63	9	11		a3	CNT	47
24	YTO	40		64	UP	27		a4	CNT	47
25	8	10		65	b	14		a5	CNT	47
26	XFR	67		66	-	34		a6	CNT	47
27	9	11		67	X	36		a7	CNT	47
28	UP	27		68	a	13		a8	CNT	47
29	b	14		69	X	36		a9	CNT	47
2a	-	34		6a	GTO	44		aa	CNT	47
2b	a	13		6b	-	34		ab	CNT	47
2c	UP	27		6c	2	02		ac	CNT	47
2d	c	16		6d	3	03		ad	CNT	47
30	-	34		70	CNT	47				
31	DN	25		71	CNT	47				
32	GTO	44		72	CNT	47				
33	SUB	77		73	CNT	47				
34	-	34		74	CNT	47				
35	0	00		75	CNT	47				
36	0	00		76	CNT	47				
37	XFR	67		77	CNT	47				
38	8	10		78	CNT	47				
39	X	36		79	CNT	47				
3a	YTO	40		7a	CNT	47				
3b	8	10		7b	CNT	47				
3c	XFR	67		7c	CNT	47				
3d	9	11		7d	CNT	47				

Intermediate Results

N

n

K

X

F(X)

```
b0  CNT  47     Plus
                Page
b1  CNT  47
b2  CNT  47
b3  CNT  47
b4  CNT  47
b5  CNT  47
b6  CNT  47
b7  CNT  47
b8  CNT  47
b9  CNT  47
ba  CNT  47
bb  CNT  47
bc  CNT  47
bd  CNT  47

c0  CNT  47
c1  CNT  47
c2  CNT  47
c3  CNT  47
c4  CNT  47
c5  CNT  47
c6  CNT  47
c7  CNT  47
c8  CNT  47
c9  CNT  47
ca  CNT  47
cb  CNT  47
cc  CNT  47
cd  CNT  47

d0  CNT  47
d1  CNT  47
d2  CNT  47
d3  CNT  47
d4  CNT  47
d5  CNT  47
d6  CNT  47
d7  CNT  47
d8  CNT  47
d9  CNT  47
da  CNT  47
db  CNT  47
dc  CNT  47
dd  CNT  47
```

```
00  UP    27
01   0    00
02  XTO   23
03   f    15
04  XTO   23
05   e    12
06  RDN   31
07  XTO   23
08   d    17
09   1    01
0a  AC+   60
0b  RDN   31
0c   f    15
0d  XEY   30

10   d    17
11  X<Y   52
12   2    02
13   1    01
14  RCL   61
15  DIV   35
16  DN    25
17   X    36
18   1    01
19  UP    27
1a  CHS   32
1b  AC-   63
1c  GTO   44
1d   0    00

20   c    16
21  DN    25
22  RTN   77
23  UP    27
24  XFR   67
25   9    11
26  XEY   30
27   -    34
28  DN    25
29   X    36
2a  XFR   67
2b   9    11
2c  DIV   35
2d  DIV   35

30  UP    27
31   1    01
32   -    34
33  DN    25
34  DIV   35
35   b    14
36  UP    27
37   a    13
38   X    36
39  XFR   67
3a   9    11
3b  DIV   35
3c  DN    25
3d  XEY   30
```

Minus
Page

```
40   √    76
41  XEY   30
42  PNT   45   s
43  PNT   45
44  PNT   45
45  PNT   45
46  PNT   45
47  PNT   45
48  GTO   44
49   +    33
4a   0    00
4b   0    00
4c  END   46
```

t-DISTRIBUTION

This program evaluates the t-distribution integral for a given value of t and γ degrees of freedom; i.e., the program evaluates:

$$P(t, \gamma) = \int_{-\infty}^{t} \frac{\Gamma(\frac{\gamma+1}{2})\ (1+\frac{X^2}{\gamma})^{-\frac{\gamma+1}{2}}}{\sqrt{\gamma\pi}\quad \Gamma(\frac{\gamma}{2})}\ dx$$

The series used to evaluate the integral are:

γ EVEN

$$P(t,\gamma) = \sin\theta \left\{1+\frac{1}{2}\cos^2\theta+\frac{1\cdot 3}{2\cdot 4}\cos^4\theta+\ldots+\frac{1\cdot 3\ \ldots\ (\gamma-3)}{2\cdot 4\ \ldots\ (\gamma-2)}\cos^{\gamma-2}\theta\right\}$$

γ ODD

$$P(t,\gamma) = \frac{2\theta}{\pi} + \frac{2\cos\theta}{\pi}\left[\sin\theta\left\{1+\frac{2}{3}\cos^3\theta+\ldots+\frac{2\cdot 4\ \ldots\ (\gamma-3)}{1\cdot 3\ \ldots\ (\gamma-2)}\cos^{\gamma-2}\theta\right\}\right]$$

where:

$$\theta = \tan^{-1}\frac{t}{\sqrt{\gamma}}$$

Reference: Handbook of Mathematical Functions, Abramowitz & Stegun, National Bureau of Standards (1964).

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

SET: ☐ RADIANS

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA:

(t statistic) t → Y
(degrees of freedom) γ → X

PRESS: CONTINUE

DISPLAY

$P(t,\gamma)$ ——— Z
γ ——— Y
t ——— X

To run another case

EXAMPLE

General Form:

$P(t,\gamma)$

A) P(2.201, 11) = 0.975
B) P(2.5 , 23) = 0.99
C) P(0.26 , 10) = 0.60

STAT-PAC II-7

```
00  CLR  20
01  STP  41        ENTRY
02  XTO  23
03   c   16
04  YTO  40
05   d   17
06  UP   27
07  RDN  31
08   √   76
09  DIV  35
0a  DN   25
0b  ARC  72
0c  TAN  71
0d  XTO  23

10   a   13
11  COS  73
12  UP   27
13   X   36
14  YTO  40
15   b   14
16  CLX  37
17  RDN  31
18   1   01
19  X=Y  50
1a   5   05
1b   5   05
1c  SFL  54
1d  XEY  30

20  UP   27
21   2   02
22  X=Y  50
23   4   04
24   b   14
25  IFG  43
26  UP   27
27  DN   25
28  DIV  35
29  DN   25
2a  UP   27
2b  INT  64
2c  X=Y  50
2d  SFL  54

30   .   21
31  DN   25
32   3   03
33   -   34
34  UP   27
35  DN   25
36   1   01
37   +   33
38  RDN  31
39  AC+  60
3a  RCL  61
3b  DIV  35
3c   b   14
3d   X   36
```

```
40  DN   25
41   X   36
42   1   01
43   +   33
44  UP   27
45  AC-  63
46  AC-  63
47   f   15
48  X>Y  53
49   3   03
4a   a   13
4b  DN   25
4c   a   13
4d  SIN  70

50   X   36
51  IFG  43
52   6   06
53   3   03
54  UP   27
55  DN   25
56   a   13
57  COS  73
58   X   36
59   a   13
5a   +   33
5b   2   02
5c  UP   27
5d   π   56

60  DIV  35
61  DN   25
62   X   36
63  CLX  37
64  UP   27
65  DN   25
66   1   01
67   -   34
68  |Y|  55
69   2   02
6a  DIV  35
6b  CLX  37
6c  RDN  31
6d   +   33

70   c   16
71  UP   27
72   d   17
73  PNT  45
74  CNT  47
75  END  46
```

θ

Y

t

BETA DISTRIBUTION

This program determines the Beta Distribution which is given by:

$$f(X) = \frac{\Gamma(\alpha + \beta + 2)\ X^{\alpha}\ (1 - X)^{\beta}}{\Gamma(\alpha + 1)\ \Gamma\ (\beta + 1)}$$

where

$0 < X < 1$

$-1 < \beta$

$-1 < \alpha$

The program also calculates the mean (μ) and variance ($\sigma 2$) of the distribution. These are given by:

$$\mu = \frac{\alpha + 1}{\alpha + \beta + 2}$$

$$\sigma^2 = \frac{(\alpha + 1)(\beta + 1)}{(\alpha + \beta + 2)^2 (\alpha + \beta + 3)}$$

The program uses program 09100-70024, GAMMA FUNCTION, as a subroutine thus restricting α and β to values less than 68.

Reference: Handbook of Tables for Probability and Statistics, Chemical Rubber Company, 1966.

USER INSTRUCTIONS

PRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM: Side A followed by Side B.

→ PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
1 ——— X

ENTER DATA:

$\beta \longrightarrow$ Z
$\alpha \longrightarrow$ Y
X $\longrightarrow$ Z

PRESS: CONTINUE

DISPLAY

f(X) ——— Z
$\sigma 2$ ——— Y
μ ——— X

To run another case

EXAMPLE

$\beta = .5$
$\alpha = .5$
$X = .5$

$f(X) = 1.2732$
$\sigma^2 = .0625$
$\mu = .5$

Error Cases

If $\beta < -1$

DISPLAY

β ——— Z
-1 ——— Y
-1 ——— X

If $\alpha < -1$

DISPLAY

α ——— Z
-1 ——— Y
-1 ——— X

If $X < 0$

DISPLAY

X ——— Z
0 ——— Y
0 ——— X

If $X > 1$

DISPLAY

X ——— Z
1 ——— Y
1 ——— X

If an error display occurs:

PRESS: END

PRESS: CONTINUE

ENTER DATA:

```
00  CLR  20    Plus    40  UP   27    80  6    06
01   1   01    Page    41  a    13    81  1    01
02  STP  41    ENTRY   42  +    33    82  -    34
03  PNT  45            43  b    14    83  YTO  40
04  PNT  45            44  +    33    84  d    17
05  XTO  23            45  GTO  44    85  UP   27
06   c   16            46  SUB  77    86  DN   25
07  YTO  40            47  8    10    87  INT  64
08   b   14            48  1    01    88  -    34
09  DN   25            49  XFR  67    89  CLX  37
0a  YTO  40            4a  9    11    8a  GTO  44
0b   a   13            4b  X    36    8b  -    34
0c  UP   27            4c  YTO  40    8c  0    00
0d   1   01            4d  9    11    8d  0    00

10  CHS  32            50  1    01    90  CNT  47
11  X>Y  53            51  UP   27    91  CNT  47
12  UP   27            52  b    14    92  CNT  47
13  STP  41            53  +    33    93  CNT  47
14  XEY  30            54  CLX  37    94  CNT  47
15  DN   25            55  XTO  23    95  CNT  47
16  X>Y  53            56  e    12    96  CNT  47
17  UP   27            57  XTO  23    97  CNT  47
18  STP  41            58  f    15    98  CNT  47
19   c   16            59  GTO  44    99  CNT  47
1a  UP   27            5a  SUB  77    9a  CNT  47
1b   0   00            5b  8    10    9b  CNT  47
1c  X>Y  53            5c  1    01    9c  CNT  47
1d  UP   27            5d  XFR  67    9d  CNT  47

20  STP  41            60  9    11    a0  CNT  47
21   1   01            61  XEY  30    a1  CNT  47
22  X<Y  52            62  DIV  35    a2  CNT  47
23  UP   27            63  YTO  40    a3  CNT  47
24  STP  41            64  9    11    a4  CNT  47
25  DN   25            65  1    01    a5  CNT  47
26  LN   65            66  UP   27    a6  CNT  47
27  UP   27            67  a    13    a7  CNT  47
28   b   14            68  +    33    a8  CNT  47
29   X   36            69  CLX  37    a9  CNT  47
2a  DN   25            6a  XTO  23    aa  CNT  47
2b  EXP  74            6b  e    12    ab  CNT  47
2c  UP   27            6c  XTO  23    ac  CNT  47
2d   1   01            6d  f    15    ad  CNT  47

30  UP   27            70  GTO  44
31   c   16            71  SUB  77
32   -   34            72  8    10
33  DN   25            73  1    01
34  LN   65            74  XFR  67
35  UP   27            75  9    11
36   a   13            76  XEY  30
37   X   36            77  DIV  35
38  DN   25            78  YTO  40
39  EXP  74            79  -    34
3a   X   36            7a  f    15
3b  YTO  40            7b  GTO  44
3c   9   11            7c  -    34
3d   2   02            7d  6    06
```

$(1-X)^{\beta}X^{\alpha}$

β

α

X

+	−
$\Gamma(\lambda)$	σ^2
	f(X)

```
b0  CNT  47    Plus
b1  CNT  47    Page
b2  CNT  47
b3  CNT  47
b4  CNT  47
b5  CNT  47
b6  CNT  47
b7  CNT  47
b8  CNT  47
b9  CNT  47
ba  CNT  47
bb  CNT  47
bc  CNT  47
bd  CNT  47

c0  CNT  47
c1  CNT  47
c2  CNT  47
c3  CNT  47
c4  CNT  47
c5  CNT  47
c6  CNT  47
c7  CNT  47
c8  CNT  47
c9  CNT  47
ca  CNT  47
cb  CNT  47
cc  CNT  47
cd  CNT  47

d0  CNT  47
d1  CNT  47
d2  CNT  47
d3  CNT  47
d4  CNT  47
d5  CNT  47
d6  CNT  47
d7  CNT  47
d8  CNT  47
d9  CNT  47
da  CNT  47
db  CNT  47
dc  CNT  47
dd  END  46
```

Minus Page

```
00  X>Y   53
01  CLX   37
02  1     01
03  +     33
04  5     05
05  +     33
06  DN    25
07  UP    27
08  X     36
09  AC+   60
0a  2     02
0b  XEY   30
0c  DIV   35
0d  3     03

10  -     34
11  e     12
12  DIV   35
13  2     02
14  DIV   35
15  +     33
16  e     12
17  DIV   35
18  7     07
19  DIV   35
1a  1     01
1b  -     34
1c  e     12
1d  DIV   35

20  6     06
21  DIV   35
22  5     05
23  +     33
24  6     06
25  0     00
26  DIV   35
27  f     15
28  DIV   35
29  -     34
2a  UP    27
2b  .     21
2c  5     05
2d  +     33

30  f     15
31  LN    65
32  X     36
33  DN    25
34  +     33
35  π     56
36  UP    27
37  +     33
38  DN    25
39  LN    65
3a  UP    27
3b  2     02
3c  DIV   35
3d  DN    25

40  +     33
41  DN    25
42  EXP   74
43  UP    27
44  f     15
45  UP    27
46  d     17
47  X>Y   53
48  5     05
49  2     02
4a  X<Y   52
4b  5     05
4c  9     11
4d  GTO   44

50  6     06
51  3     03
52  1     01
53  +     33
54  DN    25
55  X     36
56  GTO   44
57  4     04
58  5     05
59  DN    25
5a  DIV   35
5b  UP    27
5c  1     01
5d  -     34

60  GTO   44
61  4     04
62  6     06
63  DN    25
64  CNT   47
65  RTN   77
66  b     14
67  UP    27
68  a     13
69  +     33
6a  2     02
6b  +     33
6c  DN    25
6d  UP    27

70  X     36
71  UP    27
72  1     01
73  +     33
74  DN    25
75  X     36
76  b     14
77  UP    27
78  1     01
79  +     33
7a  DN    25
7b  XEY   30
7c  DIV   35
7d  a     13

80  UP    27
81  1     01
82  +     33
83  DN    25
84  X     36
85  YTO   40
86  -     34
87  e     12
88  UP    27
89  b     14
8a  UP    27
8b  a     13
8c  +     33
8d  2     02

90  +     33
91  DN    25
92  DIV   35
93  XFR   67
94  -     34
95  f     15
96  UP    27
97  XFR   67
98  -     34
99  e     12
9a  RUP   22
9b  PNT   45      S
9c  PNT   45
9d  END   46
```

F-DISTRIBUTION

This program evaluates the integral of the F distribution density function

$$Q = \int_{F}^{\infty} \frac{\Gamma\left(\frac{V_1 + V_2}{2}\right) \quad X^{V_1/2 - 1} \left(\frac{V_1}{V_2}\right)^{V_1/2}}{\Gamma\left(\frac{V_1}{2}\right) \quad \Gamma\left(\frac{V_2}{2}\right)\left(1 + \frac{V_1}{V_2} X\right)^{\frac{V_1 + V_2}{2}}} \, dx$$

for given values of F, V_1, V_2 .

The integral is evaluated by means of the following series:

V_2 EVEN

$$Q \ (F / V_1, V_2) = 1 - (1 - X)^{V_1/2} \left[1 + \frac{V_1 X}{2} + \ldots + \frac{V_1 \ (V_1 + 2) \ldots (V_2 + V_1 - 4)}{2 \cdot 4 \ldots (V_2 - 2)} X^{\frac{V_2 - 2}{2}} \right]$$

V_2 ODD

$$Q \ (F / V_1, V_2) = X^{V_2/2} \left[1 + \frac{V_2}{2} (1 - X) + \ldots + \frac{V_2 (V_2 + 2) \ldots (V_2 + V_1 - 4)}{2 \cdot 4 \ldots (V_1 - 2)} (1 - X)^{\frac{V_1 - 2}{2}} \right]$$

V_1 and V_2 both odd

$Q(F/ V_1 , V_2) = 1 - A + B$

$$A = \begin{cases} \frac{2}{\pi} \left\{ \theta + \sin\theta \cos\theta \left[1 + 2/3 \cos^2\theta + \ldots + \frac{2 \cdot 4 \ldots (V_2 - 3)}{3 \cdot 5 \ldots (V_2 - 2)} \cos^{V_2 - 2} \theta \right] \right\} & V_2 > 1 \\ \frac{2\theta}{\pi} \quad V_2 = 1 & \end{cases}$$

$$\theta = \text{Arc Tan} \sqrt{\frac{F}{V_2}}$$

$$B = \begin{cases} \frac{2 \left(\frac{V_2 - 1}{2}\right)!}{\sqrt{\pi} \left(\frac{V_2 - 2}{2}\right)!} \sin\theta_1 \cdot \cos^{V2} \theta_1 \left\{ 1 + \frac{V_2 + 1}{3} \sin^2\theta_1 + \ldots + \frac{(V_2 + 1)(V_2 + 3) \ldots (V_2 + V_2 - 4)}{(3)(5) \ldots (V_1 - 2)} \cdot \sin^{V_2 - 3} \theta_1 \right\} & V_1 > 1 \\ = 0 \quad V_1 = 1 & \end{cases}$$

$$\theta_1 = \text{Arc Tan} \sqrt{\frac{V_1 F}{V_2}}$$

Reference: Handbook of Mathematical Functions, Abramowitz and Stegun, National Bureau of Standards (1964)

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

SET: ☐ RADIANS

PRESS: END

ENTER PROGRAM: Side A followed by Side B

→ PRESS: END

PRESS: CONTINUE

ENTER DATA:
F ratio → Z
V_1 for numerator → Y
V_2 for denominator → X

PRESS: CONTINUE

DISPLAY

$Q(F/V_1, V_2)$ — Z
0 ———— Y
0 ———— X

To calculate significance level for new data.

EXAMPLE

General Form	$Q(F/V_1, V_2)$
$Q(4.21/7, 6)$	$= 0.05$
$Q(11.4/4, 5)$	$= 0.01$
$Q(3.79/7, 7)$	$= .05$

```
00  CLR  20   Plus  Page   40   +    33      80   0    00
01  STP  41    ENTRY       41  UP    27      81   f    15
02  PNT  45                42   2    02      82  UP    27
03  PNT  45                43   -    34      83   2    02
04  XTO  23                44  CLX   37      84  DIV   35
05   f   15                45  X=Y   50      85  DN    25
06  YTO  40                46   5    05      86  INT   64
07   e   12                47   d    17      87  UP    27
08  RDN  31                48  YTO   40      88  UP    27
09  YTO  40                49   f    15      89   2    02
0a   b   14                4a  DN    25      8a   X    36
0b   X   36                4b  DIV   35      8b  LN    65
0c  DN   25                4c   d    17      8c  RUP   22
0d   +   33                4d   X    36      8d   X    36

10  DN   25                50   c    16      90  DN    25
11  DIV  35                51   X    36      91  EXP   74
12   f   15                52   1    01      92  XTO   23
13  UP   27                53   +    33      93   c    16
14   1   01                54  YTO   40      94   1    01
15  XTO  23                55   c    16      95   -    34
16   c   16                56  DN    25      96  UP    27
17   2   02                57   2    02      97  DN    25
18  DIV  35                58   -    34      98   2    02
19  DN   25                59   f    15      99   -    34
1a  UP   27                5a  GTO   44      9a  RDN   31
1b  INT  64                5b   4    04      9b   X    36
1c  X=Y  50                5c   1    01      9c  |Y|   55
1d   3   03                5d   c    16      9d  RUP   22

20   6   06                60  UP    27      a0  X<Y   52
21   e   12                61   1    01      a1   9    11
22  XEY  30                62  UP    27      a2   9    11
23   2   02                63   d    17      a3  DN    25
24  DIV  35                64  CHS   32      a4   π    56
25  DN   25                65   +    33      a5   X    36
26  UP   27                66  DN    25      a6   c    16
27  INT  64                67  LN    65      a7  DIV   35
28  X<Y  52                68  UP    27      a8   2    02
29   8   10                69   e    12      a9  DIV   35
2a   1   01                6a   X    36      aa  GTO   44
2b  SFL  54                6b   2    02      ab   -    34
2c  RCL  61                6c  DIV   35      ac   0    00
2d  AC-  63                6d  DN    25      ad   0    00

30  XEY  30                70  EXP   74
31  AC+  60                71   X    36
32   1   01                72  IFG   43
33  RUP  22                73   7    07
34   -   34                74   8    10
35  UP   27                75   1    01
36  DN   25                76  XEY   30                   F
37  YTO  40                77   -    34
38   d   17                78  CLX   37
39   e   12                79  UP    27
3a  UP   27                7a  PNT   45   S
3b   4   04                7b  PNT   45                   V1
3c   -   34                7c  GTO   44
3d   f   15                7d   0    00                   V2
```

```
b0  CNT  47    Plus
b1  CNT  47    Page
b2  CNT  47
b3  CNT  47
b4  CNT  47
b5  CNT  47
b6  CNT  47
b7  CNT  47
b8  CNT  47
b9  CNT  47
ba  CNT  47
bb  CNT  47
bc  CNT  47
bd  CNT  47

c0  CNT  47
c1  CNT  47
c2  CNT  47
c3  CNT  47
c4  CNT  47
c5  CNT  47
c6  CNT  47
c7  CNT  47
c8  CNT  47
c9  CNT  47
ca  CNT  47
cb  CNT  47
cc  CNT  47
cd  CNT  47

d0  CNT  47
d1  CNT  47
d2  CNT  47
d3  CNT  47
d4  CNT  47
d5  CNT  47
d6  CNT  47
d7  CNT  47
d8  CNT  47
d9  CNT  47
da  CNT  47
db  CNT  47
dc  CNT  47
dd  CNT  47
```

STAT-PAC II-9

Minus Page

```
00  YTO  40
01  c    16
02  YE   24
03  b    14
04  f    15
05  DIV  35
06  e    12
07  X    36
08  RDN  31
09  √    76
0a  ARC  72
0b  TAN  71
0c  UP   27
0d  SIN  70

10  YTO  40
11  -    34
12  f    15
13  UP   27
14  X    36
15  YTO  40
16  d    17
17  RCL  61
18  1    01
19  X=Y  50
1a  7    07
1b  4    04
1c  XTO  23
1d  b    14

20  f    15
21  +    33
22  4    04
23  -    34
24  e    12
25  UP   27
26  2    02
27  -    34
28  X>Y  53
29  4    04
2a  6    06
2b  YTO  40
2c  -    34
2d  e    12

30  DN   25
31  DIV  35
32  d    17
33  X    36
34  b    14
35  X    36
36  1    01
37  +    33
38  YTO  40
39  b    14
3a  DN   25
3b  2    02
3c  -    34
3d  UP   27

40  YE   24
41  -    34
42  e    12
43  GTO  44
44  2    02
45  7    07
46  IFG  43
47  9    11
48  6    06
49  RUP  22
4a  XEY  30
4b  3    03
4c  X>Y  53
4d  5    05

50  d    17
51  2    02
52  DIV  35
53  UP   27
54  DN   25
55  1    01
56  -    34
57  RDN  31
58  X    36
59  RUP  22
5a  X<Y  52
5b  5    05
5c  6    06
5d  RDN  31

60  c    16
61  DIV  35
62  1    01
63  UP   27
64  d    17
65  -    34
66  √    76
67  RUP  22
68  X    36
69  RUP  22
6a  √    76
6b  LN   65
6c  XEY  30
6d  f    15

70  X    36
71  RDN  31
72  EXP  74
73  X    36
74  b    14
75  X    36
76  YTO  40
77  c    16
78  YE   24
79  -    34
7a  f    15
7b  YTO  40
7c  e    12
7d  DN   25

80  COS  73
81  UP   27
82  X    36
83  YTO  40
84  d    17
85  f    15
86  UP   27
87  1    01
88  YTO  40
89  -    34
8a  e    12
8b  XTO  23
8c  b    14
8d  SFL  54

90  -    34
91  X<Y  52
92  3    03
93  b    14
94  CLX  37
95  XTO  23
96  b    14
97  UP   27
98  d    17
99  √    76
9a  X    36
9b  e    12
9c  SIN  70
9d  X    36

a0  e    12
a1  +    33
a2  2    02
a3  X    36
a4  π    56
a5  DIV  35
a6  c    16
a7  XEY  30
a8  -    34
a9  1    01
aa  +    33
ab  CLX  37
ac  UP   27
ad  PNT  45    S
```

b0	PNT	45
b1	GTO	44
b2	+	33
b3	0	00
b4	0	00
b5	END	46

Minus
Page

NEGATIVE BINOMIAL DISTRIBUTION

This program computes various functions associated with the negative binomial distribution which is given by:

$$f(X) = \binom{X + r - 1}{r - 1} \quad p^r q^X \qquad ,$$

where

$$X = 0, 1, 2, 3, \ldots, \text{ and } q = 1 - p.$$

Definitions:

p - is the probability of success of a given event.

q - is the probability of failure of a given event.

f(X)- is the probability that exactly X + r trials will be required to produce r successes.

The cumulative distribution F(X) is given by

$$F(X) = \sum_{\lambda = 0}^{X} f(\lambda)$$

The mean and variance of the distribution are given by:

$$\mu = \frac{r}{p}$$

$$\sigma^2 = \frac{rq}{p^2}$$

This program computes μ, σ^2, f(X), and F(X).

Reference: Handbook of Tables for Probability and Statistics, Chemical Rubber Company, 1966.

USER INSTRUCTIONS

PRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM:

PRESS: CONTINUE

DISPLAY

```
0 ———————— Z
1 ———————— Y
0 ———————— X
```

ENTER DATA:

```
r ————► Y
p ————► X
```

PRESS: CONTINUE

DISPLAY

```
2  ——————— Z
σ2 ——————— Y
μ  ——————— X
```

PRESS: CONTINUE

DISPLAY

```
F(X) ————— Z
f(X) ————— Y
X    ————— X
```

Repeat until F(X) is sufficiently near unity.

To run another case:

PRESS: END

EXAMPLE

r = 4

p = .9

Results:

```
2
.4938        σ2
4.4444       μ
```

```
.6561        F(X)
.6561        f(X)
0.           X
```

```
.9185        F(X)
.2624        f(X)
1.           X
```

```
.9841        F(X)
.0656        f(X)
2.           X
```

```
.9973        F(X)
.0131        f(X)
3.           X
```

.

.

.

STAT-PAC II-10

00	CLR	20		40	-	34		
01	XTO	23		41	a	13		
02	c	16		42	UP	27		
03	XTO	23		43	1	01		
04	9	11		44	-	34		
05	1	01		45	0	00		
06	XEY	30		46	XTO	23		
07	STP	41	ENTRY	47	e	12		
08	PNT	45		48	XTO	23		
09	PNT	45		49	f	15		
0a	XTO	23		4a	RDN	31		
0b	b	14		4b	XTO	23		
0c	YTO	40		4c	d	17		
0d	a	13		4d	1	01		
10	DIV	35		50	AC+	60		
11	AC+	60		51	RDN	31		
12	DIV	35		52	f	15		
13	UP	27		53	XEY	30		
14	1	01		54	d	17		
15	XEY	30		55	X<Y	52		
16	-	34		56	6	06		
17	DN	25		57	5	05		
18	X	36		58	RCL	61		
19	2	02		59	DIV	35		
1a	XEY	30		5a	DN	25		
1b	UP	27		5b	X	36		
1c	e	12		5c	1	01		
1d	PNT	45	s	5d	UP	27		
20	PNT	45		60	CHS	32		
21	a	13		61	AC-	63		
22	UP	27		62	GTO	44		
23	b	14		63	5	05		
24	LN	65		64	2	02		
25	X	36		65	YEX	24		
26	DN	25		66	8	10		
27	EXP	74		67	DN	25		
28	UP	27		68	X	36		
29	1	01		69	DN	25		
2a	UP	27		6a	YEX	24		
2b	b	14		6b	9	11		
2c	-	34		6c	+	33		
2d	DN	25		6d	YTO	40		
30	LN	65		70	9	11		p^rq^x
31	UP	27		71	UP	27		
32	c	16		72	c	16		F(X)
33	X	36		73	PNT	45	s	
34	DN	25		74	PNT	45		r
35	EXP	74		75	UP	27		
36	X	36		76	1	01		p
37	YTO	40		77	+	33		x
38	8	10		78	YTO	40		
39	c	16		79	c	16		}
3a	UP	27		7a	GTO	44		
3b	a	13		7b	2	02		
3c	+	33		7c	1	01		
3d	1	01		7d	END	46		

GAMMA DISTRIBUTION

This program determines the Gamma Distribution given by:

$$f(X) = \frac{X^{\alpha} e^{-\frac{X}{\beta}}}{\Gamma(\alpha + 1) \beta^{\alpha + 1}} \qquad (0 < X < \infty)$$

where α and β are parameters with limitations:

$\alpha > -1$

and

$\beta > 0$

This program incorporates program 09100-70024, GAMMA FUNCTION, and is restricted to values of $\alpha < 68.98$.

Also evaluated are the mean (μ) and variance (σ^2) of the distribution

$$\mu = \beta(\alpha + 1)$$

$$\sigma^2 = \beta^2(\alpha + 1)$$

Reference: Handbook of Tables for Probability and Statistics, Chemical Rubber Company, 1966.

USER INSTRUCTIONS

PRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM: Side A followed by Side B

→ **PRESS:** CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
1 ——— X

ENTER DATA:

$\beta \longrightarrow Z$
$\alpha \longrightarrow Y$
$X \longrightarrow X$

PRESS: CONTINUE

F(X) ——— Z
σ^2 ——— Y
μ ——— Z

To run another case

EXAMPLE

Parameters:

$\beta = 2$
$\alpha = .5$
$X = 3$

Results:

$f(X) = .15418$
$\sigma^2 = 6$
$\mu = 3$

The program has three input data checks, these being:

1. is $\alpha > -1$?

2. is $\beta > 0$?

3. is $X > 0$?

If any of these checks indicate bad data, the calculator STOPS indicating the type of error by DISPLAY.

Error 1. ($\beta < 0$)

DISPLAY

β ——— Z
0 ——— Y
0 ——— X

Error 2. ($\alpha < -1$)

DISPLAY

α ——— Z
-1 ——— Y
-1 ——— X

Error 3. ($X < 0$)

DISPLAY

X ——— Z
0 ——— Y
0 ——— X

STAT-PAC II-11

Plus
Page
ENTRY

00	CLR	20	40	UP	27	80	UP	27
01	1	01	41	YTO	40	81	+	33
02	STP	41	42	d	17	82	DN	25
03	PNT	45	43	UP	27	83	LN	65
04	PNT	45	44	DN	25	84	UP	27
05	XTO	23	45	INT	64	85	2	02
06	c	16	46	-	34	86	DIV	35
07	YTO	40	47	CLX	37	87	DN	25
08	b	14	48	X>Y	53	88	+	33
09	DN	25	49	CLX	37	89	DN	25
0a	YTO	40	4a	1	01	8a	GTO	44
0b	a	13	4b	+	33	8b	-	34
0c	UP	27	4c	5	05	8c	0	00
0d	1	01	4d	+	33	8d	0	00
10	CHS	32	50	DN	25	90	CNT	47
11	X>Y	53	51	UP	27	91	CNT	47
12	UP	27	52	X	36	92	CNT	47
13	STP	41	53	AC+	60	93	CNT	47
14	DN	25	54	2	02	94	CNT	47
15	0	00	55	XEY	30	95	CNT	47
16	X>Y	53	56	DIV	35	96	CNT	47
17	UP	27	57	3	03	97	CNT	47
18	STP	41	58	-	34	98	CNT	47
19	UP	27	59	e	12	99	CNT	47
1a	c	16	5a	DIV	35	9a	CNT	47
1b	XEY	30	5b	2	02	9b	CNT	47
1c	X>Y	53	5c	DIV	35	9c	CNT	47
1d	UP	27	5d	+	33	9d	CNT	47
20	STP	41	60	e	12	a0	CNT	47
21	a	13	61	DIV	35	a1	CNT	47
22	DIV	35	62	7	07	a2	CNT	47
23	DN	25	63	DIV	35	a3	CNT	47
24	CHS	32	64	1	01	a4	CNT	47
25	EXP	74	65	-	34	a5	CNT	47
26	UP	27	66	e	12	a6	CNT	47
27	c	16	67	DIV	35	a7	CNT	47
28	LN	65	68	6	06	a8	CNT	47
29	UP	27	69	DIV	35	a9	CNT	47
2a	b	14	6a	5	05	aa	CNT	47
2b	X	36	6b	+	33	ab	CNT	47
2c	DN	25	6c	6	06	ac	CNT	47
2d	EXP	74	6d	0	00	ad	CNT	47
30	X	36	70	DIV	35			
31	b	14	71	f	15			
32	UP	27	72	DIV	35			
33	1	01	73	-	34			
34	+	33	74	UP	27			
35	a	13	75	.	21			
36	LN	65	76	5	05			
37	X	36	77	+	33			
38	DN	25	78	f	15			
39	EXP	74	79	LN	65			
3a	DIV	35	7a	X	36			
3b	YTO	40	7b	DN	25			
3c	9	11	7c	+	33			
3d	b	14	7d	π	56			

$x^{\alpha} e^{-\frac{X}{\beta}} \beta^{\alpha+1}$

β

α

X

$\Gamma(\phi)$

b0	CNT	47	Plus Page
b1	CNT	47	
b2	CNT	47	
b3	CNT	47	
b4	CNT	47	
b5	CNT	47	
b6	CNT	47	
b7	CNT	47	
b8	CNT	47	
b9	CNT	47	
ba	CNT	47	
bb	CNT	47	
bc	CNT	47	
bd	CNT	47	
c0	CNT	47	
c1	CNT	47	
c2	CNT	47	
c3	CNT	47	
c4	CNT	47	
c5	CNT	47	
c6	CNT	47	
c7	CNT	47	
c8	CNT	47	
c9	CNT	47	
ca	CNT	47	
cb	CNT	47	
cc	CNT	47	
cd	CNT	47	
d0	CNT	47	
d1	CNT	47	
d2	CNT	47	
d3	CNT	47	
d4	CNT	47	
d5	CNT	47	
d6	CNT	47	
d7	CNT	47	
d8	CNT	47	
d9	CNT	47	
da	CNT	47	
db	CNT	47	
dc	CNT	47	
dd	CNT	47	

STAT-PAC II-11

Minus Page

00	EXP	74
01	UP	27
02	f	15
03	UP	27
04	d	17
05	X>Y	53
06	1	01
07	0	00
08	X<Y	52
09	1	01
0a	7	07
0b	GTO	44
0c	2	02
0d	1	01
10	1	01
11	+	33
12	DN	25
13	X	36
14	GTO	44
15	1	01
16	3	03
17	DN	25
18	DIV	35
19	UP	27
1a	1	01
1b	-	34
1c	GTO	44
1d	0	00
20	4	04
21	DN	25
22	XFR	67
23	9	11
24	XEY	30
25	DIV	35
26	b	14
27	UP	27
28	1	01
29	+	33
2a	a	13
2b	X	36
2c	XEY	30
2d	X	36
30	PNT	45
31	PNT	45
32	END	46

Storage

0		8	
1		9	
2		a	
3		b	
4		c	
5		d	
6		e	
7		f	

LOG NORMAL DISTRIBUTION

This program computes the value of the log-normal distribution for a given set of parameters, a, μ, σ, over a specified range of X.

Definitions:

$$f(X) = \frac{1}{(X-a)\,\sigma\sqrt{2\pi}} \quad \exp\left\{-\left[\frac{(\ln[X-a]-\mu)^2}{2\sigma^2}\right]\right\}$$

where

$$X > a$$

The program determines f(X) over a range

$$X_i \leq X \leq X_f$$

in increments of ΔX.

$X_i = X_{initial}$
$X_f = X_{final}$

Reference: Probability and Statistics for Engineers, Miller and Freund, Prentice Hall, 1965, page 77.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

→ DISPLAY

0 —— Z
0 —— Y
1 —— X

ENTER DATA:

a → Z
σ → Y
μ → X

PRESS: CONTINUE

DISPLAY

0 —— Z
0 —— Y
2 —— X

ENTER DATA:

ΔX → Z
X_f → Y
X_i → X

→ PRESS: CONTINUE

DISPLAY

f(X) —— Z
X —— Y
ΔX —— Z

When $X = X_f$

EXAMPLE

$a = 0$ $\Delta X = .1$
$\sigma = 1$ $X_f = 1$
$\mu = 1$ $X_i = .1$

Results:

X	f(X)
.1	.01708
.2	.06627
.3	.11722
.4	.15902
.5	.19030
.6	.21237
.7	.22706
.8	.23602
.9	.24063
1.0	.24197

STAT-PAC II-12

```
00  CLR  20
01  1    01
02  STP  41    ENTRY
03  PNT  45
04  PNT  45
05  AC+  60
06  DN   25
07  YTO  40
08  d    17
09  0    00
0a  UP   27
0b  UP   27
0c  2    02
0d  STP  41    ENTRY

10  PNT  45
11  PNT  45
12  XTO  23
13  c    16
14  YTO  40
15  b    14
16  DN   25
17  YTO  40
18  a    13
19  c    16
1a  UP   27
1b  d    17
1c  -    34
1d  0    00

20  X>Y  53
21  5    05
22  0    00
23  X=Y  50
24  5    05
25  0    00
26  f    15
27  XEY  30
28  LN   65
29  XEY  30
2a  -    34
2b  DN   25
2c  UP   27
2d  X    36

30  2    02
31  DIV  35
32  e    12
33  DIV  35
34  DIV  35
35  DN   25
36  CHS  32
37  EXP  74
38  UP   27
39  e    12
3a  DIV  35
3b  c    16
3c  UP   27
3d  d    17

40  -    34
41  DN   25
42  DIV  35
43  π    56
44  √    76
45  DIV  35
46  2    02
47  √    76
48  DIV  35
49  c    16
4a  UP   27
4b  a    13
4c  PNT  45    s
4d  PNT  45

50  a    13
51  UP   27
52  c    16
53  +    33
54  YTO  40
55  c    16
56  b    14
57  X>Y  53
58  1    01
59  b    14
5a  X=Y  50
5b  1    01
5c  b    14
5d  GTO  44

60  0    00
61  0    00
62  END  46
```

Δx

x_f

x_i

a

σ

μ

ERROR FUNCTION

This program computes Erf (X) as defined below:

$$\mathrm{Erf}\,(X) = \frac{2}{\sqrt{\pi}} \int_0^X e^{-t^2}\,dt$$

The program applies the approximating expansion given in the reference.

$$\mathrm{Erf}\,(X) = 1 - \frac{1}{[\,1 + a_1X + a_2X^2 + a_3X^3 + a_4X^4 + a_5X^5 + a_6X^6\,]^{16}}$$

where

a_1 = .0705230784
a_2 = .0422820123
a_3 = .0092705272
a_4 = .0001520143
a_5 = .0002765672
a_6 = .0000430638

The absolute error resulting from this approximating function is less than $3 \cdot 10^{-7}$.

Reference: Approximations for Digital Computers, C. Hastings, Jr. Princeton University Press, 1955.

USER INSTRUCTIONS	EXAMPLE

DEPRESS: X on the 9120A

PRESS: END

ENTER PROGRAM

→ PRESS: CONTINUE

DISPLAY

```
0 ———— Z
0 ———— Y
1 ———— X
```

ENTER DATA: X ⟶ X

PRESS: CONTINUE

DISPLAY

```
Erf(X) —— Z
Erf(X) —— Y
Erf(X) —— X
```

To run another case

1)
X = 2.3

Erf(2.3) = .99885680

2)

Since $\text{Erf}\left(\frac{X}{\sqrt{2}}\right) = 2 \int_0^X f(t)\,dt$

where f(t) is the normal probability density.

The Erf program can be used to determine normal probability integrals.

Thus

$$2 \int_0^1 f(t)dt = \text{Erf}\left(\frac{1}{\sqrt{2}}\right)$$

$$= .68268936$$

Thus

$$\int_0^1 f(t)dt = .34134468$$

00	CLR	20	
01	1	01	
02	STP	41	ENTRY
03	PNT	45	
04	PNT	45	
05	XTO	23	
06	f	15	
07	UP	27	
08	.	21	
09	0	00	
0a	7	07	
0b	0	00	
0c	5	05	
0d	2	02	
10	3	03	
11	0	00	
12	7	07	
13	8	10	
14	4	04	
15	XEY	30	
16	X	36	
17	UP	27	
18	X	36	
19	.	21	
1a	0	00	
1b	4	04	
1c	2	02	
1d	2	02	
20	8	10	
21	2	02	
22	0	00	
23	1	01	
24	2	02	
25	3	03	
26	XEY	30	
27	X	36	
28	RDN	31	
29	+	33	
2a	RUP	22	
2b	XEY	30	
2c	f	15	
2d	X	36	
30	.	21	
31	0	00	
32	0	00	
33	9	11	
34	2	02	
35	7	07	
36	0	00	
37	5	05	
38	2	02	
39	7	07	
3a	2	02	
3b	XEY	30	
3c	X	36	
3d	RDN	31	
40	+	33	
41	RUP	22	
42	XEY	30	
43	f	15	
44	X	36	
45	.	21	
46	0	00	
47	0	00	
48	0	00	
49	1	01	
4a	5	05	
4b	2	02	
4c	0	00	
4d	1	01	
50	4	04	
51	3	03	
52	XEY	30	
53	X	36	
54	RDN	31	
55	+	33	
56	RUP	22	
57	XEY	30	
58	f	15	
59	X	36	
5a	.	21	
5b	0	00	
5c	0	00	
5d	0	00	
60	2	02	
61	7	07	
62	6	06	
63	5	05	
64	6	06	
65	7	07	
66	2	02	
67	XEY	30	
68	X	36	
69	RDN	31	
6a	+	33	
6b	RUP	22	
6c	XEY	30	
6d	f	15	
70	X	36	
71	.	21	
72	0	00	
73	0	00	
74	0	00	
75	0	00	
76	4	04	
77	3	03	
78	0	00	
79	6	06	
7a	3	03	
7b	8	10	
7c	XEY	30	
7d	X	36	
80	DN	25	
81	+	33	
82	1	01	
83	+	33	
84	DN	25	
85	LN	65	
86	UP	27	
87	1	01	
88	6	06	
89	X	36	
8a	DN	25	
8b	EXP	74	
8c	XEY	30	
8d	1	01	
90	XEY	30	
91	DIV	35	
92	1	01	
93	XEY	30	
94	-	34	
95	UP	27	
96	DN	25	
97	DN	25	
98	PNT	45	
99	PNT	45	
9a	END	46	

x

JOINT NORMAL PROBABILITY DENSITY FUNCTION

Two random variables X and Y are said to have a Bivariate Normal Distribution if their joint density function is given by

$$f(X, Y) = \frac{1}{2\pi s_X s_Y \sqrt{1 - r^2}} \quad e^{-P(X, Y)}$$

where

$$P(X, Y) = \frac{1}{2(1 - r^2)} \left\{ \frac{(X - \overline{X})^2}{s_X{}^2} - 2r \frac{(X - \overline{X})(Y - \overline{Y})}{s_X s_Y} + \frac{(Y - \overline{Y})^2}{s_Y{}^2} \right\}$$

r = Correlation Coefficient Between X and Y

$\overline{X}$ = Sample Mean of X

$\overline{Y}$ = Sample Mean of Y

s_X = Sample Standard Deviation of X

s_Y = Sample Standard Deviation of Y

X and Y are random variables

This program evaluates $f(X, Y)$ for given values of r, $\overline{X}$, $\overline{Y}$, s_X, s_Y, X, and Y.

Reference: Introduction to Mathematical Statistics, R. V. Hogg, A. T. Craig, 1959, page 218, Macmillan.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0 ——— Z
1 ——— Y
0 ——— X

ENTER DATA:

$\bar{Y} \longrightarrow Y$
$\bar{X} \longrightarrow X$

PRESS: CONTINUE

DISPLAY

0 ——— Z
2 ——— Y
0 ——— X

ENTER DATA:

$s_Y \longrightarrow Y$
$s_X \longrightarrow X$

PRESS: CONTINUE

DISPLAY

0 ——— Z
3 ——— Y
0 ——— X

ENTER DATA:

$r \longrightarrow Z$
$Y \longrightarrow Y$
$X \longrightarrow X$

PRESS: CONTINUE

DISPLAY

$f(X, Y) \rightarrow Z$
$f(X, Y) \rightarrow Y$
$f(X, Y) \rightarrow X$

To run another case

EXAMPLE

$\bar{X}$ = -1
$\bar{Y}$ = 1

s_X = 1.5
s_Y = .5

r = .7

Y = 2

X = 1

Results:

f(X, Y) = .0400399

Step	Key	Code	X	Y	Z
00	CLR	20			
01	1	01			
02	XEY	30			
03	STP	41		ENTRY	
04	PNT	45			
05	PNT	45			
06	XTO	23			
07	d	17			
08	YTO	40			
09	c	16			
0a	CLR	20			
0b	2	02			
0c	XEY	30			
0d	STP	41		ENTRY	
10	PNT	45			
11	PNT	45			
12	XTO	23			
13	b	14			
14	YTO	40			
15	a	13			
16	CLR	20			
17	3	03			
18	XEY	30			
19	STP	41		ENTRY	
1a	PNT	45			
1b	PNT	45			
1c	YEX	24			
1d	d	17			
20	XEY	30			
21	-	34			
22	YEX	24			
23	d	17			
24	c	16			
25	-	34			
26	YTO	40			
27	c	16			
28	d	17			
29	X	36			
2a	DN	25			
2b	YTO	40			
2c	e	12			
2d	X	36			
30	2	02			
31	X	36			
32	b	14			
33	DIV	35			
34	a	13			
35	DIV	35			
36	d	17			
37	UP	27			
38	X	36			
39	b	14			
3a	DIV	35			
3b	DIV	35			
3c	DN	25			
3d	XEY	30			
40	-	34			
41	c	16			
42	UP	27			
43	X	36			
44	a	13			
45	DIV	35			
46	DIV	35			
47	DN	25			
48	+	33			
49	e	12			
4a	UP	27			
4b	X	36			
4c	1	01			
4d	XEY	30			
50	-	34			
51	2	02			
52	XEY	30			
53	X	36			
54	RDN	31			
55	CHS	32			
56	DIV	35			
57	DN	25			
58	EXP	74			
59	XEY	30			
5a	√	76			
5b	DIV	35			
5c	2	02			
5d	DIV	35			
60	π	56			
61	DIV	35			
62	b	14			
63	DIV	35			
64	a	13			
65	DIV	35			
66	DN	25			
67	UP	27			
68	UP	27			
69	PNT	45	s		
6a	PNT	45			
6b	PNT	45			
6c	PNT	45			
6d	GTO	44			
70	0	00			
71	0	00			
72	END	46			

Storage

0		8	
1		9	
2		a	s_Y
3		b	s_X
4		c	$\overline{Y}$, $Y - \overline{Y}$
5		d	$\overline{X}$, Y, $(X - \overline{X})$
6		e	r
7		f	

CUMULATIVE BINOMIAL DISTRIBUTION

This program determines the cumulative binomial distribution given by:

$$P\{X \leq r\} = \sum_{x=r}^{n} B(r, n, p)$$

where:

$$B(r, n, p) = {}_nC_r\, p^r\, (1-p)^{n-r}$$

p = Probability of a Success
r = Number of Successes
n = Number of Trials

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA:

p → Z
n → Y
r → X

PRESS: CONTINUE

DISPLAY

P(X≤r) ——— Z
n ——— Y
r ——— X

PRESS: CONTINUE

DISPLAY

1-p ——— Z
p ——— Y
0 ——— X

To run another case

EXAMPLE

If the probability of a successful trial is .25 (p = .25) and if 8 trials are to be performed, (n = 8), what is the probability of at least 5 successes, (r = 5) .

$P(X \leq 5) = .0273$

STAT-PAC II-15

Step	Key	Code	X	Y	Z
00	CLR	20			
01	STP	41	ENTRY		
02	XTO	23			
03	a	13			
04	YTO	40			
05	b	14			
06	RUP	22			
07	XTO	23			
08	c	16			
09	1	01			
0a	RDN	31			
0b	X=Y	50			
0c	3	03			
0d	9	11			
10	XEY	30			
11	CLX	37			
12	X=Y	50			
13	4	04			
14	0	00			
15	DN	25			
16	c	16			
17	-	34			
18	DIV	35			
19	YTO	40			
1a	d	17			
1b	b	14			
1c	XEY	30			
1d	a	13			
20	-	34			
21	RUP	22			
22	+	33			
23	RDN	31			
24	XEY	30			
25	AC+	60			
26	RCL	61			
27	DIV	35			
28	DN	25			
29	X	36			
2a	d	17			
2b	X	36			
2c	1	01			
2d	+	33			
30	UP	27			
31	CHS	32			
32	AC+	60			
33	f	15			
34	XEY	30			
35	CLX	37			
36	X<Y	52			
37	2	02			
38	6	06			
39	c	16			
3a	LN	65			
3b	XEY	30			
3c	b	14			
3d	X	36			

Step	Key	Code	X	Y	Z
40	DN	25			
41	EXP	74			
42	X	36			
43	b	14			
44	UP	27			
45	a	13			
46	PNT	45	s		
47	PNT	45			
48	c	16			
49	UP	27			
4a	1	01			
4b	XEY	30			
4c	-	34			
4d	UP	27			
50	0	00			
51	PNT	45	s		
52	PNT	45			
53	PNT	45			
54	PNT	45			
55	PNT	45			
56	GTO	44			
57	0	00			
58	0	00			
59	END	46			

Storage

0		8	
1		9	
2		a	r
3		b	n
4		c	p
5		d	
6		e	
7		f	

SECTION III TEST STATISTICS

t STATISTIC FOR MEANS OF TWO SAMPLES

This program calculates the value of the t statistic from the equation

$$t = \left| \frac{\overline{X} - \overline{Y}}{\sqrt{\frac{1}{n_X} + \frac{1}{n_y}} \sqrt{\frac{\sum(X_i - \overline{X})^2 + \sum(Y_i - \overline{Y})^2}{n_X + n_y - 2}}} \right|$$

where $X_1, X_2, \cdots X_{n_X}$ and $Y_1, Y_2, \cdots Y_{n_y}$ are samples from normal populations with means μ_X, and μ_Y, both with same variance σ^2. The number of degrees of freedom $n = (n_X + n_y - 2)$. The sample means are calculated by the equations.

$$\overline{X} = \frac{\sum X_i}{n_X} \qquad \overline{Y} = \frac{\sum Y_i}{n_y}$$

The sample estimates of the standard deviations are also displayed. These are calculated by the equations

$$S_X = \sqrt{\frac{\sum(X_i - \overline{X})^2}{n_X - 1}} \qquad S_y = \sqrt{\frac{\sum(Y_i - \overline{Y})^2}{n_y - 1}}$$

This program may be used to test the null hypothesis $\mu_X = \mu_Y$ where we don't assume knowledge of σ^2.

Reference: Statistical Theory and Methodology in Science and Engineering
by K. A. Brownlee
Second Edition

John Wiley and Sons, Inc. 1965

USER INSTRUCTIONS

DEPRESS: X Y on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

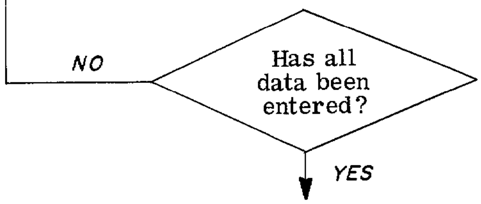
DISPLAY

0 ——— Z
i ——— Y
0 ——— X

(i indicates point to be entered)

ENTER DATA: $X_i \longrightarrow X$

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

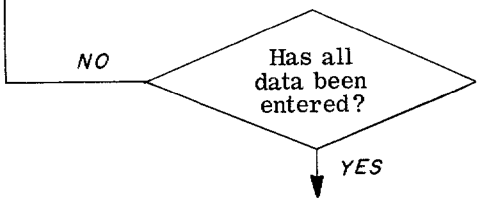
DISPLAY

0 ——— Z
j ——— Y
0 ——— X

(j indicates point to be entered)

ENTER DATA: $Y_j \longrightarrow X$

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

0 ——— Z
$\bar{Y}$ ——— Y
$\bar{X}$ ——— X

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

0 ——— Z
S_Y ——— Y
S_X ——— X

PRESS: CONTINUE

DISPLAY

0 ——— Z
t ——— Y
n ——— X

PRESS: CONTINUE To restart a new problem

EXAMPLE

X 79, 84, 108, 114, 120, 103, 122, 120

Y 91, 103, 90, 113, 108, 87, 100, 80, 99, 54

$\bar{Y}$ = 92.50
$\bar{X}$ = 106.25

S_Y = 16.82
S_X = 16.64

t = 1.73
n = 16.00

STAT-PAC III-1

00	CLR	20	
01	1	01	
02	XTO	23	
03	a	13	
04	XEY	30	
05	STP	41	ENTRY
06	IFG	43	
07	1	01	
08	c	16	
09	PNT	45	
0a	PNT	45	
0b	YTO	40	
0c	d	17	
0d	UP	27	
10	X	36	
11	AC+	60	
12	d	17	
13	UP	27	
14	1	01	
15	+	33	
16	0	00	
17	UP	27	
18	RDN	31	
19	GTO	44	
1a	0	00	
1b	5	05	
1c	d	17	
1d	RDN	31	
20	RCL	61	
21	RUP	22	
22	DIV	35	
23	YTO	40	
24	e	12	
25	X	36	
26	e	12	
27	X	36	
28	DN	25	
29	-	34	
2a	YTO	40	
2b	f	15	
2c	a	13	
2d	UP	27	
30	1	01	
31	X<Y	52	
32	4	04	
33	3	03	
34	RCL	61	
35	XTO	23	
36	c	16	
37	YTO	40	
38	b	14	
39	d	17	
3a	XTO	23	
3b	a	13	
3c	CLR	20	
3d	1	01	
40	GTO	44	
41	0	00	
42	4	04	
43	0	00	
44	UP	27	
45	e	12	
46	UP	27	
47	b	14	
48	PNT	45	s
49	PNT	45	
4a	c	16	
4b	UP	27	
4c	a	13	
4d	UP	27	
50	1	01	
51	-	34	
52	DN	25	
53	DIV	35	
54	d	17	
55	UP	27	
56	1	01	
57	-	34	
58	f	15	
59	XEY	30	
5a	DIV	35	
5b	RUP	22	
5c	√	76	
5d	RUP	22	
60	√	76	
61	UP	27	
62	0	00	
63	RDN	31	
64	XEY	30	
65	PNT	45	s
66	PNT	45	
67	1	01	
68	UP	27	
69	a	13	
6a	DIV	35	
6b	1	01	
6c	UP	27	
6d	d	17	
70	DIV	35	
71	RDN	31	
72	+	33	
73	YE	24	
74	a	13	
75	2	02	
76	-	34	
77	DN	25	
78	+	33	
79	YTO	40	
7a	d	17	
7b	c	16	
7c	UP	27	
7d	f	15	
80	+	33	
81	DN	25	
82	XEY	30	
83	DIV	35	
84	b	14	
85	UP	27	
86	e	12	
87	-	34	
88	a	13	
89	√	76	
8a	RUP	22	
8b	√	76	
8c	X	36	
8d	DN	25	
90	DIV	35	
91	|Y|	55	
92	d	17	
93	UP	27	
94	0	00	
95	RDN	31	
96	PNT	45	s
97	PNT	45	
98	PNT	45	
99	PNT	45	
9a	GTO	44	
9b	0	00	
9c	0	00	
9d	END	46	

1, n_1

n

PAIRED OBSERVATION TEST

Given a table of paired observations as illustrated below,

sample	X_i	Y_i
1	X_1	Y_1
2	X_2	Y_2
3	X_3	Y_3
.	.	.
.	.	.
.	.	.
n	X_n	Y_n

this program determines the statistics associated with the difference, D, defined by

$$D_i = X_i - Y_i$$

The following statistics are determined;

$$\overline{D} = \frac{1}{n} \sum_{i=1}^{n} D_i$$

$$S_D = \sqrt{\frac{\Sigma D_i^2 - \frac{(\Sigma D_i)^2}{n}}{n - 1}}$$

$$S_{\overline{D}} = \frac{S_D}{\sqrt{n}}$$

$$t \text{ (test statistic)} = \frac{\overline{D}}{S_{\overline{D}}}$$

$$\gamma\text{(degrees of freedom)} = n - 1$$

Application

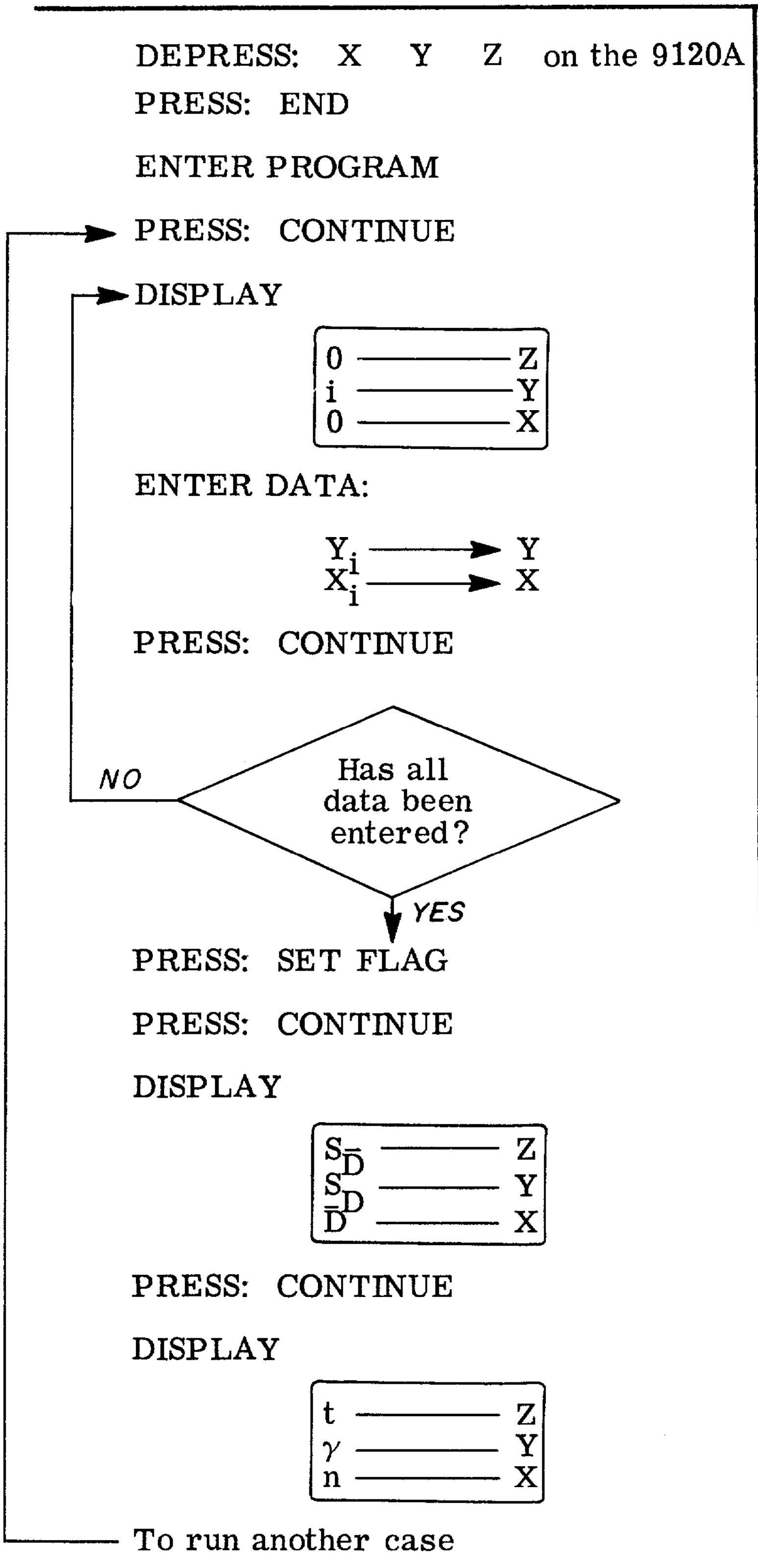
$H_o : = \mu_x = \mu_y$
$H_A : = \mu_x \neq \mu_y$
α = confidence level (.05)
$t = \frac{\overline{D}}{S_{\overline{D}}}$

Conclusion:

If $t > t_{(\frac{\alpha}{2}, \; n - 1)}$, we reject the hypothesis H_o .

Reference: Statistics in Research, by Ostle, Iowa State University Press, 1966.

USER INSTRUCTIONS



DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0 ——— Z
i ——— Y
0 ——— X

ENTER DATA:

$Y_i \longrightarrow Y$
$X_i \longrightarrow X$

PRESS: CONTINUE

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

$S_{\bar{D}}$ ——— Z
S_D ——— Y
$\bar{D}$ ——— X

PRESS: CONTINUE

DISPLAY

t ——— Z
γ ——— Y
n ——— X

To run another case

EXAMPLE

Sample	X	Y
1	14.	17.
2	17.5	20.7
3	17.	21.6
4	17.5	20.9
5	15.4	17.2

Results

$S_{\bar{D}}$ = .4472

S_D = 1.

$\bar{D}$ = −3.2

t = −7.1554

γ = 4

n = 5

STAT-PAC III-2

00	CLR	20	
01	1	01	
02	XTO	23	
03	d	17	
04	XEY	30	
05	STP	41	ENTRY
06	IFG	43	
07	2	02	
08	1	01	
09	PNT	45	
0a	PNT	45	
0b	XEY	30	
0c	-	34	
0d	DN	25	
10	UP	27	
11	X	36	
12	AC+	60	
13	d	17	
14	UP	27	
15	1	01	
16	+	33	
17	YTO	40	
18	d	17	
19	0	00	
1a	UP	27	
1b	RDN	31	
1c	GTO	44	
1d	0	00	
20	5	05	
21	d	17	
22	UP	27	
23	1	01	
24	-	34	
25	YTO	40	
26	d	17	
27	f	15	
28	XEY	30	
29	DIV	35	
2a	YTO	40	
2b	c	16	
2c	f	15	
2d	UP	27	
30	X	36	
31	d	17	
32	DIV	35	
33	e	12	
34	XEY	30	
35	-	34	
36	d	17	
37	UP	27	
38	1	01	
39	-	34	
3a	DN	25	
3b	DIV	35	
3c	DN	25	
3d	√	76	
40	XTO	23	
41	b	14	
42	UP	27	
43	d	17	
44	√	76	
45	DIV	35	
46	b	14	
47	UP	27	
48	c	16	
49	PNT	45	**S**
4a	PNT	45	
4b	XEY	30	
4c	DN	25	
4d	XEY	30	
50	DIV	35	
51	d	17	
52	UP	27	
53	1	01	
54	-	34	
55	d	17	
56	PNT	45	
57	PNT	45	
58	END	46	

Storage

0		8	
1		9	
2		a	
3		b	S_D
4		c	$\overline{D}$
5		d	n
6		e	ΣD_i^2
7		f	ΣD_i

t STATISTIC - TEST A MEAN $\overline{X}$ EQUAL TO μ_o

This program calculates the test statistic t used in testing a sample mean $\overline{X}$ equal to a value μ_o.

Hypothesis

$$H_o: \quad \mu = \mu_o$$

$$H_A: \quad \mu \neq \mu_o$$

test statistic

$$t = \frac{\overline{X} - \mu_o}{S_{\overline{X}}}$$

where

$\overline{X}$ = Sample Mean

S_X = Sample Standard Deviation

$S_{\overline{X}} = \frac{S_X}{\sqrt{n}}$

α is the level of significance

and

n is the number of observations in the sample

Conclusion:

$$\text{Reject } H_o \text{ if } t > \left| t_{(n-1,\ \alpha/2)} \right|$$

Reference: Statistics in Research, by Ostle, Iowa State University Press, 1966.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

→ PRESS: CONTINUE

DISPLAY

0 —— Z
0 —— Y
1 —— X

ENTER DATA:

$n \longrightarrow Z$
$S_X \longrightarrow Y$
$\bar{X} \longrightarrow X$

PRESS: CONTINUE

DISPLAY

0 —— Z
0 —— Y
2 —— X

ENTER DATA: $\mu_o \longrightarrow X$

PRESS: CONTINUE

DISPLAY

t —— Z
μ_o —— Y
3 —— X

← To run another case

EXAMPLE

$\bar{X}$ = 175
S_X = 25
n = 15

Test to see if $\mu = \mu_o$ where $\mu_o = 188$ and $\alpha = .05$

$H_o : \mu = 188$
$H_A : \mu \neq 188$
$\alpha = .05$

Results: t = -2.01395

Conclusion: Since $t_{(14, .025)} = 2.14$, the test statistic t (-2.01395) is not greater than $|t_{(n-1, \alpha/2)}|$ (2.14) and the hypothesis H_o is accepted.

```
00  CLR  20
01   1   01
02  STP  41   ENTRY
03  PNT  45
04  PNT  45
05  RUP  22
06   √   76
07  XEY  30
08  RDN  31
09  DIV  35
0a  DN   25
0b  AC+  60
0c   0   00
0d  UP   27

10  UP   27
11   2   02
12  STP  41
13  XTO  23
14   d   17
15  RDN  31
16  RCL  61
17  RDN  31
18  XEY  30
19   -   34
1a  DN   25
1b  XEY  30
1c  DIV  35
1d   d   17

20  UP   27
21   3   03
22  PNT  45
23  PNT  45
24  END  46
```

μ_0

$\overline{x}$

$s_{\overline{x}}$

χ^2 STATISTIC
TESTING A POPULATION VARIANCE EQUAL TO A VALUE σ_0^2

This program determines the test statistic necessary to test a variance against a given value. The equations used are:

$$X^2 = \frac{(n-1)\,s^2}{\sigma_o^2} = \chi^2 \quad \text{(Chi-Squared)}$$

where

n = Number of data observations in sample
s^2 = Sample variance
σ_o^2 = Given value to be tested against

Hypothesis:

H_o: $\sigma^2 = \sigma_o^2$
H_A: a. $\sigma^2 < \sigma_o^2$
b. $\sigma^2 > \sigma_0^2$

Level of significance: α

Critical regions: $X^2 < X^2_{n-1,\ 1-\alpha/2}$, $X^2 > X^2_{n-1,\alpha/2}$

Reference: Introduction to Statistical Inference, Jerome C. R. Li, Edwards Brothers, Inc., 1959.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

→ PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
1 ——— X

ENTER DATA:

$n \longrightarrow Z$
$s^2 \longrightarrow Y$
$\sigma_o^2 \longrightarrow X$

PRESS: CONTINUE

DISPLAY

X^2 ——— Z
X^2 ——— Y
X^2 ——— X

To run another case

EXAMPLE

A sample of 10 observations is drawn from a population. The observations;

4.8, 3.2, 3.6, 4.8, 6.1, 5.6, 4.7, 5.3, 5.1, and 7.5 ,

have a sample variance

$s^2 = 1.52622$

Question: Is the population variance equal to 4 ?
($\sigma_o^2 = 4$)

Hypothesis:

H_o: $\sigma^2 = 4$

H_A: $\sigma^2 < 4$ $(X^2 < X^2_{n-1,\ \alpha/2})$

$\sigma^2 > 4$ $(X^2 > X^2_{n-1,\ \alpha/2})$

Level of significance $\alpha = 5\%$

Results: $X^2 = 3.43400$

Conclusion: $X^2_{9,\ 9.5} = 2.70039$,
$X^2_{9,\ .05} = 19.0228$

The computed X^2 lies inside the critical region so the H_o is accepted.

STAT-PAC III-4

Step	Key	Code	X	Y	Z
00	CLR	20			
01	1	01			
02	STP	41	ENTRY		
03	PNT	45			
04	PNT	45			
05	DIV	35			
06	1	01			
07	XEY	30			
08	RDN	31			
09	-	34			
0a	DN	25			
0b	X	36			
0c	DN	25			
0d	UP	27			
10	UP	27			
11	PNT	45			
12	END	46			

χ^2 CHI SQUARE EVALUATION
EXPECTED VALUES EQUAL ($E_i = E$)

This program calculates the value of χ^2 by the equation

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E)^2}{E}$$

where

O_i — observed frequency

E — expected frequency of O_i is $E = \frac{\sum_{i=1}^{n} O_i}{n}$

Reference: Mathematical Statistics
by John E. Freund

Prentice - Hall 1962

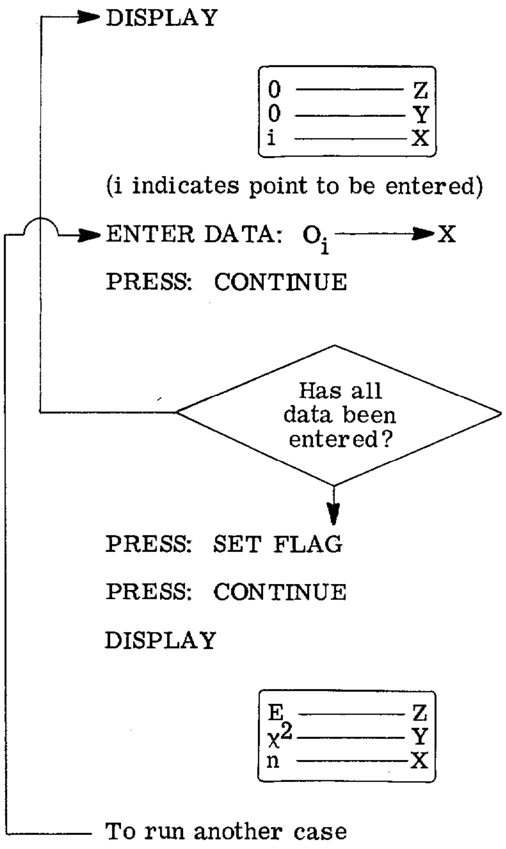
USER INSTRUCTIONS

DEPRESS: X on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE



EXAMPLES

The table shows the observed and expected frequencies in tossing a die 120 times. Calculate χ^2 for testing if the die is fair.

FACE	i	1	2	3	4	5	6
O_i Observed Frequency		25	17	15	23	24	16
E_i Expected Frequency		20	20	20	20	20	20

E = 20
χ^2 = 5.0
n = 6

```
00  CLR   20              40   0    00
01  1     01              41   END  46
02  XTO   23
03  d     17
04  STP   41     ENTRY
05  IFG   43
06  1     01
07  b     14
08  PNT   45
09  PNT   45
0a  UP    27
0b  X     36
0c  AC+   60
0d  0     00

10  UP    27
11  d     17
12  UP    27
13  1     01
14  +     33
15  YTO   40
16  d     17
17  DN    25
18  GTO   44
19  0     00
1a  4     04
1b  YE    24
1c  d     17
1d  1     01

20  -     34
21  YTO   40
22  d     17
23  RCL   61
24  UP    27
25  d     17
26  DIV   35
27  YTO   40
28  f     15
29  X     36
2a  f     15
2b  X     36
2c  RDN   31
2d  -     34

30  f     15
31  DIV   35
32  d     17
33  RUP   22
34  PNT   45
35  RUP   22
36  PNT   45
37  RUP   22
38  PNT   45     S
39  PNT   45
3a  PNT   45
3b  PNT   45
3c  GTO   44
3d  0     00
```

χ^2 2 x 2 CONTINGENCY TABLE

This program calculates χ^2 for a 2 x 2 contingency table containing observed frequencies. The table is shown below.

OBSERVED RESULTS

	I	II	TOTALS
GROUP A	a	b	a + b
GROUP B	c	d	c + d

(Note there is only one degree of freedom.) The equation used to evaluate χ^2 is

$$\chi^2 = \frac{(ad-bc)^2(a+b+c+d)}{(a+b)(c+d)(a+c)(b+d)}$$

Reference: Mathematical Statistics
by John E. Freund

Prentice - Hall 1962

USER INSTRUCTIONS

DEPRESS: X Y on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0 —— Z
0 —— Y
1 —— X

DATA:

a → Y
b → X

PRESS: CONTINUE

DISPLAY

0 —— Z
0 —— Y
2 —— X

ENTER DATA:

c → Y
d → X

PRESS: CONTINUE

DISPLAY

0 —— Z
0 —— Y
χ^2 —— X

EXAMPLES

FREQUENCIES OBSERVED

	I	II
GROUP A	75 (a)	25 (b)
GROUP B	65 (c)	35 (d)

$\chi^2 = 2.381$

STAT-PAC III-6

```
00  CLR  20
01   1   01
02  STP  41   ENTRY
03  PNT  45
04  AC+  60
05  XTO  23
06   b   14
07  YTO  40
08   a   13
09  DN   25
0a  DN   25
0b   2   02
0c  STP  41   ENTRY
0d  PNT  45

10  AC+  60
11  XTO  23
12   d   17
13  YTO  40
14   c   16
15  RCL  61
16  XEY  30
17  AC+  60
18   X   36
19   a   13
1a  UP   27
1b   b   14
1c   +   33
1d  DN   25

20   X   36
21   c   16
22  UP   27
23   d   17
24   +   33
25  DN   25
26   X   36
27  YE   24
28   a   13
29   d   17
2a   X   36
2b   b   14
2c  UP   27
2d   c   16

30   X   36
31  DN   25
32   -   34
33  DN   25
34  UP   27
35   X   36
36   e   12
37   X   36
38   a   13
39  DIV  35
3a   0   00
3b  UP   27
3c  RUP  22
3d  PNT  45

40  PNT  45
41  END  46
```

χ^2 CHI SQUARE EVALUATION
EXPECTED VALUES UNEQUAL ($E_i \neq E_j$)

This program calculates the value of χ^2 by the equation

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_j}$$

where O_i — observed frequency

E_i — expected frequency of O_i

Reference: Mathematical Statistics
by John E. Freund

Prentice - Hall 1962

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

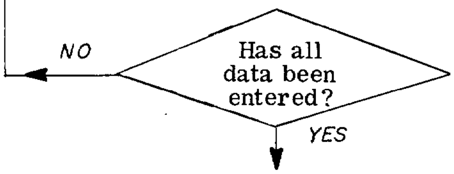
0 —— Z
0 —— Y
i —— X

(i indicates points to be entered)

ENTER DATA:

$O_i \rightarrow Y$
$E_i \rightarrow X$

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

0 —— Z
χ^2 —— Y
n —— X

To run another case

EXAMPLES

The table shows the observed and expected frequencies of some numbers. Calculate χ^2.

i	1	2	3	4	5	6
O_i - Observed Frequency	8	50	47	56	5	14
E_i - Expected Frequency	9.6	46.75	51.85	54.4	8.25	9.15

χ^2 = 4.844
n = 6

STAT-PAC III-7

```
00   CLR   20
01    1    01
02   AC+   60
03    f    15
04   STP   41      ENTRY
05   IFG   43
06    1    01
07    b    14
08   PNT   45
09   PNT   45
0a    -    34
0b   XEY   30
0c   UP    27
0d    X    36

10   DN    25
11   XEY   30
12   DIV   35
13    1    01
14   AC+   60
15    0    00
16   UP    27
17   UP    27
18   GTO   44
19    0    00
1a    3    03
1b   RCL   61
1c   UP    27
1d    1    01

20    -    34
21    0    00
22   RDN   31
23   PNT   45    S
24   PNT   45
25   PNT   45
26   PNT   45
27   GTO   44
28    0    00
29    0    00
2a   END   46
```

χ^2 - 2 x K CONTINGENCY TABLE

This program calculates χ^2 for a 2 x K contingency table containing observed frequencies. The table is shown below:

	1	2	3	4	5	TOTALS
A	a_1	a_2	a_3	. . .	a_K	N_A
B	b_1	b_2	b_3	. . .	b_K	N_B
TOTALS	N_1	N_2	N_3	. . .	N_K	N

The number of degrees of freedom $\gamma = K - 1$

The equation used to evaluate χ^2 is:

$$\chi^2 = \frac{N}{N_A}\left[\frac{a_1{}^2}{N_1} + \frac{a_2{}^2}{N_2} + \ . \ . \ . + \frac{a_K{}^2}{N_K}\right] + \frac{N}{N_B}\left[\frac{b_1{}^2}{N_1} + \frac{b_2{}^2}{N_2} + \ . \ . \ . + \frac{b_K{}^2}{N_K}\right] - N$$

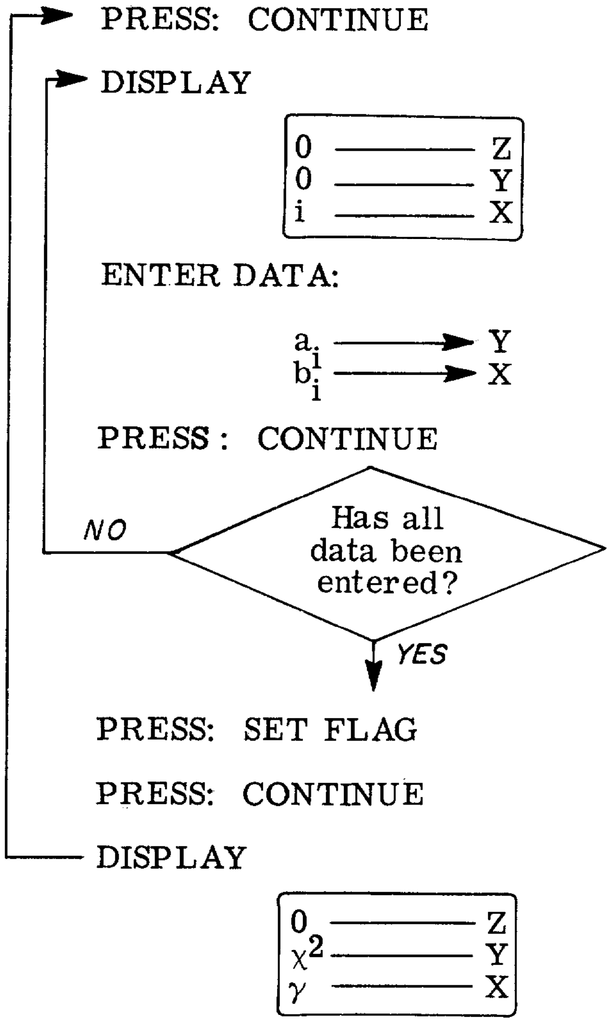
Reference: Mathematical Statistics, John E. Freund, Prentice-Hall, 1962.

USER INSTRUCTIONS

DEPRESS: X Y on the 9120A

PRESS: END

ENTER PROGRAM



PRESS: CONTINUE

DISPLAY

0 —— Z
0 —— Y
i —— X

ENTER DATA:

$a_i \rightarrow Y$
$b_i \rightarrow X$

PRESS: CONTINUE

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

0 —— Z
χ^2 —— Y
γ —— X

EXAMPLE

(2 x 3) Table

	I	II	III
A	2	5	4
B	3	8	7

$\chi^2 = .022$

$\gamma = 2$

STAT-PAC III-8

```
00  CLR  20
01  XTO  23
02   d   17
03  XTO  23
04   c   16
05   1   01
06  XTO  23
07   b   14
08  STP  41        ENTRY
09  IFG  43
0a   3   03
0b   d   17
0c  PNT  45
0d  PNT  45

10  AC+  60
11  UP   27
12  DN   25
13   +   33
14  YTO  40
15   a   13
16  XEY  30
17  DN   25
18  RDN  31
19   X   36
1a   a   13
1b  DIV  35
1c  XEY  30
1d  YEX  24

20   d   17
21   +   33
22  YTO  40
23   d   17
24  DN   25
25  RDN  31
26   X   36
27   a   13
28  DIV  35
29  XEY  30
2a  YE   24
2b   c   16
2c   +   33
2d  YTO  40

30   c   16
31   b   14
32  UP   27
33   1   01
34   +   33
35  YTO  40
36   b   14
37  CLX  37
38  UP   27
39  RUP  22
3a  GTO  44
3b   0   00
3c   8   10
3d  RCL  61

40   +   33
41  YTO  40
42   a   13
43   d   17
44   X   36
45   e   12
46  DIV  35
47   c   16
48  UP   27
49   f   15
4a  DIV  35
4b   a   13
4c   X   36
4d   -   34

50  DN   25
51   +   33
52   b   14
53  UP   27
54   2   02
55   -   34
56  CLX  37
57  RDN  31
58  PNT  45        S
59  PNT  45
5a  PNT  45
5b  PNT  45
5c  GTO  44
5d   0   00

60   0   00
61  END  46
```

t STATISTIC FOR TESTING CORRELATION COEFFICIENT

This program calculates the test statistic necessary in testing the hypothesis that correlation coefficient ρ is equal to zero. The equations used are:

$$t_{n-2} = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

where:

n = number of data points

r = estimate of the correlation coefficient

Application:

H_0 : $\rho = 0$

H_A : $\rho \neq 0$

α : Confidence level

t : $\dfrac{r\sqrt{n-2}}{\sqrt{1-r^2}}$

Conclusion:

If $t > t_{(1-\frac{\alpha}{2}\ ,\ n-2)}$, we reject the hypothesis H_0.

Reference: Statistics in Research, B. Ostle, Iowa State Press, 1966.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
1 ——— X

ENTER DATA

r → Y
n → X

PRESS: CONTINUE

DISPLAY

t ——— Z
r ——— Y
n ——— X

To run another case

EXAMPLE

Given the data and results from the second example of Program STAT-PAC IV-1 LINEAR REGRESSION AND CORRELATION COEFFICIENT, determine if $\rho = 0$.

$n = 5$
$r = .9$

Result:

$t_3 = 3.57624$

Choosing $\alpha = .05$, $t_{.975,\,3} = 3.18$.

Thus the test statistic $t > t_{(1-\frac{\alpha}{2}\,,\ n-2)}$

$(3.57 > 3.18)$

and we reject H_0 .

STAT-PAC III-9

Step	Key	Code	X	Y	Z
00	CLR	20			
01	1	01			
02	STP	41	ENTRY		
03	AC+	60			
04	UP	27			
05	2	02			
06	-	34			
07	DN	25			
08	√	76			
09	X	36			
0a	RUP	22			
0b	XEY	30			
0c	DN	25			
0d	UP	27			
10	X	36			
10	1	01			
12	XEY	30			
13	-	34			
14	DN	25			
15	√	76			
16	DIV	35			
17	UP	27			
18	RCL	61			
19	PNT	45			
1a	PNT	45			
1b	END	46			

r

n

BARTLETT'S χ^2 STATISTIC FOR VARIANCE HOMOGENEITY

This program computes a χ^2 statistic defined as:

$$\chi^2 = \frac{f \ln S^2 - \sum_{i=1}^{K} f_i \ln S_i^2}{1 + \frac{1}{3(K-1)} \left[\left(\sum_{i=1}^{K} \frac{1}{f_i} \right) - \frac{1}{\Sigma f_i} \right]} \qquad (1)$$

where:

S_i^2 = A Sample Variance

f_i = The Degrees of Freedom Associated With S_i^2

K = Number of Sample Variances

$S^2 = \frac{\sum_{i=1}^{K} f_i S_i^2}{f}$

$f = \sum_{i=1}^{K} f_i$

This χ^2 statistic has an <u>approximate</u> χ^2 distribution with K - 1 degrees of freedom.

This statistic may be used to test the hypothesis that the K sample variances $S_1^2, S_2^2, \ldots, S_K^2$ are all estimates of the same population variance σ^2.

$H_0 = S_1^2, S_2^2, \ldots, S_K^2$ are <u>each</u> estimates of σ^2.

$H_A = S_1^2, S_2^2, \ldots, S_K^2$ are not <u>all</u> estimates of σ^2.

Statistic: χ^2 as defined in (1)

Conclusion: If the computed χ^2 exceeds χ_α^2, the hypothesis H_0 is rejected.

Reference: Statistical Theory with Engineering Applications, A. Hald, John Wiley & Sons, 1960.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

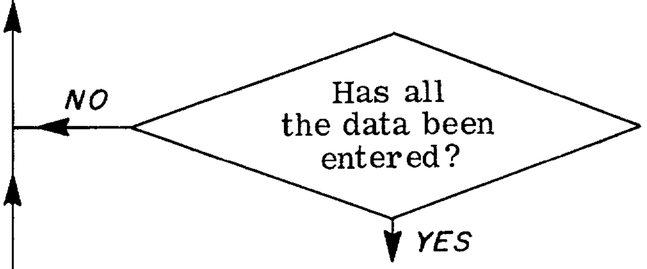
DISPLAY

0 ——— Z
i ——— Y
0 ——— X

ENTER DATA:

$S_i^2 \longrightarrow Y$
$f_i \longrightarrow X$

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

0 ——— Z
χ^2 ——— Y
K-1 ——— X (d.f.)

To run another case

EXAMPLE

The variances in the table represent variances in gas mileage measured during road testing. f_i is obtained from the number of cars in each sample road test. The samples represent slight changes in topology and local climatic conditions.

i	f_i	S_i^2
1	20	5.5
2	15	4.3
3	15	4.9
4	18	5.7
5	20	5.1

Results

$\chi^2 = .3804$

d.f. = 4

Application

H_o: The 5 samples variances each represent the population variance.

H_A: The 5 samples variances do not <u>all</u> represent the population variance

α : .05

Conclusion:

$\chi^2_{.05} = 9.4877$,

χ^2_{computed} does not

exceed $\chi^2_{.05}$.

The hypothesis H_o is not rejected.

00	CLR	20	
01	XTO	23	
02	d	17	
03	XTO	23	
04	c	16	
05	XTO	23	
06	b	14	
07	1	01	
08	XEY	30	
09	STP	41	ENTRY
0a	IFG	43	
0b	4	04	
0c	0	00	
0d	PNT	45	
10	PNT	45	
11	UP	27	
12	1	01	
13	AC+	60	
14	XEY	30	
15	DIV	35	
16	c	16	
17	XEY	30	
18	+	33	
19	YTO	40	
1a	c	16	
1b	XEY	30	
1c	1	01	
1d	XEY	30	
20	DIV	35	
21	DN	25	
22	XEY	30	
23	X	36	
24	RDN	31	
25	YE	24	
26	d	17	
27	+	33	
28	YTO	40	
29	d	17	
2a	RUP	22	
2b	DIV	35	
2c	LN	65	
2d	X	36	
30	b	14	
31	+	33	
32	YTO	40	
33	b	14	
34	0	00	
35	RDN	31	
36	f	15	
37	XEY	30	
38	1	01	
39	+	33	
3a	0	00	
3b	GTO	44	
3c	0	00	
3d	9	11	
40	d	17	
41	UP	27	
42	e	12	
43	DIV	35	
44	DN	25	
45	LN	65	
46	UP	27	
47	e	12	
48	X	36	
49	b	14	
4a	-	34	
4b	YTO	40	
4c	a	13	
4d	c	16	
50	UP	27	
51	1	01	
52	UP	27	
53	e	12	
54	DIV	35	
55	DN	25	
56	-	34	
57	3	03	
58	DIV	35	
59	f	15	
5a	UP	27	
5b	1	01	
5c	-	34	
5d	DN	25	
60	DIV	35	
61	1	01	
62	+	33	
63	a	13	
64	XEY	30	
65	DIV	35	
66	f	15	
67	UP	27	
68	1	01	
69	-	34	
6a	0	00	
6b	RDN	31	
6c	PNT	45	S
6d	PNT	45	
70	GTO	44	
71	0	00	
72	0	00	
73	END	46	

Numerator

$\Sigma f_i \ln S_i$

$\Sigma \frac{1}{f_i}$

$\Sigma f_i \; S_i^2$

Σf_i

n

CALCULATED Y_c, RESIDUAL, AND ACCUMULATED RESIDUAL OF LINEAR REGRESSION

This program calculates the quantities defined below for the linear regression model

$$Y = mX + b$$

given m, b and n the number of data points;

$$Y_c = \text{calculated } Y = mX_i + b$$

$$e_i = \text{residual at } X_i = Y_i - Y_c$$

$$\text{accumulated residual} = \sum_{i=1}^{n} e_i$$

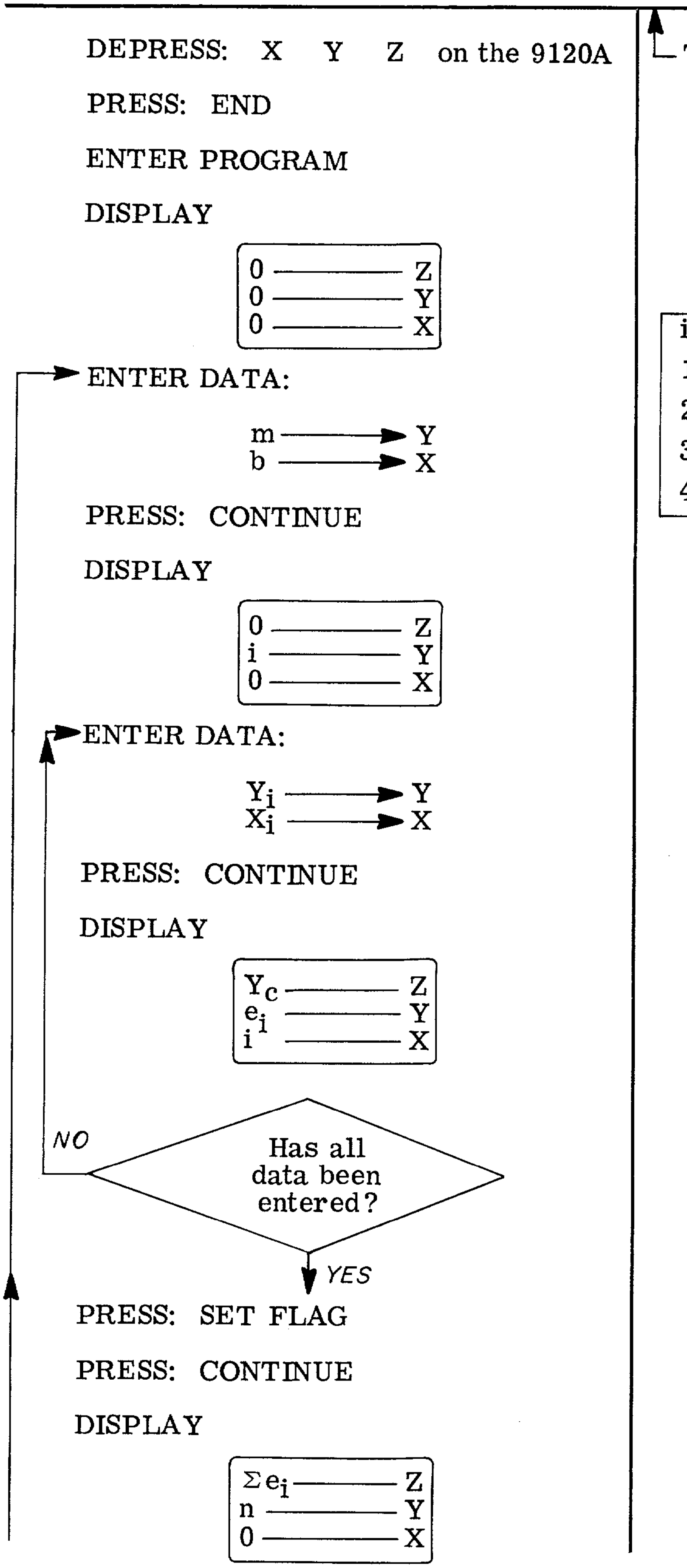
This program is to be used with programs STAT-PAC III-12 and III-13.

This program was written by Mr. Guy D. Nell, Clifford Shaw and Associates, Investment Advisors, Dallas, Texas.

Reference: Statistics - An Introductory Analysis, Taro Yamane, Harper & Row, 1964.

USER INSTRUCTIONS

USER INSTRUCTIONS (Con't)



DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

DISPLAY

ENTER DATA:

PRESS: CONTINUE

DISPLAY

ENTER DATA:

PRESS: CONTINUE

DISPLAY

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

EXAMPLE

m = .7789
b = 8.0737

	Given		Results	
i	X	Y	Y_c	e_i
1	1	9.1	8.85	-.25
2	.5	8.2	8.46	.26
3	2	9.5	9.63	.13
4	1	9.0	8.85	-.15

Σe_i = -.00015
n = 4

STAT-PAC III-11

Step	Key	Code	X	Y	Z
00	CLR	20			
01	STP	41	ENTRY		
02	PNT	45			
03	PNT	45			
04	XTO	23			
05	a	13			
06	YTO	40			
07	b	14			
08	CLR	20			
09	1	01			
0a	XEY	30			
0b	STP	41	ENTRY		
0c	IFG	43			
0d	2	02			
10	d	17			
11	PNT	45			
12	PNT	45			
13	UP	27			
14	b	14			
15	X	36			
16	a	13			
17	+	33			
18	RUP	22			
19	XEY	30			
1a	DN	25			
1b	-	34			
1c	1	01			
1d	AC+	60			
20	f	15			
21	PNT	45	s		
22	PNT	45			
23	UP	27			
24	0	00			
25	RDN	31			
26	XEY	30			
27	1	01			
28	+	33			
29	0	00			
2a	GTO	44			
2b	0	00			
2c	b	14			
2d	RCL	61			
30	UP	27			
31	0	00			
32	PNT	45	s		
33	PNT	45			
34	PNT	45			
35	GTO	44			
36	0	00			
37	0	00			
38	END	46			

Register

0		8	
1		9	
2		a	b
3		b	M
4		c	
5		d	
6		e	
7		f	

STATISTIC FOR DURBIN-WATSON TEST

This program computes the Durbin-Watson statistic d, defined as:

$$d = \frac{\sum_{i=1}^{n-1} (e_{i+1} - e_i)^2}{\sum_{i=1}^{n} e_i^2}$$

where the data values $\{e_i\}$ are the residuals calculated by program STAT-PAC III-11.

This test statistic is used to test for any serial correlation of the ε_i term associated with the linear regression model

$$Y_i = mX_i + b + \varepsilon_i$$

The error term ε_i is in general a random regression disturbance term which is independent of X and Y. In many business and economic cases, the ε_i may indeed be dependent thus reducing the usefulness of the above linear model.

This program was written by Mr. Guy D. Nell, Clifford Shaw & Associates, Investment Advisors, Dallas, Texas.

Reference: Statistics - An Introductory Analysis, Taro Yamane, Harper & Row, 1964, page 809.

USER INSTRUCTIONS

DEPRESS: X Y on the 9120A

PRESS: END

ENTER PROGRAM

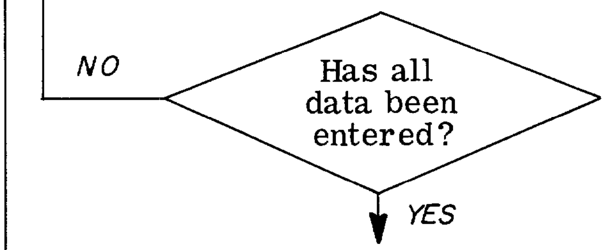
PRESS: CONTINUE

DISPLAY

0 —— Z
i —— Y
0 —— X

ENTER DATA: $e_i \longrightarrow X$

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

0 —— Z
d —— Y
n —— X

To run another case

EXAMPLE

Using the e_i (residuals) from the data of program STAT-PAC III-11, determine the Durbin-Watson statistic d.

i	e_i
1	-.2474
2	.26315
3	.1315
4	-.1474

Results:

d = 2.0993

This statistic is now used to test for serial correlation as described on page 811 of the reference.

00	CLR	20	
01	1	01	
02	XTO	23	
03	b	14	
04	XEY	30	
05	STP	41	ENTRY
06	PNT	45	
07	PNT	45	
08	XEY	30	
09	RDN	31	
0a	XTO	23	
0b	a	13	
0c	UP	27	
0d	X	36	
10	DN	25	
11	AC+	60	
12	1	01	
13	UP	27	
14	b	14	
15	+	33	
16	YTO	40	
17	b	14	
18	DN	25	
19	RUP	22	
1a	STP	41	ENTRY
1b	IFG	43	
1c	3	03	
1d	d	17	
20	PNT	45	
21	PNT	45	
22	UP	27	
23	X	36	
24	UP	27	
25	YE	24	
26	a	13	
27	XEY	30	
28	-	34	
29	DN	25	
2a	UP	27	
2b	X	36	
2c	DN	25	
2d	XEY	30	
30	AC+	60	
31	0	00	
32	UP	27	
33	1	01	
34	UP	27	
35	b	14	
36	+	33	
37	YTO	40	
38	b	14	
39	0	00	
3a	GTO	44	
3b	1	01	
3c	a	13	
3d	RCL	61	
40	DIV	35	
41	b	14	
42	UP	27	
43	1	01	
44	-	34	
45	0	00	
46	RDN	31	
47	PNT	45	S
48	PNT	45	
49	GTO	44	
4a	0	00	
4b	0	00	
4c	END	46	

STORAGE

0		8	
1		9	
2		a	e_t
3		b	n
4		c	
5		d	
6		e	$\Sigma(e_{t+1} - e_t)^2$
7		f	Σe_t^2

STATISTIC FOR SERIAL CORRELATION COEFFICIENT
(ORDER 1)

This program determines the serial correlation coefficient of order 1, given by:

$$r_1 = \frac{\sum_{i=2}^{n} \chi_i \chi_{i-1}}{\sum_{i=1}^{n} \chi_i^2}$$

where the data set $\{\chi_i\}$ is any time ordered set of data.

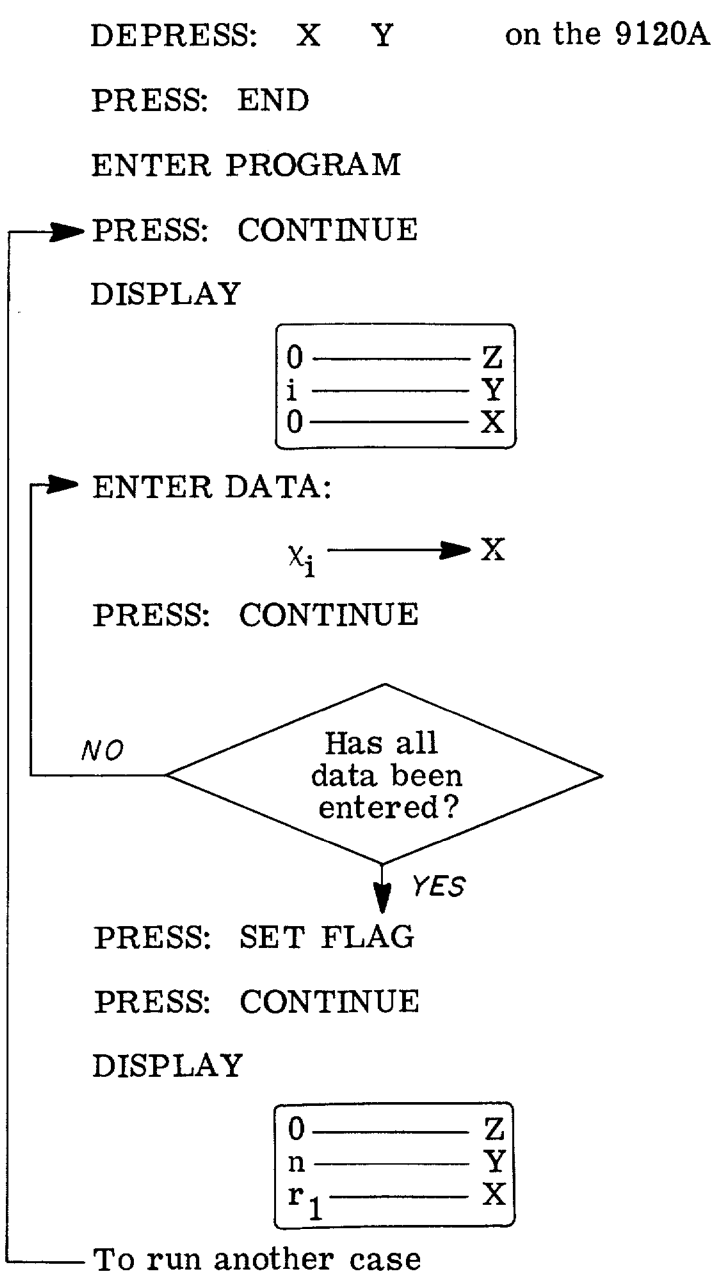
This statistic is used to test the statistical independence of a series of successive observations.

The circular correlation coefficient can be obtained by entering one extra data value, χ_{n+1}, equal to χ_1 .

This program was written by Mr. Guy D. Nell, Clifford Shaw & Associates, Investment Advisors, Dallas, Texas.

Reference: Statistics – An Introductory Analysis, Taro Yamane, Harper & Row, 1964, page 866.

USER INSTRUCTIONS



DEPRESS: X Y on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0 —— Z
i —— Y
0 —— X

ENTER DATA:

$\chi_i \longrightarrow X$

PRESS: CONTINUE

Has all data been entered?

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

0 —— Z
n —— Y
r_1 —— X

To run another case

EXAMPLE

1) Determine the serial correlation coefficient of the residuals computed by program STAT-PAC III-11 .

i	$\chi_i = e_i$
1	-.2474
2	.26315
3	.1315
4	-.1474

Result: $r_1 = -.3376$

2) Determine the circular correlation coefficient of the residuals of example 1 .

i	$\chi_i = e_i$
1	-.2474
2	.26315
3	.1315
4	-.1474
5	-.2474

Result: $r_{circular} = -.0792$

STAT-PAC III-13

00	CLR	20	
01	1	01	
02	XTO	23	
03	b	14	
04	XEY	30	
05	STP	41	ENTRY
06	PNT	45	
07	PNT	45	
08	XEY	30	
09	RDN	31	
0a	XTO	23	
0b	a	13	
0c	1	01	
0d	UP	27	
10	b	14	
11	+	33	
12	YTO	40	
13	b	14	
14	0	00	
15	UP	27	
16	RDN	31	
17	STP	41	ENTRY
18	IFG	43	
19	3	03	
1a	6	06	
1b	PNT	45	
1c	PNT	45	
1d	UP	27	
20	YE	24	
21	a	13	
22	XEY	30	
23	X	36	
24	UP	27	
25	X	36	
26	DN	25	
27	AC+	60	
28	0	00	
29	UP	27	
2a	1	01	
2b	UP	27	
2c	b	14	
2d	+	33	
30	0	00	
31	YTO	40	
32	b	14	
33	GTO	44	
34	1	01	
35	7	07	
36	1	01	
37	-	34	
38	UP	27	
39	RCL	61	
3a	DIV	35	
3b	0	00	
3c	RDN	31	
3d	PNT	45	
40	PNT	45	
41	END	46	

e_{t-1}

n

$\Sigma e_{t-1} e_t$

Σe_{t-1}^2

SPEARMAN'S RANK CORRELATION COEFFICIENT

The statistic to measure the degree of association between two variables X and Y, when the distributions of X and Y are unknown, is called a distribution - free statistic. Spearman's rank correlation coefficient was developed to measure the association between X and Y and depends only upon the ranks (or order) of the X and Y observations. This statistic is given by:

$$r_s = 1 - \frac{6 \Sigma d^2}{n(n^2-1)}$$

where

n = number of paired observations (X_i, Y_i)
d = $(X_i - Y_i)$

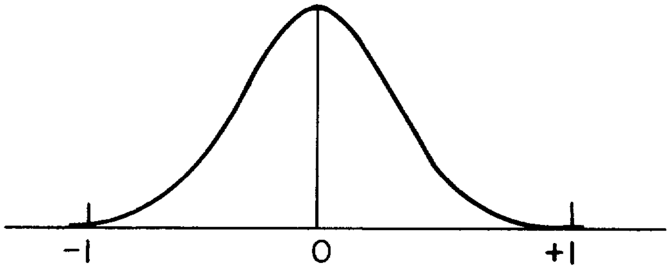
The variance of the Spearman's coefficient is:

$$\sigma_r^2 = \frac{1}{n-1}$$

The test statistic is defined by:

$$Z = \frac{r_s}{\sigma_r}$$

For $n > 20$, the statistic r_s is approximately normal bounded by ± 1. Its distribution is in fact a truncated normal distribution as seen below.



Use of r_s statistic

We want to test the dependence between X and Y. To do this, we test the null hypothesis that the population rank correlation coefficient ρ_s is 0. The Z statistic is a standardized normal variable with unit variance.

The statistic r_s can also be applied subjectively since it possesses properties similar to the regression coefficient r. That is

$$-1 \leq r_s \leq 1$$

where (+1) indicates complete agreement in order of the ranks and (-1) indicates complete agreement in the opposite order of the ranks.

Reference: Statistics - An Introductory Analysis, Taro Yamane, Harper & Row, 1967, page 467.

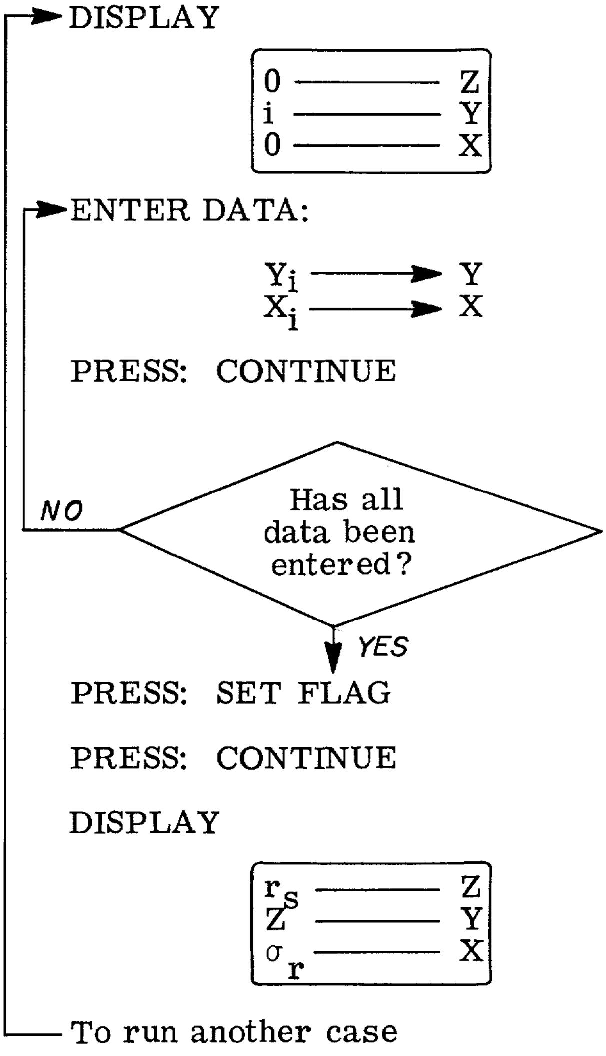
USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE



DISPLAY

ENTER DATA:

$Y_i \rightarrow Y$
$X_i \rightarrow X$

PRESS: CONTINUE

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

To run another case

EXAMPLE

See next page.

EXAMPLE

The following table lists the average grade attained in a student's senior year and the number of extra curricular activity events the student participated in. The data was taken at Panther High School, El Paso, Texas.

Student	X Average Senior Year Grade	Y Number of Activities	Rank X	Rank Y
1	82	18	9	5
2	67	11	18	11
3	91	25	4	1
4	98	17	1	6
5	74	10	15	12
6	52	9	20	13
7	86	13	6	9
8	81	14	10	8
9	75	15	14	7
10	83	23	8	2
11	89	21	5	3
12	92	8	3	14
13	95	20	2	4
14	79	2	12	19
15	73	6	16	16
16	78	5	13	17
17	65	0	19	20
18	84	12	7	10
19	69	7	17	15
20	80	4	11	18

Results:

r_s = .6150
Z = 2.6809
σ_r = .2294

Conclusion: Since Z represents a standard normal variate, a Z value of 2.68 means that r_s is 2.68 standard deviations from 0 in the sampling distribution of r_s. Thus, the null hypothesis that ρ_s is equal to zero is rejected.

00	CLR	20	
01	1	01	
02	XEY	30	
03	STP	41	ENTRY
04	IFG	43	
05	1	01	
06	a	13	
07	PNT	45	
08	PNT	45	
09	-	34	
0a	DN	25	
0b	UP	27	
0c	X	36	
0d	1	01	
10	AC+	60	
11	UP	27	
12	f	15	
13	+	33	
14	0	00	
15	UP	27	
16	RDN	31	
17	GTO	44	
18	0	00	
19	3	03	
1a	f	15	
1b	UP	27	
1c	X	36	
1d	1	01	
20	-	34	
21	f	15	
22	X	36	
23	e	12	
24	UP	27	
25	6	06	
26	X	36	
27	DN	25	
28	XEY	30	
29	DIV	35	
2a	1	01	
2b	XEY	30	
2c	-	34	
2d	f	15	
30	UP	27	
31	1	01	
32	-	34	
33	XEY	30	
34	DIV	35	
35	DN	25	
36	√	76	
37	DIV	35	
38	PNT	45	s
39	PNT	45	
3a	GTO	44	
3b	0	00	
3c	0	00	
3d	END	46	

Σd2

n

F – TEST FOR THE RATIO OF TWO SAMPLE VARIANCES (SNEDECOR)

This program determines the probability that an experimental value of $F(v_1, v_2)$ is greater than or equal to $F_T(v_1, v_2)$, F_T meaning F theoretical. This probability α is given by:

$$\alpha = P_r\,(F \geq F_T)$$

$$\alpha = \frac{\Gamma\left(\frac{v_1+v_2}{2}\right)}{\Gamma\left(\frac{v_1}{2}\right)\Gamma\left(\frac{v_2}{2}\right)} \; v_1^{\frac{v_1}{2}} \; v_2^{\frac{v_2}{2}} \int_{F_T}^{\infty} \frac{F^{\frac{v_1}{2}-1}\, dF}{(v_2 + v_1 F)^{\frac{(v_1+v_2)}{2}}}$$

where $F(v_1, v_2) = \frac{s_1^2}{s_2^2}$ = ratio of two sample variances

This integral is evaluated by the formulas:

1. for $\frac{v_1 F}{v_2} < 1$,

$$a = \arctan\sqrt{\frac{v_1 F}{v_2}} \leq \frac{\pi}{4}$$

$$\alpha = 1 - \frac{\Gamma(m+n)}{\Gamma(m)\Gamma(n)} \frac{\cos^{2m} a \sin^{2n} a}{n} \left[1 + \sum_{j=0}^{\infty} \frac{(m+n)\ldots(m+n+j-1)\sin^{2j} a}{(n+1)\ldots(n+j)}\right]$$

2. for $\frac{v_1 F}{v_2} > 1$,

$$a = \arctan\sqrt{\frac{v_2}{v_1 F}} < \frac{\pi}{4}$$

$$\alpha = \frac{\Gamma(m+n)}{\Gamma(m)\Gamma(n)} \frac{\cos^{2n} a \sin^{2m} a}{m} \left[1 + \sum_{j=0}^{\infty} \frac{(m+n)\ldots(m+n+j-1)\sin^{2j} a}{(m+1)\ldots(m+j)}\right]$$

In both cases:

$$m = \frac{v_1}{2}, \quad n = \frac{v_2}{2}$$

This program determines α when F, v_1, and v_2 are given.

This program was written by professors Marcel Gauthier and Jean-Claude Warmoes, of École Polytéchnique, Université de Montréal, Canada.

Reference: Le Contrôle Statistique des Fabrications, René Cavé, Éditions Eyrolles.

USER INSTRUCTIONS

SET: ☐ RADIANS

DEPRESS: X Y Z on the 9120A

→ PRESS: END

ENTER PROGRAM 1

PRESS: CONTINUE

DISPLAY

0 ———— Z
0 ———— Y
0 ———— X

ENTER DATA:

$v_1 \longrightarrow Z$
$v_2 \longrightarrow Y$
$F \longrightarrow X$

PRESS: CONTINUE

DISPLAY

1 ———— Z
1 ———— Y
1 ———— X

PRESS: END

ENTER PROGRAM 2

PRESS: CONTINUE

DISPLAY

———— Z
$P(F \geq F_T)$ — Y
———— X

To run another case:
Repeat USER INSTRUCTIONS.

EXAMPLE

v_1	v_2	F_{exp}	Result
12	3	5.2	.10
1	1	1	.50
60	120	1.66	.01
9	30	2.57	.025
4	60	5.31	.001

STAT-PAC III-15 Program 1

```
00  CLR  20
01  STP  41   ENTRY
02  PNT  45
03  PNT  45
04  XTO  23
05  a    13
06  2    02
07  DIV  35
08  YTO  40
09  c    16
0a  DN   25
0b  2    02
0c  DIV  35
0d  YTO  40

10  d    17
11  c    16
12  DIV  35
13  a    13
14  X    36
15  1    01
16  X>Y  53
17  2    02
18  6    06
19  d    17
1a  UP   27
1b  YE   24
1c  c    16
1d  YTO  40

20  d    17
21  DN   25
22  1    01
23  XEY  30
24  DIV  35
25  SFL  54
26  DN   25
27  √    76
28  ARC  72
29  TAN  71
2a  SIN  70
2b  UP   27
2c  X    36
2d  YTO  40

30  a    13
31  1    01
32  XTO  23
33  e    12
34  XTO  23
35  b    14
36  UP   27
37  f    15
38  +    33
39  YTO  40
3a  f    15
3b  d    17
3c  +    33
3d  UP   27

40  DN   25
41  c    16
42  +    33
43  1    01
44  -    34
45  DN   25
46  XEY  30
47  DIV  35
48  b    14
49  X    36
4a  a    13
4b  X    36
4c  YTO  40
4d  b    14

50  e    12
51  +    33
52  YTO  40
53  e    12
54  b    14
55  XEY  30
56  DIV  35
57  1    01
58  EEX  26
59  CHS  32
5a  4    04
5b  X>Y  53
5c  6    06
5d  5    05

60  1    01
61  UP   27
62  GTO  44
63  3    03
64  7    07
65  e    12
66  LN   65
67  XTO  23
68  b    14
69  a    13
6a  LN   65
6b  UP   27
6c  d    17
6d  X    36

70  LN   65
71  -    34
72  a    13
73  √    76
74  ARC  72
75  SIN  70
76  COS  73
77  LN   65
78  UP   27
79  c    16
7a  X    36
7b  2    02
7c  X    36
7d  b    14

80  +    33
81  DN   25
82  +    33
83  YTO  40
84  b    14
85  IFG  43
86  8    10
87  b    14
88  9    11
89  XTO  23
8a  a    13
8b  1    01
8c  UP   27
8d  UP   27

90  END  46
```

Program 2

```
00   CLR   20        40   +     33
01   9     11        41   YTO   40
02   UP    27        42   b     14
03   a     13        43   0     00
04   X<Y   52        44   XTO   23
05   SFL   54        45   f     15
06   CNT   47        46   c     16
07   d     17        47   UP    27
08   UP    27        48   GTO   44
09   UP    27        49   0     00
0a   1     01        4a   a     13
0b   X=Y   50        4b   +     33
0c   2     02        4c   YTO   40
0d   b     14        4d   e     12

10   X>Y   53        50   f     15
11   2     02        51   CHS   32
12   3     03        52   YE    24
13   -     34        53   b     14
14   DN    25        54   +     33
15   UP    27        55   YTO   40
16   UP    27        56   b     14
17   LN    65        57   0     00
18   YE    24        58   XTO   23
19   f     15        59   f     15
1a   +     33        5a   c     16
1b   YTO   40        5b   UP    27
1c   f     15        5c   d     17
1d   DN    25        5d   +     33

20   GTO   44        60   GTO   44
21   0     00        61   0     00
22   a     13        62   a     13
23   π     56        63   f     15
24   √     76        64   YE    24
25   LN    65        65   b     14
26   UP    27        66   +     33
27   f     15        67   DN    25
28   +     33        68   EXP   74
29   YTO   40        69   IFG   43
2a   f     15        6a   7     07
2b   e     12        6b   3     03
2c   UP    27        6c   UP    27
2d   1     01        6d   1     01

30   X<Y   52        70   XEY   30
31   6     06        71   -     34
32   3     03        72   DN    25
33   X=Y   50        73   UP    27
34   4     04        74   UP    27
35   b     14        75   PNT   45
36   +     33        76   PNT   45
37   YTO   40        77   PNT   45
38   e     12        78   PNT   45
39   UP    27        79   PNT   45
3a   f     15        7a   PNT   45
3b   CHS   32        7b   END   46
3c   UP    27
3d   b     14
```

r — DISTRIBUTION

Given n pairs of values (x_i, y_i). Program STAT-PAC IV-1 allows us to determine the line of best fit $Y = mX + b$ as well as the correlation coefficient r_{exp} (r_{exp} = r experimental).

$$r_{exp} = \frac{\text{cov}(x, y)}{\sqrt{\text{var } x \text{ var } y}} \quad \text{on } v = (n - 2) \quad \text{df}$$

The present program evaluates

$$\alpha = \text{Probability} \quad \left[|r_{exp}| \geqslant r_\alpha \right]$$

by using the relationships:

$$t_{(v = n - 2)} = \sqrt{\frac{n - 2}{1 - r^2}} \cdot r$$

$$t^2 (v) = \frac{r^2}{1 - r^2} (n - 2) = F(1, v)$$

where r_α is a given theoretical or expected value.

Consequently the series approximation for the F distribution (STAT-PAC III-15) can be used here.

This program was written by professors Marcel Gauthier and Jean-Claude Warmoes of École Polytéchnique, Université de Montréal, Canada.

Reference: Le Contrôle Statistique des Fabrications, Rene Cave, Editions Eyrolles.

USER INSTRUCTIONS

SET: ☐ RADIANS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM 1

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA:

v → Y (n - 2)
r_{exp} → X

PRESS: CONTINUE

DISPLAY

1 ——— Z
1 ——— Y
1 ——— X

PRESS: END

ENTER PROGRAM 2

PRESS: CONTINUE

DISPLAY

——— Z
$P_r[|r_{exp}| \geq r_\alpha]$ ——— Y
——— X

To run another case:
Repeat USER INSTRUCTIONS.

EXAMPLE

Determine $P_r [|r_{exp}| \geq r_\alpha]$

v	r_{exp}	Result
1	.9877	.10
10	.5760	.05
20	.4921	.02
45	.3721	.01
100	.3211	.001

STAT-PAC III-16 Program 1

00	CLR	20	
01	STP	41	ENTRY
02	PNT	45	
03	PNT	45	
04	UP	27	
05	X	36	
06	1	01	
07	XEY	30	
08	-	34	
09	XEY	30	
0a	DIV	35	
0b	DN	25	
0c	X	36	
0d	1	01	
10	RDN	31	
11	XTO	23	
12	a	13	
13	2	02	
14	DIV	35	
15	YTO	40	
16	c	16	
17	DN	25	
18	2	02	
19	DIV	35	
1a	YTO	40	
1b	d	17	
1c	c	16	
1d	DIV	35	
20	a	13	
21	X	36	
22	1	01	
23	IFG	43	
24	3	03	
25	3	03	
26	d	17	
27	UP	27	
28	YEX	24	
29	c	16	
2a	YTO	40	
2b	d	17	
2c	DN	25	
2d	1	01	
30	XEY	30	
31	DIV	35	
32	SFL	54	
33	DN	25	
34	√	76	
35	ARC	72	
36	TAN	71	
37	SIN	70	
38	UP	27	
39	X	36	
3a	YTO	40	
3b	a	13	
3c	1	01	
3d	XTO	23	
40	e	12	
41	XTO	23	
42	b	14	
43	UP	27	
44	f	15	
45	+	33	
46	YTO	40	
47	f	15	
48	d	17	
49	+	33	
4a	UP	27	
4b	DN	25	
4c	c	16	
4d	+	33	
50	1	01	
51	-	34	
52	DN	25	
53	XEY	30	
54	DIV	35	
55	b	14	
56	X	36	
57	a	13	
58	X	36	
59	YTO	40	
5a	b	14	
5b	e	12	
5c	+	33	
5d	YTO	40	
60	e	12	
61	b	14	
62	XEY	30	
63	DIV	35	
64	1	01	
65	EEX	26	
66	CHS	32	
67	4	04	
68	X>Y	53	
69	7	07	
6a	2	02	
6b	1	01	
6c	UP	27	
6d	GTO	44	
70	4	04	
71	2	02	
72	e	12	
73	LN	65	
74	XTO	23	
75	b	14	
76	a	13	
77	LN	65	
78	UP	27	
79	d	17	
7a	X	36	
7b	LN	65	
7c	-	34	
7d	a	13	
80	√	76	
81	ARC	72	
82	SIN	70	
83	COS	73	
84	LN	65	
85	UP	27	
86	c	16	
87	X	36	
88	2	02	
89	X	36	
8a	b	14	
8b	+	33	
8c	DN	25	
8d	+	33	
90	YTO	40	
91	b	14	
92	IFG	43	
93	9	11	
94	8	10	
95	9	11	
96	XTO	23	
97	a	13	
98	1	01	
99	UP	27	
9a	UP	27	
9b	END	46	

STAT-PAC III-16 Program 2

```
00  CLR  20      40   +   33
01   9   11      41  YTO  40
02  UP   27      42   b   14
03   a   13      43   0   00
04  X<Y  52      44  XTO  23
05  SFL  54      45   f   15
06  CNT  47      46   c   16
07   d   17      47  UP   27
08  UP   27      48  GTO  44
09  UP   27      49   0   00
0a   1   01      4a   a   13
0b  X=Y  50      4b   +   33
0c   2   02      4c  YTO  40
0d   b   14      4d   e   12

10  X>Y  53      50   f   15
11   2   02      51  CHS  32
12   3   03      52  YEX  24
13   -   34      53   b   14
14  DN   25      54   +   33
15  UP   27      55  YTO  40
16  UP   27      56   b   14
17  LN   65      57   0   00
18  YEX  24      58  XTO  23
19   f   15      59   f   15
1a   +   33      5a   c   16
1b  YTO  40      5b  UP   27
1c   f   15      5c   d   17
1d  DN   25      5d   +   33

20  GTO  44      60  GTO  44
21   0   00      61   0   00
22   a   13      62   a   13
23   π   56      63   f   15
24   √   76      64  YEX  24
25  LN   65      65   b   14
26  UP   27      66   +   33
27   f   15      67  DN   25
28   +   33      68  EXP  74
29  YTO  40      69  IFG  43
2a   f   15      6a   7   07
2b   e   12      6b   3   03
2c  UP   27      6c  UP   27
2d   1   01      6d   1   01

30  X<Y  52      70  XEY  30
31   6   06      71   -   34
32   3   03      72  DN   25
33  X=Y  50      73  UP   27
34   4   04      74  UP   27
35   b   14      75  PNT  45
36   +   33      76  PNT  45
37  YTO  40      77  PNT  45
38   e   12      78  PNT  45
39  UP   27      79  PNT  45
3a   f   15      7a  PNT  45
3b  CHS  32      7b  END  46
3c  UP   27
3d   b   14
```

SECTION IV CURVE FITTING

LINEAR REGRESSION AND CORRELATION COEFFICIENT

This program calculates the equation of the straight line of best fit of a set of data points. The best fit is determined by minimizing the sum of the squares of the deviations of the data points from the line.

The program calculates m and b for the equation:

$$Y = mX + b$$

The program also calculates a correlation coefficient r, an indication of goodness of fit. Note: $-1 \le r \le 1$ where the sign corresponds to the slope m. If $r = 0$, there is no correlation, and if $r = \pm 1$, there is perfect correlation or a perfect fit.

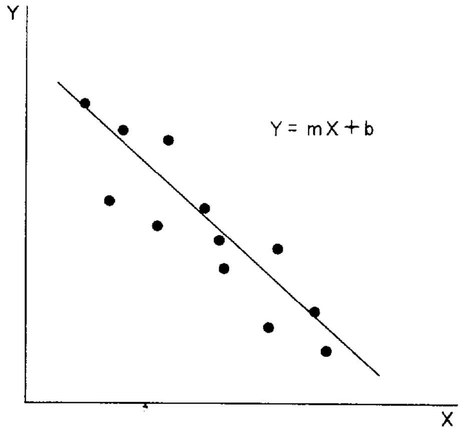
The defining equations are:

$$m = \frac{\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^{n} (X_i - \overline{X})^2}$$

$$b = \overline{Y} - m\overline{X}$$

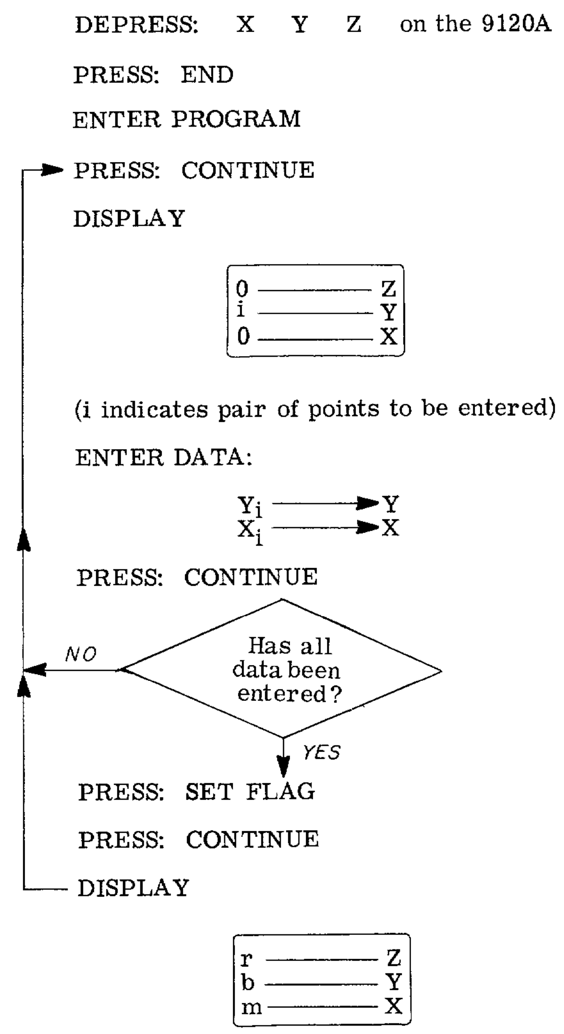
$$\text{where } \overline{Y} = \frac{\sum_{i=1}^{n} Y_i}{n} \quad \text{and} \quad \overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

$$r = \frac{\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\sum_{i=1}^{n} (X_i - \overline{X})^2 \sum_{i=1}^{n} (Y_i - \overline{Y})^2}} = \frac{n\sum_{i=1}^{n} X_iY_i - (\sum_{i=1}^{n} X_i)(\sum_{i=1}^{n} Y_i)}{\sqrt{\left[n\sum_{i=1}^{n} X_i{}^2 - (\sum_{i=1}^{n} X_i)^2\right]\left[n\sum_{i=1}^{n} Y_i{}^2 - (\sum_{i=1}^{n} Y_i)^2\right]}}$$



Reference: Mathematical Statistics
by John E. Freund
Prentice-Hall, 1962

USER INSTRUCTIONS



DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0 ———— Z
i ———— Y
0 ———— X

(i indicates pair of points to be entered)

ENTER DATA:

$Y_i \rightarrow Y$
$X_i \rightarrow X$

PRESS: CONTINUE

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

r ———— Z
b ———— Y
m ———— X

EXAMPLES

X	Y
26	92
30	85
44	78
50	81
62	54
68	51
74	40

r = -.96
b = 121.04
m = -1.03

Y = -1.03X + 121.04

X	Y
0	1
1	3
2	2
3	4
4	5

r = .9
b = 1.2
m = .9

Y = .9X + 1.2

STAT-PAC IV-1

```
00  CLR  20
01  XTO  23
02   d   17
03  XTO  23
04   c   16
05  XTO  23
06   b   14
07   1   01
08  XTO  23
09   a   13
0a  XEY  30
0b  STP  41        ENTRY
0c  IFG  43
0d   3   03

10   a   13
11  PNT  45
12  PNT  45
13  AC+  60
14  UP   27
15   X   36
16  XEY  30
17  YE   24
18   d   17
19   +   33
1a  YEX  24
1b   d   17
1c  DN   25
1d   X   36

20   b   14
21   +   33
22  YTO  40
23   b   14
24  RUP  22
25  UP   27
26   X   36
27   c   16
28   +   33
29  YTO  40
2a   c   16
2b   a   13
2c  UP   27
2d   1   01

30   +   33
31  YTO  40
32   a   13
33   0   00
34  RDN  31
35  XEY  30
36   0   00
37  GTO  44
38   0   00
39   b   14
3a   a   13
3b  UP   27
3c   1   01
3d   -   34

40  DN   25
41  XTO  23
42   a   13
43   e   12
44  UP   27
45   a   13
46  DIV  35
47  YE   24
48   f   15
49  DIV  35
4a  YTO  40
4b   e   12
4c   X   36
4d   e   12

50   X   36
51   d   17
52  XEY  30
53   -   34
54  YTO  40
55   d   17
56   c   16
57  UP   27
58   f   15
59  UP   27
5a   X   36
5b   a   13
5c   X   36
5d  DN   25

60   -   34
61  YTO  40
62   c   16
63   b   14
64  UP   27
65   f   15
66  UP   27
67   e   12
68   X   36
69   a   13
6a   X   36
6b  DN   25
6c   -   34
6d   d   17

70  UP   27
71  DN   25
72   √   76
73  DIV  35
74   c   16
75   √   76
76  DIV  35
77   d   17
78  RUP  22
79  XEY  30
7a  DIV  35
7b  YE   24
7c   e   12
7d   e   12

80   X   36
81   f   15
82  XEY  30
83   -   34
84   e   12
85  PNT  45
86  PNT  45
87  END  46
```

LINEAR REGRESSION WITH MEANS, RMS VALUES, VARIANCES, AND CORRELATION COEFFICIENT

This program fits a curve of the form $Y = mX + b$, solving for the slope (m) and intercept (b), given the input data $(X_1, Y_1), (X_2, Y_2), \ldots (X_n, Y_n)$ for any number of sets (n).

The means ($\overline{Y}$ and $\overline{X}$), rms values (Yrms and Xrms), standard deviations (σY and σX), and correlation coefficient (r) are also found.

The following equations are used:

1) $\overline{Y} = \frac{\Sigma Y}{n}$; $\overline{X} = \frac{\Sigma X}{n}$ (means).

2) $N = \overline{XY} - \overline{X}\overline{Y}$ (intermediate value).

3) $Yrms = \sqrt{\overline{Y^2}} = \sqrt{\frac{\Sigma Y^2}{n}}$; $Xrms = \sqrt{\overline{X^2}} = \sqrt{\frac{\Sigma X^2}{n}}$

4) $\sigma Y = \sqrt{\overline{Y^2} - (\overline{Y})^2}$; $\sigma X = \sqrt{\overline{X^2} - (\overline{X})^2}$ (standard deviations).

5) $m = \frac{N}{(\sigma X)^2}$ (slope).

6) $b = \overline{Y} - m\overline{X}$ (intercept).

7) $r = \frac{m\sigma X}{\sigma Y}$ (correlation coefficient).

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

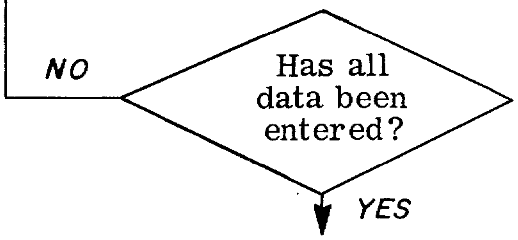
0 —— Z
n —— Y
0 —— X

n indicates the data set (X_n, Y_n) to be entered.

ENTER DATA:

$Y_n \longrightarrow Z$
$X_n \longrightarrow X$

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

n —— Z
$\overline{Y}$ —— Y
$\overline{X}$ —— X

PRESS: CONTINUE

DISPLAY

n —— Z
Yrms —— Y
Xrms —— X

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

n —— Z
σY —— Y
σX —— X

PRESS: CONTINUE

DISPLAY

r —— Z
b —— Y
m —— X

To run another case

EXAMPLE

Data:

(n = 5)

X	Y
1	3
0	1
-.5	0
2	5
-2	-3

Solution:

$\overline{Y}$ = 1.2
$\overline{X}$ = .1

Yrms = 2.966
Xrms = 1.360

σY = 2.713
σX = 1.356

m = 2
b = 1
r = 1

```
00  CLR   20
01  XTO   23
02   a    13
03  XTO   23
04   b    14
05  XTO   23
06   c    16
07  XTO   23
08   d    17
09   1    01
0a  AC+   60
0b   f    15
0c  XEY   30
0d  CLX   37

10  STP   41      ENTRY
11  IFG   43
12   3    03
13   b    14
14  PNT   45
15  PNT   45
16  UP    27
17  YEX   24
18   a    13
19   +    33
1a  YE    24
1b   a    13
1c   X    36
1d  RDN   31

20  YE    24
21   b    14
22   +    33
23  YE    24
24   b    14
25  DN    25
26  UP    27
27  YE    24
28   c    16
29   +    33
2a  YE    24
2b   c    16
2c   X    36
2d  RDN   31

30  YE    24
31   d    17
32   +    33
33  YE    24
34   d    17
35  CLX   37
36  RDN   31
37   X    36
38  GTO   44
39   0    00
3a   9    11
3b  UP    27
3c   1    01
3d  AC-   63
```

```
40   c    16
41  UP    27
42   f    15
43  DIV   35
44  YTO   40
45   c    16
46  UP    27
47   a    13
48  XEY   30
49  DIV   35
4a  YTO   40
4b   a    13
4c  RDN   31
4d  PNT   45    s

50  PNT   45
51   X    36
52   e    12
53  RUP   22
54  DIV   35
55  RDN   31
56  XEY   30
57   -    34
58  YTO   40
59   e    12
5a   d    17
5b  RUP   22
5c  DIV   35
5d  YTO   40

60   d    17
61  DN    25
62   √    76
63  UP    27
64   b    14
65  UP    27
66   f    15
67  DIV   35
68  YTO   40
69   b    14
6a  RDN   31
6b   √    76
6c  PNT   45
6d  PNT   45    s

70   b    14
71  UP    27
72   a    13
73  UP    27
74   X    36
75  DN    25
76   -    34
77   e    12
78  XEY   30
79  DIV   35
7a  YTO   40
7b   e    12
7c   √    76
7d  UP    27
```

```
80   c    16
81  UP    27
82   X    36
83   d    17
84  XEY   30
85   -    34
86   f    15
87  RDN   31
88   √    76
89  XEY   30
8a  PNT   45    s
8b  PNT   45
8c  UP    27
8d   e    12

90   X    36
91  DN    25
92  XEY   30
93  DIV   35
94   a    13
95  UP    27
96   e    12
97   X    36
98   c    16
99  XEY   30
9a   -    34
9b   e    12
9c  PNT   45
9d  END   46
```

MULTIPLE LINEAR REGRESSION
THREE VARIABLE

This program fits any number of data points (X_i, Y_i, Z_i) to a linear, two variable equation of the form:

$$Z = a_o + a_1 X + a_2 Y$$

where X and Y are the independent variables.

Development:

The constants a_o, a_1, and a_2 of the equation may be found by solving simultaneously the following normal equations which represent the least square plane (approximating plane) formed by the data points.

$$\sum Z = a_o n + a_1 \sum X + a_2 \sum Y$$

$$\sum XZ = a_o \sum X + a_1 \sum X^2 + a_2 \sum XY$$

$$\sum YZ = a_o \sum Y + a_1 \sum XY + a_2 \sum Y^2$$

In the program the constant a_2 is found from solving the equations by matrix algebra. Therefore,

$$a_2 = \frac{n(\sum X^2 \sum YZ - \sum XZ \sum XY) - \sum X(\sum X \sum YZ - \sum Y \sum XZ) + \sum Z(\sum X \sum XY - \sum Y \sum X^2)}{D}$$

Where D (the determinant) = $\begin{vmatrix} n & \sum X & \sum Y \\ \sum X & \sum X^2 & \sum XY \\ \sum Y & \sum XY & \sum Y^2 \end{vmatrix}$

After finding a_2, the solution is reduced to two equations in two unknowns which are:

$$M = a_o n + a_1 \sum X$$

$$N = a_o \sum X + a_1 \sum X^2$$

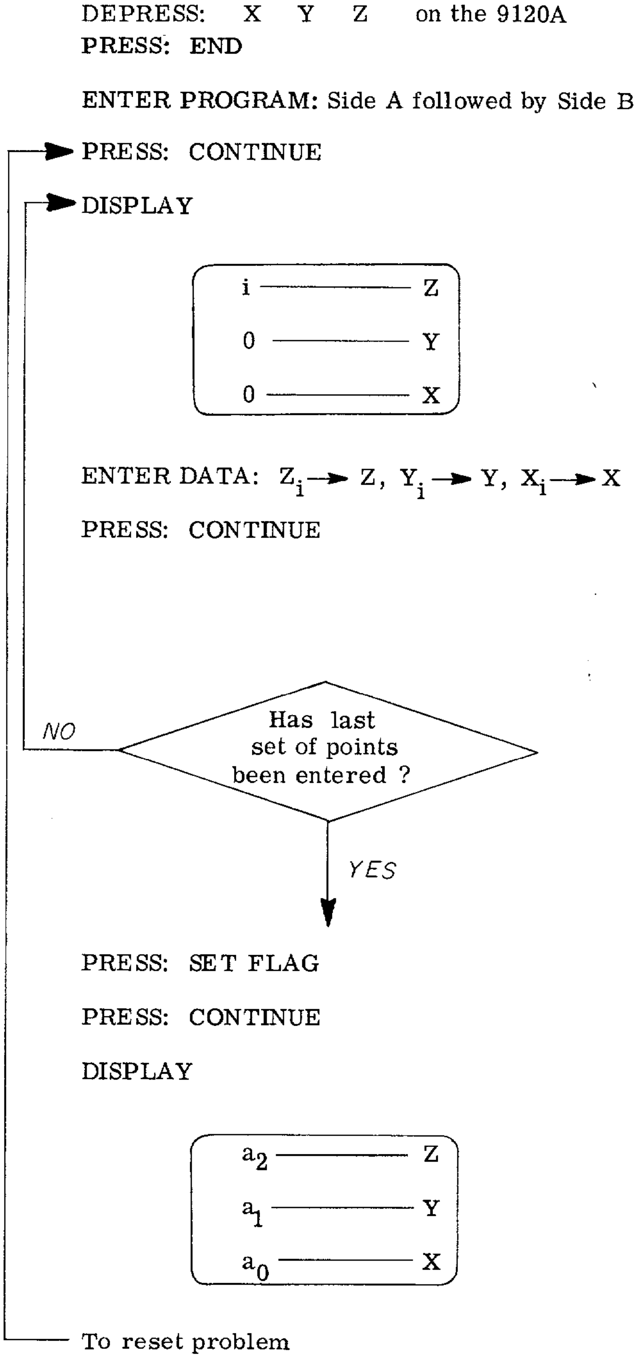
where $M = (\sum Z - a_2 \sum Y)$ $\qquad$ $N = (\sum XZ - a_2 \sum XY)$

These two equations are then solved for a_o and a_1.

Reference:

Introduction to the Theory of Statistics
Mood and Graybill
McGraw-Hill, 1963

USER INSTRUCTIONS



DEPRESS: X Y Z on the 9120A
PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

DISPLAY

i —————— Z
0 —————— Y
0 —————— X

ENTER DATA: $Z_i \rightarrow Z$, $Y_i \rightarrow Y$, $X_i \rightarrow X$

PRESS: CONTINUE

Has last set of points been entered ?

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

a_2 —————— Z
a_1 —————— Y
a_0 —————— X

To reset problem

EXAMPLES

(A) Equation of the form:

$$Z = a_o + a_1 X + a_2 Y$$

Input data:

X	Y	Z
1	0	3
0	1	4
1	1	6
3	4	19
2	2	11

Solution:

$$Z = 1 + 2X + 3Y$$

(B) Equation of the form :

$$Z = a_o + a_1 (\log X) + a_2 (\log Y)$$

Note to enter data ;
ENTER: $Z_i \rightarrow X$
PRESS: ↑
ENTER: $Y_i \rightarrow X$
PRESS: log X
PRESS: ↑
ENTER: $X_i \rightarrow X$
PRESS: log X

Input data:

X	Y	Z
1	2	4.6505
1	1	4.5
4	3.63	5.9841
10	5	6.8495
8	16	6.9082
13	7	7.1504
3	10	5.9542

Solution:

$$Z = 4.5 + 2 (\log X) + .4999 (\log Y)$$

00	CLR	20	Plus Page
01	XTO	23	
02	a	13	
03	XTO	23	
04	b	14	
05	XTO	23	
06	c	16	
07	XTO	23	
08	9	11	
09	XTO	23	
0a	-	34	
0b	f	15	
0c	XTO	23	
0d	-	34	
10	e	12	
11	XTO	23	
12	8	10	
13	1	01	
14	XTO	23	
15	d	17	
16	RDN	31	
17	STP	41	ENTRY
18	IFG	43	
19	7	07	
1a	1	01	
1b	PNT	45	
1c	PNT	45	
1d	YTO	40	
20	-	34	
21	d	17	
22	X	36	
23	XEY	30	
24	YE	24	
25	a	13	
26	+	33	
27	YE	24	
28	a	13	
29	RDN	31	
2a	AC+	60	
2b	UP	27	
2c	X	36	
2d	XEY	30	
30	YE	24	
31	-	34	
32	e	12	
33	+	33	
34	YE	24	
35	-	34	
36	e	12	
37	DN	25	
38	X	36	
39	c	16	
3a	+	33	
3b	YTO	40	
3c	c	16	
3d	RUP	22	
40	UP	27	
41	X	36	
42	XEY	30	
43	YE	24	
44	8	10	
45	+	33	
46	YE	24	
47	8	10	
48	XFR	67	
49	-	34	
4a	d	17	
4b	X	36	
4c	UP	27	
4d	X	36	
50	RUP	22	
51	YE	24	
52	9	11	
53	+	33	
54	YE	24	
55	9	11	
56	b	14	
57	+	33	
58	YTO	40	
59	b	14	
5a	RUP	22	
5b	YE	24	
5c	-	34	
5d	f	15	
60	+	33	
61	YTO	40	
62	-	34	
63	f	15	
64	1	01	
65	UP	27	
66	d	17	
67	+	33	
68	YTO	40	
69	d	17	
6a	CLX	37	
6b	UP	27	
6c	GTO	44	
6d	1	01	
70	7	07	
71	YE	24	
72	-	34	
73	f	15	
74	d	17	
75	UP	27	
76	1	01	
77	-	34	
78	YTO	40	
79	d	17	
7a	GTO	44	
7b	-	34	
7c	0	00	
7d	0	00	
80	CNT	47	
81	CNT	47	
82	CNT	47	
83	CNT	47	
84	CNT	47	
85	CNT	47	
86	CNT	47	
87	CNT	47	
88	CNT	47	
89	CNT	47	
8a	CNT	47	
8b	CNT	47	
8c	CNT	47	
8d	CNT	47	
90	CNT	47	
91	CNT	47	
92	CNT	47	
93	CNT	47	
94	CNT	47	
95	CNT	47	
96	CNT	47	
97	CNT	47	
98	CNT	47	
99	CNT	47	
9a	CNT	47	
9b	CNT	47	
9c	CNT	47	
9d	CNT	47	
a0	CNT	47	
a1	CNT	47	
a2	CNT	47	
a3	CNT	47	
a4	CNT	47	
a5	CNT	47	
a6	CNT	47	
a7	CNT	47	
a8	CNT	47	
a9	CNT	47	
aa	CNT	47	
ab	CNT	47	
ac	CNT	47	
ad	CNT	47	

ΣZ^2

ΣYZ

ΣXY

ΣY

ΣXZ

n

ΣZ

ΣX

```
b0 CNT 47
b1 CNT 47
b2 CNT 47
b3 CNT 47
b4 CNT 47
b5 CNT 47
b6 CNT 47
b7 CNT 47
b8 CNT 47
b9 CNT 47
ba CNT 47
bb CNT 47
bc CNT 47
bd CNT 47

c0 CNT 47
c1 CNT 47
c2 CNT 47
c3 CNT 47
c4 CNT 47
c5 CNT 47
c6 CNT 47
c7 CNT 47
c8 CNT 47
c9 CNT 47
ca CNT 47
cb CNT 47
cc CNT 47
cd CNT 47

d0 CNT 47
d1 CNT 47
d2 CNT 47
d3 CNT 47
d4 CNT 47
d5 CNT 47
d6 CNT 47
d7 CNT 47
d8 CNT 47
d9 CNT 47
da CNT 47
db CNT 47
dc CNT 47
dd CNT 47
```

STAT-PAC IV-3

Minus Page

```
00  DN   25
01  f    15
02  X    36
03  CHS  32
04  X    36
05  RUP  22
06  YE   24
07  -    34
08  e    12
09  YTO  40
0a  -    34
0b  e    12
0c  X    36
0d  d    17

10  X    36
11  RDN  31
12  +    33
13  a    13
14  UP   27
15  X    36
16  d    17
17  X    36
18  DN   25
19  -    34
1a  UP   27
1b  YE   24
1c  -    34
1d  e    12

20  YTO  40
21  -    34
22  e    12
23  b    14
24  X    36
25  X    36
26  DN   25
27  -    34
28  b    14
29  UP   27
2a  f    15
2b  X    36
2c  a    13
2d  X    36

30  2    02
31  X    36
32  DN   25
33  +    33
34  YTO  40
35  -    34
36  f    15
37  YE   24
38  9    11
39  UP   27
3a  DN   25
3b  f    15
3c  X    36
3d  CHS  32

40  X    36
41  RUP  22
42  XEY  30
43  d    17
44  X    36
45  XFR  67
46  -    34
47  e    12
48  X    36
49  RDN  31
4a  +    33
4b  RUP  22
4c  XEY  30
4d  b    14

50  X    36
51  e    12
52  X    36
53  RDN  31
54  -    34
55  f    15
56  RUP  22
57  X    36
58  a    13
59  X    36
5a  RDN  31
5b  +    33
5c  d    17
5d  RUP  22

60  X    36
61  c    16
62  X    36
63  RDN  31
64  -    34
65  f    15
66  RUP  22
67  X    36
68  b    14
69  X    36
6a  DN   25
6b  +    33
6c  XFR  67
6d  -    34

70  f    15
71  DIV  35
72  YTO  40
73  9    11
74  UP   27
75  DN   25
76  b    14
77  X    36
78  e    12
79  XEY  30
7a  -    34
7b  a    13
7c  YTO  40
7d  a    13

80  RUP  22
81  X    36
82  c    16
83  XEY  30
84  -    34
85  YTO  40
86  c    16
87  f    15
88  UP   27
89  d    17
8a  YTO  40
8b  d    17
8c  YE   24
8d  -    34

90  e    12
91  YTO  40
92  -    34
93  f    15
94  XTO  23
95  b    14
96  CLR  20
97  YE   24
98  -    34
99  f    15
9a  c    16
9b  UP   27
9c  d    17
9d  DIV  35

a0  RUP  22
a1  XEY  30
a2  DIV  35
a3  DN   25
a4  IFG  43
a5  b    14
a6  4    04
a7  AC+  60
a8  d    17
a9  YE   24
aa  b    14
ab  UP   27
ac  a    13
ad  RUP  22
```

Y

ΣX^2

ΣY^2

```
b0  SFL  54
b1  GTO  44
b2   9   11
b3   d   17
b4  AC-  63
b5  XEY  30
b6  YE   24
b7   e   12
b8   f   15
b9  DIV  35
ba   e   12
bb  XEY  30
bc   X   36
bd  RDN  31

c0   -   34
c1  RUP  22
c2  YE   24
c3   9   11
c4  RUP  22
c5  PNT  45
c6  PNT  45
c7  END  46
```

MULTIPLE LINEAR REGRESSION FOUR VARIABLE

$$X_1 = a_1 + a_2X_2 + a_3X_3 + a_4X_4 \qquad (1)$$

This program determines the coefficients of the normal equations required for determining the least squares regression equation (1). An option is available for correcting erroneously inputted data.

Given a set of data points $(X_{1i}, X_{2i}, X_{3i}, X_{4i})$ the normal equations are defined as:

$$a_1N + a_2\Sigma X_2 + a_3\Sigma X_3 + a_4\Sigma X_4 = \Sigma X_1$$

$$a_1\Sigma X_2 + a_2\Sigma X_2^2 + a_3\Sigma X_2X_3 + a_4\Sigma X_2X_4 = \Sigma X_1X_2$$

$$a_1\Sigma X_3 + a_2\Sigma X_2X_3 + a_3\Sigma X_3^2 + a_4\Sigma X_3X_4 = \Sigma X_1X_3$$

$$a_1\Sigma X_4 + a_2\Sigma X_2X_4 + a_3\Sigma X_3X_4 + a_4\Sigma X_4^2 = \Sigma X_1X_4$$

The quantities, N, ΣX_2, ΣX_3, ΣX_4, ΣX_1, ΣX_2^2, ΣX_3^2, ΣX_4^2, ΣX_1X_2, ΣX_1X_3, ΣX_1X_4, ΣX_2X_3, ΣX_2X_4, ΣX_3X_4 are computed and outputted in a manner convenient for immediate application of program STAT-PAC IV-13, Simultaneous Solution of Four Equations in Four Unknowns.

Reference: Theory and Problems of Statistics, Murray Spiegel, Shaum's Outline Series, McGraw-Hill Book Company, 1961, page 273.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

DISPLAY

0 ——— Z
i ——— Y
0 ——— X

ENTER DATA:

$X_2 \longrightarrow Y$
$X_1 \longrightarrow X$

PRESS: CONTINUE

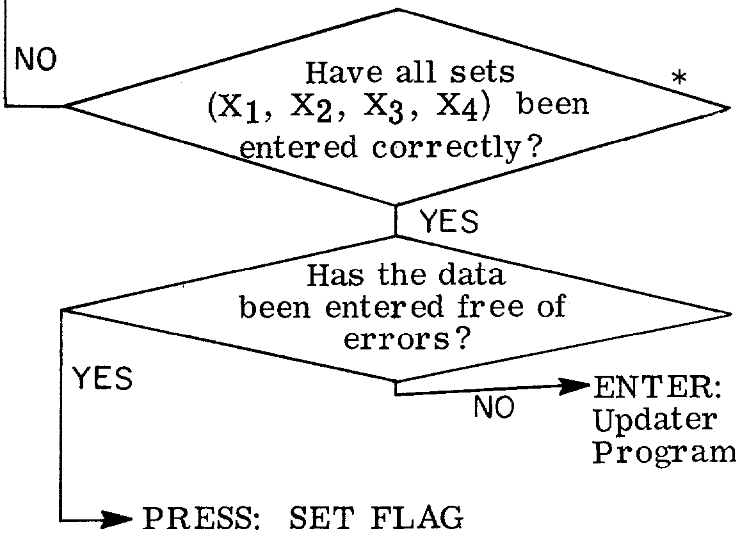
DISPLAY

0 ——— Z
i ——— Y
0 ——— X

ENTER DATA:

$X_4 \longrightarrow Y$
$X_3 \longrightarrow X$

PRESS: CONTINUE



USER INSTRUCTIONS (Con't)

PRESS: CONTINUE

The coefficients are now printed out ready for entry into program STAT-PAC IV-13. These are printed as follows:

N	Equation 1
ΣX_2	
ΣX_3	
ΣX_4	
ΣX_1	
ΣX_2	Equation 2
ΣX_2^2	
ΣX_2X_3	
ΣX_2X_4	
ΣX_1X_2	
ΣX_3	Equation 3
ΣX_2X_3	
ΣX_3^2	
ΣX_3X_4	
ΣX_1X_3	
ΣX_4	Equation 4
ΣX_2X_4	
ΣX_3X_4	
ΣX_4^2	
ΣX_1X_4	

To run another case: Repeat USER INSTRUCTIONS.

*If an error has been made in entering any of the sets (X_1, X_2, X_3, X_4), re-enter the correct sets and run the UPDATER program STAT-PAC IV-5, Normal Equation Coefficient Updater. DO NOT alter any of the storage registers as you load the UPDATER program.

EXAMPLE

$$X_1 = a_1 + a_2X_2 + a_3X_3 + a_4X_4$$

X_1	X_2	X_3	X_4
4	1	1	1
7.5	1.5	2	3
8	-1	3	5
13	2	4	6

Results:

$$\left.\begin{array}{lcr} N & = & 4.0 \\ \Sigma X_2 & = & 3.5 \\ \Sigma X_3 & = & 10.0 \\ \Sigma X_4 & = & 15.0 \\ \Sigma X_1 & = & 32.5 \end{array}\right\} \text{Equation 1}$$

$$\left.\begin{array}{lcr} \Sigma X_2 & = & 3.5 \\ \Sigma X_2^2 & = & 8.25 \\ \Sigma X_2X_3 & = & 9.0 \\ \Sigma X_2X_4 & = & 12.5 \\ \Sigma X_1X_2 & = & 33.25 \end{array}\right\} \text{Equation 2}$$

$$\left.\begin{array}{lcr} \Sigma X_3 & = & 10.0 \\ \Sigma X_2X_3 & = & 9.0 \\ \Sigma X_3^2 & = & 30.0 \\ \Sigma X_3X_4 & = & 46.0 \\ \Sigma X_1X_3 & = & 95.0 \end{array}\right\} \text{Equation 3}$$

$$\left.\begin{array}{lcr} \Sigma X_4 & = & 15.0 \\ \Sigma X_2X_4 & = & 12.5 \\ \Sigma X_3X_4 & = & 46.0 \\ \Sigma X_4^2 & = & 71.0 \\ \Sigma X_1X_4 & = & 144.5 \end{array}\right\} \text{Equation 4}$$

Upon inputting these coefficients to program STAT-PAC IV-13, the regression coefficients a_1, a_2, a_3, and a_4 are determined.

$$a_1 = a_2 = a_3 = a_4 = 1$$

STAT-PAC IV-4

Plus Page

```
00  CLR  20
01  XTO  23
02   d   17
03  XTO  23
04   c   16
05  XTO  23
06   b   14
07  XTO  23
08   a   13
09  XTO  23
0a   9   11
0b  XTO  23
0c   8   10
0d  XTO  23

10   7   07
11  XTO  23
12   6   06
13  XTO  23
14   5   05
15  XTO  23
16   4   04
17  XTO  23
18   3   03
19  ARC  72
1a  GTO  44
1b   -   34
1c   0   00
1d   0   00

20  CNT  47
21  CNT  47
22  CNT  47
23  CNT  47
24  CNT  47
25  CNT  47
26  CNT  47
27  CNT  47
28  CNT  47
29  CNT  47
2a  CNT  47
2b  CNT  47
2c  CNT  47
2d  CNT  47

30  CNT  47
31  CNT  47
32  CNT  47
33  CNT  47
34  CNT  47
35  CNT  47
36  CNT  47
37  CNT  47
38  CNT  47
39  CNT  47
3a  CNT  47
3b  CNT  47
3c  CNT  47
3d  CNT  47
```

```
40  CNT  47
41  CNT  47
42  CNT  47
43  CNT  47
44  CNT  47
45  CNT  47
46  CNT  47
47  CNT  47
48  CNT  47
49  CNT  47
4a  CNT  47
4b  CNT  47
4c  CNT  47
4d  CNT  47

50  CNT  47
51  CNT  47
52  CNT  47
53  CNT  47
54  CNT  47
55  CNT  47
56  CNT  47
57  CNT  47
58  CNT  47
59  CNT  47
5a  CNT  47
5b  CNT  47
5c  CNT  47
5d  CNT  47

60  CNT  47
61  CNT  47
62  CNT  47
63  CNT  47
64  CNT  47
65  CNT  47
66  CNT  47
67  CNT  47
68  CNT  47
69  CNT  47
6a  CNT  47
6b  CNT  47
6c  CNT  47
6d  CNT  47

70  CNT  47
71  CNT  47
72  CNT  47
73  CNT  47
74  CNT  47
75  CNT  47
76  CNT  47
77  CNT  47
78  CNT  47
79  CNT  47
7a  CNT  47
7b  CNT  47
7c  CNT  47
7d  CNT  47
```

```
80  CNT  47
81  CNT  47
82  CNT  47
83  CNT  47
84  CNT  47
85  CNT  47
86  CNT  47
87  CNT  47
88  CNT  47
89  CNT  47
8a  CNT  47
8b  CNT  47
8c  CNT  47
8d  CNT  47

90  CNT  47
91  CNT  47
92  CNT  47
93  CNT  47
94  CNT  47
95  CNT  47
96  CNT  47
97  CNT  47
98  CNT  47
99  CNT  47
9a  CNT  47
9b  CNT  47
9c  CNT  47
9d  CNT  47

a0  CNT  47
a1  CNT  47
a2  CNT  47
a3  CNT  47
a4  CNT  47
a5  CNT  47
a6  CNT  47
a7  CNT  47
a8  CNT  47
a9  CNT  47
aa  CNT  47
ab  CNT  47
ac  CNT  47
ad  CNT  47
```

x_1	Σx_4^2 (x_3)
$\bar{x}_2$	Σx_3^2
$\Sigma x_3 x_4$ (x_3)	Σx_2^2
$\Sigma x_2 x_4$	Σx_4
$\Sigma x_2 x_3$	Σx_3
$\Sigma x_1 x_4$	Σx_2
$\Sigma x_1 x_3$	Σx_1
$\Sigma x_1 x_2$	N

Step	Key	Code
b0	CNT	47
b1	CNT	47
b2	CNT	47
b3	CNT	47
b4	CNT	47
b5	CNT	47
b6	CNT	47
b7	CNT	47
b8	CNT	47
b9	CNT	47
ba	CNT	47
bb	CNT	47
bc	CNT	47
bd	CNT	47
c0	CNT	47
c1	CNT	47
c2	CNT	47
c3	CNT	47
c4	CNT	47
c5	CNT	47
c6	CNT	47
c7	CNT	47
c8	CNT	47
c9	CNT	47
ca	CNT	47
cb	CNT	47
cc	CNT	47
cd	CNT	47
d0	CNT	47
d1	CNT	47
d2	CNT	47
d3	CNT	47
d4	CNT	47
d5	CNT	47
d6	CNT	47
d7	CNT	47
d8	CNT	47
d9	CNT	47
da	CNT	47
db	CNT	47
dc	CNT	47
dd	CNT	47

Plus Page

STAT-PAC IV-4

```
00  XTO  23   Minus
01  2    02   Page
02  f    15
03  UP   27
04  1    01
05  +    33
06  0    00
07  STP  41   ENTRY
08  IFG  43
09  a    13
0a  2    02
0b  PNT  45
0c  XTO  23
0d  0    00

10  YTO  40
11  1    01
12  UP   27
13  1    01
14  AC+  60
15  DN   25
16  RDN  31
17  YE   24
18  d    17
19  +    33
1a  YTO  40
1b  d    17
1c  XEY  30
1d  DN   25

20  X    36
21  RDN  31
22  YE   24
23  7    07
24  +    33
25  YTO  40
26  7    07
27  RUP  22
28  UP   27
29  X    36
2a  DN   25
2b  YE   24
2c  a    13
2d  +    33

30  YTO  40
31  a    13
32  0    00
33  UP   27
34  f    15
35  UP   27
36  0    00
37  STP  41   ENTRY
38  PNT  45
39  PNT  45
3a  YTO  40
3b  -    34
3c  f    15
3d  RDN  31

40  YE   24
41  b    14
42  +    33
43  YTO  40
44  b    14
45  XEY  30
46  DN   25
47  UP   27
48  X    36
49  RDN  31
4a  YE   24
4b  8    10
4c  +    33
4d  YE   24

50  8    10
51  DN   25
52  X    36
53  XEY  30
54  YE   24
55  2    02
56  +    33
57  YE   24
58  2    02
59  DN   25
5a  UP   27
5b  X    36
5c  RDN  31
5d  YE   24

60  9    11
61  +    33
62  YTO  40
63  9    11
64  RUP  22
65  YE   24
66  c    16
67  +    33
68  YTO  40
69  c    16
6a  RDN  31
6b  XFR  67
6c  0    00
6d  RUP  22

70  X    36
71  RDN  31
72  YE   24
73  6    06
74  +    33
75  YTO  40
76  6    06
77  XFR  67
78  1    01
79  RUP  22
7a  X    36
7b  RDN  31
7c  YE   24
7d  4    04

80  +    33
81  YTO  40
82  4    04
83  YE   24
84  0    00
85  XFR  67
86  -    34
87  f    15
88  X    36
89  RDN  31
8a  YE   24
8b  5    05
8c  +    33
8d  YTO  40

90  5    05
91  XFR  67
92  1    01
93  RUP  22
94  X    36
95  DN   25
96  YE   24
97  3    03
98  +    33
99  YTO  40
9a  3    03
9b  0    00
9c  UP   27
9d  GTO  44

a0  0    00
a1  2    02
a2  f    15
a3  UP   27
a4  d    17
a5  UP   27
a6  c    16
a7  PNT  45   S
a8  b    14
a9  UP   27
aa  e    12
ab  RUP  22
ac  PNT  45
ad  a    13   S
```

x_4

```
b0   UP    27       Minus
b1   XFR   67       Page
b2    4    04
b3   UP    27
b4   XFR   67
b5    3    03
b6   PNT   45   s
b7   XFR   67
b8    7    07
b9   UP    27
ba    c    16
bb   RUP   22
bc   PNT   45   s
bd   XFR   67

c0    9    11
c1   UP    27
c2   XFR   67
c3    2    02
c4   UP    27
c5   XFR   67
c6    6    06
c7   PNT   45   s
c8    b    14
c9   UP    27
ca   XFR   67
cb    3    03
cc   RUP   22
cd   PNT   45

d0   XFR   67
d1    8    10
d2   UP    27
d3   XFR   67
d4    5    05
d5   UP    27
d6    0    00
d7   PNT   45   s
d8   PNT   45
d9   PNT   45
da    0    00
db   UP    27
dc   UP    27
dd   STP   41
```

NORMAL EQUATION COEFFICIENT UPDATER (CORRECTOR)

This program is to be used to correct the summations computed by program STAT-PAC IV-4. To correct for erroneous data entered into the above program, simply repeat the same erroneous entries into this UPDATER program.

USER INSTRUCTIONS

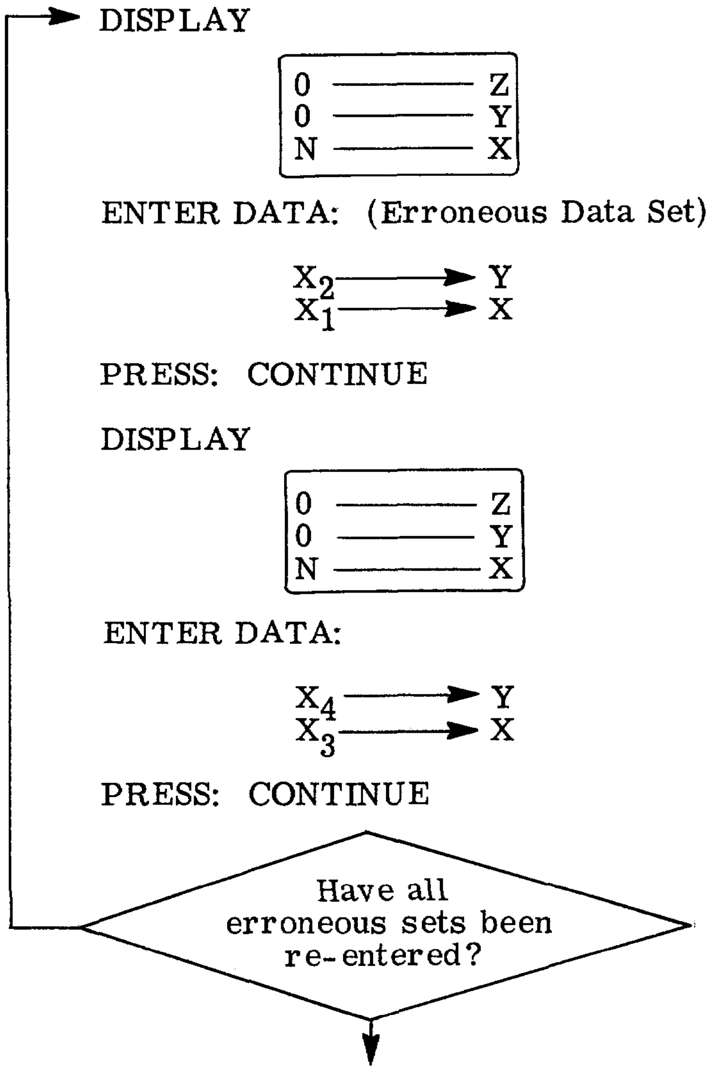
DEPRESS: X Y Z on the 9120A

PRESS: GO TO (-)(0)(0)

ENTER PROGRAM:

PRESS: GO TO (-)(0)(0)

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

The coefficients are now printed out ready for entry into program STAT-PAC IV-13. These are printed out as follows:

See program STAT-PAC IV-4.

```
00  d    17   Minus   40  YTO  40       80  YE   24
01  UP   27   Page    41  b    14       81  0    00
02  UP   27           42  XEY  30       82  XFR  67
03  f    15           43  DN   25       83  -    34
04  CNT  47           44  UP   27       84  f    15
05  STP  41   ENTRY   45  X    36       85  X    36
06  IFG  43           46  RDN  31       86  RDN  31
07  9    11           47  YE   24       87  YE   24
08  d    17           48  8    10       88  5    05
09  PNT  45           49  -    34       89  -    34
0a  XTO  23           4a  YE   24       8a  YTO  40
0b  0    00           4b  8    10       8b  5    05
0c  YTO  40           4c  DN   25       8c  XFR  67
0d  1    01           4d  X    36       8d  1    01

10  UP   27           50  XEY  30       90  RUP  22
11  1    01           51  YE   24       91  X    36
12  AC-  63           52  2    02       92  DN   25
13  DN   25           53  -    34       93  YE   24
14  RDN  31           54  YE   24       94  3    03
15  YE   24           55  2    02       95  -    34
16  d    17           56  DN   25       96  YTO  40
17  -    34           57  UP   27       97  3    03
18  YTO  40           58  X    36       98  0    00
19  d    17           59  RDN  31       99  UP   27
1a  XEY  30           5a  YE   24       9a  GTO  44
1b  DN   25           5b  9    11       9b  0    00
1c  X    36           5c  -    34       9c  0    00
1d  RDN  31           5d  YTO  40       9d  f    15

20  YE   24           60  9    11       a0  UP   27
21  7    07           61  RUP  22       a1  d    17
22  -    34           62  YE   24       a2  UP   27
23  YTO  40           63  c    16       a3  c    16
24  7    07           64  -    34       a4  PNT  45   S
25  RUP  22           65  YTO  40       a5  b    14
26  UP   27           66  c    16       a6  UP   27
27  X    36           67  RDN  31       a7  e    12
28  DN   25           68  XFR  67       a8  RUP  22
29  YE   24           69  0    00       a9  PNT  45   S
2a  a    13           6a  RUP  22       aa  a    13
2b  -    34           6b  X    36       ab  UP   27
2c  YTO  40           6c  RDN  31       ac  XFR  67
2d  a    13           6d  YE   24       ad  4    04

30  0    00           70  6    06
31  UP   27           71  -    34
32  UP   27           72  YTO  40
33  f    15           73  6    06
34  STP  41   ENTRY   74  XFR  67
35  PNT  45           75  1    01
36  PNT  45           76  RUP  22
37  YTO  40           77  X    36
38  -    34           78  RDN  31
39  f    15           79  YE   24
3a  RDN  31           7a  4    04
3b  YE   24           7b  -    34
3c  b    14           7c  YTO  40
3d  -    34           7d  4    04
```

X_1	ΣX_4^2
X_2	ΣX_3^2
ΣX_3X_4	ΣX_2^2
ΣX_2X_4	ΣX_4
ΣX_2X_3	ΣX_3
ΣX_1X_4	ΣX_2
ΣX_1X_3	ΣX_1
ΣX_1X_2	N

```
b0  UP   27
b1  XFR  67
b2   3   03   s
b3  PNT  45
b4  XFR  67
b5   7   07
b6  UP   27
b7   c   16
b8  RUP  22
b9  PNT  45   s
ba  XFR  67
bb   9   11
bc  UP   27
bd  XFR  67

c0   2   02
c1  UP   27
c2  XFR  67
c3   6   06
c4  PNT  45   s
c5   b   14
c6  UP   27
c7  XFR  67
c8   3   03
c9  RUP  22
ca  PNT  45   s
cb  XFR  67
cc   8   10
cd  UP   27

d0  XFR  67
d1   5   05
d2  UP   27
d3   0   00
d4  PNT  45   s
d5  PNT  45
d6  PNT  45
d7   0   00
d8  UP   27
d9  UP   27
da  STP  41
```

LEAST SQUARES FIT - POWER CURVE

This program computes the least squares fit and correlation coefficient of N pairs of data points for a power curve of the form:

$$Y = aX^{b} \qquad (a > 0)$$

The equation is linearized into $\ln Y = b \ln X + \ln a$

where $$b = \frac{N\sum(\ln X \ln Y) - \sum \ln X \sum \ln Y}{N\sum(\ln X)^2 - (\sum \ln X)^2}$$

and $$r = \frac{N\sum \ln X \ln Y - (\sum \ln X)(\sum \ln Y)}{\sqrt{\left[N\sum(\ln X)^2 - (\sum \ln X)^2\right]\left[N\sum(\ln Y)^2 - (\sum \ln Y)^2\right]}}$$

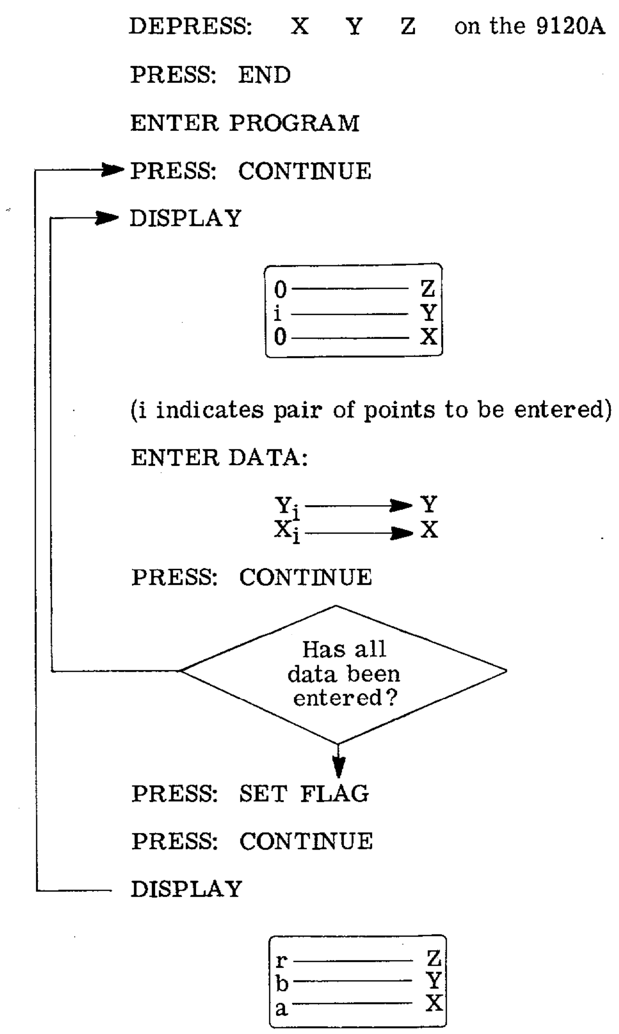
$$\ln a = \frac{\sum \ln Y}{N} - \frac{\sum \ln X}{N} b$$

Note: $X_i > 0$ and $Y_i > 0$, $i = 1, \cdots, N$

Reference: Statistical Theory and Methodology in Science and Engineering
by K. A. Brownlee

John Wiley and Sons 1965

USER INSTRUCTIONS



DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0 ——— Z
i ——— Y
0 ——— X

(i indicates pair of points to be entered)

ENTER DATA:

$Y_i \longrightarrow Y$
$X_i \longrightarrow X$

PRESS: CONTINUE

Has all data been entered?

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

r ——— Z
b ——— Y
a ——— X

EXAMPLES

$Y = aX^b$

X	Y
1.0001	25.58
3.16	14.55
10	9.26
31.6	5.63
100	3.48
316	2.12
1000	1.7

r = -0.9964
b = -0.4022
a = 23.5871

$Y = 23.5871X^{-.4022}$

X	Y
1	3
2	4.2574
3	5.2248
4	6.0417
5	6.7624

r = 1.000
b = .505
a = 3.000

$Y = 3X^{.505}$

STAT-PAC IV-6

```
00  CLR  20
01  XTO  23
02   d   17
03  XTO  23
04   c   16
05  XTO  23
06   b   14
07   1   01
08  XTO  23
09   a   13
0a  XEY  30
0b  STP  41      ENTRY
0c  IFG  43
0d   4   04

10   0   00
11  PNT  45
12  PNT  45
13  LN   65
14  XEY  30
15  LN   65
16  XEY  30
17  AC+  60
18  UP   27
19   X   36
1a  XEY  30
1b  YE   24
1c   d   17
1d   +   33

20  YE   24
21   d   17
22  DN   25
23   X   36
24   b   14
25   +   33
26  YTO  40
27   b   14
28  RUP  22
29  UP   27
2a   X   36
2b   c   16
2c   +   33
2d  YTO  40

30   c   16
31   a   13
32  UP   27
33   1   01
34   +   33
35  YTO  40
36   a   13
37  CLX  37
38  RDN  31
39  XEY  30
3a   0   00
3b  GTO  44
3c   0   00
3d   b   14
```

```
40   a   13
41  UP   27
42   1   01
43   -   34
44  YTO  40
45   a   13
46   e   12
47  UP   27
48   a   13
49  DIV  35
4a  YE   24
4b   f   15
4c  DIV  35
4d  YTO  40

50   e   12
51   X   36
52   e   12
53   X   36
54   d   17
55  XEY  30
56   -   34
57  YTO  40
58   d   17
59   c   16
5a  UP   27
5b   f   15
5c  UP   27
5d   X   36

60   a   13
61   X   36
62  DN   25
63   -   34
64  YTO  40
65   c   16
66   b   14
67  UP   27
68   f   15
69  UP   27
6a   e   12
6b   X   36
6c   a   13
6d   X   36

70  DN   25
71   -   34
72   d   17
73  UP   27
74  DN   25
75   √   76
76  DIV  35
77   c   16
78   √   76
79  DIV  35
7a   d   17
7b  RUP  22
7c  XEY  30
7d  DIV  35
```

```
80  YE   24
81   e   12
82   e   12
83   X   36
84   f   15
85  XEY  30
86   -   34
87   e   12
88  XEY  30
89  EXP  74
8a  PNT  45
8b  PNT  45
8c  END  46
```

LEAST SQUARES FIT - EXPONENTIAL

This program computes the least squares fit and a correlation coefficient of n pairs of data points for an exponential function of the form:

$$y = ae^{bx} \qquad (a > 0)$$

The equation is linearized into

$$\ln y = \ln a + bx$$

or

$$Y = A + bx$$

Using a linear regression method,

$$b = \frac{n\sum xY - \sum x \sum Y}{n\sum x^2 - (\sum x)^2}$$

$$A = \frac{\sum Y - b\sum x}{n}$$

$$a = e^A$$

the correlation coefficient is given by

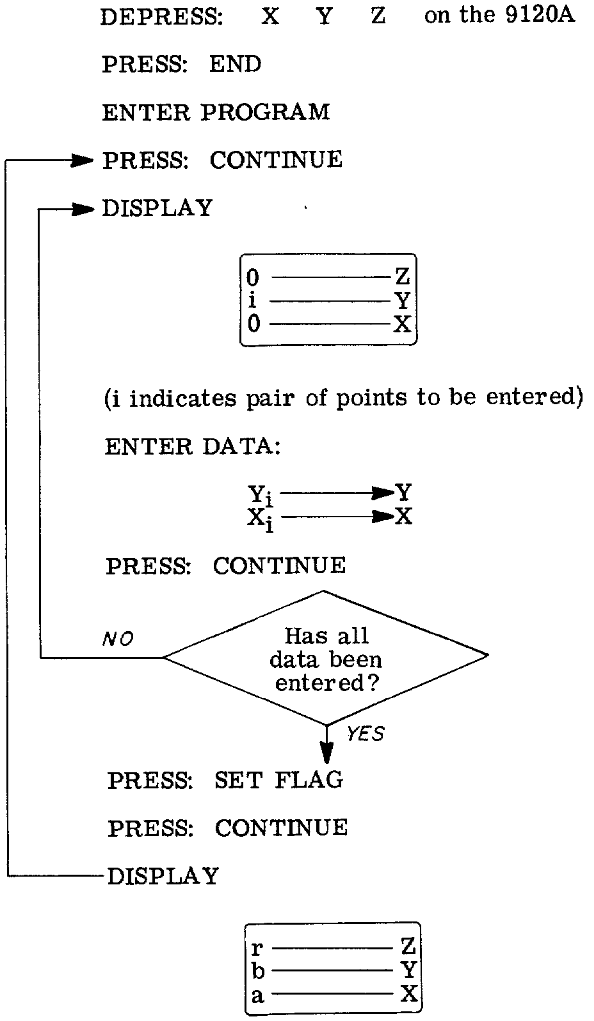
$$r = \frac{n\sum xY - \sum x \sum Y}{\sqrt{\left[n\sum x^2 - (\sum x)^2\right]\left[n\sum Y^2 - (\sum Y)^2\right]}}$$

Note: $Y_i > 0 \quad i = 1, \cdots, n$

Reference: Statistical Theory and Methodology in Science and Engineering
by K.A. Brownlee

John Wiley and Sons 1965

USER INSTRUCTIONS



DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0 ——— Z
i ——— Y
0 ——— X

(i indicates pair of points to be entered)

ENTER DATA:

$Y_i \longrightarrow Y$
$X_i \longrightarrow X$

PRESS: CONTINUE

Has all data been entered?

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

r ——— Z
b ——— Y
a ——— X

EXAMPLES

GENERAL FORM: $Y = ae^{bX}$

X	Y
.5	7.12
1.2	11.67
3.1	44.75
7.4	935.64

r = 1.000
b = .707
a = 4.998
$Y = 4.998e^{.707X}$

X	Y
.72	2.16
1.31	1.61
1.95	1.16
2.58	.85

r = - 1.000
b = -.503
a = 3.103
$Y = 3.103e^{-.503X}$

```
00  CLR   20
01  XTO   23
02  a     13
03  XTO   23
04  b     14
05  XTO   23
06  c     16
07  1     01
08  XEY   30
09  YTO   40
0a  d     17
0b  STP   41     ENTRY
0c  IFG   43
0d  3     03

10  9     11
11  PNT   45
12  PNT   45
13  XEY   30
14  LN    65
15  AC+   60
16  UP    27
17  X     36
18  XEY   30
19  YE    24
1a  b     14
1b  +     33
1c  YE    24
1d  b     14

20  DN    25
21  X     36
22  c     16
23  +     33
24  YTO   40
25  c     16
26  RUP   22
27  UP    27
28  X     36
29  a     13
2a  +     33
2b  YTO   40
2c  a     13
2d  CLX   37

30  UP    27
31  d     17
32  UP    27
33  1     01
34  +     33
35  0     00
36  GTO   44
37  0     00
38  9     11
39  d     17
3a  UP    27
3b  1     01
3c  -     34
3d  YTO   40

40  d     17
41  c     16
42  X     36
43  UP    27
44  RCL   61
45  X     36
46  DN    25
47  -     34
48  YTO   40
49  c     16
4a  d     17
4b  UP    27
4c  a     13
4d  X     36

50  e     12
51  UP    27
52  X     36
53  DN    25
54  -     34
55  YTO   40
56  a     13
57  d     17
58  UP    27
59  b     14
5a  X     36
5b  f     15
5c  UP    27
5d  X     36

60  DN    25
61  -     34
62  a     13
63  X     36
64  DN    25
65  √     76
66  UP    27
67  c     16
68  XEY   30
69  DIV   35
6a  YTO   40
6b  b     14
6c  c     16
6d  UP    27

70  a     13
71  DIV   35
72  e     12
73  UP    27
74  DN    25
75  X     36
76  f     15
77  XEY   30
78  -     34
79  d     17
7a  DIV   35
7b  b     14
7c  RDN   31
7d  EXP   74

80  PNT   45
81  PNT   45
82  END   46
```

THE LEAST SQUARE PARABOLA

Development:

The least square parabola approximating the set of points $(X_1, Y_1) \ldots\ldots (X_i, Y_i)$ has the equation:

$$Y = a_o + a_1 x + a_2 x^2$$

where the constants a_o, a_1, and a_2 are determined by solving simultaneously the following normal equations:

$$\Sigma Y = a_o n + a_1 \Sigma X + a_2 \Sigma X^2$$
$$\Sigma XY = a_o \Sigma X + a_1 \Sigma X^2 + a_2 \Sigma X^3$$
$$\Sigma X^2 Y = a_o \Sigma X^2 + a_1 \Sigma X^3 + a_2 \Sigma X^4$$

In the program the constant a_2 is found by matrix algebra; the determinate (D) involved in the solution is:

$$D = \begin{vmatrix} n & \Sigma X & \Sigma X^2 \\ \Sigma X & \Sigma X^2 & \Sigma X^3 \\ \Sigma X^2 & \Sigma X^3 & \Sigma X^4 \end{vmatrix}$$

The equation for a_2 is therefore:

$$a_2 = \frac{\left(n(\Sigma X^2 \Sigma X^2 Y - \Sigma X^3 \Sigma XY) - \Sigma X(\Sigma X \Sigma X^2 Y - \Sigma X^2 \Sigma XY) + \Sigma Y \left[\Sigma X \Sigma X^3 - (\Sigma X^2)^2\right]\right)}{D}$$

After finding a_2 the solution is reduced to two equations in two unknowns which are:

$$N = a_o n + a_1 \Sigma X$$
$$M = a_o \Sigma X + a_1 \Sigma X^2$$

where $M = \Sigma XY - a_2 \Sigma X^3$ and $N = \Sigma Y - a_2 \Sigma X^2$

These equations are then solved for a_o and a_1.

NOTE: Curves with the following equations may also be fitted with this program:

$$Y = a_o + a_1 X \qquad (1)$$
$$\log Y = a_o + a_1 X \qquad (2)$$
$$\log Y = a_o + a_1 X + a_2 X^2 \qquad (3)$$
$$Y = a_o + a_1 \log X \qquad (4)$$
$$\log Y = a_o + a_1 \log X \qquad (5)$$
$$\log Y = a_o + a_1 (\log X) + a_2 (\log X)^2 \qquad (6)$$

An equation of the form of (6) is solved in the examples.

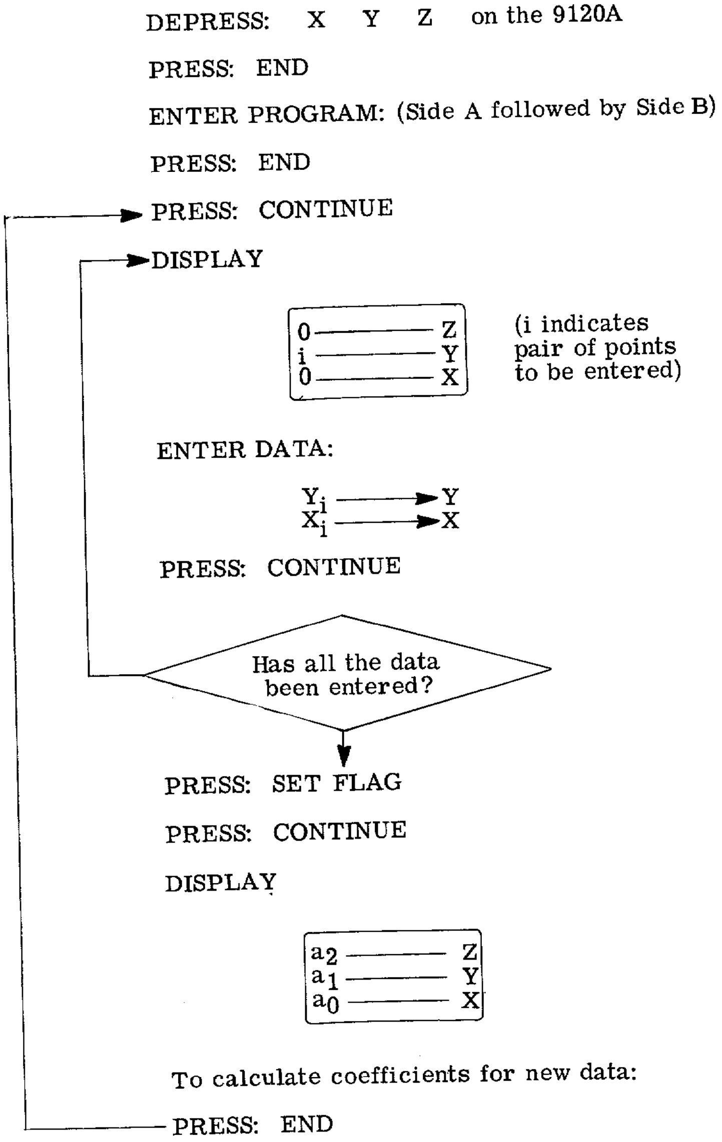
The general form, representing all of these equations, which can be fitted is:

$$f(Y) = a_o + a_1 f(X) + a_2 f^2 (X)$$

Reference:

Publisher -- McGraw-Hill
Authors -- Alexander M. Mood & Franklin A. Graybill
Introduction into the Theory of Statistics -- 2nd Edition (1961)

USER INSTRUCTIONS



DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM: (Side A followed by Side B)

PRESS: END

PRESS: CONTINUE

DISPLAY

0 — Z
i — Y
0 — X

(i indicates pair of points to be entered)

ENTER DATA:

$Y_i \longrightarrow Y$
$X_i \longrightarrow X$

PRESS: CONTINUE

Has all the data been entered?

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

a_2 — Z
a_1 — Y
a_0 — X

To calculate coefficients for new data:

PRESS: END

EXAMPLES

(A) Equation of the form: $Y = a_o + a_1 X + a_2 X^2$

Data :

X	Y
3	29
0	2
5	67
2	16
1.5	11
4	46
1	7

Solution: $Y = 2 + 3X + 2X^2$

(B) Equation of the form:

$$\log Y = a_o + a_1 \log X + a_2 (\log X)^2$$

Note: Data to be entered is $\log Y_i$, $\log X_i$; therefore to enter data sets:

ENTER: $Y_i \rightarrow X$
PRESS: log X
PRESS: ↑
ENTER: $X_i \rightarrow Y$
PRESS: log X

Data:

X	Y
1	2.7183
2	35.1595
3	245.3746
4	1188.7946
5	4530.5750

Solution: $\log Y = .43 + 3.0 \log X + 2.30 (\log X)^2$

Step	Key	Code		Step	Key	Code	Step	Key	Code
00	CLR	20	Plus	40	a	13	80	X	36
01	XTO	23	Page	41	RUP	22	81	RDN	31
02	c	16		42	X	36	82	-	34
03	XTO	23		43	XEY	30	83	f	15
04	b	14		44	YE	24	84	RUP	22
05	XTO	23		45	-	34	85	X	36
06	a	13		46	e	12	86	2	02
07	XTO	23		47	+	33	87	X	36
08	-	34		48	YTO	40	88	a	13
09	f	15		49	-	34	89	X	36
0a	XTO	23		4a	e	12	8a	DN	25
0b	-	34		4b	1	01	8b	+	33
0c	e	12		4c	UP	27	8c	YE	24
0d	1	01		4d	d	17	8d	-	34
10	XTO	23		50	+	33	90	f	15
11	d	17		51	YTO	40	91	UP	27
12	RDN	31		52	d	17	92	DN	25
13	RDN	31		53	CLX	37	93	b	14
14	STP	41	ENTRY	54	UP	27	94	X	36
15	IFG	43		55	GTO	44	95	d	17
16	5	05		56	1	01	96	X	36
17	8	10		57	3	03	97	f	15
18	PNT	45		58	XFR	67	98	RUP	22
19	PNT	45		59	-	34	99	XEY	30
1a	AC+	60		5a	e	12	9a	GTO	44
1b	X	36		5b	UP	27	9b	-	34
1c	XEY	30		5c	f	15	9c	0	00
1d	YE	24		5d	UP	27	9d	0	00
20	c	16		60	X	36	a0	CNT	47
21	+	33		61	b	14	a1	CNT	47
22	YE	24		62	RUP	22	a2	CNT	47
23	c	16		63	X	36	a3	CNT	47
24	UP	27		64	RUP	22	a4	CNT	47
25	DN	25		65	CHS	32	a5	CNT	47
26	X	36		66	X	36	a6	CNT	47
27	DN	25		67	YE	24	a7	CNT	47
28	YE	24		68	d	17	a8	CNT	47
29	-	34		69	1	01	a9	CNT	47
2a	f	15		6a	-	34	aa	CNT	47
2b	+	33		6b	YE	24	ab	CNT	47
2c	YE	24		6c	d	17	ac	CNT	47
2d	-	34		6d	d	17	ad	CNT	47
30	f	15		70	RUP	22			
31	DN	25		71	X	36			
32	X	36		72	DN	25			
33	XEY	30		73	+	33			
34	YE	24		74	a	13			
35	b	14		75	UP	27			
36	+	33		76	X	36			
37	YE	24		77	d	17			
38	b	14		78	X	36			
39	X	36		79	DN	25			
3a	a	13		7a	-	34			
3b	XEY	30		7b	b	14			
3c	+	33		7c	UP	27			
3d	YTO	40		7d	X	36			

ΣX^3

ΣX^2

ΣXY

n

ΣX

ΣY

b0	CNT	47	Plus Page
b1	CNT	47	
b2	CNT	47	
b3	CNT	47	
b4	CNT	47	
b5	CNT	47	
b6	CNT	47	
b7	CNT	47	
b8	CNT	47	
b9	CNT	47	
ba	CNT	47	
bb	CNT	47	
bc	CNT	47	
bd	CNT	47	
c0	CNT	47	
c1	CNT	47	
c2	CNT	47	
c3	CNT	47	
c4	CNT	47	
c5	CNT	47	
c6	CNT	47	
c7	CNT	47	
c8	CNT	47	
c9	CNT	47	
ca	CNT	47	
cb	CNT	47	
cc	CNT	47	
cd	CNT	47	
d0	CNT	47	
d1	CNT	47	
d2	CNT	47	
d3	CNT	47	
d4	CNT	47	
d5	CNT	47	
d6	CNT	47	
d7	CNT	47	
d8	CNT	47	
d9	CNT	47	
da	CNT	47	
db	CNT	47	
dc	CNT	47	
dd	CNT	47	

```
00   X     36    Minus   40   YTO   40
01   X     36    Page    41   a     13
02   RDN   31            42   CLR   20
03   -     34            43   XFR   67
04   e     12            44   -     34
05   RUP   22            45   e     12
06   X     36            46   UP    27
07   a     13            47   c     16
08   X     36            48   UP    27
09   RDN   31            49   d     17
0a   +     33            4a   DIV   35
0b   d     17            4b   RUP   22
0c   RUP   22            4c   XEY   30
0d   X     36            4d   DIV   35

10   c     16            50   DN    25
11   X     36            51   IFG   43
12   RDN   31            52   6     06
13   -     34            53   2     02
14   f     15            54   AC+   60
15   XTO   23            55   b     14
16   -     34            56   YE    24
17   e     12            57   -     34
18   RUP   22            58   e     12
19   X     36            59   UP    27
1a   b     14            5a   a     13
1b   X     36            5b   RUP   22
1c   RDN   31            5c   SFL   54
1d   +     33            5d   GTO   44

20   DN    25            60   4     04
21   RDN   31            61   a     13
22   X     36            62   AC-   63
23   e     12            63   XEY   30
24   X     36            64   YE    24
25   RDN   31            65   e     12
26   -     34            66   f     15
27   RUP   22            67   DIV   35
28   YE    24            68   e     12
29   -     34            69   XEY   30
2a   f     15            6a   X     36
2b   RDN   31            6b   RDN   31
2c   DIV   35            6c   -     34
2d   b     14            6d   RUP   22

30   XEY   30            70   YE    24
31   X     36            71   -     34
32   RDN   31            72   f     15
33   -     34            73   RUP   22
34   YE    24            74   PNT   45
35   c     16            75   PNT   45
36   a     13            76   PNT   45
37   RUP   22            77   PNT   45
38   XTO   23            78   PNT   45
39   -     34            79   PNT   45
3a   f     15            7a   END   46
3b   X     36
3c   RDN   31
3d   -     34
```

ΣX^4

ΣX^2Y

CUBIC REGRESSION (LEAST SQUARES)

This program determines the best fit least squares cubic for a set of N points (X_i, Y_i). The cubic is given by:

$$y = a_0 + a_1X + a_2X^2 + a_3X^3 \ .$$

The normal equation required for determining the coefficients are:

$$\begin{aligned}\Sigma Y &= a_0N + a_1\Sigma X + a_2 \ \Sigma X^2 + a_3\Sigma X^3\\ \Sigma XY &= a_0\Sigma X + a_1\Sigma X^2 + a_2\Sigma X^3 + a_3\Sigma X^4\\ \Sigma X^2Y &= a_0\Sigma X^2 + a_1\Sigma X^3 + a_2\Sigma X^4 + a_3\Sigma X^5\\ \Sigma X^3Y &= a_0\Sigma X^3 + a_1\Sigma X^4 + a_2\Sigma X^5 + a_3\Sigma X^6\end{aligned}$$

This program determines the summations (Σ) which are then input to program STAT-PAC IV-13, Simultaneous Solution of Four Linear Equations in Four Unknowns, (With Printer), for determination of a_0, a_1, a_2 and a_3 .

The data points and the cubic regression curve may be plotted by the use of program STAT-PAC X-11.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: GO TO (-)(0)(0)

ENTER PROGRAM :

PRESS: GO TO (-)(0)(0)

PRESS: CONTINUE

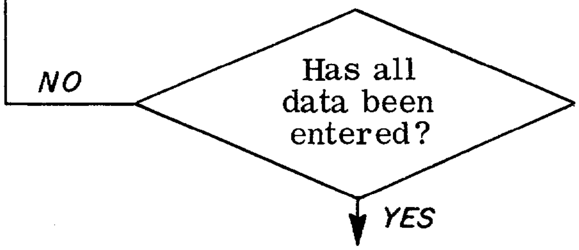
DISPLAY

0 ———— Z
i ———— Y
0 ———— X

ENTER DATA:

$Y_i \longrightarrow Y$
$X_i \longrightarrow X$

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

N ———— Z
ΣX ———— Y
ΣX^2 ———— X

PRESS: CONTINUE

DISPLAY

ΣX^3 ———— Z
ΣY ———— Y
ΣX ———— X

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

ΣX^2 ———— Z
ΣX^3 ———— Y
ΣX^4 ———— X

PRESS: CONTINUE

DISPLAY

ΣXY ———— Z
ΣX^2 ———— Y
ΣX^3 ———— X

PRESS: CONTINUE

DISPLAY

ΣX^4 ———— Z
ΣX^5 ———— Y
$\Sigma X^2 Y$ ———— X

PRESS: CONTINUE

DISPLAY

ΣX^3 ———— Z
ΣX^4 Y
ΣX^5 ———— X

PRESS: CONTINUE

DISPLAY

ΣX^6 ———— Z
$\Sigma X^3 Y$ ———— Y
0 ———— X

To run another case

EXAMPLE

X	Y
-2	-23
-1	2
0	5
1	4
2	17

Results

$N = 5$
$\Sigma X = 0$
$\Sigma X^2 = 10$
$\Sigma X^3 = 0$
$\Sigma Y = 5$
$\Sigma X = 0$
$\Sigma X^2 = 10$
$\Sigma X^3 = 0$
$\Sigma X^4 = 34$
$\Sigma XY = 82$
$\Sigma X^2 = 10$
$\Sigma X^3 = 0$
$\Sigma X^4 = 34$
$\Sigma X^5 = 0$
$\Sigma X^2Y = -18$
$\Sigma X^3 = 0$
$\Sigma X^4 = 34$
$\Sigma X^5 = 0$
$\Sigma X^6 = 130$
$\Sigma X^3Y = 322$

EXAMPLE (Con't)

Upon inputting these values in to program STAT-PAC IV-13, the coefficients are found to be:

$a_0 = 5$
$a_1 = -2$
$a_2 = -2$
$a_3 = 3$

STAT-PAC IV-9

9100B ONLY

00	CLR	20	Minus Page
01	XTO	23	
02	d	17	
03	XTO	23	
04	c	16	
05	XTO	23	
06	b	14	
07	XTO	23	
08	a	13	
09	XTO	23	
0a	9	11	
0b	XTO	23	
0c	8	10	
0d	XTO	23	
10	7	07	
11	XTO	23	
12	-	34	
13	f	15	
14	XTO	23	
15	-	34	
16	e	12	
17	1	01	
18	XEY	30	
19	STP	41	ENTRY
1a	IFG	43	
1b	7	07	
1c	d	17	
1d	PNT	45	
20	PNT	45	
21	AC+	60	
22	UP	27	
23	X	36	
24	XEY	30	
25	YEX	24	
26	c	16	
27	+	33	
28	YEX	24	
29	c	16	
2a	XEY	30	
2b	X	36	
2c	XEY	30	
2d	YEX	24	
30	b	14	
31	+	33	
32	YEX	24	
33	b	14	
34	XEY	30	
35	X	36	
36	XEY	30	
37	YEX	24	
38	a	13	
39	+	33	
3a	YEX	24	
3b	a	13	
3c	XEY	30	
3d	X	36	
40	XEY	30	
41	YEX	24	
42	-	34	
43	f	15	
44	+	33	
45	YEX	24	
46	-	34	
47	f	15	
48	XEY	30	
49	X	36	
4a	XEY	30	
4b	YEX	24	
4c	-	34	
4d	e	12	
50	+	33	
51	YEX	24	
52	-	34	
53	e	12	
54	DN	25	
55	X	36	
56	XEY	30	
57	YEX	24	
58	9	11	
59	+	33	
5a	YEX	24	
5b	9	11	
5c	XEY	30	
5d	X	36	
60	XEY	30	
61	YEX	24	
62	8	10	
63	+	33	
64	YEX	24	
65	8	10	
66	XEY	30	
67	X	36	
68	DN	25	
69	YEX	24	
6a	7	07	
6b	+	33	
6c	YTO	40	
6d	7	07	
70	0	00	
71	UP	27	
72	d	17	
73	UP	27	
74	1	01	
75	+	33	
76	YTO	40	
77	d	17	
78	+	33	
79	0	00	
7a	GTO	44	
7b	1	01	
7c	9	11	
7d	d	17	
80	UP	27	
81	f	15	
82	UP	27	
83	c	16	
84	PNT	45	s
85	b	14	
86	UP	27	
87	e	12	
88	UP	27	
89	f	15	
8a	PNT	45	s
8b	c	16	
8c	UP	27	
8d	b	14	
90	UP	27	
91	a	13	
92	PNT	45	s
93	XFR	67	
94	9	11	
95	UP	27	
96	c	16	
97	UP	27	
98	b	14	
99	PNT	45	s
9a	a	13	
9b	UP	27	
9c	XFR	67	
9d	-	34	
a0	f	15	
a1	UP	27	
a2	XFR	67	
a3	8	10	
a4	PNT	45	s
a5	b	14	
a6	UP	27	
a7	a	13	
a8	UP	27	
a9	XFR	67	
aa	-	34	
ab	f	15	
ac	PNT	45	s
ad	XFR	67	
	+		+

ΣX^2Y

ΣXY

ΣX^4

ΣX^3

ΣX^2

n

ΣY

ΣX^3Y

ΣX

b0	-	34	Minus Page
b1	e	12	
b2	UP	27	
b3	XFR	67	
b4	7	07	
b5	UP	27	
b6	0	00	
b7	PNT	45	s
b8	PNT	45	
b9	PNT	45	
ba	PNT	45	
bb	PNT	45	
bc	PNT	45	
bd	GTO	44	
c0	0	00	
c1	0	00	
c2	END	46	

Σx^6

Σx^5

LINEAR LEAST SQUARES: Y = A + BX (WITH ERROR STATISTICS)

This program computes the linear regression coefficients A and B, the root mean square error (ERMS), the error in each coefficient, s_A, s_B, and the "t" statistic for each coefficient t_A, t_B. Given a set of data points X_i, Y_i, i = 1, n, the following formulas are employed:

$$A = \frac{\begin{vmatrix} \Sigma Y & \Sigma X \\ \Sigma XY & \Sigma X^2 \end{vmatrix}}{\begin{vmatrix} n & \Sigma X \\ \Sigma X & \Sigma X^2 \end{vmatrix}}$$

$$B = \frac{\Sigma Y - nA}{\Sigma X}$$

$$\Sigma\ (\text{Residuals})^2 = \Sigma Y^2 - 2A\Sigma Y + 2AB\Sigma X - 2B\Sigma XY + nA^2 + B^2\Sigma X^2$$

$$\text{ERMS} = \sqrt{\frac{\Sigma(\text{Residuals})^2}{n - 2}}$$

$$s_B = \frac{(\text{ERMS})^2}{\Sigma X^2 - \frac{(\Sigma X)^2}{n}}$$

$$s_A = \Sigma X^2 \cdot \frac{s_B}{n}$$

$$t_A = \frac{A}{s_A}$$

$$t_B = \frac{B}{s_B}$$

This program was written by Mr. Patrick Ward of the U. S. Army Ballistic Research Laboratories, Aberdeen Proving Grounds, Maryland.

Reference: Introduction to the Theory of Statistics, Mood and Graybill, 2nd Edition, McGraw-Hill, page 351.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM 1:

PRESS: CONTINUE

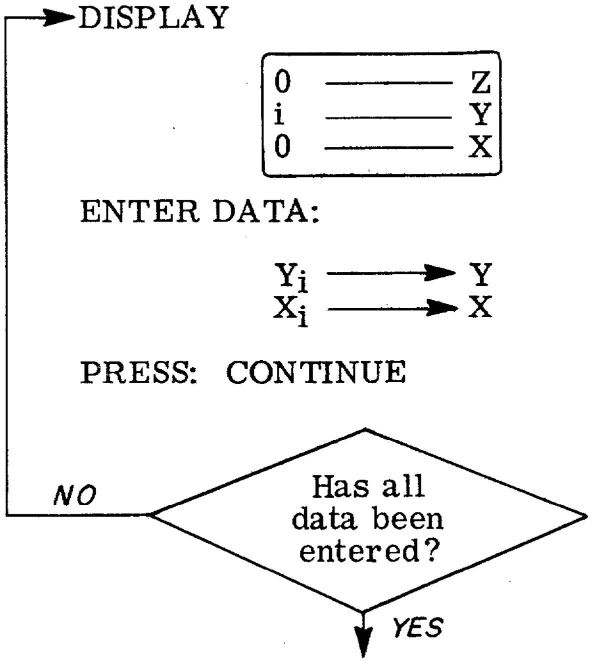
DISPLAY

0 ———— Z
i ———— Y
0 ———— X

ENTER DATA:

$Y_i \longrightarrow Y$
$X_i \longrightarrow X$

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

1 ———— Z
B ———— Y
A ———— X

PRESS: END

ENTER PROGRAM 2: (do not alter the DISPLAY)

PRESS: CONTINUE

DISPLAY

ERMS —— Z
s_B ———— Y
s_A ———— X

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

0 ———— Z
t_B ———— Y
t_A ———— X

To run another case: repeat USER INSTRUCTIONS

EXAMPLE

Y_1 = 3
X_1 = 0

Y_2 = 0
X_2 = 1

Y_3 = -13
X_3 = 2

Y_4 = -36
X_4 = 3

Y_5 = -69
X_5 = 4

Results: B = -18
A = 13

ERMS= 10.80123
s_B = 3.41565
s_A = 8.36660

t_B = -5.26986
t_A = 1.55380

STAT-PAC IV-10

00	CLR	20	Program 1
01	XTO	23	
02	d	17	
03	XTO	23	
04	9	11	
05	XTO	23	
06	8	10	
07	XTO	23	
08	c	16	
09	1	01	
0a	XEY	30	
0b	STP	41	ENTRY
0c	IFG	43	
0d	4	04	
10	0	00	
11	PNT	45	
12	PNT	45	
13	AC+	60	
14	UP	27	
15	X	36	
16	XEY	30	
17	YE	24	
18	d	17	
19	+	33	
1a	YEX	24	
1b	d	17	
1c	DN	25	
1d	X	36	
20	XEY	30	
21	YE	24	
22	9	11	
23	+	33	
24	YEX	24	
25	9	11	
26	DN	25	
27	DN	25	
28	X	36	
29	XEY	30	
2a	YEX	24	
2b	8	10	
2c	+	33	
2d	YEX	24	
30	8	10	
31	c	16	
32	UP	27	
33	1	01	
34	+	33	
35	YTO	40	
36	c	16	
37	+	33	
38	0	00	
39	UP	27	
3a	RDN	31	
3b	GTO	44	
3c	0	00	
3d	b	14	
40	1	01	
41	-	34	
42	d	17	
43	X	36	
44	f	15	
45	UP	27	
46	X	36	
47	DN	25	
48	-	34	
49	YTO	40	
4a	a	13	
4b	f	15	
4c	YE	24	
4d	9	11	
50	UP	27	
51	DN	25	
52	YEX	24	
53	9	11	
54	RUP	22	
55	X	36	
56	e	12	
57	UP	27	
58	d	17	
59	X	36	
5a	DN	25	
5b	XEY	30	
5c	-	34	
5d	a	13	
60	DIV	35	
61	YTO	40	
62	a	13	
63	c	16	
64	X	36	
65	e	12	
66	XEY	30	
67	-	34	
68	f	15	
69	DIV	35	
6a	YTO	40	
6b	b	14	
6c	a	13	
6d	UP	27	
70	1	01	
71	RDN	31	
72	PNT	45	
73	PNT	45	
74	END	46	

00	UP	27	Program 2
01	e	12	
02	X	36	
03	a	13	
04	RUP	22	
05	X	36	
06	f	15	
07	X	36	
08	DN	25	
09	-	34	
0a	b	14	
0b	UP	27	
0c	YE	24	
0d	9	11	
10	X	36	
11	DN	25	
12	+	33	
13	2	02	
14	CHS	32	
15	X	36	
16	a	13	
17	UP	27	
18	X	36	
19	c	16	
1a	X	36	
1b	DN	25	
1c	+	33	
1d	b	14	
20	UP	27	
21	X	36	
22	d	17	
23	X	36	
24	XEY	30	
25	YE	24	
26	8	10	
27	+	33	
28	DN	25	
29	+	33	
2a	c	16	
2b	UP	27	
2c	2	02	
2d	-	34	

ΣY^2

ΣXY

a

b

n

ΣX^2

ΣY

ΣX

```
30  DN    25
31  DIV   35
32  YTO   40
33   e    12
34   c    16
35  UP    27
36   d    17
37   X    36
38   f    15
39  UP    27
3a   X    36
3b  DN    25
3c   -    34
3d   d    17

40  UP    27
41   e    12
42   X    36
43  DN    25
44  XEY   30
45  DIV   35
46  UP    27
47   c    16
48  DIV   35
49   e    12
4a  XEY   30
4b  DIV   35
4c   e    12
4d   √    76

50  RDN   31
51   √    76
52  XEY   30
53   √    76
54  PNT   45    s
55  PNT   45
56  UP    27
57   a    13
58  XEY   30
59  DIV   35
5a   b    14
5b  RUP   22
5c  DIV   35
5d   0    00

60  RDN   31
61  XEY   30
62  PNT   45
63  PNT   45
64  END   46
```

MULTIPLE LINEAR REGRESSION (3 VARIABLE)
STATISTICS FAMILY

This program consists of 3 parts computing statistics associated with the regression model:

$$Z = a_0 + a_1X + a_2Y \qquad (X_1 = a_0 + a_1X_2 + a_2X_3)$$

PART a —— determines the correlation coefficients between X, Y, and Z using the equations:

$$r_{XY} = r_{23} = \frac{\Sigma XY - \frac{\Sigma X \Sigma Y}{n}}{nS_X S_Y}$$

$$r_{XZ} = r_{12} = \frac{\Sigma XZ - \frac{\Sigma X \Sigma Z}{n}}{n\, S_X S_Z}$$

$$r_{YZ} = r_{13} = \frac{\Sigma YZ - \frac{\Sigma Y \Sigma Z}{n}}{n\, S_Y S_Z}$$

$$S_X^2 = \frac{1}{n}\left\{\Sigma X^2 - \frac{(\Sigma X)^2}{n}\right\}$$

$$S_Y^2 = \frac{1}{n}\left\{\Sigma Y^2 - \frac{(\Sigma Y)^2}{n}\right\}$$

$$S_Z^2 = \frac{1}{n}\left\{\Sigma Z^2 - \frac{(\Sigma Z)^2}{n}\right\}$$

<u>This program must be run first in the sequence.</u>

PART b —— computes the coefficients of linear multiple correlation given by:

$$R_{Z,XY} = R_{1.23} = \left[\frac{r_{13}^2 + r_{12}^2 - 2r_{13}r_{12}r_{23}}{1 - r_{23}^2}\right]^{\frac{1}{2}}$$

$$R_{X,YZ} = R_{2.13} = \left[\frac{r_{12}^2 + r_{23}^2 - 2r_{13}r_{12}r_{23}}{1 - r_{13}^2}\right]^{\frac{1}{2}}$$

$$R_{Y,XZ} = R_{3.12} = \left[\frac{r_{13}^2 + r_{23}^2 - 2r_{13}r_{12}r_{23}}{1 - r_{12}^2}\right]^{\frac{1}{2}}$$

A coefficient of linear multiple correlation will have a value between 0 and 1. Zero indicates no linear relationship between the variables whereas 1 indicates a perfect dependence.

PART c —— determines the constants a_0, a_1, and a_2 from the equations defined by program STAT-PAC IV-3. PART c can be run immediately following PART a if the coefficients of linear multiple correlation are not desired.

Reference: Theory and Problems of Statistics, M.R. Spiegel, McGraw-Hill, 1961, page 271.

USER INSTRUCTIONS

PART a

DEPRESS: $\boxed{X}$ $\boxed{Y}$ $\boxed{Z}$ on the 9120A

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

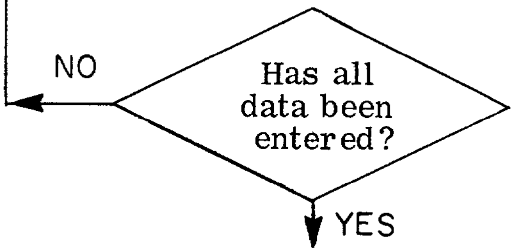
DISPLAY

i ——— Z
0 ——— Y
0 ——— X

ENTER DATA:

$Z_i \longrightarrow Z$
$Y_i \longrightarrow Y$
$X_i \longrightarrow X$

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

S_Z ——— Z
S_Y ——— Y
S_X ——— X

PRESS: CONTINUE

DISPLAY

r_{YZ} ——— Z
r_{XZ} ——— Y
r_{XY} ——— X

USER INSTRUCTIONS (Con't)

To rerun PART a

PRESS: END

EXAMPLE

i	X	Y	Z
1	1	0	3
2	0	1	4
3	1	1	6
4	3	4	19
5	2	2	11

results:

S_Z = 5.8856
S_Y = 1.3565
S_X = 1.0198

r_{YZ} = .9820
r_{XZ} = .9263
r_{XY} = .8386

USER INSTRUCTIONS

PART b

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM:

PRESS: CONTINUE

DISPLAY

$R_{X,YZ}$ —— Z
$R_{Y,XZ}$ —— Y
$R_{Z,XY}$ —— X

EXAMPLE (Con't)

$R_{X,YZ} = 1.0000$
$R_{Y,XZ} = 1.0000$
$R_{Z,XY} = 1.0000$

USER INSTRUCTIONS

PART c

DEPRESS: $\boxed{X}$ $\boxed{Y}$ $\boxed{Z}$ on the 9120A

PRESS: GO TO (-)(0)(0)

ENTER PROGRAM

PRESS: GO TO (-)(0)(0)

PRESS: CONTINUE

DISPLAY

a_2 ———— Z
a_1 ———— Y
a_0 ———— X

EXAMPLE (Con't)

a_2 = 3.0000
a_1 = 2.0000
a_0 = 1.0000

00	CLR	20	Plus
01	XTO	23	Page
02	a	13	
03	XTO	23	
04	b	14	
05	XTO	23	
06	c	16	
07	XTO	23	
08	9	11	
09	XTO	23	
0a	-	34	
0b	f	15	
0c	XTO	23	
0d	-	34	
10	e	12	
11	XTO	23	
12	8	10	
13	1	01	
14	XTO	23	
15	d	17	
16	RDN	31	
17	STP	41	ENTRY
18	IFG	43	
19	7	07	
1a	1	01	
1b	PNT	45	
1c	PNT	45	
1d	YTO	40	
20	-	34	
21	d	17	
22	X	36	
23	XEY	30	
24	YE	24	
25	a	13	
26	+	33	
27	YE	24	
28	a	13	
29	RDN	31	
2a	AC+	60	
2b	UP	27	
2c	X	36	
2d	XEY	30	
30	YE	24	
31	-	34	
32	e	12	
33	+	33	
34	YE	24	
35	-	34	
36	e	12	
37	DN	25	
38	X	36	
39	c	16	
3a	+	33	
3b	YTO	40	
3c	c	16	
3d	RUP	22	

40	UP	27
41	X	36
42	XEY	30
43	YE	24
44	8	10
45	+	33
46	YE	24
47	8	10
48	XFR	67
49	-	34
4a	d	17
4b	X	36
4c	UP	27
4d	X	36
50	RUP	22
51	YE	24
52	9	11
53	+	33
54	YE	24
55	9	11
56	b	14
57	+	33
58	YTO	40
59	b	14
5a	RUP	22
5b	YE	24
5c	-	34
5d	f	15
60	+	33
61	YTO	40
62	-	34
63	f	15
64	1	01
65	UP	27
66	d	17
67	+	33
68	YTO	40
69	d	17
6a	CLX	37
6b	UP	27
6c	GTO	44
6d	1	01
70	7	07
71	d	17
72	UP	27
73	1	01
74	-	34
75	YTO	40
76	d	17
77	XFR	67
78	-	34
79	e	12
7a	GTO	44
7b	-	34
7c	0	00
7d	0	00

80	CNT	47
81	CNT	47
82	CNT	47
83	CNT	47
84	CNT	47
85	CNT	47
86	CNT	47
87	CNT	47
88	CNT	47
89	CNT	47
8a	CNT	47
8b	CNT	47
8c	CNT	47
8d	CNT	47
90	CNT	47
91	CNT	47
92	CNT	47
93	CNT	47
94	CNT	47
95	CNT	47
96	CNT	47
97	CNT	47
98	CNT	47
99	CNT	47
9a	CNT	47
9b	CNT	47
9c	CNT	47
9d	CNT	47
a0	CNT	47
a1	CNT	47
a2	CNT	47
a3	CNT	47
a4	CNT	47
a5	CNT	47
a6	CNT	47
a7	CNT	47
a8	CNT	47
a9	CNT	47
aa	CNT	47
ab	CNT	47
ac	CNT	47
ad	CNT	47

ΣZ^2

ΣYZ

ΣXY

ΣY

ΣXZ

n

ΣZ

ΣX

b0	CNT	47	Plus
b1	CNT	47	Page
b2	CNT	47	
b3	CNT	47	
b4	CNT	47	
b5	CNT	47	
b6	CNT	47	
b7	CNT	47	
b8	CNT	47	
b9	CNT	47	
ba	CNT	47	
bb	CNT	47	
bc	CNT	47	
bd	CNT	47	
c0	CNT	47	
c1	CNT	47	
c2	CNT	47	
c3	CNT	47	
c4	CNT	47	
c5	CNT	47	
c6	CNT	47	
c7	CNT	47	
c8	CNT	47	
c9	CNT	47	
ca	CNT	47	
cb	CNT	47	
cc	CNT	47	
cd	CNT	47	
d0	CNT	47	
d1	CNT	47	
d2	CNT	47	
d3	CNT	47	
d4	CNT	47	
d5	CNT	47	
d6	CNT	47	
d7	CNT	47	
d8	CNT	47	
d9	CNT	47	
da	CNT	47	
db	CNT	47	
dc	CNT	47	
dd	CNT	47	

Y

ΣX^2

ΣY^2

STAT-PAC IV-11 Part a 9100B ONLY

Minus Page

```
00  UP   27
01  f    15
02  GTO  44
03  SUB  77
04  8    10
05  b    14
06  YTO  40
07  -    34
08  d    17
09  XFR  67
0a  8    10
0b  UP   27
0c  e    12
0d  GTO  44

10  SUB  77
11  8    10
12  b    14
13  YTO  40
14  -    34
15  c    16
16  XFR  67
17  -    34
18  f    15
19  UP   27
1a  b    14
1b  GTO  44
1c  SUB  77
1d  8    10

20  b    14
21  YTO  40
22  -    34
23  b    14
24  XFR  67
25  -    34
26  c    16
27  XEY  30
28  UP   27
29  XFR  67
2a  -    34
2b  d    17
2c  PNT  45
2d  PNT  45

30  a    13
31  UP   27
32  f    15
33  UP   27
34  b    14
35  X    36
36  d    17
37  DIV  35
38  DN   25
39  -    34
3a  d    17
3b  DIV  35
3c  XFR  67
3d  -    34

40  d    17
41  DIV  35
42  XFR  67
43  -    34
44  b    14
45  DIV  35
46  YTO  40
47  -    34
48  a    13
49  c    16
4a  UP   27
4b  f    15
4c  UP   27
4d  e    12

50  X    36
51  d    17
52  DIV  35
53  DN   25
54  -    34
55  d    17
56  DIV  35
57  XFR  67
58  -    34
59  d    17
5a  DIV  35
5b  XFR  67
5c  -    34
5d  c    16

60  DIV  35
61  YTO  40
62  -    34
63  d    17
64  XFR  67
65  9    11
66  UP   27
67  e    12
68  UP   27
69  b    14
6a  X    36
6b  d    17
6c  DIV  35
6d  DN   25

70  -    34
71  d    17
72  DIV  35
73  XFR  67
74  -    34
75  c    16
76  DIV  35
77  XFR  67
78  -    34
79  b    14
7a  DIV  35
7b  XFR  67
7c  -    34
7d  d    17

80  YTO  40
81  -    34
82  c    16
83  UP   27
84  XFR  67
85  -    34
86  a    13
87  PNT  45
88  PNT  45
89  STP  41
8a  STP  41
8b  UP   27
8c  X    36
8d  d    17

90  DIV  35
91  DN   25
92  -    34
93  d    17
94  UP   27
95  0    00
96  -    34
97  DN   25
98  DIV  35
99  DN   25
9a  √    76
9b  UP   27
9c  RTN  77
9d  END  46
```

STAT-PAC IV-11 Part b 9100B ONLY

```
00  GTO  44    Minus
01  SUB  77    Page
02  2    02
03  c    16
04  GTO  44
05  SUB  77
06  4    04
07  6    06
08  YTO  40
09  -    34
0a  9    11
0b  GTO  44
0c  SUB  77
0d  4    04

10  6    06
11  YTO  40
12  -    34
13  8    10
14  GTO  44
15  SUB  77
16  4    04
17  6    06
18  XFR  67
19  -    34
1a  8    10
1b  UP   27
1c  XFR  67
1d  -    34

20  9    11
21  PNT  45    S
22  PNT  45
23  GTO  44
24  SUB  77
25  2    02
26  c    16
27  0    00
28  UP   27
29  UP   27
2a  STP  41
2b  STP  41
2c  f    15
2d  UP   27

30  YE   24
31  -    34
32  d    17
33  YTO  40
34  f    15
35  e    12
36  UP   27
37  YE   24
38  -    34
39  c    16
3a  YTO  40
3b  e    12
3c  d    17
3d  UP   27

40  YE   24
41  -    34
42  a    13
43  YTO  40
44  d    17
45  RTN  77
46  f    15
47  UP   27
48  X    36
49  e    12
4a  UP   27
4b  X    36
4c  DN   25
4d  +    33

50  e    12
51  UP   27
52  f    15
53  X    36
54  d    17
55  X    36
56  2    02
57  X    36
58  DN   25
59  -    34
5a  d    17
5b  UP   27
5c  X    36
5d  1    01

60  XEY  30
61  -    34
62  DN   25
63  DIV  35
64  DN   25
65  √    76
66  UP   27
67  f    15
68  UP   27
69  YE   24
6a  d    17
6b  YE   24
6c  e    12
6d  YTO  40

70  f    15
71  DN   25
72  RTN  77
73  END  46
```

Step	Key	Code	
00	YE	24	Minus Page
01	-	34	
02	f	15	
03	UP	27	
04	DN	25	
05	f	15	
06	X	36	
07	CHS	32	
08	X	36	
09	RUP	22	
0a	YE	24	
0b	-	34	
0c	e	12	
0d	YTO	40	
10	-	34	
11	e	12	
12	X	36	
13	d	17	
14	X	36	
15	RDN	31	
16	+	33	
17	a	13	
18	UP	27	
19	X	36	
1a	d	17	
1b	X	36	
1c	DN	25	
1d	-	34	
20	UP	27	
21	YE	24	
22	-	34	
23	e	12	
24	YTO	40	
25	-	34	
26	e	12	
27	b	14	
28	X	36	
29	X	36	
2a	DN	25	
2b	-	34	
2c	b	14	
2d	UP	27	
30	f	15	
31	X	36	
32	a	13	
33	X	36	
34	2	02	
35	X	36	
36	DN	25	
37	+	33	
38	YTO	40	
39	-	34	
3a	f	15	
3b	YE	24	
3c	9	11	
3d	UP	27	
40	DN	25	
41	f	15	
42	X	36	
43	CHS	32	
44	X	36	
45	RUP	22	
46	XEY	30	
47	d	17	
48	X	36	
49	XFR	67	
4a	-	34	
4b	e	12	
4c	X	36	
4d	RDN	31	
50	+	33	
51	RUP	22	
52	XEY	30	
53	b	14	
54	X	36	
55	e	12	
56	X	36	
57	RDN	31	
58	-	34	
59	f	15	
5a	RUP	22	
5b	X	36	
5c	a	13	
5d	X	36	
60	RDN	31	
61	+	33	
62	d	17	
63	RUP	22	
64	X	36	
65	c	16	
66	X	36	
67	RDN	31	
68	-	34	
69	f	15	
6a	RUP	22	
6b	X	36	
6c	b	14	
6d	X	36	
70	DN	25	
71	+	33	
72	XFR	67	
73	-	34	
74	f	15	
75	DIV	35	
76	YTO	40	
77	9	11	
78	UP	27	
79	DN	25	
7a	b	14	
7b	X	36	
7c	e	12	
7d	XEY	30	
80	-	34	
81	a	13	
82	YTO	40	
83	a	13	
84	RUP	22	
85	X	36	
86	c	16	
87	XEY	30	
88	-	34	
89	YTO	40	
8a	c	16	
8b	f	15	
8c	UP	27	
8d	d	17	
90	YTO	40	
91	d	17	
92	YE	24	
93	-	34	
94	e	12	
95	YTO	40	
96	-	34	
97	f	15	
98	XTO	23	
99	b	14	
9a	CLR	20	
9b	YE	24	
9c	-	34	
9d	f	15	
a0	c	16	
a1	UP	27	
a2	d	17	
a3	DIV	35	
a4	RUP	22	
a5	XEY	30	
a6	DIV	35	
a7	DN	25	
a8	IFG	43	
a9	b	14	
aa	8	10	
ab	AC+	60	
ac	d	17	
ad	YE	24	

Minus Page

```
b0    b     14
b1    UP    27
b2    a     13
b3    RUP   22
b4    SFL   54
b5    GTO   44
b6    a     13
b7    3     03
b8    AC-   63
b9    XEY   30
ba    YE    24
bb    e     12
bc    f     15
bd    DIV   35

c0    e     12
c1    XEY   30
c2    X     36
c3    RDN   31
c4    -     34
c5    RUP   22
c6    YE    24
c7    9     11
c8    RUP   22
c9    PNT   45
ca    PNT   45
cb    END   46
```

GOMPERTZ CURVE FIT

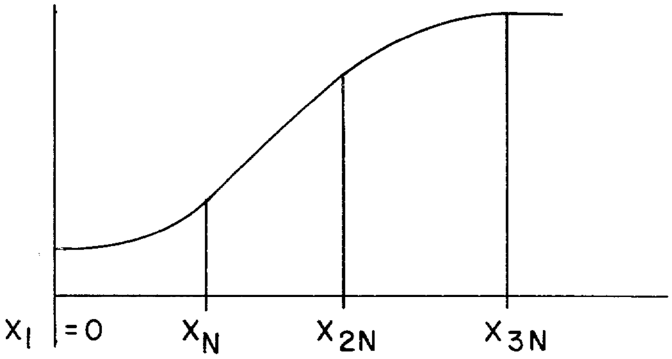
This program fits a GOMPERTZ curve to observed data. The GOMPERTZ curve is given by the equation:

$$Y_c = Ka^{(b^X)}$$

or

$$\log Y_c = \log K + b^X \log a \quad .$$

The usual geometric form of the curve is given below:



3N Observations:

Notice that the curve is divided into 3 sections, each having N observations.

The b, log K, and log a are determined by computing the quantities:

$$\Sigma_1 = \sum_{i=1}^{N} \log Y_i$$

$$\Sigma_2 = \sum_{i=N+1}^{2N} \log Y_i$$

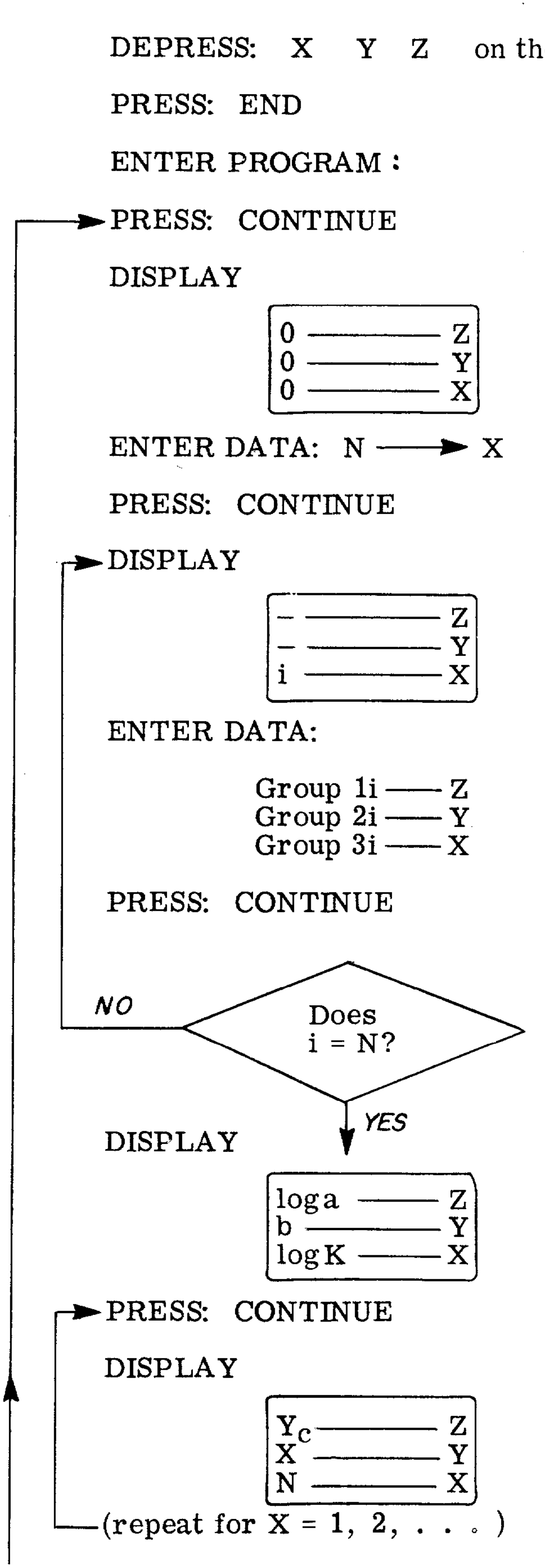
$$\Sigma_3 = \sum_{i=2N+1}^{3N} \log Y_i$$

$$b = \left(\frac{\Sigma_3 - \Sigma_2}{\Sigma_2 - \Sigma_1} \right)^{\frac{1}{N}}$$

$$\log K = \frac{1}{N} \left[\frac{\Sigma_1 \Sigma_3 - (\Sigma_2)^2}{\Sigma_1 + \Sigma_3 - 2\Sigma_2}\right]$$

$$\log a = (\Sigma_2 - \Sigma_1) \frac{(b - 1)}{(b^N - 1)^2}$$

USER INSTRUCTIONS



DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM :

PRESS: CONTINUE

DISPLAY

ENTER DATA: N → X

PRESS: CONTINUE

DISPLAY

ENTER DATA:

Group 1i —— Z
Group 2i —— Y
Group 3i —— X

PRESS: CONTINUE

DISPLAY

PRESS: CONTINUE

DISPLAY

(repeat for X = 1, 2, . . .)

USER INSTRUCTIONS (Con't)

To run another case:

PRESS: END

EXAMPLE

The following table shows the kilowatt requirements of a group of atomic power plants.

Year	X	July Kilowatts	
1954	0	226	Group 1
1955	1	244	
1956	2	265	
1957	3	287	
1958	4	290	
1959	5	317	Group 2
1960	6	316	
1961	7	362	
1962	8	378	
1963	9	417	
1964	10	442	Group 3
1965	11	461	
1966	12	526	
1967	13	566	
1968	14	649	

N = 5

Group 11	226	Input data
Group 21	317	
Group 31	442	
Group 12	244	
Group 22	316	
Group 32	461	
⋮	265	
	362	
	526	
	287	
	378	
	566	
	290	
	417	
	649	

EXAMPLE (Con't)

Results:

```
log a =    .49712
b     =   1.04514
log K =   1.87293

Yc    = 234.44858
X     =   0.00000
N     =   5.00000

      = 246.88231
      =   1.00000
      =   5.00000

      = 260.58266
      =   2.00000
      =   5.00000

      = 275.71470
      =   3.00000
      =   5.00000

      = 292.46981
      =   4.00000
      =   5.00000

      = 311.07049
      =   5.00000
      =   5.00000

      = 331.77638
      =   6.00000
      =   5.00000

      = 354.89147
      =   7.00000
      =   5.00000

              .
              .
              .
```

```
00  CLR   20
01  XTO   23
02   d    17
03  STP   41    ENTRY
04  XTO   23
05   a    13
06   1    01
07  XTO   23
08   c    16
09  STP   41    ENTRY
0a  PNT   45
0b  PNT   45
0c  LN    65
0d  XEY   30

10  LN    65
11  AC+   60
12   d    17
13  RUP   22
14  LN    65
15   +    33
16  YTO   40
17   d    17
18   c    16
19  UP    27
1a   a    13
1b  X=Y   50
1c   2    02
1d   6    06

20   1    01
21   +    33
22  DN    25
23  GTO   44
24   0    00
25   7    07
26   d    17
27  UP    27
28   e    12
29   X    36
2a   f    15
2b  UP    27
2c   X    36
2d  DN    25

30   -    34
31   a    13
32  DIV   35
33   f    15
34  UP    27
35   +    33
36   e    12
37  XEY   30
38   -    34
39   d    17
3a   +    33
3b  DN    25
3c  DIV   35
3d  YTO   40
```

```
40   b    14
41   e    12
42  UP    27
43   f    15
44   -    34
45  UP    27
46   d    17
47   -    34
48  DN    25
49  DIV   35
4a  RDN   31
4b  LN    65
4c  XEY   30
4d   a    13

50  DIV   35
51  DN    25
52  EXP   74
53  XTO   23
54   c    16
55  LN    65
56  UP    27
57   a    13
58   X    36
59  DN    25
5a  EXP   74
5b  UP    27
5c   1    01
5d   -    34

60  DN    25
61  UP    27
62   X    36
63  DN    25
64  DIV   35
65   c    16
66  UP    27
67   1    01
68   -    34
69  DN    25
6a   X    36
6b  YTO   40
6c   d    17
6d  DN    25

70  EXP   74
71  LOG   75
72  UP    27
73  CLX   37
74  XTO   23
75   e    12
76   c    16
77  UP    27
78   b    14
79  EXP   74
7a  LOG   75
7b  PNT   45    S
7c  PNT   45
7d  DN    25
```

```
80  LN    65
81  UP    27
82   e    12
83   X    36
84  DN    25
85  EXP   74
86  UP    27
87   d    17
88   X    36
89   b    14
8a   +    33
8b  DN    25
8c  EXP   74
8d  UP    27

90   e    12
91  UP    27
92   a    13
93  PNT   45    S
94  PNT   45
95   1    01
96  XEY   30
97  AC+   60
98   c    16
99  GTO   44
9a   8    10
9b   0    00
9c  END   46
```

N

ln K

i

ΣGrp 1

ΣGrp 3

ΣGrp 2

9100B ONLY
STAT-PAC
IV-13

PROGRAM FOR SIMULTANEOUS SOLUTION OF FOUR EQUATIONS IN FOUR UNKNOWNS WITH PRINTER

This program in its original form was written by Dr. Stefan J. Medwadowski, a Consulting Structural Engineer in San Francisco. One of his hobbies is programming the -hp- 9100A.

4x4 SYSTEM OF LINEAR ALGEBRAIC EQUATIONS

Given a system of linear algebraic equations:

$$\begin{aligned} a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + a_{14} x_4 &= p_1 \\ a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + a_{24} x_4 &= p_2 \\ a_{31} x_1 + a_{32} x_2 + a_{33} x_3 + a_{34} x_4 &= p_3 \\ a_{41} x_1 + a_{42} x_2 + a_{43} x_3 + a_{44} x_4 &= p_4 \end{aligned}$$

or a matrix notation:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \qquad \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}$$

i.e., $[a_{ij}]\ \{x_i\} = \{p_i\}$ with i, j = 1, 2, 3, 4

Such systems occur frequently in the solution of boundary value problems of structural mechanics, such as those which arise in the theory of thin elastic shells or plates.

It is assumed that the solution of the system exists; i.e., that the determinant of the a_{ij} coefficient matrix does not vanish. The coefficients a_{ij} are assumed real.

NOTE: None of the determinates of the leading submatrices may be zero, or

$$\begin{vmatrix} a_{11} \end{vmatrix} \neq 0$$

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0$$

Should one or more of these conditions exist (and should therefore the illegal operation light come on), it may be removed by re-arranging the sequence of the equations within the system. It is always possible to do this as a consequence of the postulated existence of a unique solution.

Method of Solution: Cholewski's Method

Reference: Salvadori and Baron, Numerical Method in Engineering, Prentice-Hall, 1952.

USER INSTRUCTIONS

DEPRESS: X on the 9120A

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

ENTER DATA: a_{ij} (or P_i) ⟶ X

NOTE: Data is entered row by row in the following order with a CONTINUE following each entry:

$a_{11}, a_{12}, a_{13}, a_{14}, \quad p_1$

$a_{21}, a_{22}, a_{23}, a_{24}, \quad p_2$

$a_{31}, a_{32}, a_{33}, a_{34}, \quad p_3$

$a_{41}, a_{42}, a_{43}, a_{44}, \quad p_4$

DISPLAY

```
*  ———— Z
*  ———— Y
x1 ———— X
```

PRESS: CONTINUE

DISPLAY

```
*  ———— Z
*  ———— Y
x2 ———— X
```

PRESS: CONTINUE

DISPLAY

```
*  ———— Z
*  ———— Y
x3 ———— X
```

PRESS: CONTINUE

DISPLAY

```
*  ———— Z
*  ———— Y
x4 ———— X
```

To rerun program:

Repeat USER INSTRUCTIONS

*Denotes Insignificant Display

EXAMPLE

$$
\begin{aligned}
2x_1 + x_2 + 3x_3 + x_4 &= 7\\
x_1 + 4x_2 + x_3 &= 10\\
3x_1 - 5x_3 - 2x_4 &= 10\\
8x_1 - x_2 + 4x_3 + 2x_4 &= 22
\end{aligned}
$$

$x_1 = 3$ $\quad x_3 = -1$

$x_2 = 2$ $\quad x_4 = 2$

STAT-PAC IV-13

00	CLR	20	Plus Page
01	STP	41	ENTRY
02	PNT	45	
03	RDN	31	
04	STP	41	ENTRY
05	PNT	45	
06	RUP	22	
07	DIV	35	
08	YTO	40	
09	f	15	
0a	RDN	31	
0b	STP	41	ENTRY
0c	PNT	45	
0d	RUP	22	
10	DIV	35	
11	YTO	40	
12	e	12	
13	RDN	31	
14	STP	41	ENTRY
15	PNT	45	
16	RUP	22	
17	DIV	35	
18	YTO	40	
19	b	14	
1a	RDN	31	
1b	STP	41	ENTRY
1c	PNT	45	
1d	RUP	22	
20	DIV	35	
21	YTO	40	
22	a	13	
23	STP	41	ENTRY
24	PNT	45	
25	UP	27	
26	UP	27	
27	f	15	
28	X	36	
29	STP	41	ENTRY
2a	PNT	45	
2b	XEY	30	
2c	-	34	
2d	YTO	40	
30	c	16	
31	DN	25	
32	e	12	
33	X	36	
34	STP	41	ENTRY
35	PNT	45	
36	XEY	30	
37	-	34	
38	STP	41	ENTRY
39	PNT	45	
3a	XTO	23	
3b	d	17	
3c	c	16	
3d	DIV	35	
40	YE	24	
41	d	17	
42	b	14	
43	RUP	22	
44	X	36	
45	RDN	31	
46	-	34	
47	c	16	
48	DIV	35	
49	YE	24	
4a	a	13	
4b	YTO	40	
4c	-	34	
4d	f	15	
50	DN	25	
51	X	36	
52	STP	41	ENTRY
53	PNT	45	
54	XEY	30	
55	-	34	
56	c	16	
57	DIV	35	
58	STP	41	ENTRY
59	PNT	45	
5a	RUP	22	
5b	f	15	
5c	XEY	30	
5d	X	36	
60	RDN	31	
61	YTO	40	
62	-	34	
63	e	12	
64	XEY	30	
65	STP	41	ENTRY
66	PNT	45	
67	XEY	30	
68	-	34	
69	YTO	40	
6a	c	16	
6b	d	17	
6c	X	36	
6d	e	12	
70	RUP	22	
71	X	36	
72	RDN	31	
73	+	33	
74	STP	41	ENTRY
75	PNT	45	
76	XEY	30	
77	-	34	
78	YE	24	
79	a	13	
7a	YTO	40	
7b	-	34	
7c	d	17	
7d	c	16	
80	X	36	
81	b	14	
82	RUP	22	
83	X	36	
84	RDN	31	
85	+	33	
86	STP	41	ENTRY
87	PNT	45	
88	XEY	30	
89	-	34	
8a	a	13	
8b	DIV	35	
8c	YE	24	
8d	-	34	
90	e	12	
91	YTO	40	
92	+	33	
93	1	01	
94	c	16	
95	X	36	
96	RUP	22	
97	XEY	30	
98	CNT	47	
99	CNT	47	
9a	GTO	44	
9b	-	34	
9c	0	00	
9d	0	00	
a0	CNT	47	
a1	CNT	47	
a2	CNT	47	
a3	CNT	47	
a4	CNT	47	
a5	CNT	47	
a6	CNT	47	
a7	CNT	47	
a8	CNT	47	
a9	CNT	47	
aa	CNT	47	
ab	CNT	47	
ac	CNT	47	
ad	CNT	47	

STAT-PAC IV-13

Plus
Page

```
b0  CNT  47
b1  CNT  47
b2  CNT  47
b3  CNT  47
b4  CNT  47
b5  CNT  47
b6  CNT  47
b7  CNT  47
b8  CNT  47
b9  CNT  47
ba  CNT  47
bb  CNT  47
bc  CNT  47
bd  CNT  47

c0  CNT  47
c1  CNT  47
c2  CNT  47
c3  CNT  47
c4  CNT  47
c5  CNT  47
c6  CNT  47
c7  CNT  47
c8  CNT  47
c9  CNT  47
ca  CNT  47
cb  CNT  47
cc  CNT  47
cd  CNT  47

d0  CNT  47
d1  CNT  47
d2  CNT  47
d3  CNT  47
d4  CNT  47
d5  CNT  47
d6  CNT  47
d7  CNT  47
d8  CNT  47
d9  CNT  47
da  CNT  47
db  CNT  47
dc  CNT  47
dd  CNT  47
```

00	XFR	67	Minus Page	40	+	33		80	-	34	
01	-	34		41	b	14		81	RUP	22	
02	f	15		42	RUP	22		82	YE	24	
03	XEY	30		43	X	36		83	-	34	
04	X	36		44	RDN	31		84	d	17	
05	RDN	31		45	+	33		85	c	16	
06	+	33		46	STP	41	ENTRY	86	X	36	
07	STP	41	ENTRY	47	PNT	45		87	RDN	31	
08	PNT	45		48	XEY	30		88	-	34	
09	XEY	30		49	-	34		89	YTO	40	
0a	-	34		4a	YE	24		8a	d	17	
0b	a	13		4b	-	34		8b	f	15	
0c	DIV	35		4c	f	15		8c	X	36	
0d	YTO	40		4d	STP	41	ENTRY	8d	a	13	
10	0	00		50	PNT	45		90	UP	27	
11	STP	41	ENTRY	51	RDN	31		91	e	12	
12	PNT	45		52	X	36		92	X	36	
13	UP	27		53	RDN	31		93	DN	25	
14	UP	27		54	-	34		94	+	33	
15	f	15		55	XEY	30		95	c	16	
16	X	36		56	YE	24		96	UP	27	
17	STP	41	ENTRY	57	0	00		97	b	14	
18	PNT	45		58	RDN	31		98	X	36	
19	XEY	30		59	YE	24		99	DN	25	
1a	-	34		5a	-	34		9a	+	33	
1b	YTO	40		5b	e	12		9b	DN	25	
1c	c	16		5c	X	36		9c	YE	24	
1d	d	17		5d	RDN	31		9d	-	34	
20	X	36		60	-	34		a0	e	12	
21	e	12		61	RDN	31		a1	-	34	
22	RUP	22		62	YE	24		a2	d	17	
23	X	36		63	+	33		a3	UP	27	
24	RDN	31		64	1	01		a4	XFR	67	
25	+	33		65	RUP	22		a5	-	34	
26	STP	41	ENTRY	66	c	16		a6	c	16	
27	PNT	45		67	RUP	22		a7	PNT	45	
28	XEY	30		68	X	36		a8	PNT	45	
29	-	34		69	RDN	31		a9	DN	25	
2a	DN	25		6a	-	34		aa	XEY	30	
2b	UP	27		6b	XFR	67		ab	PNT	45	s
2c	YE	24		6c	-	34		ac	DN	25	
2d	-	34		6d	f	15		ad	PNT	45	s
30	e	12		70	DIV	35					
31	YTO	40		71	a	13					
32	a	13		72	YTO	40					
33	X	36		73	c	16					
34	YE	24		74	X	36					
35	-	34		75	XEY	30					
36	d	17		76	YE	24					
37	c	16		77	1	01					
38	XEY	30		78	-	34					
39	X	36		79	YTO	40					
3a	XEY	30		7a	a	13					
3b	YE	24		7b	d	17					
3c	-	34		7c	X	36					
3d	d	17		7d	RDN	31					

```
b0   a     13        Minus
b1   PNT   45   s    Page
b2   c     16
b3   PNT   45   s
b4   PNT   45
b5   XFR   67
b6   -     34
b7   c     16
b8   PNT   45
b9   PNT   45
ba   PNT   45
bb   CLR   20
bc   STP   41
bd   STP   41

c0   CHS   32
c1   CHS   32
c2   CHS   32
c3   CHS   32
c4   CHS   32
c5   CHS   32
c6   CHS   32
c7   CHS   32
c8   CHS   32
c9   CHS   32
ca   YTO   40
cb   YTO   40
cc   YTO   40
cd   YTO   40

d0   END   46
```

LEAST SQUARES REGRESSION
$Y = c_0 X^a + c_1 X^b$

This program calculates the coefficients of the equation $Y = c_0 X^a + c_1 X^b$ of least squares fit of a set of data points. The exponents a and b are any real numbers specified by the user.

Defining Equations are:

$$c_1 = \frac{\Sigma X^{2a} \Sigma X^b Y - \Sigma X^a Y \Sigma X^{a+b}}{\Sigma X^{2b} \Sigma X^{2a} - [\Sigma X^{a+b}]^2}$$

$$c_0 = \frac{[\Sigma X^a Y - b_0 \Sigma X^{a+b}]}{\Sigma X^{2a}}$$

where all $X_i > 0$.

USER INSTRUCTIONS

DEPRESS: X Y on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER EXPONENTS:

$a \longrightarrow Y$
$b \longrightarrow X$

PRESS: CONTINUE

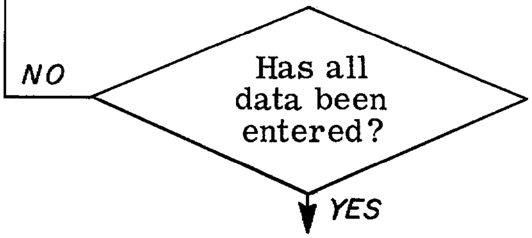
DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA:

$Y_i \longrightarrow Y$
$X_i \longrightarrow X$

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

▲ ——— Z
c_1 ——— Y
c_0 ——— X

▲Denotes Insignificant Display

USER INSTRUCTIONS (Con't)

To run another case, Repeat USER INSTRUCTIONS.

EXAMPLE

X	Y
1	9
4	-44
9	-699
16	-4056

$a = \frac{1}{2}$

$b = 3$

$$Y = c_0 X^{\frac{1}{2}} + c_1 X^3$$

Results:

$c_1 = -1$

$c_0 = 10$

STAT-PAC IV-14

```
00  CLR  20
01  STP  41   ENTRY
02  PNT  45
03  PNT  45
04  XTO  23
05   b   14
06  YTO  40
07   a   13
08  CLR  20
09  STP  41   ENTRY
0a  IFG  43
0b   4   04
0c   c   16
0d  PNT  45

10  PNT  45
11  LN   65
12  XTO  23
13   8   10
14  UP   27
15   a   13
16   X   36
17  DN   25
18  EXP  74
19   X   36
1a  UP   27
1b   X   36
1c  RDN  31
1d  AC+  60

20  RDN  31
21  XEY  30
22  DIV  35
23  UP   27
24   X   36
25  YEX  24
26   8   10
27   b   14
28   X   36
29  DN   25
2a  EXP  74
2b   X   36
2c  UP   27
2d   X   36

30  DN   25
31  YEX  24
32   c   16
33   +   33
34  YEX  24
35   c   16
36  XEY  30
37  YEX  24
38   d   17
39   +   33
3a  YEX  24
3b   d   17
3c  DN   25
3d  YEX  24

40   8   10
41   X   36
42  DN   25
43   √   76
44  YEX  24
45   9   11
46   +   33
47  YEX  24
48   9   11
49  GTO  44
4a   0   00
4b   9   11
4c  YEX  24
4d   9   11

50  YTO  40
51   9   11
52   e   12
53   X   36
54   d   17
55  UP   27
56   f   15
57   X   36
58  DN   25
59   -   34
5a  UP   27
5b  YEX  24
5c   9   11
5d  YTO  40

60   9   11
61  DN   25
62  UP   27
63   X   36
64  YEX  24
65   c   16
66   f   15
67   X   36
68  DN   25
69  YEX  24
6a   c   16
6b   -   34
6c  DN   25
6d  DIV  35

70  UP   27
71  DN   25
72  DN   25
73  YEX  24
74   9   11
75   X   36
76   e   12
77  XEY  30
78   -   34
79   f   15
7a  DIV  35
7b  RDN  31
7c  PNT  45
7d  STP  41

80   0   00
81   0   00
82   0   00
83   0   00
84   0   00
85   0   00
86   0   00
87   0   00
88   0   00
89   0   00
8a   0   00
8b   0   00
8c   0   00
8d   0   00

90   0   00
91   0   00
92   0   00
93   0   00
94   0   00
95   0   00
96   0   00
97   0   00
98   0   00
99   0   00
9a   0   00
9b   0   00
9c   0   00
9d   0   00

a0   0   00
a1   0   00
a2   0   00
a3   0   00
a4   0   00
a5   0   00
a6   0   00
a7   0   00
a8   0   00
a9   0   00
aa   0   00
ab   0   00
ac   0   00
ad   0   00
```

STAT-PAC IV-14

b0	0	00
b1	0	00
b2	0	00
b3	0	00
b4	0	00
b5	0	00
b6	0	00
b7	0	00
b8	0	00
b9	0	00
ba	0	00
bb	0	00
bc	0	00
bd	0	00
c0	0	00
c1	0	00
c2	0	00
c3	0	00
c4	0	00
c5	0	00
c6	0	00
c7	0	00
c8	0	00
c9	0	00
ca	0	00
cb	0	00
cc	0	00
cd	0	00
d0	0	00
d1	0	00
d2	0	00
d3	0	00
d4	0	00
d5	0	00
d6	0	00
d7	0	00
d8	0	00
d9	0	00
da	0	00
db	0	00
dc	0	00
dd	END	46

SECTION V HARMONIC ANALYSIS

SINE WAVE CURVE FIT

This program accepts discrete samples of a curve of the form:

$$Y(t) = c_0 + c_1 \sin \left[\omega_1 t + \phi \right]$$

where ω_1 and t are known and determines c_0, c_1, and ϕ. The function Y(t) is obtained for N discrete values of t at Δt increments for one complete cycle of Y(t).

The quantities c_0, c_1, and ϕ are obtained from the relations:

$$c_0 = \frac{1}{N} \Sigma Y_n \quad ,$$

$$A = \frac{2}{N} \sum_{k=1}^{N} Y_k \cos \frac{2\pi k}{N}$$

$$B = \frac{2}{N} \sum_{k=1}^{N} Y_k \sin \frac{2\pi k}{N}$$

$$c_1 = \sqrt{A^2 + B^2}$$

$$\phi = \arctan \frac{A}{B}$$

Restrictions: N should be chosen such that at least 8 samples are obtained for the complete sine wave cycle.

Reference: Measurement and Analysis of Random Data, J.S. Bendat, A.G. Piersol, John Wiley & Sons, 1966, page 287.

USER INSTRUCTIONS

DEPRESS: X on the 9120A

SET: ☐ RADIANS

PRESS: END

ENTER PROGRAM

→ PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA: N ⟶ X

PRESS: CONTINUE

DISPLAY

0 ——— Z
n ——— Y
0 ——— X

ENTER DATA: $Y_n \longrightarrow X$

After the Nth data value Y_N is entered:

DISPLAY

ϕ ——— Z
c_1 ——— Y
c_0 ——— X

To run another case

EXAMPLE

$$y(t) = c_0 + c_1 \sin [\omega_1 t + \phi]$$

$$\omega_1 = \frac{360^o}{24} \text{ or } \frac{2\pi}{24} \text{ radians/hr.}$$

$$\omega_1 = .262 \text{ radians/hr.}$$

$$\Delta t = 1 \text{ hr.}$$

t(hr.)	y(t)
1	1.953
2	1.998
3	1.977
4	1.888
5	1.740
6	1.540
7	1.303
8	1.046
9	.787
10	.541
11	.327
12	.159
13	.047
14	.001
15	.023
16	.111
17	.260
18	.460
19	.696
20	.953
21	1.213
22	1.459
23	1.673
24	1.841

N = 24 Data Set

$y_1 \longrightarrow y_{24}$

N = 12 Data Set

$y_2, y_4, \cdots y_{24}$

N = 8 Data Set

$y_3, y_6, \cdots y_{24}$

EXAMPLE (Con't)

Results for all these sets

$\phi = 1$
$c_1 = 1$
$c_0 = 1$

$y(t) = 1 + 1 \sin [.262t + 1]$

```
00  CLR  20              40   6    06
01  XTO  23              41   0    00
02   a   13              42  UP    27
03  XTO  23              43  GTO   44
04   b   14              44   0    00
05  STP  41    ENTRY     45   8    10
06  XTO  23              46   f    15
07   d   17              47  UP    27
08   1   01              48   d    17
09  UP   27              49  DIV   35
0a   e   12              4a  YTO   40
0b   +   33              4b   9    11
0c   0   00              4c   b    14
0d  STP  41    ENTRY     4d  UP    27

10  PNT  45              50   a    13
11  PNT  45              51  POL   62
12  UP   27              52  UP    27
13   1   01              53   2    02
14  XEY  30              54   X    36
15  AC+  60              55   d    17
16  XEY  30              56  DIV   35
17   e   12              57  XEY   30
18  UP   27              58  YEX   24
19   d   17              59   9    11
1a  DIV  35              5a  YTO   40
1b   π   56              5b   9    11
1c   X   36              5c  XEY   30
1d   2   02              5d  RUP   22

20   X   36              60  PNT   45
21  DN   25              61  RUP   22
22  XTO  23              62  PNT   45
23   c   16              63  RUP   22
24  COS  73              64  PNT   45    S
25  XEY  30              65  PNT   45
26   X   36              66  PNT   45
27  RDN  31              67  PNT   45
28  YEX  24              68  PNT   45
29   b   14              69  PNT   45
2a   +   33              6a  GTO   44
2b  YTO  40              6b   0    00
2c   b   14              6c   0    00
2d  DN   25              6d  END   46

30   c   16
31  SIN  70
32   X   36
33  DN   25
34  YEX  24
35   a   13
36   +   33
37  YTO  40
38   a   13
39   d   17
3a  UP   27
3b   e   12
3c  X=Y  50
3d   4   04
```

C_0

$\Sigma Y_n \sin^{\pi n/N}$

$\Sigma Y_n \cos^{2\pi n/N}$

$N_{2\pi n/N}$

n

ΣY_n

FOURIER ANALYSIS
(12 ORDINATE SCHEME)

This program computes the first <u>four</u> sine and cosine terms of the Fourier Series obtained from 12 observed ordinates. The 12 ordinates, μ_0, μ_1, μ_2, . . . μ_{11} represent the value of some function f(X) evaluated at $X = 0, \frac{\pi}{6}, \frac{2\pi}{6}, \ldots \frac{11\pi}{6}$ respectively.

This program computes a_0, a_1, a_2, a_3, a_4, b_1, b_2, b_3 and b_4

where

$$f(X) = a_0 + a_1\cos X + a_2\cos 2X + a_3\cos 3X + a_4\cos 4X + \ldots$$
$$+ b_1\sin X + b_2\sin 2X + b_3\sin 3X + b_4\sin 4X + \ldots$$

The following formulas are applied:

$$V_1 = \mu_1 + \mu_{11} \qquad W_1 = \mu_1 - \mu_{11}$$
$$V_2 = \mu_2 + \mu_{10} \qquad W_2 = \mu_2 - \mu_{10}$$
$$V_3 = \mu_3 + \mu_9 \qquad W_3 = \mu_3 - \mu_9$$
$$V_4 = \mu_4 + \mu_8 \qquad W_4 = \mu_4 - \mu_8$$
$$V_5 = \mu_5 + \mu_7 \qquad W_5 = \mu_5 - \mu_7$$

$$a_0 = \frac{1}{12}\left[\mu_0 + V_1 + V_2 + V_3 + V_4 + V_5 + \mu_6 \right]$$

$$a_1 = \frac{1}{6}\left[\mu_0 + \frac{\sqrt{3}}{2} V_1 + \frac{1}{2} V_2 - \frac{1}{2} V_4 - \frac{\sqrt{3}}{2} V_5 - \mu_6 \right]$$

$$a_2 = \frac{1}{6}\left[\mu_0 + \frac{1}{2} V_1 - \frac{1}{2} V_2 - V_3 - \frac{1}{2} V_4 + \frac{1}{2} V_5 + \mu_6 \right]$$

$$a_3 = \frac{1}{6}\left[\mu_0 - V_2 + V_4 - \mu_6 \right]$$

$$a_4 = \frac{1}{6}\left[\mu_0 - \frac{1}{2} V_1 - \frac{1}{2} V_2 + V_3 - \frac{1}{2} V_4 - \frac{1}{2} V_5 + \mu_6 \right]$$

$$b_1 = \frac{1}{6}\left[\frac{1}{2} W_1 + \frac{\sqrt{3}}{2} W_2 + W_3 + \frac{\sqrt{3}}{2} W_4 + \frac{1}{2} W_5 \right]$$

$$b_2 = \frac{1}{6}\left[\frac{\sqrt{3}}{2} W_1 + \frac{\sqrt{3}}{2} W_2 - \frac{\sqrt{3}}{2} W_4 - \frac{\sqrt{3}}{2} W_5 \right]$$

$$b_3 = \frac{1}{6}\left[W_1 - W_3 + W_5 \right]$$

$$b_4 = \frac{1}{6}\left[\frac{\sqrt{3}}{2} W_1 - \frac{\sqrt{3}}{2} W_2 + \frac{\sqrt{3}}{2} W_4 - \frac{\sqrt{3}}{2} W_5 \right]$$

Reference: The Calculus of Observations, Sir Edmund Whittaker and G. Robinson, Dover Publications, 1967, page 267.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM 1: Side A followed by Side B

PRESS: CONTINUE

DISPLAY

0 ———— Z
0 ———— Y
1 ———— X

ENTER DATA:

$\mu_0 \longrightarrow Z$
$\mu_1 \longrightarrow Y$
$\mu_2 \longrightarrow X$

PRESS: CONTINUE

DISPLAY

0 ———— Z
0 ———— Y
2 ———— X

ENTER DATA:

$\mu_3 \longrightarrow Z$
$\mu_4 \longrightarrow Y$
$\mu_5 \longrightarrow X$

PRESS: CONTINUE

DISPLAY

0 ———— Z
0 ———— Y
3 ———— X

ENTER DATA:

$\mu_6 \longrightarrow Z$
$\mu_7 \longrightarrow Y$
$\mu_8 \longrightarrow X$

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

0 ———— Z
0 ———— Y
4 ———— X

ENTER DATA:

$\mu_9 \longrightarrow Z$
$\mu_{10} \longrightarrow Y$
$\mu_{11} \longrightarrow X$

PRESS: CONTINUE

DISPLAY

a_i ———— Z
i ———— Y
i ———— X

When the display is:

DISPLAY

5 ———— Z
5 ———— Y
5 ———— X

PRESS: GO TO (-)(0)(0)

ENTER PROGRAM 2

PRESS: GO TO (-)(0)(0)

PRESS: CONTINUE

DISPLAY

b_i ———— Z
i ———— Y
i ———— X

To run another case, repeat USER INSTRUCTIONS

EXAMPLE

$\mu_0 = 0$
$\mu_1 = .262$
$\mu_2 = .524$
$\mu_3 = .786$
$\mu_4 = 1.047$
$\mu_5 = 1.309$
$\mu_6 = 0$
$\mu_7 = -1.309$
$\mu_8 = -1.047$
$\mu_9 = -.786$
$\mu_{10} = -.524$
$\mu_{11} = -.262$

Results:

$a_0 = 0$
$a_1 = 0$
$a_2 = 0$
$a_3 = 0$
$a_4 = 0$

$b_1 = .977$
$b_2 = -.453$
$b_3 = .262$
$b_4 = -.151$

Or

$f(X) = .977 \sin X - .453 \sin 2X + .262 \sin 3X - .151 \sin 4X \ldots$

The μ_i and resulting Fourier series are plotted in the example of program STAT-PAC X-18.

STAT-PAC V-2 Program 1

00	CLR	20	Plus Page	40	AC+	60		80	CNT	47
01	1	01		41	YTO	40		81	CNT	47
02	STP	41	ENTRY	42	5	05		82	CNT	47
03	PNT	45		43	DN	25		83	CNT	47
04	AC+	60		44	4	04		84	CNT	47
05	XTO	23		45	STP	41	ENTRY	85	CNT	47
06	d	17		46	PNT	45		86	CNT	47
07	YTO	40		47	PNT	45		87	CNT	47
08	c	16		48	AC+	60		88	CNT	47
09	0	00		49	ARC	72		89	CNT	47
0a	RDN	31		4a	GTO	44		8a	CNT	47
0b	0	00		4b	-	34		8b	CNT	47
0c	AC+	60		4c	0	00		8c	CNT	47
0d	YTO	40		4d	0	00		8d	CNT	47
10	b	14		50	CNT	47		90	CNT	47
11	DN	25		51	CNT	47		91	CNT	47
12	2	02		52	CNT	47		92	CNT	47
13	STP	41	ENTRY	53	CNT	47		93	CNT	47
14	AC+	60		54	CNT	47		94	CNT	47
15	PNT	45		55	CNT	47		95	CNT	47
16	XTO	23		56	CNT	47		96	CNT	47
17	a	13		57	CNT	47		97	CNT	47
18	YTO	40		58	CNT	47		98	CNT	47
19	9	11		59	CNT	47		99	CNT	47
1a	0	00		5a	CNT	47		9a	CNT	47
1b	RDN	31		5b	CNT	47		9b	CNT	47
1c	0	00		5c	CNT	47		9c	CNT	47
1d	AC+	60		5d	CNT	47		9d	CNT	47
20	YTO	40		60	CNT	47		a0	CNT	47
21	8	10		61	CNT	47		a1	CNT	47
22	DN	25		62	CNT	47		a2	CNT	47
23	3	03		63	CNT	47		a3	CNT	47
24	STP	41	ENTRY	64	CNT	47		a4	CNT	47
25	PNT	45		65	CNT	47		a5	CNT	47
26	AC+	60		66	CNT	47		a6	CNT	47
27	YE	24		67	CNT	47		a7	CNT	47
28	9	11		68	CNT	47		a8	CNT	47
29	+	33		69	CNT	47		a9	CNT	47
2a	YTO	40		6a	CNT	47		aa	CNT	47
2b	7	07		6b	CNT	47		ab	CNT	47
2c	-	34		6c	CNT	47		ac	CNT	47
2d	-	34		6d	CNT	47		ad	CNT	47
30	YE	24		70	CNT	47				
31	9	11		71	CNT	47				
32	a	13		72	CNT	47				
33	XEY	30		73	CNT	47				
34	+	33		74	CNT	47				
35	YTO	40		75	CNT	47				
36	a	13		76	CNT	47				
37	-	34		77	CNT	47				
38	-	34		78	CNT	47				
39	YTO	40		79	CNT	47				
3a	6	06		7a	CNT	47				
3b	0	00		7b	CNT	47				
3c	RDN	31		7c	CNT	47				
3d	0	00		7d	CNT	47				

```
b0  CNT  47    Plus
b1  CNT  47    Page
b2  CNT  47
b3  CNT  47
b4  CNT  47
b5  CNT  47
b6  CNT  47
b7  CNT  47
b8  CNT  47
b9  CNT  47
ba  CNT  47
bb  CNT  47
bc  CNT  47
bd  CNT  47

c0  CNT  47
c1  CNT  47
c2  CNT  47
c3  CNT  47
c4  CNT  47
c5  CNT  47
c6  CNT  47
c7  CNT  47
c8  CNT  47
c9  CNT  47
ca  CNT  47
cb  CNT  47
cc  CNT  47
cd  CNT  47

d0  CNT  47
d1  CNT  47
d2  CNT  47
d3  CNT  47
d4  CNT  47
d5  CNT  47
d6  CNT  47
d7  CNT  47
d8  CNT  47
d9  CNT  47
da  CNT  47
db  CNT  47
dc  CNT  47
dd  CNT  47
```

Step	Key	Code	X	Y	Z
00	YE	24	Minus Page		
01	c	16			
02	+	33			
03	YTO	40			
04	-	34			
05	f	15			
06	-	34			
07	-	34			
08	YE	24			
09	c	16			
0a	d	17			
0b	XEY	30			
0c	+	33			
0d	YTO	40			
10	d	17			
11	-	34			
12	-	34			
13	YTO	40			
14	-	34			
15	e	12			
16	DN	25			
17	0	00			
18	AC+	60			
19	UP	27			
1a	RCL	61			
1b	+	33			
1c	YTO	40			
1d	f	15			
20	DN	25			
21	XFR	67			
22	8	10			
23	XEY	30			
24	+	33			
25	YTO	40			
26	e	12			
27	-	34			
28	-	34			
29	YTO	40			
2a	8	10			
2b	f	15			
2c	UP	27			
2d	1	01			
30	2	02			
31	DIV	35			
32	0	00			
33	UP	27			
34	PNT	45	S		
35	PNT	45			
36	XFR	67			
37	-	34			
38	f	15			
39	XTO	23			
3a	f	15			
3b	UP	27			
3c	3	03			
3d	√	76			
40	X	36			
41	2	02			
42	DIV	35			
43	b	14			
44	+	33			
45	d	17			
46	UP	27			
47	2	02			
48	DIV	35			
49	DN	25			
4a	+	33			
4b	XFR	67			
4c	7	07			
4d	UP	27			
50	2	02			
51	DIV	35			
52	DN	25			
53	-	34			
54	a	13			
55	UP	27			
56	3	03			
57	√	76			
58	X	36			
59	2	02			
5a	DIV	35			
5b	DN	25			
5c	-	34			
5d	XFR	67			
60	5	05			
61	-	34			
62	6	06			
63	DIV	35			
64	1	01			
65	UP	27			
66	PNT	45	S		
67	PNT	45			
68	XFR	67			
69	5	05			
6a	UP	27			
6b	b	14			
6c	+	33			
6d	f	15			
70	UP	27			
71	2	02			
72	DIV	35			
73	DN	25			
74	+	33			
75	d	17			
76	UP	27			
77	2	02			
78	DIV	35			
79	DN	25			
7a	-	34			
7b	e	12			
7c	-	34			
7d	XFR	67			
80	7	07			
81	UP	27			
82	2	02			
83	DIV	35			
84	DN	25			
85	-	34			
86	a	13			
87	UP	27			
88	2	02			
89	DIV	35			
8a	DN	25			
8b	+	33			
8c	6	06			
8d	DIV	35			
90	2	02			
91	UP	27			
92	PNT	45			
93	PNT	45	S		
94	b	14			
95	UP	27			
96	d	17			
97	-	34			
98	XFR	67			
99	7	07			
9a	+	33			
9b	XFR	67			
9c	5	05			
9d	-	34			
a0	6	06			
a1	DIV	35			
a2	3	03			
a3	UP	27			
a4	PNT	45			
a5	PNT	45			
a6	b	14	S		
a7	UP	27			
a8	f	15			
a9	UP	27			
aa	2	02			
ab	DIV	35			
ac	DN	25			
ad	-	34			

Step	Key	Code	x	y	z
b0	d	17			
b1	UP	27			
b2	2	02			
b3	DIV	35			
b4	DN	25			
b5	-	34			
b6	e	12			
b7	+	33			
b8	XFR	67			
b9	7	07			
ba	UP	27			
bb	2	02			
bc	DIV	35			
bd	DN	25			
c0	-	34			
c1	a	13			
c2	UP	27			
c3	2	02			
c4	DIV	35			
c5	DN	25			
c6	-	34			
c7	XFR	67			
c8	5	05			
c9	+	33			
ca	6	06			
cb	DIV	35			
cc	4	04			
cd	UP	27			
d0	PNT	45	S		
d1	PNT	45			
d2	XFR	67			
d3	6	06			
d4	XTO	23			
d5	e	12			
d6	XFR	67			
d7	8	10			
d8	XTO	23			
d9	d	17			
da	5	05			
db	UP	27			
dc	UP	27			
dd	END	46			

STAT-PAC V-2 Program 2

00	XFR	67	Minus Page	40	UP	27		80	END	46
01	-	34		41	a	13				
02	e	12		42	X	36				
03	XTO	23		43	DN	25				
04	f	15		44	-	34				
05	XFR	67		45	e	12				
06	9	11		46	UP	27				
07	XTO	23		47	a	13				
08	b	14		48	X	36				
09	3	03		49	DN	25				
0a	√	76		4a	-	34				
0b	UP	27		4b	6	06				
0c	2	02		4c	DIV	35				
0d	DIV	35		4d	2	02				
10	YTO	40		50	UP	27				
11	a	13		51	PNT	45	s			
12	c	16		52	PNT	45				
13	UP	27		53	c	16				
14	2	02		54	UP	27				
15	DIV	35		55	d	17				
16	f	15		56	-	34				
17	UP	27		57	e	12				
18	a	13		58	+	33				
19	X	36		59	6	06				
1a	DN	25		5a	DIV	35				
1b	+	33		5b	3	03				
1c	d	17		5c	UP	27				
1d	+	33		5d	PNT	45	s			
20	b	14		60	PNT	45				
21	UP	27		61	c	16				
22	a	13		62	UP	27				
23	X	36		63	a	13				
24	DN	25		64	X	36				
25	+	33		65	UP	27				
26	e	12		66	f	15				
27	UP	27		67	X	36				
28	2	02		68	DN	25				
29	DIV	35		69	-	34				
2a	DN	25		6a	b	14				
2b	+	33		6b	UP	27				
2c	6	06		6c	a	13				
2d	DIV	35		6d	X	36				
30	1	01		70	DN	25				
31	UP	27		71	+	33				
32	PNT	45	s	72	e	12				
33	PNT	45		73	UP	27				
34	c	16		74	a	13				
35	UP	27		75	X	36				
36	a	13		76	DN	25				
37	X	36		77	-	34				W1
38	UP	27		78	6	06				
39	f	15		79	DIV	35				W3
3a	X	36		7a	4	04				W5
3b	DN	25		7b	UP	27				
3c	+	33		7c	PNT	45				W2
3d	b	14		7d	PNT	45	s			

FOURIER SERIES [SAMPLED X(t)]

This program calculates the first 15 Fourier sine or cosine coefficients associated with a discrete (sampled) time function X(t).

Definitions:

$X(t) = X(k\Delta t)$, $k = 1, 2, \ldots , N$

Δt = sampling interval chosen such that the highest frequency present in X(t) is

$$f_{max} \leqq \frac{1}{2\Delta t}$$

$$X_k = A_0 + \sum_{q=1}^{N/2} A_q \cos\left(\frac{2\pi qk}{N}\right) + \sum_{q=1}^{N/2\ -1} B_q \sin\left(\frac{2\pi qk}{N}\right) \qquad (1)$$

$$A_q = \frac{2}{N} \sum_{k=1}^{N} X_k \cos \frac{2\pi qk}{N} , \quad q = 1, 2, \ldots , \frac{N}{2} - 1 \qquad (2)$$

$$B_q = \frac{2}{N} \sum_{k=1}^{N} X_k \sin \frac{2\pi qk}{N} , \quad q = 1, 2, \ldots , \frac{N}{2} - 1 \qquad (3)$$

This program computes either A_q, eq. (2) or B_q, eq. (3) for $q = 1, 2, \ldots 15$.

Assumptions:

$N\Delta t = T$ (period of X(t)).

N is even

At points where X(t) is discontinuous, the midpoint value should be used for the input value.

Reference: Measurement and Analysis of Random Data, J.S. Bendat, A.G. Piersol, John Wiley & Sons, 1966, page 287.

USER INSTRUCTIONS

SET: ☐ RADIANS

DEPRESS: X Y Z on the 9120A

PRESS: GO TO (-)(0)(0)

ENTER PROGRAM

PRESS: GO TO (-)(4)(b)

SET: PROGRAM

PRESS: (sin X) or (cos X) for appropriate series.

SET: ☐ RUN

PRESS: GO TO (-)(0)(0)

PRESS: CONTINUE

DISPLAY

0 ———— Z
0 ———— Y
0 ———— X

ENTER DATA: N ——→ X

PRESS: CONTINUE

DISPLAY

N ———— Z
k ———— Y
0 ———— X

ENTER DATA: X_k ——→ X

After X_N has been entered, the results are displayed and printed.

DISPLAYS

B_1 ———— Z
B_2 ———— Y
B_3 ———— X

CONT:

B_4 ———— Z
B_5 ———— Y
B_6 ———— X

CONT:

USER INSTRUCTIONS (Con't)

B_7 ———— Z
B_8 ———— Y
B_9 ———— X

CONT:

B_{10} ———— Z
B_{11} ———— Y
B_{12} ———— X

CONT:

B_{13} ———— Z
B_{14} ———— Y
B_{15} ———— X

To run another case:

EXAMPLE

Determine the sine terms for the following function: [sin x → (-)(4)(b)].

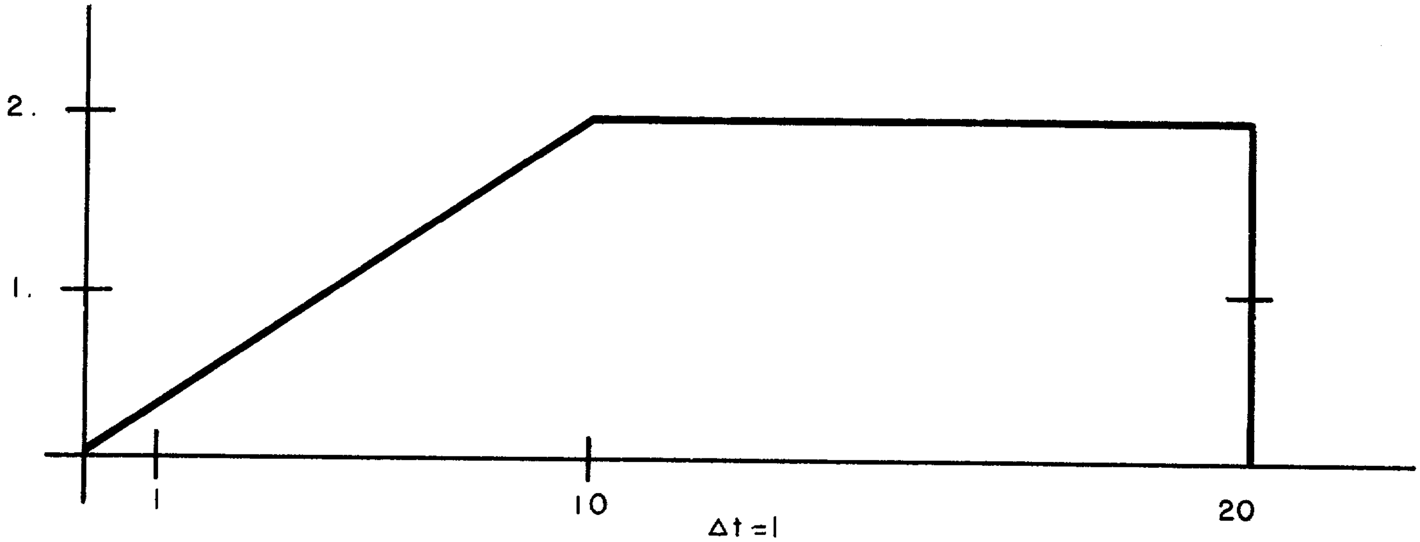
N = 20

k Δt	X(t)
1	.2
2	.4
3	.6
4	.8
5	1.0
6	1.2
7	1.4
8	1.6
9	1.8
10	2.0
11	2.0
12	2.0
13	2.0
14	2.0
15	2.0
16	2.0
17	2.0
18	2.0
19	2.0
20	1.0

EXAMPLE (Con't)

Results:

B_1 = -.63138
B_2 = -.30777
B_3 = -.19626
B_4 = -.13764
B_5 = -.10000
B_6 = -.07265
B_7 = -.05095
B_8 = -.03249
B_9 = -.01584
B_{10} = 0.00000
B_{11} = .01584
B_{12} = .03249
B_{13} = .05095
B_{14} = .07265
B_{15} = .10000



STAT-PAC V-3

00	CLR	20	Minus Page
01	XTO	23	
02	1	01	
03	XTO	23	
04	2	02	
05	XTO	23	
06	3	03	
07	XTO	23	
08	4	04	
09	XTO	23	
0a	5	05	
0b	XTO	23	
0c	6	06	
0d	XTO	23	
10	7	07	
11	XTO	23	
12	8	10	
13	XTO	23	
14	9	11	
15	XTO	23	
16	a	13	
17	XTO	23	
18	b	14	
19	XTO	23	
1a	c	16	
1b	XTO	23	
1c	d	17	
1d	XTO	23	
20	-	34	
21	f	15	
22	STP	41	ENTRY
23	XTO	23	
24	-	34	
25	e	12	
26	UP	27	
27	UP	27	
28	2	02	
29	XEY	30	
2a	DIV	35	
2b	XFR	67	
2c	-	34	
2d	f	15	
30	XEY	30	
31	+	33	
32	YTO	40	
33	-	34	
34	f	15	
35	DIV	35	
36	0	00	
37	STP	41	ENTRY
38	PNT	45	
39	PNT	45	
3a	RUP	22	
3b	DIV	35	
3c	2	02	
3d	X	36	

40	1	01
41	RUP	22
42	YTO	40
43	0	00
44	XFR	67
45	-	34
46	f	15
47	X	36
48	π	56
49	X	36
4a	DN	25
4b	SIN	70
4c	X	36
4d	f	15
50	+	33
51	YE	24
52	1	01
53	YE	24
54	2	02
55	YE	24
56	3	03
57	YE	24
58	4	04
59	YE	24
5a	5	05
5b	YE	24
5c	6	06
5d	YE	24
60	7	07
61	YE	24
62	8	10
63	YE	24
64	9	11
65	YE	24
66	a	13
67	YE	24
68	b	14
69	YE	24
6a	c	16
6b	YE	24
6c	d	17
6d	YE	24
70	e	12
71	YE	24
72	f	15
73	XFR	67
74	0	00
75	XEY	30
76	1	01
77	+	33
78	1	01
79	6	06
7a	X>Y	53
7b	4	04
7c	2	02
7d	XFR	67

80	-	34
81	f	15
82	XEY	30
83	2	02
84	DIV	35
85	XFR	67
86	-	34
87	e	12
88	X	36
89	X>Y	53
8a	2	02
8b	6	06
8c	XFR	67
8d	-	34
90	d	17
91	UP	27
92	UP	27
93	PNT	45
94	PNT	45
95	RCL	61
96	XEY	30
97	UP	27
98	d	17
99	PNT	45
9a	c	16
9b	UP	27
9c	b	14
9d	UP	27
a0	a	13
a1	PNT	45
a2	XFR	67
a3	9	11
a4	UP	27
a5	XFR	67
a6	8	10
a7	UP	27
a8	XFR	67
a9	7	07
aa	PNT	45
ab	XFR	67
ac	6	06
ad	UP	27

q	B_8
B_{15}	B_7
B_{14}	B_6
B_{13}	B_5
B_{12}	B_4
B_{11}	B_3
B_{10}	B_2
B_9	B_1

Minus Page

```
b0  XFR  67
b1  5    05
b2  UP   27
b3  XFR  67
b4  4    04
b5  PNT  45
b6  XFR  67
b7  3    03
b8  UP   27
b9  XFR  67
ba  2    02
bb  UP   27
bc  XFR  67
bd  1    01

c0  PNT  45
c1  PNT  45
c2  XFR  67
c3  -    34
c4  d    17
c5  UP   27
c6  UP   27
c7  PNT  45
c8  PNT  45
c9  PNT  45
ca  PNT  45
cb  GTO  44
cc  0    00
cd  0    00

d0  CHS  32
d1  CHS  32
d2  CHS  32
d3  CHS  32
d4  CHS  32
d5  CHS  32
d6  CHS  32
d7  CHS  32
d8  CHS  32
d9  CHS  32
da  YTO  40
db  YTO  40
dc  YTO  40
dd  YTO  40
```

N

(O) K $\cdot \frac{2}{N}$

SECTION VI SAMPLING THEORY

SAMPLE SIZE n FOR CONTINUOUS DATA

This program calculates the necessary sample size (n) required to achieve a desired precision (d), assuming a small risk α that the precision may not be reached. The equations required are:

$$n_o = \frac{t^2 s^2}{d^2}$$

$$n = \frac{n_o}{1 + \frac{n_o}{N}}$$

where

t = t statistic associated with α ($\alpha = .05$, $t = 1.96$)
d = Chosen margin of error
s^2 = Sample variance
N = Population size in sampling units

The quantity n represents the sample size required to ensure:

$$P_r (|\bar{y} - \bar{Y}| \geq d) = \alpha$$

where

$\bar{y}$ = Sample unit mean

$\bar{Y}$ = Population unit mean

Reference: Sampling Techniques, Cochran, Wiley, 1965, page 75.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
1 ——— X

ENTER DATA:

d → Z
s^2 → Y
t → X

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
2 ——— X

ENTER DATA: N → X

PRESS: CONTINUE

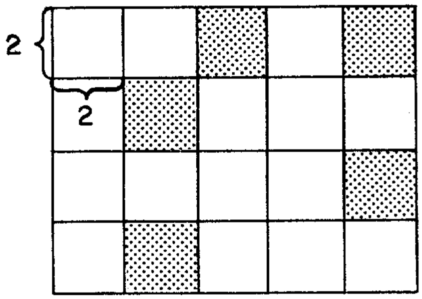
DISPLAY

n ——— Z
N ——— Y
0 ——— X

To run another case

EXAMPLE

Ottumwa National Forest



In the spring of 1969, a complete deer population head count was made of the Ottumwa National Forest. This forest is 8 miles by 10 miles. The sampling unit was a square with 2 mile sides. The population unit mean was 40 deer. ($\overline{Y} = 40$). The population unit variance was determined to be 25. ($s^2 = 25$)

In order to determine the deer population following the 1969 fall harvesting, how many units (2 mile squares) must be sampled in spring 1970 within a unit margin of error of 10% $\overline{Y}$? (d = 4)

t = 1.96
d = 4
s^2 = 25
N = 4 x 5 = 20

Results:

n = 4.6

Thus 5 squares must be sampled to obtain a new deer population.

STAT-PAC VI-1

Step	Key	Code
00	CLR	20
01	1	01
02	STP	41
03	PNT	45
04	PNT	45
05	X	36
06	X	36
07	XEY	30
08	RUP	22
09	UP	27
0a	X	36
0b	DN	25
0c	DIV	35
0d	YTO	40
10	d	17
11	CLR	20
12	2	02
13	STP	41
14	XTO	23
15	e	12
16	YEX	24
17	d	17
18	UP	27
19	DN	25
1a	DIV	35
1b	1	01
1c	+	33
1d	DN	25
20	DIV	35
21	UP	27
22	RCL	61
23	PNT	45
24	PNT	45
25	END	46

ENTRY

n_0

N

O

SAMPLE SIZE (n) IN SAMPLING FOR PROPORTIONS

This program calculates the necessary sample size (n) to achieve a desired precision d, assuming a small risk α that the desired precision may not be reached. $[P_r\ (|p - P| \geq d) = \alpha\ ]$

The equations used are:

$$n_o = \frac{t^2(p - p^2)}{d^2} = \frac{t^2\ pq}{d^2}$$

$$n = \frac{n_o}{1 + \frac{n_o}{N}}$$

where,

t = t statistic associated with α (α = .05, t = 1.96)
d = Desired precision
p = Sample proportion
N = Population size
q = $1 - p$
P = Population proportion

Reference: Sampling Techniques, Cochran, Wiley, 1965, page 75.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

→ PRESS: CONTINUE

DISPLAY

0 — Z
0 — Y
1 — X

ENTER DATA:

d → Z
p → Y
t → X

PRESS: CONTINUE

DISPLAY

0 — Z
0 — Y
2 — X

ENTER DATA: N → X

PRESS: CONTINUE

DISPLAY

n — Z
N — Y
0 — X

To run another case

EXAMPLE

Last year a small sample of TV watching habits was taken in Livingston, Montana. This sample indicated that 70% of the Livingston TV sets were used for more than 4 hours per day.

How large a sample is necessary to accurately determine what proportion of TV owners are using their sets over 4 hours per day?

N = (total number of sets)= 1500
p = .7 (estimated proportion)
d = 0.1 (desired precision)
α = .05 (t = 1.96)
$\boxed{n = 77.}$

STAT-PAC VI-2

Step	Key	Code	X	Y	Z
00	CLR	20			
01	1	01			
02	STP	41	ENTRY		
03	PNT	45			
04	PNT	45			
05	XTO	23			
06	f	15			
07	DN	25			
08	UP	27			
09	X	36			
0a	XEY	30			
0b	-	34			
0c	DN	25			
0d	RDN	31			
10	X	36			
11	DN	25			
12	DIV	35			
13	f	15			
14	UP	27			
15	X	36			
16	DN	25			
17	X	36			
18	YTO	40			
19	d	17			
1a	CLR	20			
1b	2	02			
1c	STP	41	ENTRY		
1d	XTO	23			
20	e	12			
21	YEX	24			
22	d	17			
23	UP	27			
24	DN	25			
25	DIV	35			
26	1	01			
27	+	33			
28	DN	25			
29	DIV	35			
2a	e	12			
2b	UP	27			
2c	0	00			
2d	PNT	45			
30	PNT	45			
31	END	46			

n_0

N

t

NEYMAN ALLOCATION IN STRATIFIED RANDOM SAMPLING

STAT-PAC
VI-3

This program calculates the optimum allocation of a fixed number of samples between the N strata.

The equation used

$n_h = n \frac{N_h s_h}{\Sigma N_h s_h}$

n = Number of Items to be Allocated

N_h = The Number of Items in Strata h

s_h = Standard Deviation of Strata h

n_h = Number of Items Allocated in Strata h

N = Number of Strata (N ≤ 6)

Reference: Sampling Techniques by Cochran, Wiley, 1965.

USER INSTRUCTIONS

SET: ☐ FIXED POINT

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM A

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA:

N ——► Y
n ——► X

PRESS: CONTINUE

DISPLAY

0 ——— Z
h ——— Y
0 ——— X

ENTER DATA:

s_h ——► Y
N_h ——► X

PRESS: CONTINUE

DISPLAY

1 ——— Z
1 ——— Y
1 ——— X

(When this display appears, all data has been entered.)

PRESS: END

ENTER PROGRAM B

USER INSTRUCTIONS (Con't)

PRESS: CONTINUE

DISPLAY

h ——— Z
h ——— Y
n_h ——— X

To run a new case, return to the beginning of the USER INSTRUCTIONS.

EXAMPLE

See next page.

EXAMPLE

The following table shows the stratification of all farms in Milliken County, and the average sugar beet tonnage/acre per farm in each stratum for the year 1968.

Stratum (h)	Farm Size (acres)	Average Tonnage/Acre	Standard Deviation s_h	Number for farms N_h
1	0 - 50	9	4.1	21
2	50 - 100	13	6.3	62
3	100 - 150	16	8.2	104
4	150 - 200	17.5	10.1	181
5	200 - 250	20	9.0	75
6	250 ——	19.6	5.8	14

Great Mountain Sugar would like to determine if its 1969 "Increased Tonnage Incentives" plan has been effective by taking samples of 100 Milliken County farms. How should they distribute the 100 samples?

$N = 6$

$n = 100$

N_h	s_h
$N_1 = 21,$	$s_1 = 4.1$
$N_2 = 62,$	$s_2 = 6.3$
$N_3 = 104,$	$s_3 = 8.2$
$N_4 = 181,$	$s_4 = 10.1$
$N_5 = 75,$	$s_5 = 9.0$
$N_6 = 14,$	$s_6 = 5.8$

Results:

$n_1 = 2$

$n_2 = 10$

$n_3 = 22$

$n_4 = 47$

$n_5 = 17$

$n_6 = 2$

STAT-PAC VI-3 Program A

Step	Key	Code	X	Y	Z
00	CLR	20			
01	STP	41		ENTRY	
02	PNT	45			
03	PNT	45			
04	AC+	60			
05	DN	25			
06	1	01			
07	XEY	30			
08	STP	41		ENTRY	
09	PNT	45			
0a	PNT	45			
0b	X	36			
0c	YTO	40			
0d	d	17			
10	0	00			
11	RDN	31			
12	DN	25			
13	2	02			
14	XEY	30			
15	STP	41		ENTRY	
16	PNT	45			
17	PNT	45			
18	X	36			
19	YTO	40			
1a	c	16			
1b	0	00			
1c	RDN	31			
1d	DN	25			
20	3	03			
21	XEY	30			
22	e	12			
23	X<Y	52			
24	6	06			
25	2	02			
26	0	00			
27	STP	41		ENTRY	
28	PNT	45			
29	PNT	45			
2a	X	36			
2b	YTO	40			
2c	b	14			
2d	0	00			
30	RDN	31			
31	4	04			
32	XEY	30			
33	e	12			
34	X<Y	52			
35	6	06			
36	2	02			
37	0	00			
38	STP	41		ENTRY	
39	PNT	45			
3a	PNT	45			
3b	X	36			
3c	YTO	40			
3d	a	13			
40	0	00			
41	RDN	31			
42	5	05			
43	XEY	30			
44	e	12			
45	X<Y	52			
46	6	06			
47	2	02			
48	0	00			
49	STP	41		ENTRY	
4a	PNT	45			
4b	PNT	45			
4c	X	36			
4d	YTO	40			
50	9	11			
51	0	00			
52	RDN	31			
53	6	06			
54	XEY	30			
55	e	12			
56	X<Y	52			
57	6	06			
58	2	02			
59	0	00			
5a	STP	41		ENTRY	
5b	PNT	45			
5c	PNT	45			
5d	X	36			
60	YTO	40			
61	8	10			
62	YEX	24			
63	8	10			
64	YTO	40			
65	8	10			
66	d	17			
67	+	33			
68	DN	25			
69	YEX	24			
6a	9	11			
6b	YTO	40			
6c	9	11			
6d	+	33			
70	c	16			
71	+	33			
72	b	14			
73	+	33			
74	a	13			
75	+	33			
76	f	15			
77	DIV	35			
78	YTO	40			
79	f	15			
7a	1	01			
7b	UP	27			
7c	UP	27			
7d	STP	41			
80	0	00			
81	0	00			
82	0	00			
83	0	00			
84	0	00			
85	0	00			
86	0	00			
87	0	00			
88	0	00			
89	0	00			
8a	0	00			
8b	0	00			
8c	0	00			
8d	0	00			
90	0	00			
91	0	00			
92	0	00			
93	0	00			
94	0	00			
95	0	00			
96	0	00			
97	0	00			
98	0	00			
99	0	00			
9a	0	00			
9b	0	00			
9c	0	00			
9d	0	00			
a0	0	00			
a1	0	00			
a2	0	00			
a3	0	00			
a4	0	00			
a5	0	00			
a6	0	00			
a7	0	00			
a8	0	00			
a9	0	00			
aa	0	00			
ab	0	00			
ac	0	00			
ad	0	00			

Storage

0		8	N_6S_6
1		9	N_5S_5
2		a	N_4S_4
3		b	N_3S_3
4		c	N_2S_2
5		d	N_1S_1
6		e	N
7		f	$n / \frac{\Sigma}{n}$

STAT-PAC VI-3 Program A

```
b0   0    00                   10  DIV  35          50  PNT  45   s
b1   0    00                   11  DN   25          51  PNT  45
b2   0    00                   12  PNT  45   s      52  PNT  45
b3   0    00                   13  PNT  45          53  PNT  45
b4   0    00                   14  e    12          54  PNT  45
b5   0    00                   15  X=Y  50          55  PNT  45
b6   0    00                   16  5    05          56  CLR  20
b7   0    00                   17  6    06          57  END  46
b8   0    00                   18  3    03
b9   0    00                   19  UP   27
ba   0    00                   1a  b    14
bb   0    00                   1b  UP   27
bc   0    00                   1c  f    15
bd   0    00                   1d  DIV  35

c0   0    00                   20  DN   25
c1   0    00                   21  PNT  45   s
c2   0    00                   22  PNT  45
c3   0    00                   23  e    12
c4   0    00                   24  X=Y  50
c5   0    00                   25  5    05
c6   0    00                   26  6    06
c7   0    00                   27  4    04
c8   0    00                   28  UP   27
c9   0    00                   29  a    13
ca   0    00                   2a  UP   27
cb   0    00                   2b  f    15
cc   0    00                   2c  DIV  35
cd   0    00                   2d  DN   25

d0   0    00                   30  PNT  45   s
d1   0    00                   31  PNT  45
d2   0    00                   32  e    12
d3   0    00                   33  X=Y  50
d4   0    00                   34  5    05
d5   0    00                   35  6    06
d6   0    00                   36  5    05
d7   0    00                   37  YEX  24
d8   0    00                   38  9    11
d9   0    00                   39  XEY  30
da   0    00                   3a  UP   27
db   0    00                   3b  f    15
dc   0    00                   3c  DIV  35
dd   END  46                   3d  DN   25

00   1    01   Program B       40  PNT  45   s
01   UP   27                   41  PNT  45
02   d    17                   42  e    12
03   UP   27                   43  X=Y  50
04   f    15                   44  5    05
05   DIV  35                   45  6    06
06   DN   25                   46  6    06
07   PNT  45   s               47  YEX  24
08   PNT  45                   48  8    10
09   2    02                   49  XEY  30
0a   UP   27                   4a  UP   27
0b   c    16                   4b  f    15
0c   UP   27                   4c  DIV  35
0d   f    15                   4d  DN   25
```

SECTION VII ANALYSIS OF VARIANCE

ONE WAY ANALYSIS OF VARIANCE m x n

This program separates the total variance in a table of data into a portion due to chance and a portion due to differences between and population means underlying each column of sample data. It then calculates the variance ratio.

$$F = \frac{nm(m - 1) \sum_{j=1}^{n} (X_j - \bar{X})}{(n - 1) \sum_{i=1}^{m} \sum_{j=1}^{n} (X_{ij} - \bar{X}_j)}$$

with

$\nu_1 = n - 1$ degrees of freedom

$\nu_2 = n\,(m - 1)$ degrees of freedom

where

$$\bar{X} = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} X_{ij}$$

$$\bar{X}_j = \frac{1}{n} \sum_{j=1}^{n} X_{ij}$$

The equation used by the program is:

$$F = \frac{nm(m - 1) \left\{ \sum_{j=1}^{n} \left[\sum_{i=1}^{m} X_{ij} \right]^2 - \frac{1}{mn} \left[\sum_{j=1}^{n} \sum_{i=1}^{m} X_{ij} \right]^2 \right\}}{(n - 1) \left\{ \sum_{j=1}^{n} \sum_{i=1}^{m} X^2_{ij} - \frac{1}{mn} \left[\sum_{j=1}^{n} \sum_{i=1}^{m} X_{ij} \right]^2 - \frac{1}{m} \sum_{j=1}^{n} \left[\sum_{i=1}^{m} X_{ij} \right]^2 + \frac{1}{mn} \left[\sum_{j=1}^{n} \sum_{i=1}^{m} X_{ij} \right]^2 \right\}}$$

Reference: Freund, John E., Mathematical Statistics, Prentice Hall (1962)

USER INSTRUCTIONS

DEPRESS: X on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

→ DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA: n Columns → Y, m Rows → X

PRESS: CONTINUE

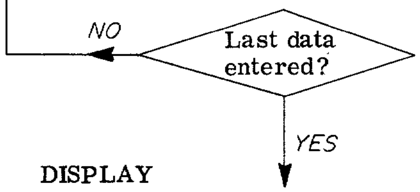
→ DISPLAY

0 ——— Z
j ——— Y jth Column
i ——— X ith Row

Enter data Column by Column

ENTER DATA: $X_{ij} \longrightarrow X$

PRESS: CONTINUE



DISPLAY

F ——— Z F=variance ratio
ν_1 ——— Y ν_1=degrees of freedom in numerator
ν_2 ——— X ν_2=degrees of freedom in denominator

← PRESS: CONTINUE for new case

EXAMPLE

General form

$$\begin{array}{c|cccccc} & \multicolumn{6}{c}{j \text{ Columns}} \\ & 1 & 2 & . & . & . & n \\ \hline 1 & X_{11} & & & & & X_{1n} \\ 2 & & & & & & \\ . & & & & & & \\ i \text{ Rows} & & & X_{ij} & & & \\ . & & & & & & \\ . & & & & & & \\ m & X_{m1} & & & & & X_{mn} \end{array}$$

Columns

Rows	172	203	161
	185	172	149
	165	187	183
	194	183	156
	212	179	144

F = 5.01
ν_1 = 2
ν_2 = 12

STAT-PAC VII-1

```
00  CLR  20
01  XTO  23
02  9    11
03  XTO  23
04  b    14
05  STP  41   ENTRY
06  XEY  30
07  PNT  45
08  XEY  30
09  PNT  45
0a  PNT  45
0b  YTO  40
0c  c    16
0d  UP   27

10  1    01
11  +    33
12  YTO  40
13  d    17
14  XEY  30
15  CLX  37
16  RDN  31
17  XTO  23
18  a    13
19  STP  41   ENTRY
1a  PNT  45
1b  PNT  45
1c  YEX  24
1d  b    14

20  +    33
21  YEX  24
22  b    14
23  UP   27
24  X    36
25  AC+  60
26  DN   25
27  a    13
28  UP   27
29  1    01
2a  +    33
2b  d    17
2c  X>Y  53
2d  1    01

30  5    05
31  DN   25
32  b    14
33  UP   27
34  X    36
35  DN   25
36  YEX  24
37  9    11
38  +    33
39  YEX  24
3a  9    11
3b  CLX  37
3c  RDN  31
3d  1    01

40  -    34
41  XTO  23
42  a    13
43  CLX  37
44  XTO  23
45  b    14
46  X<Y  52
47  1    01
48  8    10
49  d    17
4a  UP   27
4b  1    01
4c  -    34
4d  YTO  40

50  d    17
51  RCL  61
52  UP   27
53  X    36
54  c    16
55  DIV  35
56  d    17
57  DIV  35
58  DN   25
59  -    34
5a  YTO  40
5b  e    12
5c  RDN  31
5d  YEX  24

60  9    11
61  d    17
62  DIV  35
63  DN   25
64  XEY  30
65  -    34
66  e    12
67  XEY  30
68  -    34
69  XEY  30
6a  DIV  35
6b  c    16
6c  X    36
6d  UP   27

70  1    01
71  -    34
72  YTO  40
73  f    15
74  DN   25
75  DIV  35
76  d    17
77  UP   27
78  1    01
79  -    34
7a  DN   25
7b  X    36
7c  UP   27
7d  c    16

80  X    36
81  f    15
82  XEY  30
83  CNT  47
84  RUP  22
85  PNT  45
86  RUP  22
87  PNT  45
88  RUP  22
89  PNT  45
8a  PNT  45
8b  PNT  45
8c  PNT  45
8d  END  46
```

TWO WAY ANALYSIS OF VARIANCE (m x 4)

This program analyzes the total statistical variance in a table of data by separating the total variance into two parts, the variance among rows of data, and the variance between columns of data, and comparing each to the variance due to random influence. In a table of four columns and m rows it calculates the variance ratio between columns.

$$F_c = \frac{m \sum_{j=1}^{4} (\bar{X}_j - \bar{X})^2 \Big/ 3}{\sum_{j=1}^{4} \sum_{i=1}^{m} (X_{ij} - \bar{X}_j - \bar{X}_i + \bar{X})^2 \Big/ (m-1)(3)}$$

with $V_1 = 3$ degrees of freedom

and $V_2 = 3(m-1)$ degrees of freedom

and the variance ratio between rows:

$$F_r = \frac{4 \sum_{i=1}^{m} (\bar{X}_i - \bar{X})^2 \Big/ (m-1)}{\sum_{j=1}^{4} \sum_{i=1}^{m} (X_{ij} - \bar{X}_j - \bar{X}_i + \bar{X})^2 \Big/ 3(m-1)}$$

with $V_1 = m - 1$ degrees of freedom
$V_2 = 3(m-1)$ degrees of freedom

where:

$$\bar{X}_j = \frac{1}{m} \sum_{i=1}^{m} X_{ij} \qquad \bar{X}_i = \frac{1}{4} \sum_{j=1}^{4} X_{ij}$$

$$\bar{X} = \frac{1}{4m} \sum_{j=1}^{4} \sum_{i=1}^{m} X_{ij}$$

V_1 = degrees of freedom in numerator
V_2 = degrees of freedom in denominator

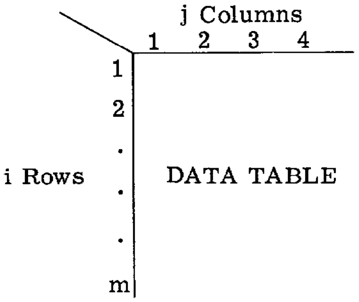
The equations used by the program are:

$$F_r = \frac{4(m-1)\left\{\frac{1}{4} \sum_{i=1}^{m} \left[\sum_{j=1}^{4} X_{ij}\right]^2 - \frac{1}{4m}\left[\sum_{j=1}^{4} \sum_{i=1}^{m} X_{ij}\right]^2\right\}}{3\left\{\sum_{j=1}^{4} \sum_{i=1}^{m} X^2_{ij} - \frac{1}{4m}\left[\sum_{j=1}^{4} \sum_{i=1}^{m} X_{ij}\right]^2 - \frac{1}{m} \sum_{j=1}^{4}\left[\sum_{i=1}^{m} X_{ij}\right]^2 - \frac{1}{4} \sum_{i=1}^{m}\left[\sum_{j=1}^{4} X_{ij}\right]^2 + \frac{1}{2m}\left[\sum_{j=1}^{4} \sum_{i=1}^{m} X_{ij}\right]^2\right\}}$$

$$F_c = \frac{3m\left\{\frac{1}{m} \sum_{j=1}^{4} \left[\sum_{i=1}^{m} X_{ij}\right]^2 - \frac{1}{4m}\left[\sum_{j=1}^{4} \sum_{i=1}^{m} X_{ij}\right]^2\right\}}{(m-1)\left\{\sum_{j=1}^{4} \sum_{i=1}^{m} X_{ij}^2 - \frac{1}{4m}\left[\sum_{j=1}^{4} \sum_{i=1}^{m} X_{ij}\right]^2 - \frac{1}{m} \sum_{j=1}^{4}\left[\sum_{i=1}^{m} X_{ij}\right]^2 - \frac{1}{4} \sum_{i=1}^{m}\left[\sum_{j=1}^{4} X_{ij}\right]^2 + \frac{1}{2m}\left[\sum_{j=1}^{4} \sum_{i=1}^{m} X_{ij}\right]^2\right\}}$$

Reference: Freund, John E., Mathematical Statistics, Prentice Hall (1962)

USER INSTRUCTIONS



DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: END

PRESS: CONTINUE

DISPLAY

0 —— Z
0 —— Y
0 —— X

ENTER DATA: m → X number of rows in data table

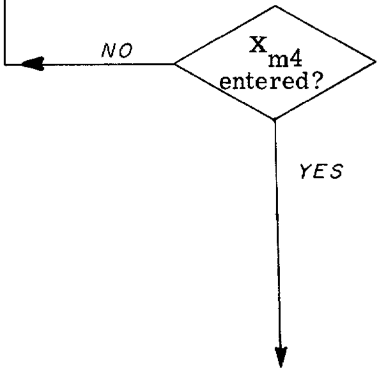
PRESS: CONTINUE

DISPLAY

0 —— Z
i —— Y
j —— X

ENTER DATA: X_{ij} → X j = column
i = row
(data entered row by row)

PRESS: CONTINUE



USER INSTRUCTIONS (Con't)

DISPLAY

F_c —— Z F ratio between columns
V_1 —— Y V_1 degrees of freedom in numerator
V_2 —— X V_2 degrees of freedom in denominator

PRESS: CONTINUE

DISPLAY

F_r —— Z F ratio between rows
V_1 —— Y V_1 degrees of freedom in numerator
V_2 —— X V_2 degrees of freedom in denominator

To re-run program:

PRESS: END

EXAMPLE

General form

$$\begin{array}{c|cccc} & \multicolumn{4}{c}{j \text{ Columns}} \\ & 1 & 2 & 3 & 4 \\ \hline 1 & X_{11} & \cdots & & X_{14} \\ 2 & \cdot & & & \\ \vdots & \cdot & X_{ij} & & \\ m & X_{m1} & \cdots & & X_{m4} \end{array}$$

i Rows

	Columns			
Rows	58.2	49.1	60.1	75.8
	56.2	54.1	70.9	58.2
	65.3	51.6	39.2	48.7

$F_c = 0.43$ $V_1 = 3$ $V_2 = 6$

$F_r = 0.92$ $V_1 = 2$ $V_2 = 6$

```
00  CLR  20   Plus
01  XTO  23   Page
02  a    13
03  XTO  23
04  b    14
05  XTO  23
06  c    16
07  XTO  23
08  9    11
09  XTO  23
0a  8    10
0b  STP  41   ENTRY
0c  RUP  22
0d  YTO  40

10  d    17
11  1    01
12  STP  41   ENTRY
13  PNT  45
14  XTO  23
15  7    07
16  YE   24
17  a    13
18  +    33
19  YE   24
1a  a    13
1b  UP   27
1c  X    36
1d  AC+  60

20  CLX  37
21  RDN  31
22  2    02
23  STP  41   ENTRY
24  PNT  45
25  YE   24
26  7    07
27  +    33
28  YE   24
29  7    07
2a  YE   24
2b  b    14
2c  +    33
2d  YE   24

30  b    14
31  UP   27
32  X    36
33  AC+  60
34  CLX  37
35  RDN  31
36  3    03
37  STP  41   ENTRY
38  PNT  45
39  YE   24
3a  7    07
3b  +    33
3c  YE   24
3d  7    07

40  YE   24
41  c    16
42  +    33
43  YE   24
44  c    16
45  UP   27
46  X    36
47  AC+  60
48  CLX  37
49  RDN  31
4a  4    04
4b  STP  41   ENTRY
4c  PNT  45
4d  PNT  45

50  YE   24
51  7    07
52  +    33
53  YE   24
54  7    07
55  YE   24
56  8    10
57  +    33
58  YE   24
59  8    10
5a  UP   27
5b  X    36
5c  AC+  60
5d  YE   24

60  7    07
61  DN   25
62  YE   24
63  9    11
64  UP   27
65  X    36
66  CLX  37
67  RDN  31
68  +    33
69  GTO  44
6a  -    34
6b  0    00
6c  0    00
6d  CNT  47

70  CNT  47
71  CNT  47
72  CNT  47
73  CNT  47
74  CNT  47
75  CNT  47
76  CNT  47
77  CNT  47
78  CNT  47
79  CNT  47
7a  CNT  47
7b  CNT  47
7c  CNT  47
7d  CNT  47

80  CNT  47
81  CNT  47
82  CNT  47
83  CNT  47
84  CNT  47
85  CNT  47
86  CNT  47
87  CNT  47
88  CNT  47
89  CNT  47
8a  CNT  47
8b  CNT  47
8c  CNT  47
8d  CNT  47

90  CNT  47
91  CNT  47
92  CNT  47
93  CNT  47
94  CNT  47
95  CNT  47
96  CNT  47
97  CNT  47
98  CNT  47
99  CNT  47
9a  CNT  47
9b  CNT  47
9c  CNT  47
9d  CNT  47

a0  CNT  47
a1  CNT  47
a2  CNT  47
a3  CNT  47
a4  CNT  47
a5  CNT  47
a6  CNT  47
a7  CNT  47
a8  CNT  47
a9  CNT  47
aa  CNT  47
ab  CNT  47
ac  CNT  47
ad  CNT  47
```

ΣX_{ij}

M

ΣX_{ij}

b0	CNT	47	Plus Page
b1	CNT	47	
b2	CNT	47	
b3	CNT	47	
b4	CNT	47	
b5	CNT	47	
b6	CNT	47	
b7	CNT	47	
b8	CNT	47	
b9	CNT	47	
ba	CNT	47	
bb	CNT	47	
bc	CNT	47	
bd	CNT	47	
c0	CNT	47	
c1	CNT	47	
c2	CNT	47	
c3	CNT	47	
c4	CNT	47	
c5	CNT	47	
c6	CNT	47	
c7	CNT	47	
c8	CNT	47	
c9	CNT	47	
ca	CNT	47	
cb	CNT	47	
cc	CNT	47	
cd	CNT	47	
d0	CNT	47	
d1	CNT	47	
d2	CNT	47	
d3	CNT	47	
d4	CNT	47	
d5	CNT	47	
d6	CNT	47	
d7	CNT	47	
d8	CNT	47	
d9	CNT	47	
da	CNT	47	
db	CNT	47	
dc	CNT	47	
dd	CNT	47	

00	YE	24	Minus Page
01	9	11	
02	1	01	
03	-	34	
04	CLX	37	
05	X<Y	52	
06	6	06	
07	8	10	
08	RCL	61	
09	AC-	63	
0a	UP	27	
0b	X	36	
0c	4	04	
0d	DIV	35	
10	d	17	
11	DIV	35	
12	DN	25	
13	-	34	
14	AC+	60	
15	a	13	
16	UP	27	
17	X	36	
18	b	14	
19	UP	27	
1a	X	36	
1b	DN	25	
1c	+	33	
1d	c	16	
20	UP	27	
21	X	36	
22	DN	25	
23	+	33	
24	UP	27	
25	YE	24	
26	8	10	
27	DN	25	
28	UP	27	
29	X	36	
2a	DN	25	
2b	+	33	
2c	d	17	
2d	DIV	35	
30	f	15	
31	-	34	
32	UP	27	
33	YE	24	
34	9	11	
35	4	04	
36	DIV	35	
37	f	15	
38	-	34	
39	YTO	40	
3a	a	13	
3b	e	12	
3c	RUP	22	
3d	-	34	
40	XEY	30	
41	RUP	22	
42	-	34	
43	YTO	40	
44	b	14	
45	DN	25	
46	DIV	35	
47	d	17	
48	UP	27	
49	1	01	
4a	-	34	
4b	DN	25	
4c	XTO	23	
4d	d	17	
50	X	36	
51	UP	27	
52	3	03	
53	X	36	
54	XEY	30	
55	PNT	45	s
56	PNT	45	
57	a	13	
58	UP	27	
59	b	14	
5a	DIV	35	
5b	3	03	
5c	X	36	
5d	UP	27	
60	d	17	
61	X	36	
62	XEY	30	
63	PNT	45	s
64	PNT	45	
65	GTO	44	
66	6	06	
67	c	16	
68	GTO	44	
69	+	33	
6a	1	01	
6b	1	01	
6c	END	46	

10

TWO WAY ANALYSIS OF VARIANCE WITH REPLICATES
(THREE WAY)

This program analyzes the total statistical variance in a table of data by separating the total variance into three parts, the variance among rows, the variance between columns, and the variance due to interactions.

Computational Equations

$$SST = \sum_{i}^{k} \sum_{j}^{n} \sum_{r}^{m} X_{ijr}^{2} - \frac{1}{knm} \left[\sum_{i}^{k} \sum_{j}^{n} \sum_{r}^{m} X_{ijr} \right]^{2}$$

$$SSA = \frac{1}{nm} \sum_{i}^{k} \left[\sum_{j}^{n} \sum_{r}^{m} X_{ijr} \right]^{2} - \frac{1}{knm} \left[\sum_{i}^{k} \sum_{j}^{n} \sum_{r}^{m} X_{ijr} \right]^{2}$$

$$SSB = \frac{1}{km} \sum_{j}^{n} \left[\sum_{r}^{m} \sum_{i}^{k} X_{ijr} \right]^{2} - \frac{1}{knm} \left[\sum_{i}^{k} \sum_{j}^{n} \sum_{r}^{m} X_{ijr} \right]^{2}$$

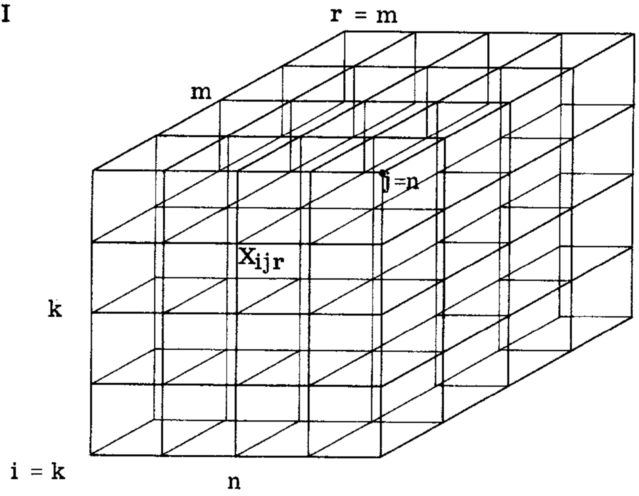
$$SSI = \frac{1}{m} \sum_{i}^{k} \sum_{j}^{n} \left[\sum_{r}^{m} X_{ijr} \right]^{2} - \frac{1}{nm} \sum_{i}^{k} \left[\sum_{j}^{n} \sum_{r}^{m} X_{ijr} \right]^{2} - \frac{1}{km} \sum_{j}^{n} \left[\sum_{i}^{k} \sum_{r}^{m} X_{ijr} \right]^{2} + \frac{1}{kmn} \left[\sum_{i}^{k} \sum_{j}^{n} \sum_{r}^{m} X_{ijr} \right]^{2}$$

$$F_R = \frac{SSA}{(k-1)} \quad \frac{kn(m-1)}{SSE}$$

$$F_C = \frac{SSB}{(n-1)} \quad \frac{kn(m-1)}{SSE}$$

$$F_I = \frac{SSI}{(k-1)(n-1)} \quad \frac{kn\,(m-1)}{SSE}$$

$$SSE = SST - SSA - SSB - SSI$$



(n ≤ 8)

Reference: Mathematical Statistics, John E. Freund, Prentice Hall, 1963

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A
PRESS: END

ENTER PROGRAM 1: Side A followed by Side B

PRESS: END

PRESS: CONTINUE

DISPLAY

0 ———— Z
0 ———— Y
0 ———— X

ENTER DATA: rows $k \rightarrow Z$, columns $n \rightarrow Y$, repetitions $m \rightarrow X$

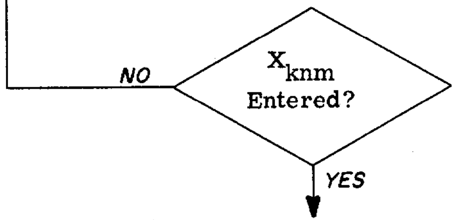
PRESS: CONTINUE

DISPLAY

i ———— Z row
j ———— Y column
r ———— X repetition

ENTER DATA: $X_{ijr} \rightarrow X$

PRESS: CONTINUE



DISPLAY

I ———— Z
I ———— Y
I ———— X

Intermediate results, DO NOT ALTER.

PRESS: GO TO (-)(0)(0)

ENTER PROGRAM 2:

PRESS: GO TO (-)(0)(0)

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

F_{row} ———— Z
V_1 ———— Y
V_2 ———— X

PRESS: CONTINUE

DISPLAY

F_{column} ———— Z
V_1 ———— Y
V_2 ———— X

PRESS: CONTINUE

DISPLAY

$F_{interaction}$ — Z
V_1 ———— Y
V_2 ———— X

EXAMPLE

n = 3 columns

k = 4 rows

58. 2	56. 2	65. 3
52. 6	41. 2	60. 8
49. 1	54. 1	51. 6
42. 8	50. 5	48. 4
60. 1	70. 9	39. 2
58. 3	73. 2	40. 7
75. 8	58. 2	48. 7
71. 5	51. 0	41. 4

m = 2 repetition

Results:

F_{row} = 4. 42
V_1 = 3
V_2 = 12

F_{column} = 9. 39
V_1 = 2
V_2 = 12

$F_{interaction}$ = 14. 93
V_1 = 6
V_2 = 12

Step	Key	Code	X	Y	Z	Step	Key	Code	X	Y	Z	Step	Key	Code	X	Y	Z
00	CLR	20	Plus		Page	40	CNT	47				80	CNT	47			
01	STP	41		ENTRY		41	CNT	47				81	CNT	47			
02	PNT	45				42	CNT	47				82	CNT	47			
03	PNT	45				43	CNT	47				83	CNT	47			
04	XTO	23				44	CNT	47				84	CNT	47			
05	d	17				45	CNT	47				85	CNT	47			
06	YTO	40				46	CNT	47				86	CNT	47			
07	c	16				47	CNT	47				87	CNT	47			
08	DN	25				48	CNT	47				88	CNT	47			
09	YTO	40				49	CNT	47				89	CNT	47			
0a	b	14				4a	CNT	47				8a	CNT	47			
0b	CLR	20				4b	CNT	47				8b	CNT	47			
0c	XTO	23				4c	CNT	47				8c	CNT	47			
0d	a	13				4d	CNT	47				8d	CNT	47			
10	XTO	23				50	CNT	47				90	CNT	47			
11	9	11				51	CNT	47				91	CNT	47			
12	XTO	23				52	CNT	47				92	CNT	47			
13	8	10				53	CNT	47				93	CNT	47			
14	XTO	23				54	CNT	47				94	CNT	47			
15	7	07				55	CNT	47				95	CNT	47			
16	XTO	23				56	CNT	47				96	CNT	47			
17	6	06				57	CNT	47				97	CNT	47			
18	XTO	23				58	CNT	47				98	CNT	47			
19	5	05				59	CNT	47				99	CNT	47			
1a	XTO	23				5a	CNT	47				9a	CNT	47			
1b	4	04				5b	CNT	47				9b	CNT	47			
1c	XTO	23				5c	CNT	47				9c	CNT	47			
1d	3	03				5d	CNT	47				9d	CNT	47			
20	XTO	23				60	CNT	47				a0	CNT	47			
21	-	34				61	CNT	47				a1	CNT	47			
22	f	15				62	CNT	47				a2	CNT	47			
23	XTO	23				63	CNT	47				a3	CNT	47			
24	-	34				64	CNT	47				a4	CNT	47			
25	e	12				65	CNT	47				a5	CNT	47			
26	XTO	23				66	CNT	47				a6	CNT	47			
27	1	01				67	CNT	47				a7	CNT	47			
28	XTO	23				68	CNT	47				a8	CNT	47			
29	0	00				69	CNT	47				a9	CNT	47			
2a	GTO	44				6a	CNT	47				aa	CNT	47			
2b	-	34				6b	CNT	47				ab	CNT	47			
2c	0	00				6c	CNT	47				ac	CNT	47			
2d	0	00				6d	CNT	47				ad	CNT	47			
30	CNT	47				70	CNT	47									
31	CNT	47				71	CNT	47									
32	CNT	47				72	CNT	47									
33	CNT	47				73	CNT	47									
34	CNT	47				74	CNT	47									
35	CNT	47				75	CNT	47									
36	CNT	47				76	CNT	47									
37	CNT	47				77	CNT	47									
38	CNT	47				78	CNT	47									
39	CNT	47				79	CNT	47									
3a	CNT	47				7a	CNT	47									
3b	CNT	47				7b	CNT	47									
3c	CNT	47				7c	CNT	47									
3d	CNT	47				7d	CNT	47									

Storage

0		8	
1		9	
2		a	
3		b	
4		c	
5		d	
6		e	
7		f	

STAT-PAC VII-3 Program 1

Step	Key	Code
b0	CNT	47
b1	CNT	47
b2	CNT	47
b3	CNT	47
b4	CNT	47
b5	CNT	47
b6	CNT	47
b7	CNT	47
b8	CNT	47
b9	CNT	47
ba	CNT	47
bb	CNT	47
bc	CNT	47
bd	CNT	47
c0	CNT	47
c1	CNT	47
c2	CNT	47
c3	CNT	47
c4	CNT	47
c5	CNT	47
c6	CNT	47
c7	CNT	47
c8	CNT	47
c9	CNT	47
ca	CNT	47
cb	CNT	47
cc	CNT	47
cd	CNT	47
d0	CNT	47
d1	CNT	47
d2	CNT	47
d3	CNT	47
d4	CNT	47
d5	CNT	47
d6	CNT	47
d7	CNT	47
d8	CNT	47
d9	CNT	47
da	CNT	47
db	CNT	47
dc	CNT	47
dd	CNT	47

Plus Page

STAT-PAC VII-3 Program 1

Minus Page

Step	Key	Code
00	XTO	23
01	2	02
02	1	01
03	+	33
04	YTO	40
05	9	11
06	GTO	44
07	SUB	77
08	9	11
09	c	16
0a	YE	24
0b	5	05
0c	+	33
0d	YE	24
10	5	05
11	GTO	44
12	SUB	77
13	9	11
14	c	16
15	YE	24
16	4	04
17	+	33
18	YE	24
19	4	04
1a	GTO	44
1b	SUB	77
1c	9	11
1d	c	16
20	YE	24
21	3	03
22	+	33
23	YE	24
24	3	03
25	GTO	44
26	SUB	77
27	9	11
28	c	16
29	YE	24
2a	2	02
2b	+	33
2c	YE	24
2d	2	02
30	GTO	44
31	SUB	77
32	9	11
33	c	16
34	YE	24
35	8	10
36	+	33
37	YTO	40
38	8	10
39	GTO	44
3a	SUB	77
3b	9	11
3c	c	16
3d	YE	24
40	-	34
41	e	12
42	+	33
43	YE	24
44	-	34
45	e	12
46	GTO	44
47	SUB	77
48	9	11
49	c	16
4a	YE	24
4b	-	34
4c	f	15
4d	+	33
50	YE	24
51	-	34
52	f	15
53	XFR	67
54	1	01
55	UP	27
56	X	36
57	XFR	67
58	0	00
59	+	33
5a	YTO	40
5b	0	00
5c	CLX	37
5d	XTO	23
60	1	01
61	RDN	31
62	YE	24
63	9	11
64	b	14
65	X>Y	53
66	0	00
67	2	02
68	XFR	67
69	5	05
6a	UP	27
6b	X	36
6c	XFR	67
6d	4	04
70	UP	27
71	X	36
72	DN	25
73	+	33
74	XFR	67
75	3	03
76	UP	27
77	X	36
78	DN	25
79	+	33
7a	XFR	67
7b	2	02
7c	UP	27
7d	X	36
80	DN	25
81	+	33
82	XFR	67
83	8	10
84	UP	27
85	X	36
86	DN	25
87	+	33
88	XFR	67
89	-	34
8a	e	12
8b	UP	27
8c	X	36
8d	DN	25
90	+	33
91	XFR	67
92	-	34
93	f	15
94	UP	27
95	X	36
96	DN	25
97	+	33
98	DN	25
99	UP	27
9a	UP	27
9b	STP	41
9c	DN	25
9d	c	16
a0	X=Y	50
a1	5	05
a2	3	03
a3	1	01
a4	+	33
a5	a	13
a6	UP	27
a7	d	17
a8	X=Y	50
a9	c	16
aa	5	05
ab	1	01
ac	+	33
ad	YTO	40

STAT-PAC VII-3 Program 1

b0	a	13	Minus Page
b1	XFR	67	
b2	9	11	
b3	RDN	31	
b4	STP	41	ENTRY
b5	PNT	45	
b6	PNT	45	
b7	UP	27	
b8	X	36	
b9	AC+	60	
ba	YE	24	
bb	7	07	
bc	+	33	
bd	YE	24	
c0	7	07	
c1	DN	25	
c2	GTO	44	
c3	a	13	
c4	5	05	
c5	DN	25	
c6	XFR	67	
c7	7	07	
c8	YE	24	
c9	1	01	
ca	+	33	
cb	YE	24	
cc	1	01	
cd	UP	27	
d0	X	36	
d1	RDN	31	
d2	YE	24	
d3	6	06	
d4	+	33	
d5	YE	24	
d6	6	06	
d7	CLX	37	
d8	XTO	23	
d9	7	07	
da	XTO	23	
db	a	13	
dc	RUP	22	
dd	RTN	77	

STAT-PAC VII-3   Program 2

```
00  YTO  40   Minus
01  1    01   Page
02  f    15
03  UP   27
04  X    36
05  d    17
06  DIV  35
07  c    16
08  DIV  35
09  b    14
0a  DIV  35
0b  e    12
0c  XEY  30
0d  -    34

10  YTO  40
11  e    12
12  XTO  23
13  f    15
14  XFR  67
15  0    00
16  UP   27
17  c    16
18  DIV  35
19  d    17
1a  DIV  35
1b  f    15
1c  -    34
1d  YTO  40

20  5    05
21  XFR  67
22  1    01
23  UP   27
24  b    14
25  DIV  35
26  d    17
27  DIV  35
28  f    15
29  -    34
2a  YTO  40
2b  4    04
2c  XFR  67
2d  6    06

30  UP   27
31  d    17
32  DIV  35
33  XFR  67
34  5    05
35  -    34
36  XFR  67
37  1    01
38  UP   27
39  d    17
3a  DIV  35
3b  b    14
3c  DIV  35
3d  DN   25

40  -    34
41  YTO  40
42  3    03
43  RCL  61
44  XFR  67
45  5    05
46  -    34
47  XFR  67
48  4    04
49  -    34
4a  XFR  67
4b  3    03
4c  -    34
4d  d    17

50  UP   27
51  1    01
52  -    34
53  c    16
54  X    36
55  b    14
56  X    36
57  DN   25
58  XEY  30
59  YTO  40
5a  d    17
5b  DIV  35
5c  YTO  40
5d  2    02

60  XFR  67
61  5    05
62  X    36
63  b    14
64  UP   27
65  1    01
66  -    34
67  YTO  40
68  b    14
69  DN   25
6a  DIV  35
6b  UP   27
6c  d    17
6d  PNT  45   s

70  PNT  45
71  CNT  47
72  XFR  67
73  2    02
74  UP   27
75  XFR  67
76  4    04
77  X    36
78  c    16
79  UP   27
7a  1    01
7b  -    34
7c  YTO  40
7d  c    16

80  DN   25
81  DIV  35
82  UP   27
83  d    17
84  PNT  45   s
85  PNT  45
86  CNT  47
87  XFR  67
88  2    02
89  UP   27
8a  XFR  67
8b  3    03
8c  X    36
8d  b    14

90  UP   27
91  c    16
92  X    36
93  DN   25
94  DIV  35
95  UP   27
96  d    17
97  PNT  45   s
98  PNT  45
99  PNT  45
9a  PNT  45
9b  PNT  45
9c  PNT  45
9d  END  46
```

ANALYSIS OF VARIANCE F TEST FOR MEANS

This program operates on 2 or more columns of data containing equal numbers of items, classified according to a single criterion:

$$\begin{array}{c|ccccc} & & & K & & \\ \hline & X_{11} & X_{12} & X_{13} & \cdot\ \cdot\ \cdot & X_{1K} \\ & X_{21} & X_{22} & \cdot\ \cdot\ \cdot & & \\ & X_{31} & X_{32} & \cdot\ \cdot\ \cdot & & \\ R & \cdot & \cdot & \cdot & & \\ & \cdot & \cdot & \cdot & & \\ & \cdot & \cdot & \cdot & & \\ & X_{R1} & & & & \end{array}$$

The program generates an Analysis of Variance Table containing the sums of squares among the column means and within each group. This essentially amounts to breaking down the total variation in the data to its component parts, i.e., among and within groups. It subsequently computes the corresponding mean squares and F ratio for testing the hypothesis that the means of the columns are not significantly different.

ANOV Table

	Sum of Squares	Degrees of Freedom	Mean Squares
Among Means	$\text{Sam} = \sum_{i=1}^{K} \left[\frac{(\sum_{j=1}^{R} X_{ij})^2}{R} \right] - \frac{\left[\sum_{i=1}^{K} \sum_{j=1}^{R} X_{ij} \right]^2}{N}$	K - 1 df_1	Sam/K-1
Within Groups	$\text{Swg} = \sum_{i=1}^{K} \sum_{j=1}^{R} (X_{ij}^2) - \sum_{i=1}^{K} \left[\frac{(\sum_{j=1}^{R} X_{ij})^2}{R} \right]$	N - K df_2	Swg/N-K
Total	$\text{St} = \sum_{i=1}^{K} \sum_{j=1}^{R} (X_{ij})^2 - \frac{\left[\sum_{i=1}^{K} \sum_{j=1}^{R} X_{ij} \right]^2}{N}$	N - 1	

$$F = \frac{\dfrac{\sum_{i=1}^{K} \left[\dfrac{(\sum_{j=1}^{R} X_{ij})^2}{R} \right] - \dfrac{\left[\sum_{i=1}^{K} \sum_{j=1}^{R} X_{ij} \right]^2}{N}}{K - 1}}{\dfrac{\sum_{i=1}^{K} \sum_{j=1}^{R} (X_{ij})^2 - \sum_{i=1}^{K} \left[\dfrac{(\sum_{j=1}^{R} X_{ij})^2}{R} \right]}{N - K}} = \frac{\frac{Sam}{K-1}}{\frac{Swg}{N-K}}$$

Where:

i = Index of Rows
j = Index of Columns
K = No. of Columns
R = No. of Rows
N = Total Number of Observations

For abbreviations in the program:

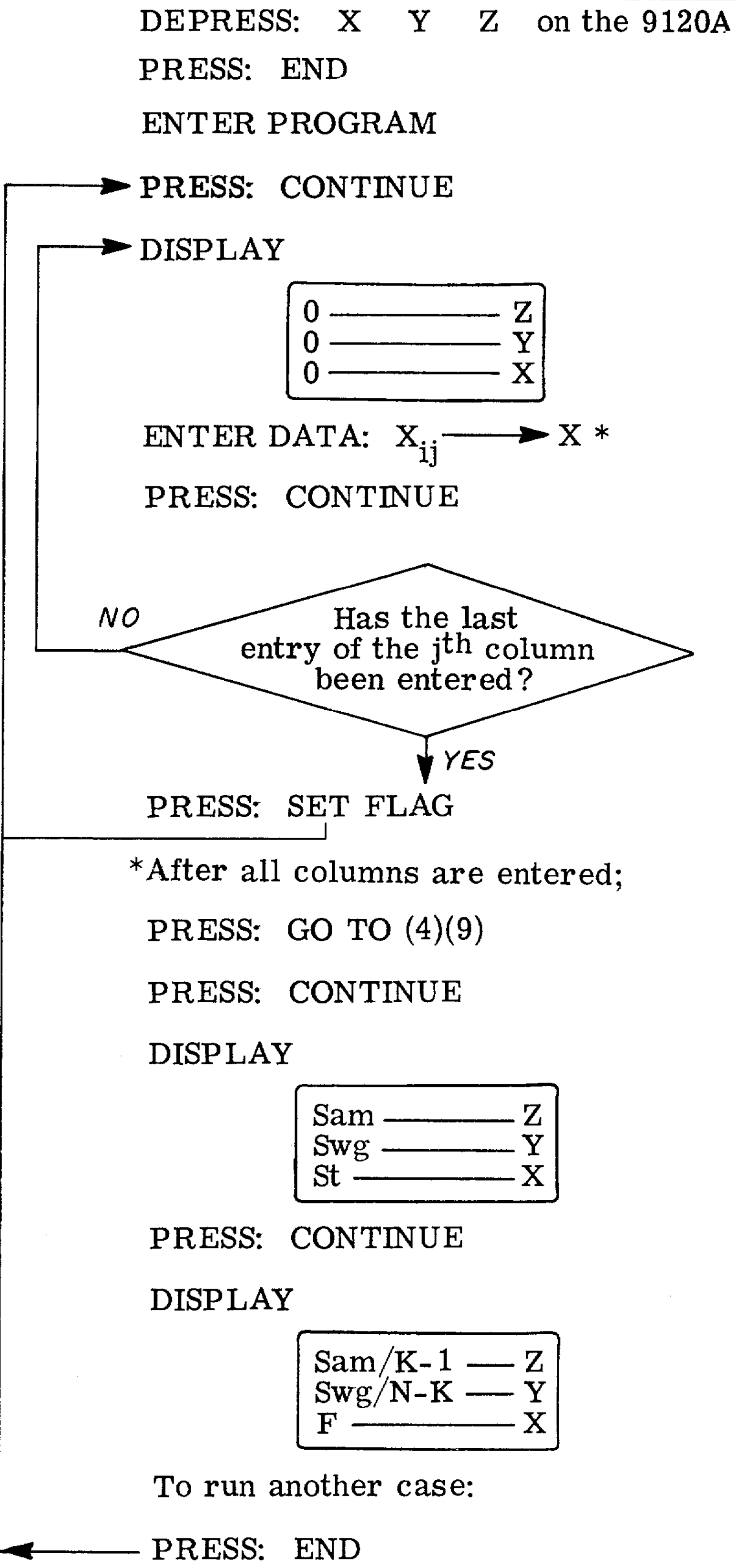
Sam = A - B
Swg = C - A

The calculated F is to be compared with the value found in a table of F percentiles at $\nu_1 = K - 1$, $\nu_2 = N - K$. The hypothesis is rejected if $F_{calc.} > F_{table}$. The latter will depend on the level of confidence used in the table (usually 95%).

Data is entered column by column beginning with column 1. Thus data is entered as $X_{11}, X_{21}, X_{31}, \ldots, X_{R1}; X_{12}, X_{22}, \ldots, X_{R2}$; etc.

This program was written by Dr. Thomas F. Brodasky of the UPJOHN COMPANY, Kalamazoo, Michigan.

USER INSTRUCTIONS



DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0 —— Z
0 —— Y
0 —— X

ENTER DATA: $X_{ij} \longrightarrow X$ *

PRESS: CONTINUE

PRESS: SET FLAG

*After all columns are entered;

PRESS: GO TO (4)(9)

PRESS: CONTINUE

DISPLAY

Sam —— Z
Swg —— Y
St —— X

PRESS: CONTINUE

DISPLAY

Sam/K-1 —— Z
Swg/N-K —— Y
F —— X

To run another case:

PRESS: END

EXAMPLE

		K		
		1	2	3
	1	2.10	2.20	3.00
	2	3.00	2.30	2.20
R	3	2.60	2.90	2.80
	4	2.50	2.60	2.80
	5	2.80	2.10	2.40

$df_1 = K - 1 = 2$
$df_2 = N - K = 12$

Answer:

Sam = 0.1373

Swg = 1.3200

St = 1.4573

Sam/K-1 = 0.0687

Swg/N-K = 0.1100

F_{calc} = 0.6242

F_{table} = 3.89 (@ 95% confidence)

STAT-PAC VII-4

00	CLR	20	
01	XTO	23	
02	9	11	
03	XTO	23	
04	a	13	
05	XTO	23	
06	b	14	
07	XTO	23	
08	c	16	
09	XTO	23	
0a	d	17	
0b	CLR	20	
0c	STP	41	ENTRY
0d	IFG	43	
10	2	02	
11	7	07	
12	PNT	45	
13	PNT	45	
14	UP	27	
15	X	36	
16	UP	27	
17	RDN	31	
18	a	13	
19	+	33	
1a	YTO	40	
1b	a	13	
1c	1	01	
1d	RUP	22	
20	AC+	60	
21	CLX	37	
22	UP	27	
23	UP	27	
24	GTO	44	
25	0	00	
26	c	16	
27	e	12	
28	UP	27	
29	b	14	
2a	+	33	
2b	YTO	40	
2c	b	14	
2d	f	15	
30	UP	27	
31	X	36	
32	e	12	
33	DIV	35	
34	c	16	
35	+	33	
36	YTO	40	
37	c	16	
38	f	15	
39	UP	27	
3a	d	17	
3b	+	33	
3c	YTO	40	
3d	d	17	
40	1	01	
41	YEX	24	
42	9	11	
43	+	33	
44	YEX	24	
45	9	11	
46	GTO	44	
47	0	00	
48	b	14	
49	d	17	
4a	UP	27	
4b	X	36	
4c	b	14	
4d	DIV	35	
50	YTO	40	
51	f	15	
52	c	16	
53	XEY	30	
54	-	34	
55	YTO	40	
56	d	17	
57	a	13	
58	UP	27	
59	c	16	
5a	-	34	
5b	RUP	22	
5c	a	13	
5d	XEY	30	
60	f	15	
61	-	34	
62	d	17	
63	RDN	31	
64	PNT	45	S
65	PNT	45	
66	1	01	
67	YEX	24	
68	9	11	
69	YTO	40	
6a	d	17	
6b	-	34	
6c	RDN	31	
6d	DIV	35	
70	YTO	40	
71	e	12	
72	a	13	
73	UP	27	
74	c	16	
75	-	34	
76	UP	27	
77	b	14	
78	XEY	30	
79	d	17	
7a	-	34	
7b	RDN	31	
7c	DIV	35	
7d	e	12	
80	UP	27	
81	RUP	22	
82	DIV	35	
83	XEY	30	
84	PNT	45	
85	PNT	45	
86	PNT	45	
87	PNT	45	
88	END	46	

SECTION VIII OPERATIONS RESEARCH

QUEUEING MODEL

The program determines P_n (the probability of a waiting line of length n) and $\overline{n}$ (the mean value of a waiting line) for a single-station Queueing problem.

The equations used are:

$$P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right)$$

$$\overline{n} = \frac{\frac{\lambda}{\mu}}{\left(1 - \frac{\lambda}{\mu}\right)}$$

where

$$\left.\begin{array}{l} \lambda = \text{Mean arrival rate} \\ \mu = \text{Mean service rate} \end{array}\right\} \quad \frac{\lambda}{\mu} < 1$$

n = Number of units in the waiting line.

Reference: Introduction to Operations Research, Churchman, Ackoff, and Arnoff, Wiley, 1964.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

→ DISPLAY

0 ——— Z
0 ——— Y
1 ——— X

ENTER DATA:

$n \longrightarrow Z$
$\lambda \longrightarrow Y$
$\mu \longrightarrow X$

PRESS: CONTINUE

DISPLAY

P_n ——— Z
$\bar{n}$ ——— Y
2 ——— X

To run another case :

PRESS: CONTINUE

EXAMPLE

If the mean arrival rate is 10 units/day, and the mean service rate is 20 units/day, what is the probability of 4 units in the waiting line?

$n = 4$
$\lambda = 10$
$\mu = 20$

Results:

$P_4 = .03125$
$\bar{n} = 1$

```
00  CLR  20
01   1   01
02  STP  41
03  PNT  45
04  PNT  45
05  DIV  35
06  YTO  40
07   e   12
08   1   01
09  XEY  30
0a   -   34
0b  YTO  40
0c   f   15
0d   e   12

10  RUP  22
11  XEY  30
12  LN   65
13   X   36
14  XEY  30
15  EXP  74
16  RUP  22
17   X   36
18  UP   27
19  RCL  61
1a  DIV  35
1b   2   02
1c  PNT  45
1d  PNT  45

20  END  46
```

ENTRY

$\frac{\lambda}{\mu}$

$(1 - \frac{\lambda}{\mu})$

QUEUEING WITH FINITE CUSTOMERS

This program computes the "average number in system, $\bar{n}$" and the "queue length, L_Q" when the following parameters are known:

N = Number of Available Customers
M = Number of Service Channels
MTTR = Mean Service Channels
MTBF = Mean Time Between Calls for Service Per Customer

The recursive equation applied is:

$$\frac{X}{1-X}(N-n+1)P_{n-1} = \begin{Bmatrix} nP_n \text{ when } 0 \le n \le M \\ MP_n \text{ when } M < n \le N \end{Bmatrix}$$

where

$$X = \frac{MTTR}{MTTR + MTBF}$$

and

P_n = Probability of n customers in the system.

The above recursive formula is iteratively applied to solve

$$\bar{n} = \frac{\Sigma nP_n}{\Sigma P_n} \quad \text{(average number in system)}$$

$$F = \frac{\frac{\bar{n}}{N} - 1}{X - 1} \quad \text{(efficiency factor)}$$

and $L_Q = N(1-F)$. (queue length)

This program varies M (number of service channels) and iteratively solves for $\bar{n}$, F, and L_Q.

For certain highly unlikely combinations of N, MTTR and MTBF, an overflow condition will occur. However the program is protected against this eventuality and it re-cycles to the initial DISPLAY when an illegal combination is attempted.

Computing times for each channel value (M) can range between 30 - 60 seconds.

This program was written by Mr. E.J. Schmidt of ITT Federal Electric, Paramus, N.J.

Reference: Finite Queueing Tables, Peck and Hazelwood, Wiley, 1958.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0 ———— Z
0 ———— Y
1 ———— X

ENTER DATA:

N ———→ Z
MTTR ——→ Y
MTBF ——→ X

PRESS: CONTINUE

DISPLAY

M ———— Z
$\bar{n}$ ———— Y
L_Q ———— X

To run another case:

EXAMPLE

Given a "system" with 625 units (customers) each having a 30,000 hour MTBF, and maintenance conditions such that the MTTR is 50 hours, find the average number in system ($\bar{n}$) and the queue length L_Q for varying M (number of channels).

Input data:

N = 625 (number of units - customers - available)

MTTR = 50 (mean time to restore)

MTBR = 30,000 (mean time between failures, at 'failure it "calls' for service")

Results:

1
31.72550
30.73671

2
1.41706
.37776

3
1.09101
.05116

4
1.04723
.00731

STAT-PAC VIII-2

00	CLR	20		40	DN	25		80	c	16	
01	XTO	23		41	YEX	24		81	X	36	
02	9	11		42	9	11		82	b	14	
03	1	01		43	+	33		83	RUP	22	
04	STP	41	ENTRY	44	YTO	40		84	a	13	
05	PNT	45		45	9	11		85	RUP	22	
06	PNT	45		46	EEX	26		86	PNT	45	S
07	DIV	35		47	9	11		87	PNT	45	
08	YTO	40		48	8	10		88	STP	41	
09	d	17		49	X<Y	52		89	c	16	
0a	DN	25		4a	0	00		8a	RDN	31	
0b	0	00		4b	0	00		8b	GTO	44	
0c	YTO	40		4c	DN	25		8c	1	01	
0d	c	16		4d	a	13		8d	1	01	
10	RUP	22		50	+	33		90	END	46	
11	1	01		51	YTO	40					
12	+	33		52	a	13					
13	YTO	40		53	DN	25					
14	b	14		54	GTO	44					
15	RUP	22		55	2	02					
16	+	33		56	2	02					
17	YTO	40		57	CLR	20					
18	f	15		58	YEX	24					
19	EEX	26		59	9	11					
1a	9	11		5a	a	13					
1b	3	03		5b	DIV	35					
1c	CHS	32		5c	YTO	40					
1d	XTO	23		5d	a	13					
20	a	13		60	c	16					
21	UP	27		61	DIV	35					
22	1	01		62	1	01					
23	UP	27		63	-	34					
24	CHS	32		64	UP	27					
25	AC+	60		65	d	17					
26	RCL	61		66	XEY	30					
27	c	16		67	+	33					
28	X<Y	52		68	RDN	31					
29	5	05		69	CHS	32					
2a	7	07		6a	X	36					
2b	DN	25		6b	.	21					
2c	d	17		6c	9	11					
2d	X	36		6d	X<Y	52					
30	f	15		70	7	07					
31	X	36		71	c	16					
32	e	12		72	XEY	30					
33	UP	27		73	DIV	35					
34	b	14		74	b	14					
35	X<Y	52		75	X	36					
36	XEY	30		76	c	16					
37	CNT	47		77	XEY	30					
38	DN	25		78	INT	64					
39	DIV	35		79	GTO	44					
3a	UP	27		7a	1	01					
3b	DN	25		7b	0	00					
3c	e	12		7c	RDN	31					
3d	X	36		7d	-	34					

10^{-95}

N

$\frac{MTTR}{MTBF}$

QUEUEING WITH INFINITE CUSTOMERS

This program computes the "queue length" of a system when the following system parameters are known:

K = Number of Service Stations

MTTR = Mean Service Time

MTBF = "Ensemble" Mean Time Between Calls for Service From Total Source of Customers

The equation applied is:

$$L_Q = \frac{1}{(K - R)\left[\dfrac{\sum\limits_{n=0}^{K-1} \dfrac{1}{n!} R^n}{\dfrac{K}{K-R} \dfrac{1}{K!} R^K} + 1\right]}$$

where,

$$R = \frac{MTTR}{MTBF}$$

L_Q is defined to be "average length of non-empty queues".

This program varies K (number of service stations) and determines L_Q for each K. The user can optionally output

A_{VS} = Average Time In System
A_{VW} = Average Time In Waiting

This program was written by Mr. E.J. Schmidt of ITT Federal Electric, Paramus, N.J.

Reference: Finite Queueing Models, Peck and Hazelwood, Wiley, 1958.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0 —— Z
0 —— Y
1 —— X

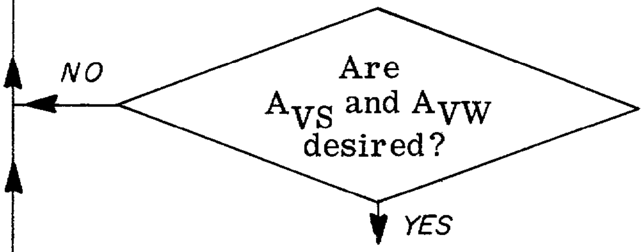
ENTER DATA:

MTTR → Y
MTBF → X

PRESS: CONTINUE

DISPLAY

0 —— Z
K —— Y
L_Q —— X



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

K —— Z
A_{VS} —— Y
A_{VW} —— X

To run another case

PRESS: END

EXAMPLE

Consider the problem posed by program STAT-PAC VIII-2. To convert the finite model for an infinite model, determine the ensemble

$$\text{MTBF} = \frac{\text{unit MTBF}}{\text{N}}$$

For the example,

Unit MTBF = 30,000
N = 625

Thus

MTTR = 50
ensemble MTBF = 48

Results:

K = 2.
L_Q = .37225 *

= 3.
= .05130

= 4.
= .00790

= 5.
= .00115

= 6.
= .00015

.
.
.

*PRESS: SET FLAG – CONTINUE to obtain:

A_{VS} = 68.61227
A_{VW} = 18.61227

00	CLR	20	
01	1	01	
02	STP	41	ENTRY
03	PNT	45	
04	PNT	45	
05	YTO	40	
06	9	11	
07	DIV	35	
08	YTO	40	
09	f	15	
0a	DN	25	
0b	INT	64	
0c	UP	27	
0d	1	01	
10	+	33	
11	YTO	40	
12	a	13	
13	XTO	23	
14	d	17	
15	XTO	23	
16	e	12	
17	XTO	23	
18	b	14	
19	XTO	23	
1a	c	16	
1b	f	15	
1c	UP	27	
1d	b	14	
20	DIV	35	
21	c	16	
22	X	36	
23	YTO	40	
24	c	16	
25	b	14	
26	UP	27	
27	e	12	
28	X=Y	50	
29	3	03	
2a	4	04	
2b	1	01	
2c	+	33	
2d	YTO	40	
30	b	14	
31	GTO	44	
32	1	01	
33	b	14	
34	IFG	43	
35	5	05	
36	5	05	
37	1	01	
38	UP	27	
39	CLX	37	
3a	AC+	60	
3b	a	13	
3c	X=Y	50	
3d	4	04	
40	7	07	
41	d	17	
42	UP	27	
43	c	16	
44	+	33	
45	YTO	40	
46	d	17	
47	RCL	61	
48	a	13	
49	X>Y	53	
4a	5	05	
4b	1	01	
4c	XTO	23	
4d	e	12	
50	SFL	54	
51	1	01	
52	GTO	44	
53	1	01	
54	7	07	
55	d	17	
56	RUP	22	
57	DIV	35	
58	a	13	
59	DIV	35	
5a	UP	27	
5b	f	15	
5c	-	34	
5d	1	01	
60	RDN	31	
61	X	36	
62	RDN	31	
63	XEY	30	
64	+	33	
65	RDN	31	
66	X	36	
67	DN	25	
68	DIV	35	
69	a	13	
6a	XEY	30	
6b	UP	27	
6c	CLX	37	
6d	RDN	31	
70	PNT	45	
71	PNT	45	
72	STP	41	
73	IFG	43	
74	7	07	
75	9	11	
76	GTO	44	
77	0	00	
78	d	17	
79	UP	27	
7a	YEX	24	
7b	9	11	
7c	YTO	40	
7d	9	11	
80	XEY	30	
81	X	36	
82	XEY	30	
83	+	33	
84	PNT	45	
85	PNT	45	
86	STP	41	
87	DN	25	
88	GTO	44	
89	0	00	
8a	d	17	
8b	END	46	

SECTION IX RELIABILITY AND QUALITY CONTROL

WEIBULL DISTRIBUTION PARAMETER CALCULATION FOR FAILURE DATA

The Weibull probability density function is given by

$$f(X) = \frac{bX^{(b-1)}}{\theta^b} e^{-(\frac{X}{\theta})^b}$$

and the cumulative distribution function is given by

$$F(X) = 1 - e^{-(\frac{X}{\theta})^b}$$

For a set of data, the Weibull parameters b and θ are to be calculated for these functions.

A common application is to use Weibull analysis for failure data where all samples are tested to failure. To use the program, list the items in order of increasing time to failure. The number of items and times to failure are entered. The parameters b, θ, and r are displayed. r is a correlation coefficient indicating goodness of fit. The time required for 10% (B_{10}) to fail is displayed and times to other failure percentages ($B_{\%}$) may be requested.

The Median Rank (M.R.) is calculated by the equation

$$M.R. = \frac{j-.3}{N+4}$$

where j = failure order number

N = number of samples tested

This is an approximation of F(X).

The cumulative distribution function is linearized into the form

$$b\text{Ln}X - b\text{Ln}\theta = \text{LnLn}\left(\frac{1}{1-F(X)}\right)$$

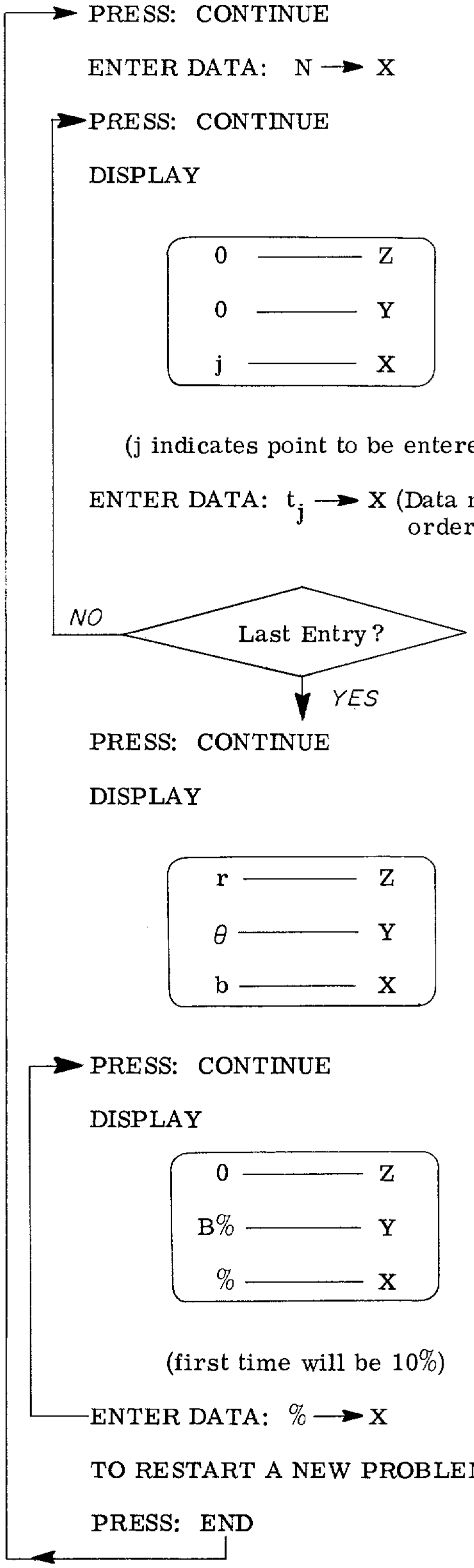
A least squares fit is performed which calculates the slope, intercept, and correlation coefficient. The solution is similar to the linear regression program 09100-70803. Thus estimates of b and θ are obtained.

USER INSTRUCTIONS

DEPRESS: X on the 9120A

PRESS: END

ENTER PROGRAM: Side A followed by Side B



PRESS: CONTINUE

ENTER DATA: N → X

PRESS: CONTINUE

DISPLAY

0 — Z
0 — Y
j — X

(j indicates point to be entered)

ENTER DATA: t_j → X (Data must be ordered)

PRESS: CONTINUE

DISPLAY

r — Z
θ — Y
b — X

PRESS: CONTINUE

DISPLAY

0 — Z
B% — Y
% — X

(first time will be 10%)

ENTER DATA: % → X

TO RESTART A NEW PROBLEM

PRESS: END

EXAMPLES

TEST DATA

Hours to failure (must be ordered)

34
60
75
95
119
158

N = 6 (number of samples)

r = .999
θ = 104.091
b = 1.953

B_{10} = 32.887
B_{90} = 159.539

STAT-PAC IX-1

```
00  CLR  20    Plus
01  XTO  23    Page
02  d    17
03  XTO  23
04  c    16
05  XTO  23
06  b    14
07  STP  41    ENTRY
08  PNT  45
09  PNT  45
0a  XTO  23
0b  8    10
0c  UP   27
0d  .    21

10  4    04
11  +    33
12  DN   25
13  XTO  23
14  9    11
15  1    01
16  XTO  23
17  a    13
18  STP  41    ENTRY
19  PNT  45
1a  PNT  45
1b  LN   65
1c  UP   27
1d  a    13

20  UP   27
21  .    21
22  3    03
23  -    34
24  XFR  67
25  9    11
26  DIV  35
27  1    01
28  XEY  30
29  -    34
2a  1    01
2b  XEY  30
2c  DIV  35
2d  DN   25

30  LN   65
31  LN   65
32  XEY  30
33  AC+  60
34  UP   27
35  X    36
36  XEY  30
37  YE   24
38  d    17
39  +    33
3a  YE   24
3b  d    17
3c  CNT  47
3d  DN   25
```

```
40  X    36
41  b    14
42  +    33
43  YTO  40
44  b    14
45  RUP  22
46  UP   27
47  X    36
48  c    16
49  +    33
4a  YTO  40
4b  c    16
4c  a    13
4d  UP   27

50  1    01
51  +    33
52  XFR  67
53  8    10
54  XEY  30
55  UP   27
56  0    00
57  RDN  31
58  X>Y  53
59  6    06
5a  4    04
5b  XTO  23
5c  a    13
5d  XEY  30

60  DN   25
61  GTO  44
62  1    01
63  8    10
64  e    12
65  UP   27
66  a    13
67  DIV  35
68  YE   24
69  f    15
6a  DIV  35
6b  YTO  40
6c  e    12
6d  X    36

70  e    12
71  X    36
72  d    17
73  XEY  30
74  -    34
75  YTO  40
76  d    17
77  c    16
78  UP   27
79  f    15
7a  GTO  44
7b  -    34
7c  0    00
7d  0    00
```

```
80  CNT  47
81  CNT  47
82  CNT  47
83  CNT  47
84  CNT  47
85  CNT  47
86  CNT  47
87  CNT  47
88  CNT  47
89  CNT  47
8a  CNT  47
8b  CNT  47
8c  CNT  47
8d  CNT  47

90  CNT  47
91  CNT  47
92  CNT  47
93  CNT  47
94  CNT  47
95  CNT  47
96  CNT  47
97  CNT  47
98  CNT  47
99  CNT  47
9a  CNT  47
9b  CNT  47
9c  CNT  47
9d  CNT  47

a0  CNT  47
a1  CNT  47
a2  CNT  47
a3  CNT  47
a4  CNT  47
a5  CNT  47
a6  CNT  47
a7  CNT  47
a8  CNT  47
a9  CNT  47
aa  CNT  47
ab  CNT  47
ac  CNT  47
ad  CNT  47
```

N

N + 4

```
b0  CNT  47     Plus
b1  CNT  47     Page
b2  CNT  47
b3  CNT  47
b4  CNT  47
b5  CNT  47
b6  CNT  47
b7  CNT  47
b8  CNT  47
b9  CNT  47
ba  CNT  47
bb  CNT  47
bc  CNT  47
bd  CNT  47

c0  CNT  47
c1  CNT  47
c2  CNT  47
c3  CNT  47
c4  CNT  47
c5  CNT  47
c6  CNT  47
c7  CNT  47
c8  CNT  47
c9  CNT  47
ca  CNT  47
cb  CNT  47
cc  CNT  47
cd  CNT  47

d0  CNT  47
d1  CNT  47
d2  CNT  47
d3  CNT  47
d4  CNT  47
d5  CNT  47
d6  CNT  47
d7  CNT  47
d8  CNT  47
d9  CNT  47
da  CNT  47
db  CNT  47
dc  CNT  47
dd  CNT  47
```

STAT-PAC IX-1

```
00  UP    27     Minus
01  X     36     Page
02  a     13
03  X     36
04  DN    25
05  -     34
06  YTO   40
07  c     16
08  b     14
09  UP    27
0a  f     15
0b  UP    27
0c  e     12
0d  X     36

10  a     13
11  X     36
12  DN    25
13  -     34
14  d     17
15  UP    27
16  DN    25
17  √     76
18  DIV   35
19  c     16
1a  √     76
1b  DIV   35
1c  d     17
1d  RUP   22

20  XEY   30
21  DIV   35
22  YE    24
23  e     12
24  e     12
25  X     36
26  f     15
27  XEY   30
28  -     34
29  e     12
2a  CHS   32
2b  DIV   35
2c  XEY   30
2d  EXP   74

30  XEY   30
31  CHS   32
32  CNT   47     s
33  RUP   22
34  PNT   45
35  RUP   22
36  PNT   45
37  RUP   22
38  PNT   45
39  PNT   45
3a  UP    27
3b  1     01
3c  XEY   30
3d  DIV   35
```

```
40  YTO   40
41  f     15
42  RUP   22
43  LN    65
44  XTO   23
45  e     12
46  1     01
47  0     00
48  XTO   23
49  d     17
4a  UP    27
4b  EEX   26
4c  2     02
4d  DIV   35

50  1     01
51  -     34
52  XEY   30
53  CHS   32
54  DIV   35
55  DN    25
56  LN    65
57  LN    65
58  UP    27
59  f     15
5a  X     36
5b  e     12
5c  +     33
5d  0     00

60  RDN   31
61  EXP   74
62  XEY   30
63  d     17
64  CNT   47
65  RUP   22
66  PNT   45
67  RUP   22
68  PNT   45
69  RUP   22
6a  PNT   45
6b  PNT   45
6c  STP   41
6d  GTO   44

70  4     04
71  8     10
72  END   46
```

WEIBULL ANALYSIS VIA FAILED AND SUSPENDED SETS

This program determines the Weibull parameters b, and θ , from a grouping of failure data into sets of failed items and suspended items.

A Weibull analysis is obtained by proper choice of a representative point from each failed set. This representative point is the latest failure in the set of failed items.

The Weibull distribution function is given by:

$$f(X) = \frac{bX^{(b-1)}}{\theta^b} \quad e^{-\left(\frac{X}{\theta}\right)^b}$$

The program outputs:

r , Goodness of Fit
b , Weibull Slope
M, Median Life
Q_5, 5%, Rank at any point X
R, Median rank at any point X
Q_{95}, 95%, Rank at any point X
θ, Time for 63.2% to fail
B_{10}, Time required for 10% to fail

The formulas applied are:

$$\lambda_K = \lambda_{K-1} + T_K \left(\frac{N + 1 - \lambda_{K-1}}{1+N - \sum_{i=0}^{K-1} S_i - \sum_{i=1}^{K-1} T_i} \right)$$

where:

λ_K = The mean order number of K^{th} representative point
S_i = Size of the (i + 1)th suspended set
T_i = Size of the i^{th} failed set
N = Total number of failed and suspended sets

The latest failure in T_K has a median rank given by

$$R_K = \frac{\lambda_K - .3}{N + .4}$$

The 5% and 95% ranks at point whose mean order number is λ is

$$Q_5 = \frac{\lambda}{N+1} + t_5 \sigma$$

$$Q_{95} = \frac{\lambda}{N+1} + t_{95} \sigma$$

t_5 and t_{95} are given by

$$t_5 = -1.645 + .2885\, \alpha_3 + .0565\, \alpha_3^2 \qquad \text{for } \alpha_3 < 0$$

$$t_5 = -1.645 + .2871\, \alpha_3 + .041\, \alpha_3^2 \qquad \text{for } \alpha_3 > 0$$

$$t_{95} = 1.645 + .2871\, \alpha_3 - .041\, \alpha_3^2 \qquad \text{for } \alpha_3 < 0$$

$$t_{95} = 1.645 + .2885\, \alpha_3 - .0565\, \alpha_3^2 \qquad \text{for } \alpha_3 > 0$$

Note that t_5 is always less than zero and t_{95} is always greater than zero.

$$\alpha_3 = \frac{2(1-2Q)}{\sigma\sqrt{N+3}} = \text{the skewness of the rank}$$

$$\sigma = \sqrt{\frac{Q(1-Q)}{N+2}}$$

The cumulative distribution

$$F(X) = 1 - e^{-\left(\frac{X}{\theta}\right)^b}$$

is linearized into the form

$$b \ln X - b \ln \theta = \ln \ln \left(\frac{1}{1 - F(X)} \right)$$

The X_i used for this linearizing are the latest failures in each failed set.

The program can optionally output the time required for Y% to fail.

Program Assumptions:

$R < 1$

$.0155\, \alpha_3^2 < 1$

The sets S_i are ordered from shortest life to longest life.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM 1: Side A followed by Side B

PRESS: CONTINUE

PRESS: (X →)(-)(e)
(X →)(-)(f)

ENTER DATA:

N → Y
F → X

(F is the number of failed sets)

PRESS: CONTINUE

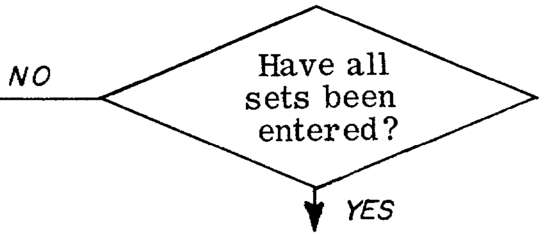
ENTER DATA:

S_i → Z (Size of the (i+1) suspended set)
T_i → Y (Size of the i^{th} failed set)
X_i → X (Latest failure in failed set)

PRESS: CONTINUE

DISPLAY

Q5	Z
R	Y
Q95	X



DISPLAY

1	Z
1	Y
1	X

USER INSTRUCTIONS (Con't)

(Y = 0 when all sets have been entered)

PRESS: END

ENTER PROGRAM 2: Side A

PRESS: GO TO (-)(0)(0)

ENTER PROGRAM 2: Side B

ENTER DATA:

N → Y
F → X

PRESS: CONTINUE

DISPLAY

M	Z
b	Y
r	X

PRESS: CONTINUE

DISPLAY

N	Z
θ	Y
0	X

PRESS: CONTINUE

DISPLAY

0	Z
B%	Y
%	X

If another B% is desired enter the % → X.

To run another case, repeat USER INSTRUCTIONS.

EXAMPLE (Con't)

Input Data

$N = 18$
$F = 4$

S_i	T_i	X_i
2.	3.	1740.
1.	2.	2700.
2.	1.	3510.
4.	3.	4304.
9	9	

Results:

Q_S	R	Q_{95}
.05654	.16592	.33118
.14569	.29675	.48201
.20883	.37670	.56538
.69452	.85640	.95876

$M = 3362.62$
$b = 2.329$
$r = .91171$
$N = 18$
$\theta = 3935.60$

$B_{10\%} = 1497.78$
$B_{50\%} = 3362.62$

STAT-PAC IX-2   Program 1

```
00  CLR  20   Plus    40  DN   25      80  0  00
01  XTO  23   Page    41  DIV  35      81  0  00
02   d   17           42   c   16      82  0  00
03  STP  41   ENTRY   43  AC+  60      83  0  00
04  RUP  22           44  RCL  61      84  0  00
05   1   01           45   .   21      85  0  00
06  RUP  22           46   3   03      86  0  00
07   +   33           47   -   34      87  0  00
08  YTO  40           48   a   13      88  0  00
09   a   13           49  INT  64      89  0  00
0a  RUP  22           4a  GTO  44      8a  0  00
0b  XTO  23           4b   -   34      8b  0  00
0c   b   14           4c   0   00      8c  0  00
0d  STP  41   ENTRY   4d   0   00      8d  0  00

10  PNT  45           50   0   00      90  0  00
11  LN   65           51   0   00      91  0  00
12  YE   24           52   0   00      92  0  00
13   -   34           53   0   00      93  0  00
14   e   12           54   0   00      94  0  00
15   +   33           55   0   00      95  0  00
16  YE   24           56   0   00      96  0  00
17   -   34           57   0   00      97  0  00
18   e   12           58   0   00      98  0  00
19  YTO  40           59   0   00      99  0  00
1a   c   16           5a   0   00      9a  0  00
1b  XEY  30           5b   0   00      9b  0  00
1c  DN   25           5c   0   00      9c  0  00
1d  UP   27           5d   0   00      9d  0  00

20   X   36           60   0   00      a0  0  00
21  XTO  23           61   0   00      a1  0  00
22   5   05           62   0   00      a2  0  00
23  XFR  67           63   0   00      a3  0  00
24   -   34           64   0   00      a4  0  00
25   f   15           65   0   00      a5  0  00
26   +   33           66   0   00      a6  0  00
27  YTO  40           67   0   00      a7  0  00
28   -   34           68   0   00      a8  0  00
29   f   15           69   0   00      a9  0  00
2a   c   16           6a   0   00      aa  0  00
2b  XEY  30           6b   0   00      ab  0  00
2c   d   17           6c   0   00      ac  0  00
2d  RUP  22           6d   0   00      ad  0  00

30   +   33           70   0   00
31  YTO  40           71   0   00
32   d   17           72   0   00
33  RCL  61           73   0   00
34   a   13           74   0   00
35  INT  64           75   0   00
36   -   34           76   0   00
37  RDN  31           77   0   00
38   X   36           78   0   00
39   f   15           79   0   00
3a  RUP  22           7a   0   00
3b   -   34           7b   0   00
3c   d   17           7c   0   00
3d   +   33           7d   0   00
```

$\alpha_3, t_5 + t_{95}$

$(W+1) \; . \; 155\alpha^2$

$R + F_L$

θ

σ

ΣS

$.576\alpha_3$

ΣA

ΣT

b0	0	00	Plus Page
b1	0	00	
b2	0	00	
b3	0	00	
b4	0	00	
b5	0	00	
b6	0	00	
b7	0	00	
b8	0	00	
b9	0	00	
ba	0	00	
bb	0	00	
bc	0	00	
bd	0	00	
c0	0	00	
c1	0	00	
c2	0	00	
c3	0	00	
c4	0	00	
c5	0	00	
c6	0	00	
c7	0	00	
c8	0	00	
c9	0	00	
ca	0	00	
cb	0	00	
cc	0	00	
cd	0	00	
d0	0	00	
d1	0	00	
d2	0	00	
d3	0	00	
d4	0	00	
d5	0	00	
d6	0	00	
d7	0	00	
d8	0	00	
d9	0	00	
da	0	00	
db	0	00	
dc	0	00	
dd	0	00	

STAT-PAC IX-2 Program 1

```
00  UP    27     Minus
01  .     21     Page
02  6     06
03  -     34
04  DN    25
05  DIV   35
06  b     14
07  XEY   30
08  +     33
09  YTO   40
0a  b     14
0b  UP    27
0c  1     01
0d  -     34

10  XEY   30
11  CHS   32
12  DIV   35
13  DN    25
14  LN    65
15  LN    65
16  UP    27
17  UP    27
18  X     36
19  YE    24
1a  8     10
1b  +     33
1c  YE    24
1d  8     10

20  DN    25
21  YE    24
22  9     11
23  +     33
24  YE    24
25  9     11
26  XFR   67
27  5     05
28  X     36
29  XFR   67
2a  7     07
2b  +     33
2c  YTO   40
2d  7     07

30  RCL   61
31  a     13
32  INT   64
33  DIV   35
34  UP    27
35  1     01
36  +     33
37  RUP   22
38  -     34
39  X     36
3a  RDN   31
3b  XEY   30
3c  DIV   35
3d  XEY   30

40  √     76
41  XTO   23
42  5     05
43  1     01
44  +     33
45  RUP   22
46  XTO   23
47  c     16
48  -     34
49  -     34
4a  2     02
4b  X     36
4c  XFR   67
4d  5     05

50  RUP   22
51  X     36
52  DN    25
53  DIV   35
54  YTO   40
55  0     00
56  .     21
57  5     05
58  7     07
59  5     05
5a  6     06
5b  XEY   30
5c  X     36
5d  UP    27

60  X     36
61  .     21
62  0     00
63  1     01
64  5     05
65  5     05
66  X     36
67  0     00
68  RUP   22
69  XTO   23
6a  6     06
6b  X>Y   53
6c  d     17
6d  9     11

70  RUP   22
71  +     33
72  YE    24
73  0     00
74  RUP   22
75  a     13
76  +     33
77  .     21
78  2     02
79  8     10
7a  8     10
7b  5     05
7c  YTO   40
7d  a     13

80  RUP   22
81  X     36
82  UP    27
83  .     21
84  0     00
85  5     05
86  6     06
87  5     05
88  XEY   30
89  X     36
8a  X     36
8b  1     01
8c  .     21
8d  6     06

90  4     04
91  5     05
92  -     34
93  DN    25
94  +     33
95  XFR   67
96  6     06
97  YE    24
98  0     00
99  X>Y   53
9a  a     13
9b  d     17
9c  YE    24
9d  0     00

a0  RUP   22
a1  -     34
a2  -     34
a3  DN    25
a4  +     33
a5  a     13
a6  UP    27
a7  INT   64
a8  -     34
a9  DN    25
aa  -     34
ab  YE    24
ac  0     00
ad  XFR   67
```

```
b0   0     00    Minus
b1   -     34    Page
b2   RUP   22
b3   XFR   67
b4   5     05
b5   X     36
b6   RUP   22
b7   X     36
b8   c     16
b9   +     33
ba   RUP   22
bb   +     33
bc   b     14
bd   YE    24

c0   b     14
c1   INT   64
c2   -     34
c3   XEY   30
c4   YE    24
c5   b     14
c6   RUP   22
c7   PNT   45    S
c8   PNT   45
c9   b     14
ca   UP    27
cb   1     01
cc   -     34
cd   YTO   40

d0   b     14
d1   X>Y   53
d2   UP    27
d3   UP    27
d4   STP   41
d5   GTO   44
d6   +     33
d7   1     01
d8   0     00
d9   RUP   22
da   -     34
db   GTO   44
dc   7     07
dd   2     02
```

STAT-PAC IX-2 Program 2

```
00  XTO  23     Plus
01   a   13     Page
02  YTO  40
03   e   12
04  XFR  67
05   -   34
06   e   12
07  UP   27
08   X   36
09  UP   27
0a  XFR  67
0b   8   10
0c   X   36
0d   a   13

10  YE   24
11   7   07
12   X   36
13  XEY  30
14  YE   24
15   7   07
16  XEY  30
17   -   34
18  YE   24
19   7   07
1a  XFR  67
1b   -   34
1c   f   15
1d   X   36

20  DN   25
21  XEY  30
22   -   34
23  XFR  67
24   7   07
25  XEY  30
26  DIV  35
27  YTO  40
28   b   14
29  XTO  23
2a   d   17
2b  XFR  67
2c   -   34
2d   e   12

30   X   36
31  XFR  67
32   8   10
33   -   34
34   a   13
35  DIV  35
36  DN   25
37  EXP  74
38  UP   27
39   .   21
3a   6   06
3b   9   11
3c   3   03
3d   1   01

40   5   05
41   X   36
42  DN   25
43  LN   65
44  UP   27
45   b   14
46  DIV  35
47  DN   25
48  EXP  74
49  XTO  23
4a   c   16
4b  XFR  67
4c   8   10
4d  UP   27

50   X   36
51  XFR  67
52   9   11
53  UP   27
54   a   13
55   X   36
56  DN   25
57  XEY  30
58   -   34
59   d   17
5a   X   36
5b  DN   25
5c   √   76
5d  YE   24

60   7   07
61  DIV  35
62   c   16
63  UP   27
64   b   14
65  RUP  22
66  PNT  45     S
67   e   12
68  UP   27
69  GTO  44
6a   -   34
6b   0   00
6c   0   00
6d  END  46
```

STAT-PAC IX-2 Program 2

```
00   c    16     Minus     40  PNT  45
01   LN   65     Page      41  STP  41
02   UP   27               42  GTO  44
03   b    14               43  2    02
04   X    36               44  3    03
05   2    02               45  END  46
06   LN   65
07   LN   65
08   -    34
09   b    14
0a   DIV  35
0b   DN   25
0c   EXP  74
0d   UP   27

10   0    00
11   PNT  45    s
12   PNT  45
13   YTO  40
14   a    13
15   1    01
16   UP   27
17   b    14
18   DIV  35
19   YTO  40
1a   d    17
1b   a    13
1c   LN   65
1d   XTO  23

20   e    12
21   1    01
22   0    00
23   XTO  23
24   f    15
25   UP   27
26   EEX  26
27   2    02
28   DIV  35
29   1    01
2a   -    34
2b   XEY  30
2c   CHS  32
2d   DIV  35

30   DN   25
31   LN   65
32   LN   65
33   UP   27
34   d    17
35   X    36
36   e    12
37   +    33
38   0    00
39   RDN  31
3a   EXP  74
3b   XEY  30
3c   f    15
3d   PNT  45    s
```

θ

b

Med. life

$\frac{1}{b}$

ln θ

%

GENERAL STATISTICS FOR QUALITY CONTROL

This program written by J.S. Madachy of Monsanto Research Corp., Miamisburg, Ohio, determines various statistics useful in quality control. Given a sample of n observations the following statistics are computed:

$\bar{X}$ = Sample Mean
s_X^2 = Sample Variance
s_X = Sample Standard Deviation
$\bar{X} + 3s_X$ = Upper 3 Sigma Limit
$\bar{X} - 3s_X$ = Lower 3 Sigma Limit
$X_{max} - \bar{X}$ = Maximum Positive Excursion From Mean
$\bar{X} - X_{min}$ = Maximum Negative Excursion From Mean
X_{max} = Maximum Observation
X_{min} = Minimum Observation
R = Range ($X_{max} - X_{min}$)

The sample mean and variance are given by:

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

$$s_X^2 = \frac{\sum_{i=1}^{n} X_i^2 - \frac{(\Sigma X_i)^2}{n}}{n - 1}$$

This program has been designed for use with the 9120A Printer but can be used without the printer by placing STOP statements in program locations containing PRINT statements as shown. Replace other PRINT statements with CONTINUE. Results appear in the Y register.

USER INSTRUCTIONS

DEPRESS: Y on the 9120A

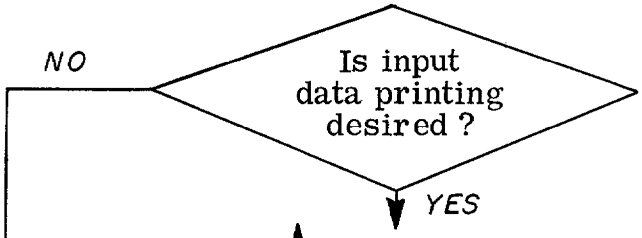
PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0 ——————— Z
0 ——————— Y
1 ——————— X



PRESS: ↑

ENTER DATA: $X_1 \longrightarrow X$

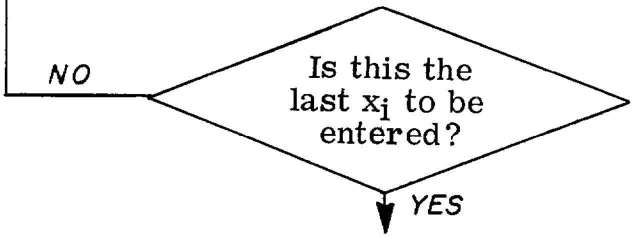
PRESS: CONTINUE

DISPLAY

1 ——————— Z
1 ——————— Y
i ——————— X

(i is input data number: i = 2, 3, . . . n)

ENTER DATA: $X_i \longrightarrow X$



PRESS: SET FLAG

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

The general statistics will now be printed in the following order:

n	# of Data Entries
$\overline{X}$	Mean Value of Data Set
s_x^2	Variance of Data Set
s_x	Standard Deviation of Data Set

$\overline{X} + 3s_x$

$\overline{X} - 3s_x$

$\text{Max } X_i - \overline{X}$

$\overline{X} - \text{Min } X_i$

$\text{Max } X_i$

$\text{Min } X_i$

Range

EXAMPLE

The following measurements represent the observed diameters of ball bearing produced by machine A.

Input data:

1.157
1.152
1.150
1.141
1.135
1.116
1.133
1.128
1.146
1.123
1.121
1.139

$n = 12$

$\overline{X} = 1.13675$

$s_x^2 = .00017$

$s_x = .01310$

EXAMPLE (Con't)

$\overline{X} + 3s_x = 1.17606$

$\overline{X} - 3s_x = 1.09744$

$\text{Max } X_i - \overline{X} = .02025$

$\overline{X} - \text{Min } X_i = .02075$

$\text{Max } X_i = 1.157$

$\text{Min } X_i = 1.116$

Range = .041

```
00  CLR  20
01   1   01
02  XTO  23
03   d   17
04  STP  41   ENTRY
05  YTO  40
06   c   16
07  RDN  31
08  X>Y  53
09   8   10
0a   8   10
0b  RUP  22
0c  XTO  23
0d   a   13

10  XTO  23
11   b   14
12  UP   27
13   X   36
14  AC+  60
15   d   17
16  UP   27
17  UP   27
18   2   02
19  XTO  23
1a   d   17
1b  STP  41   ENTRY
1c  RDN  31
1d   c   16

20  X=Y  50
21   8   10
22   8   10
23  RUP  22
24  UP   27
25   a   13
26  X>Y  53
27  YTO  40
28   a   13
29   b   14
2a  X<Y  52
2b  YTO  40
2c   b   14
2d  DN   25

30  UP   27
31   X   36
32  AC+  60
33   d   17
34  UP   27
35   1   01
36  IFG  43
37   4   04
38   3   03
39   +   33
3a  YTO  40
3b   d   17
3c  UP   27
3d  RUP  22

40  GTO  44
41   1   01
42   b   14
43  PNT  45   S
44   -   34
45   f   15
46  UP   27
47   d   17
48  DIV  35
49  PNT  45
4a  YTO  40
4b   c   16
4c   f   15
4d   X   36

50   e   12
51  XEY  30
52   -   34
53  DN   25
54  XEY  30
55  DIV  35
56  PNT  45   S
57  DN   25
58   √   76
59  UP   27
5a  PNT  45   S
5b  PNT  45
5c   +   33
5d   +   33

60   c   16
61  UP   27
62  DN   25
63   +   33
64  PNT  45   S
65  RUP  22
66   -   34
67  PNT  45   S
68  PNT  45
69   +   33
6a   a   13
6b   -   34
6c   b   14
6d  UP   27

70   c   16
71   -   34
72  PNT  45   S
73  DN   25
74  PNT  45   S
75  PNT  45
76   b   14
77  UP   27
78   a   13
79  PNT  45   S
7a  UP   27
7b  PNT  45
7c  DN   25
7d   -   34

80  PNT  45
81  PNT  45
82  PNT  45
83  PNT  45
84  PNT  45
85  GTO  44
86   0   00
87   0   00
88  RDN  31
89  IFG  43
8a   9   11
8b   6   06
8c  PNT  45
8d  RUP  22

90  X=Y  50
91   2   02
92   3   03
93  GTO  44
94   0   00
95   b   14
96  PNT  45
97  PNT  45
98  PNT  45
99  SFL  54
9a  GTO  44
9b   2   02
9c   5   05
9d  END  46
```

REPEATABILITY

This program computes parameters associated with the repeatability of a measuring (or experimental) process. The statistics are given by:

sample = Collection of n_1 repeat readings on a single piece

X = A reading on a piece

n_1 = Number of repeat readings on a piece

$\overline{X}_i$ = Average Value of the i^{th} Sample

s^2_{ei} = Variance (error) of the i^{th} Sample

n = Number of pieces tested

$\overline{\overline{X}}$ = Pooled Mean $= \frac{1}{n} \sum_{i=1}^{n} \overline{X}_i$

$$s_e = s_{\text{pooled error}} = \sqrt{\frac{\sum_{i=1}^{n} s^2_{ei}}{n}}$$

$$s_{TV} = s_{\text{true value}} = \sqrt{s^2_{\overline{X}_i} - \frac{s^2_e}{n_1}} \qquad \text{(measure of spread)}$$

$$s_{RDG} = s_{\text{reading}} = \sqrt{s^2_e + s^2_{TV}}$$

The program determines additional statistics including

$3s_e$, $3s_{TV}$, $3s_{RDG}$

$\overline{\overline{X}} + 3s_{TV}$, $\overline{\overline{X}} - 3s_{TV}$

$\overline{\overline{X}} + 3s_{RDG}$, $\overline{\overline{X}} - 3s_{RDG}$

This REPEATABILITY program was written by Mr. John Barr of Delco-Remy Division of General Motors, Anderson, Indiana.

Reference: According to Mr. Barr, "the techniques used throughout this program are derived from analysis of variance procedures. However, to my knowledge, there is no published reference available that would explain and deal with Repeatability in exactly the same manners we have used."

USER INSTRUCTIONS

DEPRESS: X on the 9120A

PRESS: END

ENTER PROGRAM A

PRESS: CONTINUE

DISPLAY

0 —— Z
0 —— Y
0 —— X

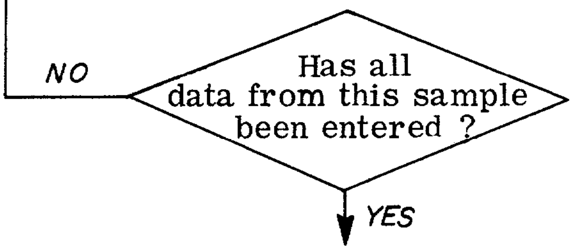
ENTER DATA: X $\longrightarrow$ X

PRESS: CONTINUE

DISPLAY

0 —— Z
j —— Y
0 —— X

(j=number of points entered in this sample)



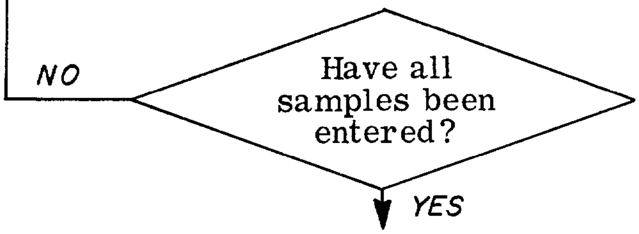
PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

n_1 —— Z
$\bar{X}_1$ —— Y
s^2_{ei} —— X

(i is sample counter)



USER INSTRUCTIONS (Con't)

PRESS: END

ENTER PROGRAM B

PRESS: CONTINUE

ENTER DATA: $n_1 \longrightarrow X$

PRESS: CONTINUE

DISPLAY

$\bar{n}$ —— Z
$\bar{\bar{X}}$ —— Y
0 —— X

PRESS: CONTINUE

DISPLAY

s_{error} —— Z
s_{TV} —— Y
s_{RDG} —— X

PRESS: CONTINUE

DISPLAY

$3s_{error}$ —— Z
$3s_{TV}$ —— Y
$3s_{RDG}$ —— X

PRESS: CONTINUE

DISPLAY

$\bar{\bar{X}}$ —— Z
$\bar{\bar{X}} - 3s_{TV}$ —— Y
$\bar{\bar{X}} + 3s_{TV}$ —— X

PRESS: CONTINUE

DISPLAY

$\bar{\bar{X}}$ —— Z
$\bar{\bar{X}} - 3s_{RDG}$ —— Y
$\bar{\bar{X}} + 3s_{RDG}$ —— X

To run another case, return to the beginning of the USER INSTRUCTIONS.

EXAMPLE

This example was supplied by Mr. Barr.

Consider an example concerning switch operating forces where we are trying to determine the part of the variation of readings due to repeatability and the part that is true variation of switches. Shown below are force measurements on ten switches where each switch was checked three times.

Switch No.	Repeat Readings I	II	III
1	4.0	4.25	4.0
2	3.5	3.5	3.5
3	4.75	5.25	5.0
4	4.5	4.75	4.5
5	3.25	3.5	3.5
6	2.5	3.0	3.0
7	3.5	3.5	3.75
8	4.0	4.0	4.0
9	2.5	2.75	2.5
10	2.75	3.25	3.0

Results:

n_1 = 3.
$\overline{X}_1$ = 4.083333
s_{e1}^2 = .02083

n_1 = 3.
$\overline{X}_2$ = 3.50000
s_{e2}^2 = 0.00000

n_1 = 3.
$\overline{X}_3$ = 5.00000
s_{e3}^2 = .06250

n_1 = 3.
$\overline{X}_4$ = 4.58333
s_{e4}^2 = .02083

n_1 = 3.
$\overline{X}_5$ = 3.41667
s_{e5}^2 = .02083

n_1 = 3.
$\overline{X}_6$ = 2.83333
s_{e6}^2 = .08333

n_1 = 3.
$\overline{X}_7$ = 3.58333
s_{e7}^2 = .02083

n_1 = 3.
$\overline{X}_8$ = 4.00000
s_{e8}^2 = 0.00000

n_1 = 3.
$\overline{X}_9$ = 2.58333
s_{e9}^2 = .02083

n_1 = 3.
$\overline{X}_{10}$ = 3.00000
s_{e10}^2 = .06250

EXAMPLE (Con't)

n = 10.
$\overline{\overline{\overline{X}}}$ = 3.65833

s_{error} = .17678
s_{TV} = .76144
s_{RDG} = .78169

$3s_{error}$ = .53033
$3s_{TV}$ = 2.28431
$3s_{RDG}$ = 2.34506

$\overline{\overline{\overline{X}}}$ = 3.65833
$\overline{\overline{\overline{X}}} - 3s_{TV}$ = 1.37403
$\overline{\overline{\overline{X}}} + 3s_{TV}$ = 5.94264

$\overline{\overline{\overline{X}}}$ = 3.65833
$\overline{\overline{\overline{X}}} - 3s_{RDG}$ = 1.31327
$\overline{\overline{\overline{X}}} + 3s_{RDG}$ = 6.00339

STAT-PAC IX-4 Program A

```
00  CLR  20              40  DN   25         80  0  00
01  STP  41   ENTRY      41  UP   27         81  0  00
02  IFG  43              42  X    36         82  0  00
03  1    01              43  UP   27         83  0  00
04  8    10              44  c    16         84  0  00
05  PNT  45              45  +    33         85  0  00
06  PNT  45              46  YTO  40         86  0  00
07  UP   27              47  c    16         87  0  00
08  X    36              48  b    14         88  0  00
09  AC+  60              49  RUP  22         89  0  00
0a  YEX  24              4a  +    33         8a  0  00
0b  7    07              4b  YTO  40         8b  0  00
0c  1    01              4c  b    14         8c  0  00
0d  +    33              4d  YEX  24         8d  0  00

10  CLX  37              50  a    13         90  0  00
11  UP   27              51  1    01         91  0  00
12  RDN  31              52  +    33         92  0  00
13  YTO  40              53  YTO  40         93  0  00
14  7    07              54  a    13         94  0  00
15  GTO  44              55  CLR  20         95  0  00
16  0    00              56  YEX  24         96  0  00
17  1    01              57  7    07         97  0  00
18  f    15              58  DN   25         98  0  00
19  XEY  30              59  YEX  24         99  0  00
1a  DIV  35              5a  8    10         9a  0  00
1b  XEY  30              5b  RDN  31         9b  0  00
1c  UP   27              5c  YEX  24         9c  0  00
1d  X    36              5d  9    11         9d  0  00

20  XTO  23              60  RUP  22         a0  0  00
21  f    15              61  PNT  45         a1  0  00
22  DN   25              62  RUP  22         a2  0  00
23  X    36              63  PNT  45         a3  0  00
24  e    12              64  RUP  22         a4  0  00
25  XEY  30              65  PNT  45   S     a5  0  00
26  -    34              66  PNT  45         a6  0  00
27  1    01              67  PNT  45         a7  0  00
28  CHS  32              68  PNT  45         a8  0  00
29  RUP  22              69  CLR  20         a9  0  00
2a  +    33              6a  GTO  44         aa  0  00
2b  RDN  31              6b  0    00         ab  0  00
2c  DIV  35              6c  1    01         ac  0  00
2d  f    15              6d  0    00         ad  0  00

30  XTO  23              70  0    00
31  9    11              71  0    00
32  YTO  40              72  0    00
33  8    10              73  0    00
34  RDN  31              74  0    00
35  YTO  40              75  0    00
36  7    07              76  0    00
37  RUP  22              77  0    00
38  XEY  30              78  0    00
39  YEX  24              79  0    00
3a  d    17              7a  0    00
3b  +    33              7b  0    00
3c  YEX  24              7c  0    00
3d  d    17              7d  0    00
```

```
b0   0    00                 10  YEX  24          50  3    03
b1   0    00                 11  e    12          51  X    36
b2   0    00                 12  YEX  24          52  YEX  24
b3   0    00                 13  c    16          53  f    15
b4   0    00                 14  a    13          54  X    36
b5   0    00                 15  DIV  35          55  RUP  22
b6   0    00                 16  XEY  30          56  X    36
b7   0    00                 17  RUP  22          57  f    15
b8   0    00                 18  CNT  47          58  RUP  22
b9   0    00                 19  RUP  22          59  CNT  47
ba   0    00                 1a  PNT  45          5a  RUP  22
bb   0    00                 1b  RUP  22          5b  PNT  45
bc   0    00                 1c  PNT  45          5c  RUP  22
bd   0    00                 1d  RUP  22          5d  PNT  45

c0   0    00                 20  PNT  45   S      60  RUP  22
c1   0    00                 21  PNT  45          61  PNT  45   S
c2   0    00                 22  YTO  40          62  PNT  45
c3   0    00                 23  c    16          63  XTO  23
c4   0    00                 24  DN   25          64  e    12
c5   0    00                 25  UP   27          65  c    16
c6   0    00                 26  X    36          66  XEY  30
c7   0    00                 27  DN   25          67  -    34
c8   0    00                 28  X    36          68  UP   27
c9   0    00                 29  b    14          69  DN   25
ca   0    00                 2a  XEY  30          6a  +    33
cb   0    00                 2b  -    34          6b  +    33
cc   0    00                 2c  1    01          6c  c    16
cd   0    00                 2d  CHS  32          6d  RDN  31

d0   0    00                 30  RUP  22          70  CNT  47
d1   0    00                 31  +    33          71  RUP  22
d2   0    00                 32  RDN  31          72  PNT  45
d3   0    00                 33  DIV  35          73  RUP  22
d4   0    00                 34  e    12          74  PNT  45
d5   0    00                 35  -    34          75  RUP  22
d6   0    00                 36  YTO  40          76  PNT  45   S
d7   0    00                 37  e    12          77  PNT  45
d8   0    00                 38  d    17          78  DN   25
d9   0    00                 39  +    33          79  e    12
da   0    00                 3a  YTO  40          7a  -    34
db   0    00                 3b  f    15          7b  UP   27
dc   0    00                 3c  √    76          7c  DN   25
dd   END  46                 3d  UP   27          7d  +    33

00   YEX  24   Program B     40  e    12
01   d    17                 41  √    76
02   a    13                 42  RUP  22
03   DIV  35                 43  √    76
04   YTO  40                 44  CNT  47
05   d    17                 45  RUP  22
06   CLR  20                 46  PNT  45
07   STP  41   ENTRY         47  RUP  22
08   PNT  45                 48  PNT  45
09   PNT  45                 49  RUP  22
0a   UP   27                 4a  PNT  45   S
0b   d    17                 4b  PNT  45
0c   XEY  30                 4c  XTO  23
0d   DIV  35                 4d  f    15
```

```
80    +    33
81    c    16
82   RDN   31
83   CNT   47
84   RUP   22
85   PNT   45
86   RUP   22
87   PNT   45
88   RUP   22
89   PNT   45
8a   PNT   45
8b   PNT   45
8c   PNT   45
8d   END   46
```

SECTION X PLOTTER PROGRAMS

9100B ONLY
STAT-PAC
X-1 ▲

HISTOGRAM GENERATION (WITH PLOT)

This program generates and plots a histogram of ten windows given a data set of positive numbers. In addition, it determines the mean (M_x) and the variance($\sigma_x 2$) of the raw data, and the mean (M_h) and the variance ($\sigma_h 2$) of the normalized histogram data. Since the raw data is normalized by the program to values $0 \leq h \leq 10$, the new mean and variance are given by

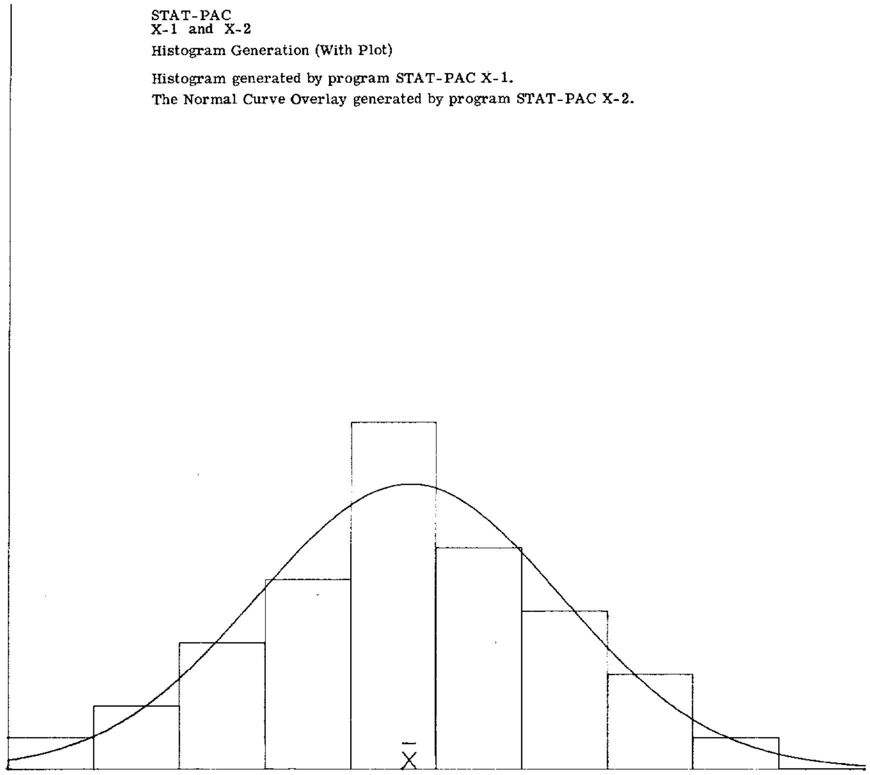
$$M_h = \frac{M_x}{W}$$

$$\sigma_h 2 = \frac{\sigma_x^2}{W^2}$$

where W is the histogram window width (normalizing factor)*. The program plots the histogram and stores M_h and σ_h^2 for use by program STAT-PAC X-2 which can be used to plot a normal curve over the histogram.

This program uses <u>Indirect Addressing</u> and is self-destructing of the registers +(0,0) through +(d,d). Thus, to rerun, the A side must be re-entered in the calculator.

NOTE: To generate a histogram with 1 cm. wide windows, place 2's in locations (-)(6)(c), (-)(7)(6), and (-)(8)(5).



STAT-PAC
X-1 and X-2
Histogram Generation (With Plot)

Histogram generated by program STAT-PAC X-1.
The Normal Curve Overlay generated by program STAT-PAC X-2.

---

* The window width W is chosen such that all normalized data entries X/W will lie between 0 and 10. Thus, if the data ranges from 0 → 200, a W of of 20 would be proper.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

Using the origin controls, locate the pen at X = 1 in., Y = 1 in.

SET: Decimal Wheel at 6 or less

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

DISPLAY

0 ———— Z
0 ———— Y
0 ———— X

ENTER DATA: W ⟶ X

PRESS: CONTINUE

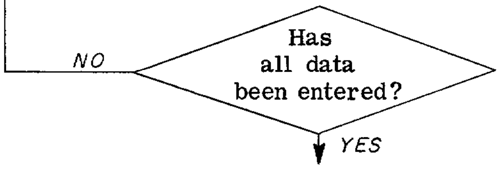
DISPLAY

N ———— Z
N ———— Y
1 ———— X

N is the number of data points entered.

ENTER DATA: $X_{N+1} \rightarrow X$

PRESS: CONTINUE



PRESS: SET FLAG

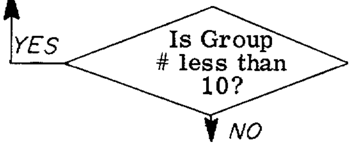
PRESS: CONTINUE

DISPLAY

% ———— Z (% of data points in Group)
K_A ———— Y (No. of data points in Group)
Group ———— X (Group #)

PRESS: CONTINUE

Plot Window



USER INSTRUCTIONS (Con't)

DISPLAY

σ_x^2 ———— Z
M_x ———— Y
0 ———— X

PRESS: CONTINUE

DISPLAY

σ_h^2 ———— Z
M_h ———— Y
1 ———— X

EXAMPLE

The data set is:

104, 92, 83, 78, 58, 135, 146, 24, 74, 85, 81, 128, 140, 113, 79, 78, 53, 42, 34, 85, 96, 110, 133, 158, 171, 108, 84, 90, 73, 11, 51, 118, 68, 139, 92, 109, 89, 124, 91, 116.

The data varies between 0 and 200 so W is chosen to be 20.

Result

Group	K_A	%
1	1	2.5
2	2	5.
3	4	10
4	6	15.
5	11	27.5
6	7	17.5
7	5	12.5
8	3	7.5
9	1	2.5
10	0	0

N = 40

$\sigma_x^2 = 1252.644$ $\sigma_h^2 = 3.132$
$M_x = 93.575$ $M_h = 4.679$

The histogram plot is given with the normal curve superimposed. The normal curve resulted from running program 09100-70904 following completion of the Histogram Generation program.

STAT-PAC X-1

00	CLR	20	Plus Page	40	CNT	47		80	CNT	47
01	XTO	23		41	CNT	47		81	CNT	47
02	9	11		42	CNT	47		82	CNT	47
03	XTO	23		43	CNT	47		83	CNT	47
04	8	10		44	CNT	47		84	CNT	47
05	XTO	23		45	CNT	47		85	CNT	47
06	7	07		46	CNT	47		86	CNT	47
07	XTO	23		47	CNT	47		87	CNT	47
08	6	06		48	CNT	47		88	CNT	47
09	XTO	23		49	CNT	47		89	CNT	47
0a	5	05		4a	CNT	47		8a	CNT	47
0b	XTO	23		4b	CNT	47		8b	CNT	47
0c	4	04		4c	CNT	47		8c	CNT	47
0d	XTO	23		4d	CNT	47		8d	CNT	47
10	3	03		50	CNT	47		90	CNT	47
11	STP	41	ENTRY	51	CNT	47		91	CNT	47
12	PNT	45		52	CNT	47		92	CNT	47
13	PNT	45		53	CNT	47		93	CNT	47
14	XTO	23		54	CNT	47		94	CNT	47
15	a	13		55	CNT	47		95	CNT	47
16	CLX	37		56	CNT	47		96	CNT	47
17	XTO	23		57	CNT	47		97	CNT	47
18	2	02		58	CNT	47		98	CNT	47
19	XTO	23		59	CNT	47		99	CNT	47
1a	0	00		5a	CNT	47		9a	CNT	47
1b	GTO	44		5b	CNT	47		9b	CNT	47
1c	c	16		5c	CNT	47		9c	CNT	47
1d	0	00		5d	CNT	47		9d	CNT	47
20	CNT	47		60	CNT	47		a0	CNT	47
21	CNT	47		61	CNT	47		a1	CNT	47
22	CNT	47		62	CNT	47		a2	CNT	47
23	CNT	47		63	CNT	47		a3	CNT	47
24	CNT	47		64	CNT	47		a4	CNT	47
25	CNT	47		65	CNT	47		a5	CNT	47
26	CNT	47		66	CNT	47		a6	CNT	47
27	CNT	47		67	CNT	47		a7	CNT	47
28	CNT	47		68	CNT	47		a8	CNT	47
29	CNT	47		69	CNT	47		a9	CNT	47
2a	CNT	47		6a	CNT	47		aa	CNT	47
2b	CNT	47		6b	CNT	47		ab	CNT	47
2c	CNT	47		6c	CNT	47		ac	CNT	47
2d	CNT	47		6d	CNT	47		ad	CNT	47
30	CNT	47		70	CNT	47				
31	CNT	47		71	CNT	47				
32	CNT	47		72	CNT	47				
33	CNT	47		73	CNT	47				
34	CNT	47		74	CNT	47				
35	CNT	47		75	CNT	47				
36	CNT	47		76	CNT	47				
37	CNT	47		77	CNT	47				
38	CNT	47		78	CNT	47				
39	CNT	47		79	CNT	47				
3a	CNT	47		7a	CNT	47				
3b	CNT	47		7b	CNT	47				
3c	CNT	47		7c	CNT	47				
3d	CNT	47		7d	CNT	47				

K_0	K_8
K_1	K_9
K_2	W
K_3	N
K_4	
K_5	
K_6	$\Sigma X^2/\sigma_h^2$
K_7	$\Sigma X/M_x/M_h$

STAT-PAC X-1

b0	CNT	47	Plus Page
b1	CNT	47	
b2	CNT	47	
b3	CNT	47	
b4	CNT	47	
b5	CNT	47	
b6	CNT	47	
b7	CNT	47	
b8	CNT	47	
b9	CNT	47	
ba	CNT	47	
bb	CNT	47	
bc	CNT	47	
bd	CNT	47	
c0	XTO	23	
c1	1	01	
c2	1	01	
c3	CHS	32	
c4	RDN	31	
c5	DN	25	
c6	1	01	
c7	+	33	
c8	UP	27	
c9	DN	25	
ca	STP	41	ENTRY
cb	CNT	47	
cc	IFG	43	
cd	d	17	
d0	d	17	
d1	PNT	45	
d2	PNT	45	
d3	UP	27	
d4	X	36	
d5	AC+	60	
d6	XEY	30	
d7	a	13	
d8	DIV	35	
d9	GTO	44	
da	-	34	
db	0	00	
dc	2	02	
dd	GTO	44	

STAT-PAC X-1

00	3	03	Minus Page	40	0	00		80	0	00	
01	0	00		41	0	00		81	XEY	30	
02	DN	25		42	0	00		82	FMT	42	
03	INT	64		43	0	00		83	DN	25	
04	UP	27		44	0	00		84	UP	27	
05	EEX	26		45	0	00		85	5	05	
06	9	11		46	0	00		86	0	00	
07	+	33		47	0	00		87	0	00	
08	YTO	40		48	0	00		88	DIV	35	
09	-	34		49	0	00		89	1	01	
0a	1	01		4a	0	00		8a	0	00	
0b	CLX	37		4b	0	00		8b	X>Y	53	
0c	CLX	37		4c	0	00		8c	3	03	
0d	YE	24		4d	0	00		8d	4	04	
10	0	00		50	YTO	40		90	0	00	
11	0	00		51	-	34		91	UP	27	
12	0	00		52	1	01		92	FMT	42	
13	0	00		53	b	14		93	DN	25	
14	0	00		54	DIV	35		94	5	05	
15	0	00		55	EEX	26		95	EEX	26	
16	0	00		56	2	02		96	3	03	
17	0	00		57	X	36		97	XEY	30	
18	0	00		58	1	01		98	FMT	42	
19	0	00		59	RUP	22		99	DN	25	
1a	0	00		5a	+	33		9a	UP	27	
1b	0	00		5b	XFR	67		9b	FMT	42	
1c	0	00		5c	-	34		9c	DN	25	
1d	0	00		5d	1	01		9d	RCL	61	
20	IFG	43		60	XEY	30		a0	UP	27	
21	2	02		61	PNT	45	S	a1	b	14	
22	a	13		62	PNT	45		a2	DIV	35	
23	CLX	37		63	RUP	22		a3	YTO	40	
24	1	01		64	UP	27		a4	f	15	
25	+	33		65	7	07		a5	X	36	
26	SFL	54		66	5	05		a6	f	15	
27	GTO	44		67	X	36		a7	X	36	
28	0	00		68	RUP	22		a8	DN	25	
29	d	17		69	XEY	30		a9	-	34	
2a	GTO	44		6a	1	01		aa	b	14	
2b	+	33		6b	-	34		ab	DIV	35	
2c	c	16		6c	5	05		ac	f	15	
2d	5	05		6d	0	00		ad	UP	27	
30	YTO	40		70	0	00					
31	b	14		71	X	36					
32	CLX	37		72	DN	25			K_A		
33	UP	27		73	FMT	42					
34	EEX	26		74	DN	25					
35	9	11		75	UP	27					
36	+	33		76	5	05					
37	YTO	40		77	0	00					
38	-	34		78	0	00					
39	4	04		79	+	33					
3a	-	34		7a	DN	25					
3b	0	00		7b	FMT	42					
3c	UP	27		7c	DN	25					
3d	YE	24		7d	XEY	30					

b0	0	00		Minus
b1	PNT	45	s	Page
b2	PNT	45		
b3	a	13		
b4	DIV	35		
b5	XEY	30		
b6	RDN	31		
b7	DIV	35		
b8	DIV	35		
b9	DN	25		
ba	XEY	30		
bb	XTO	23		
bc	f	15		
bd	YTO	40		
c0	e	12		
c1	UP	27		
c2	1	01		
c3	PNT	45	s	
c4	PNT	45		
c5	0	00		
c6	UP	27		
c7	UP	27		
c8	FMT	42		
c9	UP	27		
ca	END	46		

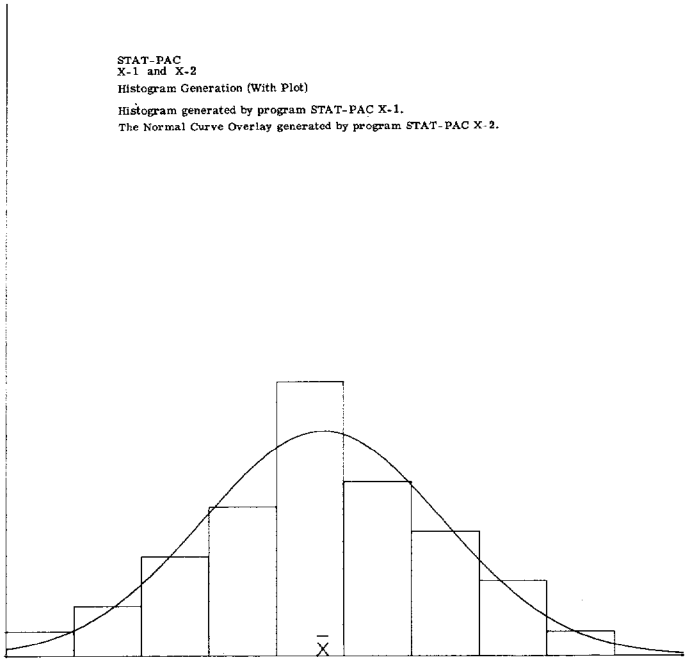
NORMAL CURVE OVERLAY

This program generates a normal curve given mean M_h and variance σ^2_h. The program determines Y from:

$$Y = \frac{1}{\sigma \sqrt{2\pi}} e^{\frac{-(h - M_h)^2}{2\sigma^2_h}}$$

by varying h from 0 to 10 in increments of 0.1. The program requires that M_h and σ^2_h be stored in the f and e registers respectively prior to execution. This program was intended to be used in conjunction with program STAT-PAC X-1 Histogram Generation (With Plot). To plot in units of centimeters, place a 2 in locations (2)(5) (2)(c), and (4)(a).

USER INSTRUCTIONS	EXAMPLE
PRESS: STOP Using the origin controls, locate the pen at X = 1 in., Y = 1 in. SET: Decimal Wheel at 6 or less PRESS: END ENTER PROGRAM PRESS: CONTINUE	M_h = 4.679 σ_h^2 = 3.132 See plot below



STAT-PAC
X-1 and X-2
Histogram Generation (With Plot)

Histogram generated by program STAT-PAC X-1.
The Normal Curve Overlay generated by program STAT-PAC X-2.

STAT-PAC X-2

```
00   e     12
01   √     76
02   UP    27
03   π     56
04   UP    27
05   2     02
06   X     36
07   DN    25
08   √     76
09   X     36
0a   YTO   40
0b   c     16
0c   0     00
0d   UP    27

10   YTO   40
11   d     17
12   f     15
13   -     34
14   DN    25
15   UP    27
16   X     36
17   e     12
18   UP    27
19   2     02
1a   X     36
1b   DN    25
1c   DIV   35
1d   DN    25

20   CHS   32
21   EXP   74
22   UP    27
23   c     16
24   DIV   35
25   7     07
26   5     05
27   0     00
28   0     00
29   X     36
2a   d     17
2b   UP    27
2c   5     05
2d   0     00

30   0     00
31   X     36
32   DN    25
33   FMT   42
34   DN    25
35   d     17
36   UP    27
37   .     21
38   1     01
39   +     33
3a   1     01
3b   0     00
3c   .     21
3d   1     01

40   X>Y   53
41   1     01
42   0     00
43   0     00
44   UP    27
45   UP    27
46   FMT   42
47   UP    27
48   f     15
49   UP    27
4a   5     05
4b   0     00
4c   0     00
4d   X     36

50   4     04
51   0     00
52   -     34
53   0     00
54   XEY   30
55   FMT   42
56   DN    25
57   UP    27
58   8     10
59   0     00
5a   +     33
5b   EEX   26
5c   2     02
5d   XEY   30

60   FMT   42
61   DN    25
62   FMT   42
63   UP    27
64   XEY   30
65   0     00
66   XEY   30
67   FMT   42
68   DN    25
69   UP    27
6a   8     10
6b   0     00
6c   -     34
6d   EEX   26

70   2     02
71   XEY   30
72   FMT   42
73   DN    25
74   FMT   42
75   UP    27
76   XEY   30
77   UP    27
78   5     05
79   0     00
7a   +     33
7b   DN    25
7c   XEY   30
7d   FMT   42

80   DN    25
81   UP    27
82   8     10
83   0     00
84   +     33
85   DN    25
86   FMT   42
87   DN    25
88   FMT   42
89   UP    27
8a   0     00
8b   UP    27
8c   UP    27
8d   FMT   42

90   UP    27
91   END   46
```

$\sigma_h \sqrt{2\pi}$

h

σ_h^2

M_h

NORMAL CURVE PLOT (GENERAL)

This program generates a normal curve given the mean and variance of the normal distribution. The curve plotted is given by:

$$Y(h) = \frac{e^{-\frac{(h - \mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}$$

where μ = Mean of distribution

σ^2 = Variance of distribution

The program is written to vary h between 0 and 10, in increments of 0.1. To decrease the plotting time, change Δh in locations (3)(d) and (4)(0).

To convert the plot from units of inches to centimeters, place 2's in locations (2)(b) and (3)(4).

To place axes on the plot with program STAT-PAC X-19, AXES PLOT, use the following input values:

$Y_{shift} = X_{shift} = 0$

$Y_{scale} = \frac{1}{15} = .06667$

$X_{scale} = 1$

$Y_{origin} = X_{origin} = 0$

$Y_{tic} = .1$

$X_{tic} = 1$

PRESS: STOP

Using the origin controls, locate the pen at X = 1 in., Y = 1 in. (2.54 cm., 2.54 cm.).

SET: Decimal wheel at 6 or less.

PRESS: END

ENTER PROGRAM

→ PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
1 ——— X

ENTER DATA:

$\sigma^2 \longrightarrow Y$
$\mu \longrightarrow X$

PRESS: CONTINUE

The normal curve is now plotted.

To run another case

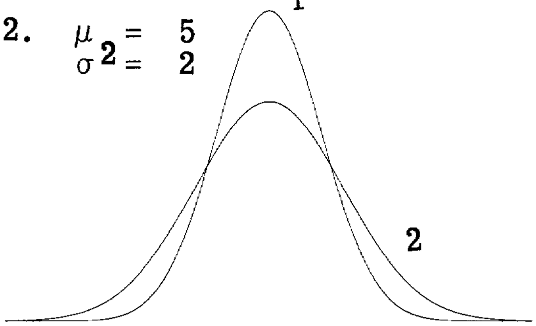
PRESS: END

STAT-PAC
X-3

Normal Curve Plot (General)

EXAMPLE

1. $\mu = 5$
   $\sigma^2 = 1$
2. $\mu = 5$
   $\sigma^2 = 2$



STAT-PAC X-3

Step	Key	Code	X	Y	Z	Step	Key	Code	X	Y	Z
00	CLR	20				40	1	01			
01	1	01				41	+	33			
02	STP	41		ENTRY		42	1	01			
03	PNT	45				43	0	00			
04	PNT	45				44	.	21			
05	AC+	60				45	1	01			
06	DN	25				46	X>Y	53			
07	√	76				47	1	01			
08	UP	27				48	6	06			
09	π	56				49	CLR	20			
0a	UP	27				4a	FMT	42			
0b	2	02				4b	UP	27			
0c	X	36				4c	END	46			
0d	DN	25									
10	√	76									
11	X	36									
12	YTO	40									
13	c	16									
14	0	00									
15	UP	27									
16	YTO	40									
17	d	17									
18	f	15									
19	-	34									
1a	DN	25									
1b	UP	27									
1c	X	36									
1d	e	12									
20	UP	27									
21	2	02									
22	X	36									
23	DN	25									
24	DIV	35									
25	DN	25									
26	CHS	32									
27	EXP	74									
28	UP	27									
29	c	16									
2a	DIV	35									
2b	7	07									
2c	5	05									
2d	0	00									
30	0	00									
31	X	36									
32	d	17									
33	UP	27									
34	5	05									
35	0	00									
36	0	00									
37	X	36									
38	DN	25									
39	FMT	42									
3a	DN	25									
3b	d	17									
3c	UP	27									
3d	.	21									

Storage

0		8	
1		9	
2		a	
3		b	
4		c	$\sqrt{2\pi}\ \sigma$
5		d	h
6		e	σ^2
7		f	M

9100B ONLY
STAT-PAC
X-4 ▲

CUMULATIVE PROBABILITY FUNCTION
INTEGRAL OF THE FORM: $F(x) = \int_a^x f(u)\, du$ WITH PLOT

This program evaluates the integral of a function f(u) between any lower limit a and a successively incremented upper limit.

A modification of Simpson's one third rule is used to perform the integration. The following equations are used:

$$\text{For } j = 0, \quad \int_a^{x_j} f(u)\, du = h/6 \left[1/3\ f(a) + 4/3\ f(x_{j+1}) + 1/3\ f(x_{j+2}) \right]$$

$$\text{For } j \geq 3, \quad \int_{x_{j-1}}^{x_j} f(u)\, du = h/24 \left[f(x_{j-3}) + 5f(x_{j-2}) + 19\ f(x_{j-1}) + 9f(x_j) \right]$$

Notes:

Due to the manner in which the program is written, the function will initially be integrated over an area formed by twice the increment; thereafter, integration will proceed by one increment at a time.

To integrate with some constant (a) as a lower limit (other than zero), in the program steps in the f(u) subroutine add a to u to form u' and form f(u'). For a lower limit of zero simply form f(u). In each case the upper limit will exceed the lower by some multiple of the increment depending on the number of times x is incremented by depression of the CONTINUE key.

To integrate from some lower limit to a specified upper limit without continually depressing the CONTINUE key, enter a Pause at step (5) (7). Then just before the incremented upper limit (shown in a flashing display) reaches the desired upper limit, depress PAUSE--this will stop the program after the next integration.

If a plotter is available a plot of the function being integrated may be obtained by inserting a plot subroutine at (-9) (0). This plot subroutine scales the calculated X and F(X) by multiplying them by X_{coef} and Y_{coef} respectively.

Reference: Numerical Analysis
by
Faiser S. Kunz
McGraw-Hill Book Company, Inc.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A
Using origin controls, position pen at (0, Y)*
PRESS: END

ENTER PROGRAM: Side A followed by Side B
PRESS: GO TO

PRESS: –

PRESS: 0

PRESS: 0

SET: PROGRAM ☐

Starting at (-0) (0) enter the program steps to form the function f(u) to be integrated; u is located in the Y register. After forming f(u) place it in the Y register. The last step of the subroutine must be:

RETURN

NOTE: Registers (-0) thru (-8) are available for programming and storage of f(u).

SET: ☐ RUN

PRESS: END

PRESS: CONTINUE

DISPLAY

0 ———— Z
0 ———— Y
0 ———— X

ENTER DATA:

h ——→ Z
Y_{coef} ——→ Y
X_{coef} ——→ X

PRESS: CONTINUE

DISPLAY

$\int_{a}^{x} f(u)\,du$ —— Z
(- a) ———— Y
0 ———— X

USER INSTRUCTIONS

Each successive "CONTINUE" increments the upper limit and evaluates the integral using the increment upper limit. The integral is plotted after each x incrementation.

If no plot is desired, place a RETURN in (-9)(0) and CONTINUES in (0)(1) and (0)(2).

EXAMPLE

Obtain the cumulative distribution function of the normally distributed random variable X of variance 1 and mean value of 0

The probability density function of X is

$$P(X) = \frac{1}{\sqrt{2\pi}} e^{-\frac{X^2}{2}}$$

The cumulative distribution function is given by

$$Q(X) = \int_{-\infty}^{X} P(u)\,du$$

The lower limit ($-\infty$) can be replaced by a = -4 without loss of accuracy.

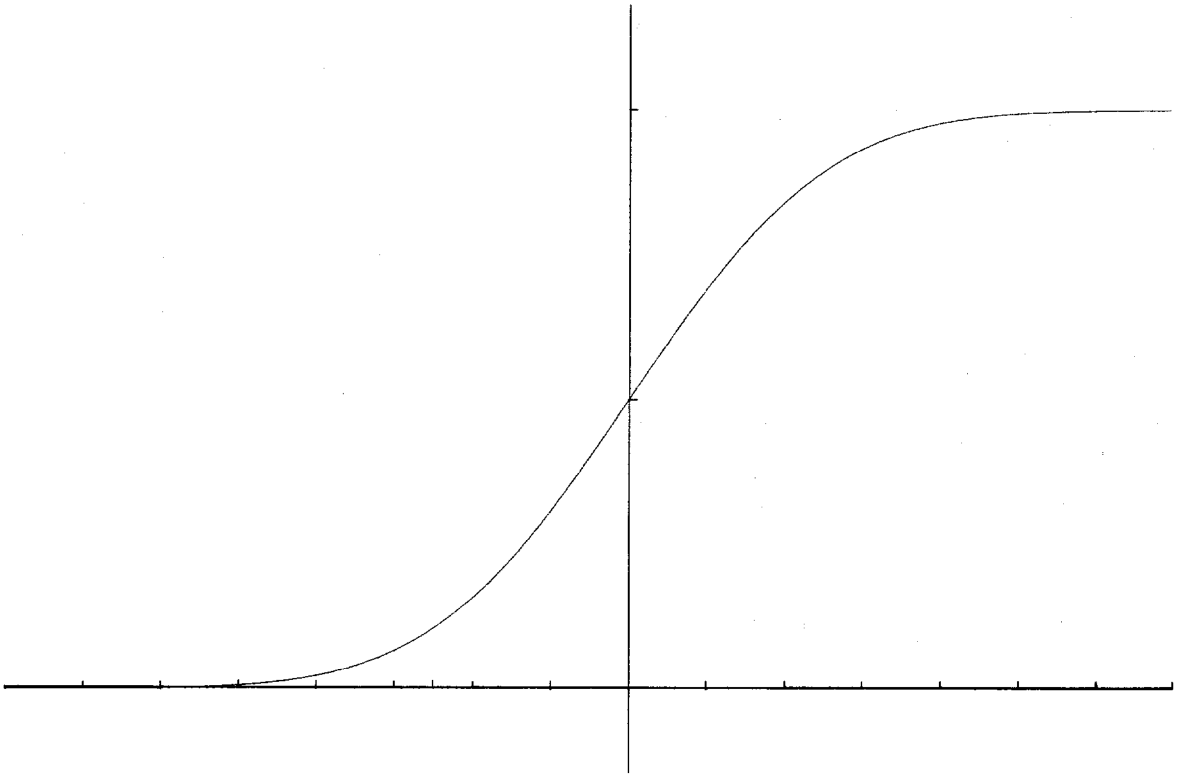
The program steps to generate P(u) are given on page 2 .

Data: Ycoef = 3760, Xcoef = 1000, h = .1

Results

X + 4	$\int_{-4}^{x} P(u)du$	X
0.5	.000201	-3.5
1.0	.001318	-3.0
1.5	.006178	-2.5
2.0	.022719	-2.0
2.5	.066775	-1.5
3.0	.158622	-1.0
3.5	.308504	-0.5
4.0	.499968	-0.0
4.5	.691432	0.5
5.0	.841314	1.0
5.5	.933162	1.5
6.0	.977218	2.0
6.5	.993758	2.5
7.0	.998618	3.0
7.5	.999736	3.5
8.0	.999937	4.0

*The Y (inches) must be determined by trial.



00	CLR	20	Plus	40	YE	24	80	GTO	44	
01	FMT	42	Page	41	b	14	81	4	04	
02	DN	25		42	RDN	31	82	a	13	
03	STP	41	ENTRY	43	X	36	83	e	12	
04	PNT	45		44	RDN	31	84	UP	27	
05	PNT	45		45	AC-	63	85	d	17	
06	XTO	23		46	DN	25	86	X	36	
07	-	34		47	1	01	87	GTO	44	
08	f	15		48	YE	24	88	1	01	
09	YTO	40		49	c	16	89	2	02	
0a	-	34		4a	XEY	30	8a	CNT	47	
0b	e	12		4b	AC+	60	8b	CNT	47	
0c	DN	25		4c	RCL	61	8c	CNT	47	
0d	YTO	40		4d	UP	27	8d	CNT	47	
10	d	17		50	d	1	90	CNT	47	
11	CLR	20		51	X	36	91	CNT	47	
12	GTO	44		52	RUP	22	92	CNT	47	
13	SUB	77		53	X	36	93	CNT	47	
14	-	34		54	d	17	94	CNT	47	
15	0	00		55	-	34	95	CNT	47	
16	0	00		56	CLX	37	96	CNT	47	
17	2	02		57	PSE	57	97	CNT	47	
18	4	04		58	CNT	47	98	CNT	47	
19	DIV	35		59	GTO	44	99	CNT	47	
1a	e	12		5a	SUB	77	9a	CNT	47	
1b	UP	27		5b	-	34	9b	CNT	47	
1c	0	00		5c	9	11	9c	CNT	47	
1d	X=Y	50		5d	0	00	9d	CNT	47	
20	6	06		60	GTO	44	a0	CNT	47	
21	3	03		61	8	10	a1	CNT	47	
22	1	01		62	3	03	a2	CNT	47	
23	X=Y	50		63	DN	25	a3	CNT	47	
24	7	07		64	YTO	40	a4	CNT	47	
25	0	00		65	c	16	a5	CNT	47	
26	2	02		66	8	10	a6	CNT	47	
27	X=Y	50		67	X	36	a7	CNT	47	
28	7	07		68	1	01	a8	CNT	47	
29	8	10		69	XEY	30	a9	CNT	47	
2a	0	00		6a	AC+	60	aa	CNT	47	
2b	XEY	30		6b	GTO	44	ab	CNT	47	
2c	9	11		6c	8	10	ac	CNT	47	
2d	RUP	22		6d	3	03	ad	CNT	47	
30	X	36		70	DN	25				
31	RDN	31		71	YTO	40				
32	AC+	60		72	b	14				
33	1	01		73	3	03				
34	9	11		74	2	02				
35	RDN	31		75	GTO	44				
36	YE	24		76	6	06				
37	a	13		77	7	07				
38	RDN	31		78	DN	25				
39	X	36		79	YTO	40				h
3a	RDN	31		7a	a	13				
3b	AC+	60		7b	8	10				j
3c	5	05		7c	X	36				
3d	RDN	31		7d	1	01				$f(X_j)$

```
b0  CNT  47     Plus
b1  CNT  47     Page
b2  CNT  47
b3  CNT  47
b4  CNT  47
b5  CNT  47
b6  CNT  47
b7  CNT  47
b8  CNT  47
b9  CNT  47
ba  CNT  47
bb  CNT  47
bc  CNT  47
bd  CNT  47

c0  CNT  47
c1  CNT  47
c2  CNT  47
c3  CNT  47
c4  CNT  47
c5  CNT  47
c6  CNT  47
c7  CNT  47
c8  CNT  47
c9  CNT  47
ca  CNT  47
cb  CNT  47
cc  CNT  47
cd  CNT  47

d0  CNT  47
d1  CNT  47
d2  CNT  47
d3  CNT  47
d4  CNT  47
d5  CNT  47
d6  CNT  47
d7  CNT  47
d8  CNT  47
d9  CNT  47
da  CNT  47
db  CNT  47
dc  CNT  47
dd  CNT  47
```

```
00   4    04  ┐
01  CHS   32  │
02   +    33  │
03  DN    25  │
04  UP    27  │
05   X    36  │
06   2    02  │
07  DIV   35  │
08  DN    25  │
09  CHS   32  │
0a  EXP   74  ├ Example
0b  UP    27  │
0c   π    56  │
0d  UP    27  │
              │
10   2    02  │
11   X    36  │
12  DN    25  │
13   √    76  │
14  DIV   35  │
15  RTN   77  ┘
```

```
00   4    04   Minus
01   CHS  32   Page
02   +    33
03   DN   25
04   UP   27
05   X    36
06   2    02
07   DIV  35   } Example
08   DN   25
09   CHS  32
0a   EXP  74
0b   UP   27
0c   π    56
0d   UP   27

10   2    02
11   X    36
12   DN   25
13   √    76
14   DIV  35
15   RTN  77
16   CNT  47
17   CNT  47
18   CNT  47
19   CNT  47
1a   CNT  47
1b   CNT  47
1c   CNT  47
1d   CNT  47

20   CNT  47
21   CNT  47
22   CNT  47
23   CNT  47
24   CNT  47
25   CNT  47
26   CNT  47
27   CNT  47
28   CNT  47
29   CNT  47
2a   CNT  47
2b   CNT  47
2c   CNT  47
2d   CNT  47

30   CNT  47
31   CNT  47
32   CNT  47
33   CNT  47
34   CNT  47
35   CNT  47
36   CNT  47
37   CNT  47
38   CNT  47
39   CNT  47
3a   CNT  47
3b   CNT  47
3c   CNT  47
3d   CNT  47

40   CNT  47
41   CNT  47
42   CNT  47
43   CNT  47
44   CNT  47
45   CNT  47
46   CNT  47
47   CNT  47
48   CNT  47
49   CNT  47
4a   CNT  47
4b   CNT  47
4c   CNT  47
4d   CNT  47

50   CNT  47
51   CNT  47
52   CNT  47
53   CNT  47
54   CNT  47
55   CNT  47
56   CNT  47
57   CNT  47
58   CNT  47
59   CNT  47
5a   CNT  47
5b   CNT  47
5c   CNT  47
5d   CNT  47

60   CNT  47
61   CNT  47
62   CNT  47
63   CNT  47
64   CNT  47
65   CNT  47
66   CNT  47
67   CNT  47
68   CNT  47
69   CNT  47
6a   CNT  47
6b   CNT  47
6c   CNT  47
6d   CNT  47

70   CNT  47
71   CNT  47
72   CNT  47
73   CNT  47
74   CNT  47
75   CNT  47
76   CNT  47
77   CNT  47
78   CNT  47
79   CNT  47
7a   CNT  47
7b   CNT  47
7c   CNT  47
7d   CNT  47

80   CNT  47
81   CNT  47
82   CNT  47
83   CNT  47
84   CNT  47
85   CNT  47
86   CNT  47
87   CNT  47
88   CNT  47
89   CNT  47
8a   CNT  47
8b   CNT  47
8c   CNT  47
8d   CNT  47

90   XFR  67
91   -    34
92   f    15
93   X    36
94   RUP  22
95   XEY  30
96   XFR  67
97   -    34
98   e    12
99   X    36
9a   XEY  30
9b   RUP  22
9c   FMT  42
9d   DN   25

a0   RTN  77
a1   END  46
```

Y_{coef}

X_{coef}

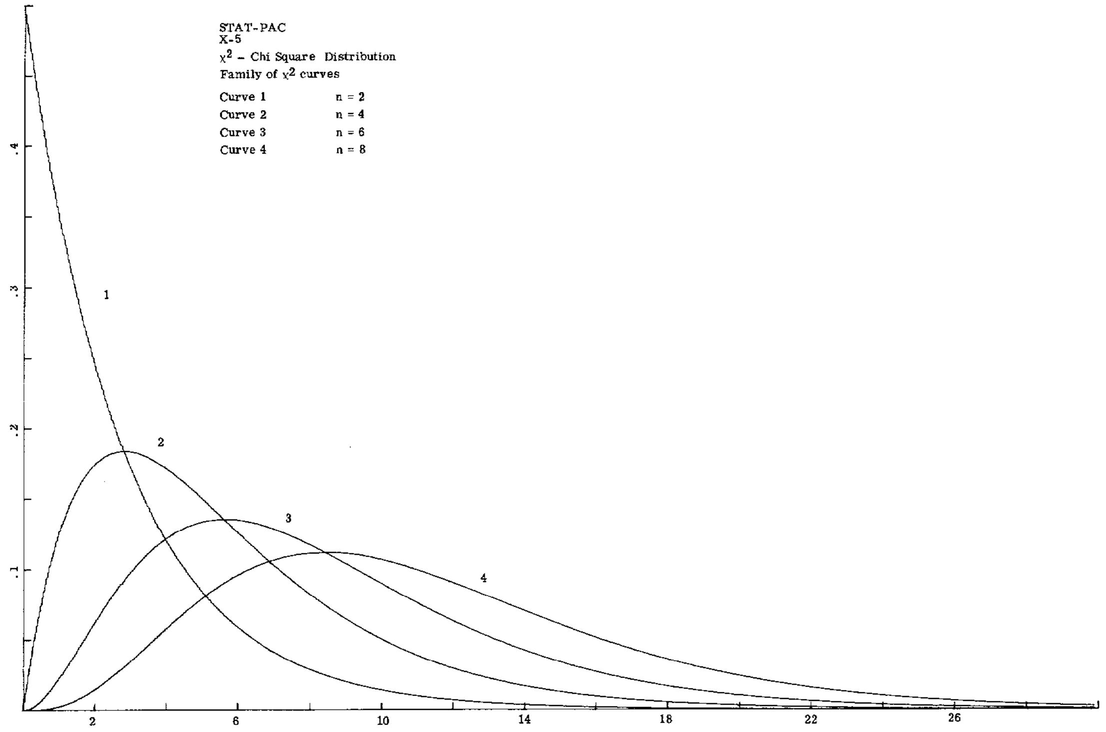
CHI-SQUARE DISTRIBUTION

This program will plot the distribution

$$Y(\chi^2) = \frac{1}{2^{\nu/2}\,\Gamma(\frac{\nu}{2})}(\chi^2)^{\frac{\nu}{2}-1}\, e^{-\frac{1}{2}\chi^2}$$

for specified even values of ν , the number of degrees of freedom desired.

The axes will also be plotted.



PRESS: STOP

Using the origin controls, locate the pen in the lower left corner of the paper.

SET: Decimal Wheel at 6 or less

PRESS: END

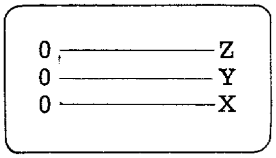
ENTER PROGRAM

PRESS: CONTINUE

The axes will now be drawn

NOTE: X axis increments ≈ 1.4
Y axis increments = .05

DISPLAY



ENTER ν : ν (Degrees of Freedom) ——► X

NOTE: $\nu = 2, 4, 6 \ldots$

PRESS: CONTINUE

The distribution will now be plotted. When it is completed, the calculator will return to accept a new ν .

If it is not desired to wait for the termination of a distribution:

PRESS: STOP

PRESS: GO TO (5)(6)

PRESS: CONTINUE

NOTE: To plot distributions for higher degrees of freedom, decrease the scaling factor in locations b-9, b-a, b-b.

Execution of this program destroys the first two registers of program steps. To rerun the program the magnetic card must be re-entered.

```
00  CLR  20
01   5   05
02   0   00
03   0   00
04   -   34
05  AC+  60
06  DN   25
07  DN   25
08  FMT  42
09  DN   25
0a  UP   27
0b  IFG  43
0c  SFL  54
0d  UP   27

10  RCL  61
11  AC-  63
12   +   33
13  AC+  60
14   7   07
15   5   05
16  IFG  43
17  SFL  54
18  INT  64
19  IFG  43
1a  SFL  54
1b   5   05
1c  EEX  26
1d   3   03

20  X<Y  52
21   4   04
22   a   13
23  DN   25
24  IFG  43
25  SFL  54
26  XEY  30
27  FMT  42
28  DN   25
29  XEY  30
2a  IFG  43
2b  SFL  54
2c  XEY  30
2d  UP   27

30   5   05
31   0   00
32   +   33
33  RDN  31
34  XEY  30
35  IFG  43
36  SFL  54
37  XEY  30
38  FMT  42
39  DN   25
3a  XEY  30
3b  IFG  43
3c  SFL  54
3d  XEY  30

40  RUP  22
41   -   34
42  DN   25
43  XEY  30
44  IFG  43
45  SFL  54
46  XEY  30
47  GTO  44
48   0   00
49   8   10
4a  IFG  43
4b   5   05
4c   6   06
4d  CLR  20

50  SFL  54
51  FMT  42
52  UP   27
53  GTO  44
54   0   00
55   1   01
56  CLR  20
57  FMT  42
58  UP   27
59  STP  41      ENTRY
5a  XTO  23
5b   1   01
5c  XEY  30
5d   1   01

60  XEY  30
61  UP   27
62   2   02
63  DIV  35
64  LN   65
65  XEY  30
66   X   36
67  XEY  30
68  EXP  74
69  XTO  23
6a   0   00
6b   1   01
6c   -   34
6d   0   00

70  X=Y  50
71   7   07
72   9   11
73  DN   25
74   X   36
75  UP   27
76  GTO  44
77   6   06
78   b   14
79  YEX  24
7a   0   00
7b  DN   25
7c   X   36
7d  YTO  40

80   0   00
81  EEX  26
82   3   03
83  CHS  32
84  UP   27
85   .   21
86   0   00
87   6   06
88   -   34
89  AC+  60
8a  RCL  61
8b   +   33
8c  YTO  40
8d   e   12

90  DN   25
91  LN   65
92  UP   27
93  UP   27
94  YEX  24
95   1   01
96  YTO  40
97   1   01
98   2   02
99   -   34
9a  DN   25
9b   X   36
9c  DN   25
9d  EXP  74

a0  UP   27
a1   e   12
a2  UP   27
a3   X   36
a4   2   02
a5  CHS  32
a6  DIV  35
a7  DN   25
a8  EXP  74
a9   X   36
aa  DN   25
ab  YEX  24
ac   0   00
ad  YTO  40
```

```
b0   0    00
b1   XEY  30
b2   DIV  35
b3   EEX  26
b4   4    04
b5   X    36
b6   e    12
b7   UP   27
b8   X    36
b9   3    03
ba   5    05
bb   0    00
bc   X    36
bd   7    07

c0   5    05
c1   0    00
c2   0    00
c3   X<Y  52
c4   c    16
c5   c    16
c6   DN   25
c7   FMT  42
c8   DN   25
c9   GTO  44
ca   8    10
cb   a    13
cc   CLR  20
cd   FMT  42

d0   UP   27
d1   GTO  44
d2   5    05
d3   6    06
d4   END  46
```

LINEAR REGRESSION AND PLOT

This program calculates the equation of the straight line of best fit of a set of data points. It also plots each data point and the straight line determined by the calculated equation. The best fit is determined by minimizing the sum of the squares of the deviations of the data points from the line.

The program calculates m and C for the equation:

$$Y = mX + C$$

The program also calculates a correlation coefficient r, an indication of goodness of fit. Note: $-1 \leq r \leq 1$ where the sign corresponds to the slope m. If $r = 0$, there is no correlation, and if $r = \pm 1$, there is perfect correlation or a perfect fit.

The defining equations are:

$$m = \frac{\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^{n} (X_i - \overline{X})^2}$$

$$C = \overline{Y} - m\overline{X}$$

$$\text{where} \quad \overline{Y} = \frac{\sum_{i=1}^{n} Y_i}{n} \quad \text{and} \quad \overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

$$r = \frac{\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\sum_{i=1}^{n} (X_i - \overline{X})^2 \sum_{i=1}^{n} (Y_i - \overline{Y})^2}} = \frac{n \sum_{i=1}^{n} X_i Y_i - (\sum_{i=1}^{n} X_i)(\sum_{i=1}^{n} Y_i)}{\sqrt{\left[n \sum_{i=1}^{n} X_i^2 - (\sum_{i=1}^{n} X_i)^2\right] \left[n \sum_{i=1}^{n} Y_i^2 - (\sum_{i=1}^{n} Y_i)^2\right]}}$$

Reference: Mathematical Statistics
by John E. Fruend
Prentice-Hall, 1962

USER INSTRUCTIONS

PRESS: STOP

Note: Use the origin controls to locate the pen in the lower left corner of the paper (on intersection of border lines if graph paper is being used).

ENTER PROGRAM A: (Starting Address is (0)(0)

SET: Decimal wheel at 6 or less

PRESS: END

PRESS: CONTINUE

ENTER TRANSLATION CONSTANTS*:

$Y_{shift} \rightarrow Y$

$X_{shift} \rightarrow X$

PRESS: CONTINUE

ENTER SCALING CONSTANTS*:

$Y_{scale} \rightarrow Y$

$X_{scale} \rightarrow X$

*See General Plotter Instructions

PRESS: CONTINUE

DISPLAY

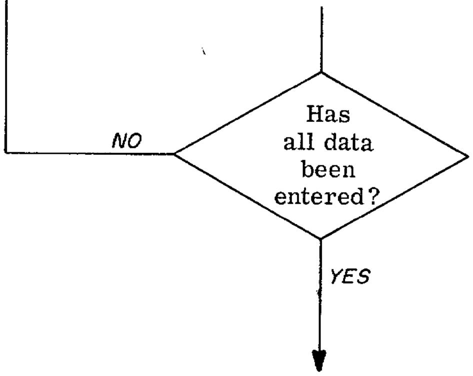
i ——— Z
0 ——— Y
0 ——— X

ENTER DATA: $Y_i \rightarrow Y$

$X_i \rightarrow X$

PRESS: CONTINUE

Note: (X_i, Y_i) is now plotted



USER INSTRUCTIONS (con't)

ENTER PROGRAM B: (Starting Address is (0)(0))

PRESS: END

PRESS: CONTINUE

DISPLAY

r ——— Z
C ——— Y
m ——— X

PRESS: CONTINUE

ENTER SCALING CONSTANTS: $Y_{scale} \rightarrow Y$

$X_{scale} \rightarrow X$

PRESS: CONTINUE

Note: The straight line of best fit is now plotted.

As the pen approaches any of the boundaries,
PRESS: STOP

EXAMPLE

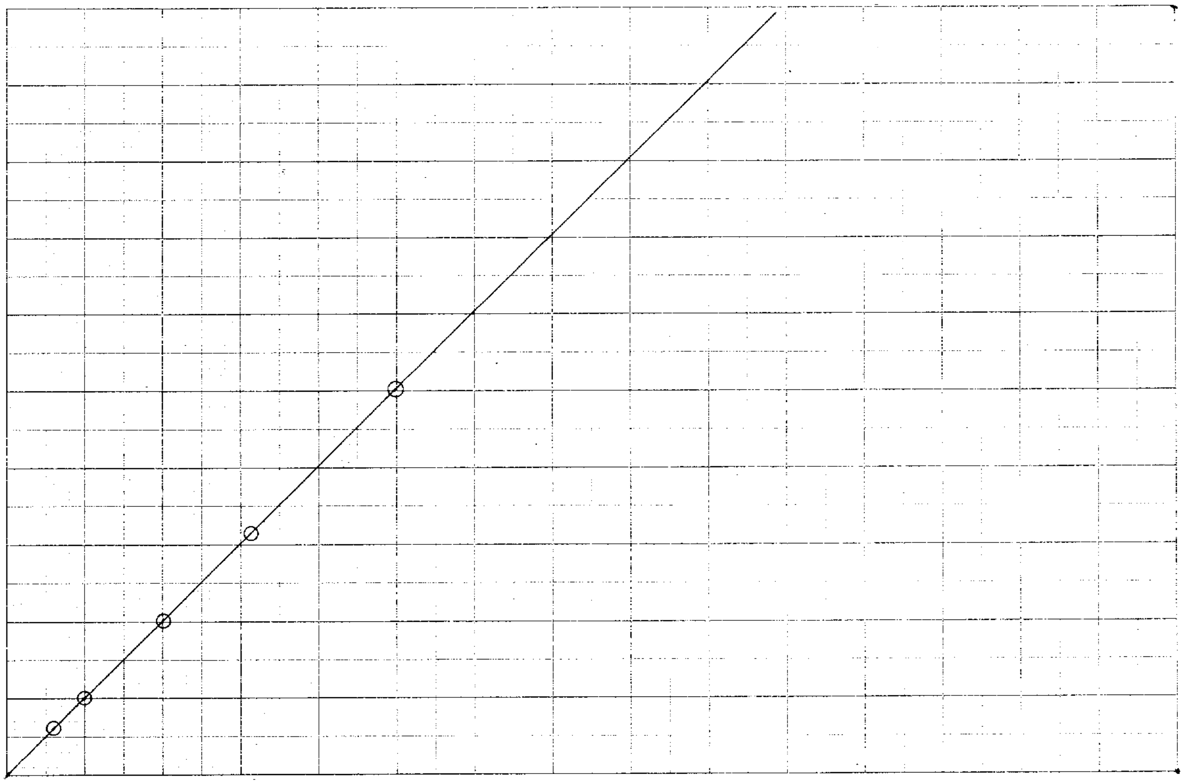
$$\left.\begin{array}{l} Y_{shift} = 0 \\ X_{shift} = 0 \end{array}\right\}$$

$$\left.\begin{array}{l} Y_{scale} = 1 \\ X_{scale} = 1 \end{array}\right\}$$

X	Y
1	1
2	2
5	5
.6	.6
π	π

Results

r = 1
c = 0
m = 1



STAT-PAC X-6 Program A

00	CLR	20	
01	XTO	23	
02	a	13	
03	XTO	23	
04	b	14	
05	XTO	23	
06	c	16	
07	XTO	23	
08	d	17	
09	STP	41	ENTRY
0a	YTO	40	
0b	9	11	
0c	XTO	23	
0d	8	10	
10	CLR	20	
11	STP	41	ENTRY
12	YTO	40	
13	7	07	
14	XTO	23	
15	0	00	
16	a	13	
17	UP	27	
18	1	01	
19	+	33	
1a	YTO	40	
1b	a	13	
1c	CLX	37	
1d	UP	27	
20	STP	41	ENTRY
21	AC+	60	
22	UP	27	
23	X	36	
24	XEY	30	
25	YEX	24	
26	d	17	
27	+	33	
28	YEX	24	
29	d	17	
2a	DN	25	
2b	X	36	
2c	XEY	30	
2d	YEX	24	
30	b	14	
31	+	33	
32	YEX	24	
33	b	14	
34	DN	25	
35	RDN	31	
36	X	36	
37	XEY	30	
38	YEX	24	
39	c	16	
3a	+	33	
3b	YEX	24	
3c	c	16	
3d	RUP	22	
40	YEX	24	
41	9	11	
42	YTO	40	
43	9	11	
44	RDN	31	
45	-	34	
46	XEY	30	
47	YEX	24	
48	8	10	
49	YTO	40	
4a	8	10	
4b	RDN	31	
4c	-	34	
4d	5	05	
50	0	00	
51	0	00	
52	X	36	
53	RUP	22	
54	X	36	
55	XEY	30	
56	YEX	24	
57	0	00	
58	YTO	40	
59	0	00	
5a	RDN	31	
5b	DIV	35	
5c	XEY	30	
5d	YEX	24	
60	7	07	
61	YTO	40	
62	7	07	
63	RDN	31	
64	DIV	35	
65	DN	25	
66	XEY	30	
67	FMT	42	
68	UP	27	
69	FMT	42	
6a	DN	25	
6b	GTO	44	
6c	1	01	
6d	6	06	
70	END	46	

STAT-PAC X-6 Program B

```
00  DN    25
01  1     01
02  -     34
03  YTO   40
04  a     13
05  e     12
06  XEY   30
07  DIV   35
08  YEX   24
09  f     15
0a  DIV   35
0b  YTO   40
0c  e     12
0d  X     36

10  e     12
11  X     36
12  d     17
13  XEY   30
14  -     34
15  YTO   40
16  d     17
17  c     16
18  UP    27
19  f     15
1a  UP    27
1b  X     36
1c  a     13
1d  X     36

20  DN    25
21  -     34
22  YTO   40
23  c     16
24  b     14
25  UP    27
26  f     15
27  UP    27
28  e     12
29  X     36
2a  a     13
2b  X     36
2c  DN    25
2d  -     34

30  d     17
31  UP    27
32  DN    25
33  √     76
34  DIV   35
35  c     16
36  √     76
37  DIV   35
38  d     17
39  RUP   22
3a  XEY   30
3b  DIV   35
3c  YTO   40
3d  a     13
```

```
40  e     12
41  X     36
42  f     15
3   XEY   30
44  -     34
45  a     13
46  STP   41
47  YTO   40
48  b     14
49  CLR   20
4a  XTO   23
4b  c     16
4c  FMT   42
4d  UP    27

50  STP   41
51  AC+   60
52  DN    25
53  f     15
54  X     36
55  5     05
56  0     00
57  0     00
58  DIV   35
59  RDN   31
5a  YEX   24
5b  8     10
5c  YTO   40
5d  8     10

60  +     33
61  a     13
62  X     36
63  b     14
64  +     33
65  RUP   22
66  YEX   24
67  9     11
68  YTO   40
69  9     11
6a  RDN   31
6b  -     34
6c  e     12
6d  DIV   35

70  DN    25
71  X     36
72  c     16
73  FMT   42
74  DN    25
75  RUP   22
76  +     33
77  YTO   40
78  c     16
79  GTO   44
7a  5     05
7b  3     03
7c  END   46
```

STAT-PAC
X-7

LINEAR REGRESSION (PRINT PLOT)

This program calculates the equation of the straight line of best fit of a set of data points. It also plots each data point and the straight line determined by the calculated equation. The best fit is determined by minimizing the sum of the squares of the deviations of the data points from the line.

The program calculates m and C for the equation:

$$Y = mX + C$$

The program also calculates a correlation coefficient r, an indication of goodness of fit. Note: $-1 \leq r \leq 1$ where the sign corresponds to the slope m. If $r = 0$, there is no correlation, and if $r = \pm 1$, there is perfect correlation or a perfect fit.

The defining equations are:

$$m = \frac{\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^{n} (X_i - \overline{X})^2}$$

$$C = \overline{Y} - m\overline{X}$$

where $$\overline{Y} = \frac{\sum_{i=1}^{n} Y_i}{n} \quad \text{and} \quad \overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

$$r = \frac{\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\sum_{i=1}^{n} (X_i - \overline{X})^2 \sum_{i=1}^{n} (Y_i - \overline{Y})^2}} = \frac{n \sum_{i=1}^{n} X_i Y_i - (\sum_{i=1}^{n} X_i)(\sum_{i=1}^{n} Y_i)}{\sqrt{\left[n \sum_{i=1}^{n} X_i^2 - (\sum_{i=1}^{n} X_i)^2\right] \left[n \sum_{i=1}^{n} Y_i^2 - (\sum_{i=1}^{n} Y_i)^2\right]}}$$

Reference: Mathematical Statistics, John E. Freund, Prentice-Hall, 1962.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: CLEAR

PRESS: STOP

Using the origin controls, locate the pen in the lower left corner of the paper.

PRESS: END

ENTER PROGRAM A

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER SHIFT DATA:

$Y_{shift} \rightarrow Y$
$X_{shift} \rightarrow X$

PRESS: CONTINUE

ENTER SCALING DATA:

$Y_{scale} \rightarrow Y$
$X_{scale} \rightarrow X$

PRESS: CONTINUE

DISPLAY

1 ——— Z
0 ——— Y
0 ——— X

(n - number of the data point to be entered)

ENTER DATA:

$Y_n \longrightarrow Y$
$X_n \longrightarrow X$

PRESS: CONTINUE

When all the data points have been entered:

USER INSTRUCTIONS (Con't)

PRESS: END

ENTER PROGRAM B

PRESS: CONTINUE

DISPLAY

r ——— Z Correlation Coefficient
c ——— Y Intercept
m ——— X Slope

ENTER SCALING DATA:

$Y_{scale} \rightarrow Y$
$X_{scale} \rightarrow X$

PRESS: CONTINUE

(The least fit straight line will now be plotted.) To terminate, PRESS: STOP .

EXAMPLE

$$\left.\begin{array}{l} Y_{shift} = 0 \\ X_{shift} = 0 \end{array}\right\}$$

$$\left.\begin{array}{l} Y_{scale} = 1 \\ X_{scale} = 1 \end{array}\right\}$$

X	Y
1	1
2	2
5	5
.6	.6
π	π

Results:

r = 1
c = 0
m = 1

For plot, see page 25.

STAT-PAC X-7 Program A

```
00  XTO  23
01  a    13
02  XTO  23
03  b    14
04  XTO  23
05  c    16
06  XTO  23
07  d    17
08  STP  41   ENTRY
09  YTO  40
0a  9    11
0b  XTO  23
0c  8    10
0d  STP  41   ENTRY

10  YTO  40
11  7    07
12  XTO  23
13  0    00
14  a    13
15  UP   27
16  1    01
17  +    33
18  YTO  40
19  a    13
1a  CLX  37
1b  UP   27
1c  STP  41   ENTRY
1d  PNT  45

20  PNT  45
21  AC+  60
22  UP   27
23  X    36
24  XEY  30
25  YEX  24
26  d    17
27  +    33
28  YEX  24
29  d    17
2a  DN   25
2b  X    36
2c  XEY  30
2d  YEX  24

30  b    14
31  +    33
32  YEX  24
33  b    14
34  DN   25
35  RDN  31
36  X    36
37  XEY  30
38  YEX  24
39  c    16
3a  +    33
3b  YEX  24
3c  c    16
3d  RUP  22
```

```
40  YEX  24
41  9    11
42  YTO  40
43  9    11
44  RDN  31
45  -    34
46  XEY  30
47  YEX  24
48  8    10
49  YTO  40
4a  8    10
4b  RDN  31
4c  -    34
4d  5    05

50  0    00
51  0    00
52  X    36
53  RUP  22
54  X    36
55  XEY  30
56  YEX  24
57  0    00
58  YTO  40
59  0    00
5a  RDN  31
5b  DIV  35
5c  XEY  30
5d  YEX  24

60  7    07
61  YTO  40
62  7    07
63  RDN  31
64  DIV  35
65  DN   25
66  XEY  30
67  FMT  42
68  UP   27
69  FMT  42
6a  DN   25
6b  GTO  44
6c  1    01
6d  4    04

70  END  46
```

Program B

```
00  DN   25
01  1    01
02  -    34
03  YTO  40
04  a    13
05  e    12
06  XEY  30
07  DIV  35
08  YEX  24
09  f    15
0a  DIV  35
0b  YTO  40
0c  e    12
0d  X    36

10  e    12
11  X    36
12  d    17
13  XEY  30
14  -    34
15  YTO  40
16  d    17
17  c    16
18  UP   27
19  f    15
1a  UP   27
1b  X    36
1c  a    13
1d  X    36

20  DN   25
21  -    34
22  YTO  40
23  c    16
24  b    14
25  UP   27
26  f    15
27  UP   27
28  e    12
29  X    36
2a  a    13
2b  X    36
2c  DN   25
2d  -    34
```

30	d	17	
31	UP	27	
32	DN	25	
33	√	76	
34	DIV	35	
35	c	16	
36	√	76	
37	DIV	35	
38	d	17	
39	RUP	22	
3a	XEY	30	
3b	DIV	35	
3c	YTO	40	
3d	a	13	
40	e	12	
41	X	36	
42	f	15	
43	XEY	30	
44	-	34	
45	a	13	
46	PNT	45	S
47	YTO	40	
48	b	14	
49	CLR	20	
4a	XTO	23	
4b	c	16	
4c	FMT	42	
4d	UP	27	
50	STP	41	ENTRY
51	AC+	60	
52	DN	25	
53	f	15	
54	X	36	
55	5	05	
56	0	00	
57	0	00	
58	DIV	35	
59	RDN	31	
5a	YEX	24	
5b	8	10	
5c	YTO	40	
5d	8	10	
60	+	33	
61	a	13	
62	X	36	
63	b	14	
64	+	33	
65	RUP	22	
66	YEX	24	
67	9	11	
68	YTO	40	
69	9	11	
6a	RDN	31	
6b	-	34	
6c	e	12	
6d	DIV	35	
70	DN	25	
71	X	36	
72	c	16	
73	FMT	42	
74	DN	25	
75	RUP	22	
76	+	33	
77	YTO	40	
78	c	16	
79	GTO	44	
7a	5	05	
7b	3	03	
7c	END	46	

POWER CURVE REGRESSION AND PLOT

This program computes the least squares fit and correlation coefficient of N pairs of data points for a power curve of the form:

$$Y = aX^b$$

The program will also plot each data point and the curve after the unknowns have been calculated.

The equation is linearized into $\ln Y = b \ln X + \ln a$

where $$b = \frac{N\sum(\ln X \ln Y) - \sum \ln X \sum \ln Y}{N\sum(\ln X)^2 - (\sum \ln X)^2}$$

and $$r = \frac{N\sum(\ln X \ln Y) - (\sum \ln X)(\sum \ln Y)}{\sqrt{\left[N\sum(\ln X)^2 - (\sum \ln X)^2\right]\left[N\sum(\ln Y)^2 - (\sum \ln Y)^2\right]}}$$

$$\ln a = \frac{\sum \ln Y}{N} - \frac{\sum \ln X}{N} b$$

Note: $X_i > 0$, $Y_i > 0$, $i = 1, \cdots, N$ and $X_{shift} > 0$

Reference: Statistical Theory and Methodology in Science and Engineering
by K. A. Brownlee
John Wiley and Sons, 1965

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A Printer

PRESS: STOP

Using the origin controls, locate the pen in the lower left corner of the paper.

ENTER PROGRAM A: (Starting Address is (0)(0))

PRESS: END

PRESS: CONTINUE

ENTER TRANSLATION CONSTANTS*:

$Y_{shift} \rightarrow Y$

$X_{shift} \rightarrow X$ Note: $X_{shift} > 0$

PRESS: CONTINUE

ENTER SCALING CONSTANTS*:

$Y_{scale} \rightarrow Y$

$X_{scale} \rightarrow X$

→ PRESS: CONTINUE

DISPLAY

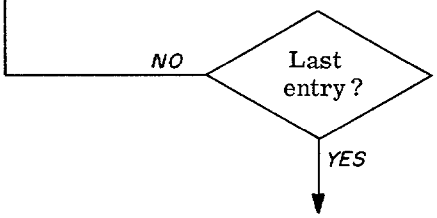
0 —— Z
0 —— Y
i —— X

Each data point will be plotted as entered.

ENTER DATA: $Y_i \rightarrow Y$

$X_i \rightarrow X$

PRESS: PRINT



PRESS: SET FLAG

PRESS: CONTINUE

ENTER PROGRAM B: (Starting Address is (0)(0))

PRESS: END

PRESS: CONTINUE

USER INSTRUCTIONS (con't)

DISPLAY

r —— Z
b —— Y
a —— X

PRESS: PRINT

PRESS: CONTINUE

The Plotter will now plot the least squares curve. To interrupt the plot or stop it when finished, PRESS: STOP.

To re-run program, return to beginning of USER INSTRUCTIONS, ENTER PROGRAM A and proceed as before.

*See general Plotter instructions.

EXAMPLE

X	Y
1.2	15.25
1.7	6.68
2.3	4.54
2.9	1.74
4.4	.87
4.8	.51
5.7	.33
7.2	.23
8.4	.17

Let:

$X_{shift} = 1.1 \quad X_{scale} = \frac{8.4-1.1}{15} \approx .5 \frac{\text{units}}{\text{inch}}$

$Y_{shift} = 0 \quad Y_{scale} = \frac{15.25-.17}{10} \approx 2 \frac{\text{units}}{\text{inch}}$

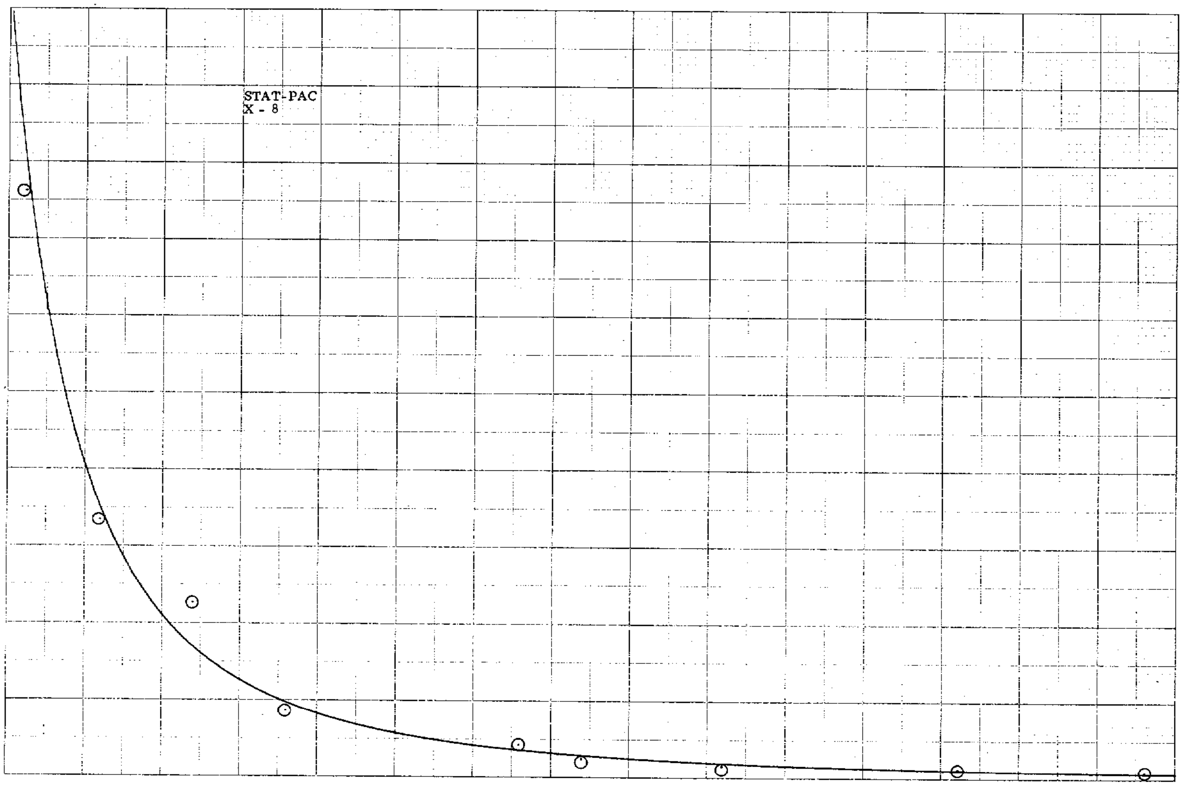
Solution:

r = -.9952

b = -2.384

a = 24.949

$Y = 24.949X^{-2.384}$


STAT-PAC
X - 8

STAT-PAC X-8 Program A

```
00  CLR  20
01  STP  41   ENTRY
02  XTO  23
03   a   13
04  YTO  40
05   b   14
06  CLR  20
07  STP  41   ENTRY
08  XTO  23
09   c   16
0a  YTO  40
0b   d   17
0c  CLR  20
0d   1   01

10  XTO  23
11   0   00
12  STP  41   ENTRY
13  LN   65
14  XEY  30
15  LN   65
16  AC+  60
17  UP   27
18   X   36
19  XEY  30
1a  YEX  24
1b   8   10
1c   +   33
1d  YEX  24

20   8   10
21  DN   25
22   X   36
23  XEY  30
24  YEX  24
25   9   11
26   +   33
27  YEX  24
28   9   11
29  DN   25
2a  RDN  31
2b   X   36
2c  XEY  30
2d  YEX  24

30   7   07
31   +   33
32  YEX  24
33   7   07
34  DN   25
35  EXP  74
36  XEY  30
37  EXP  74
38  UP   27
39   b   14
3a   -   34
3b   d   17
3c  DIV  35
3d   a   13

40  RUP  22
41  XEY  30
42   -   34
43   c   16
44  DIV  35
45   5   05
46   0   00
47   0   00
48   X   36
49  RUP  22
4a   X   36
4b  CLX  37
4c  RDN  31
4d  XEY  30

50  FMT  42
51  DN   25
52  FMT  42
53  UP   27
54  YEX  24
55   0   00
56  IFG  43
57   6   06
58   1   01
59   1   01
5a   +   33
5b  DN   25
5c  GTO  44
5d   1   01

60   0   00
61   e   12
62  UP   27
63   X   36
64  RUP  22
65  YEX  24
66   7   07
67   X   36
68  RDN  31
69   -   34
6a  YEX  24
6b   f   15
6c  DN   25
6d  STP  41

70   0   00
71   0   00
72   0   00
73   0   00
74   0   00
75   0   00
76   0   00
77   0   00
78   0   00
79   0   00
7a   0   00
7b   0   00
7c   0   00
7d   0   00

80   0   00
81   0   00
82   0   00
83   0   00
84   0   00
85   0   00
86   0   00
87   0   00
88   0   00
89   0   00
8a   0   00
8b   0   00
8c   0   00
8d   0   00

90   0   00
91   0   00
92   0   00
93   0   00
94   0   00
95   0   00
96   0   00
97   0   00
98   0   00
99   0   00
9a   0   00
9b   0   00
9c   0   00
9d   0   00

a0   0   00
a1   0   00
a2   0   00
a3   0   00
a4   0   00
a5   0   00
a6   0   00
a7   0   00
a8   0   00
a9   0   00
aa   0   00
ab   0   00
ac   0   00
ad   0   00
```

X_{shift}

Y_{shift}

X_{scale}

Y_{scale}

x

Δx

STAT-PAC X-8 Program B

```
b0   0    00     10  UP   27     50  RCL  61
b1   0    00     11  YEX  24     51  AC-  63
b2   0    00     12  9    11     52  +    33
b3   0    00     13  DN   25     53  AC+  60
b4   0    00     14  X    36     54  DN   25
b5   0    00     15  RUP  22     55  LN   65
b6   0    00     16  XEY  30     56  YEX  24
b7   0    00     17  YEX  24     57  1    01
b8   0    00     18  8    10     58  YTO  40
b9   0    00     19  YTO  40     59  1    01
ba   0    00     1a  0    00     5a  X    36
bb   0    00     1b  e    12     5b  DN   25
bc   0    00     1c  X    36     5c  EXP  74
bd   0    00     1d  DN   25     5d  YEX  24

c0   0    00     20  -    34     60  0    00
c1   0    00     21  UP   27     61  YTO  40
c2   0    00     22  DN   25     62  0    00
c3   0    00     23  f    15     63  X    36
c4   0    00     24  CHS  32     64  b    14
c5   0    00     25  DIV  35     65  -    34
c6   0    00     26  DN   25     66  d    17
c7   0    00     27  UP   27     67  DIV  35
c8   0    00     28  YEX  24     68  e    12
c9   0    00     29  e    12     69  UP   27
ca   0    00     2a  X    36     6a  a    13
cb   0    00     2b  XEY  30     6b  -    34
cc   0    00     2c  YEX  24     6c  c    16
cd   0    00     2d  0    00     6d  DIV  35

d0   0    00     30  -    34     70  5    05
d1   0    00     31  XEY  30     71  0    00
d2   0    00     32  YEX  24     72  0    00
d3   0    00     33  8    10     73  X    36
d4   0    00     34  XEY  30     74  RUP  22
d5   0    00     35  DIV  35     75  X    36
d6   0    00     36  XEY  30     76  DN   25
d7   0    00     37  EXP  74     77  XEY  30
d8   0    00     38  YEX  24     78  FMT  42
d9   0    00     39  9    11     79  DN   25
da   0    00     3a  RDN  31     7a  GTO  44
db   0    00     3b  DIV  35     7b  5    05
dc   0    00     3c  e    12     7c  0    00
dd   END  46     3d  RUP  22     7d  END  46

Program B
00   UP   27     40  XTO  23
01   X    36     41  0    00
02   XEY  30     42  YTO  40
03   YEX  24     43  1    01
04   8    10     44  STP  41
05   RDN  31     45  CLR  20
06   XEY  30     46  c    16
07   X    36     47  UP   27
08   RDN  31     48  8    10
09   -    34     49  DIV  35
0a   f    15     4a  a    13
0b   X    36     4b  XEY  30
0c   DN   25     4c  -    34
0d   √    76     4d  AC+  60
```

GENERAL POWER CURVE REGRESSION AND PLOT

This program computes the least squares fit and correlation coefficient of N pairs of data points for a power curve of the form:

$$Y = aX^b + C$$

The program will also plot each data point and the curve after the unknowns have been calculated.

The equation is linearized into $\ln Y = b \ln X + \ln a$

where

$$b = \frac{N\Sigma(\ln X \ln Y) - \Sigma \ln X \, \Sigma \ln Y}{N\Sigma(\ln X)^2 - (\Sigma \ln X)^2}$$

and

$$r = \frac{N\Sigma (\ln X \ln Y) - (\Sigma \ln X)(\Sigma \ln Y)}{\sqrt{\left[N\Sigma (\ln X)^2 - (\Sigma \ln X)^2\right] \left[N\Sigma (\ln Y)^2 - (\Sigma \ln Y)^2\right]}}$$

$$\ln a = \frac{\Sigma \ln Y}{N} - \frac{\Sigma \ln X}{N} b$$

NOTE: $X_i > 0$, $Y_i > 0$, $i = 1, \ldots, N$

C is the Y-Intercept Value at $X = 0$ or at $X = \infty$. The value for C may be entered directly if it is known. If C is unknown the user is advised to first run one of the exponential regression programs ($Y = ae^{bX}$) utilizing the first 3 or 4 minimum Y-value data points. The resulting value for <u>a</u> may then be entered as the value for C.

Reference: Statistical Theory and Methodology in Science and Engineering by K. A. Brownlee, John Wiley and Sons, 1965.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

SET: Decimal Wheel at 6 or less

PRESS: STOP

Using the origin controls, locate the pen in the lower left corner of the paper.

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

ENTER DATA:

C → Z (C ≠ 0)
Y_{shift} → Y
X_{shift} → X

PRESS: CONTINUE

ENTER DATA:

ΔY → Z *
Y_{scale} → Y
X_{scale} → X

PRESS: CONTINUE

DISPLAY

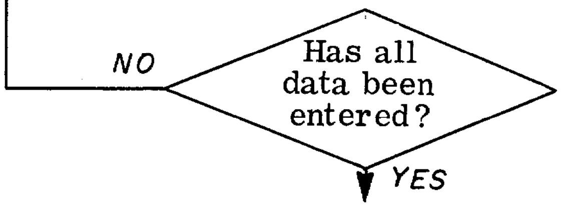
0 — Z
i — Y
0 — X

ENTER DATA:

Y_i → Y
X_i → X

NOTE: Plot points are of the form:

PRESS: CONTINUE



USER INSTRUCTIONS (Con't)

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

r — Z
b — Y
a — X

r, b, and a will be printed out

PRESS: CONTINUE

The plotter will now plot the least squares curve. To interrupt the plot, PRESS: STOP. The plot is self-terminating.

To run another case, return to the beginning of the USER INSTRUCTIONS, enter the program and proceed as before.

*ΔY is the desired tolerance on Y.

EXAMPLE

$C = 10$
$Y_{shift} = 0$
$X_{shift} = 0$
$\Delta Y = 3$
$Y_{scale} = 30$
$X_{scale} = 1$

i	X	Y
1	1	18
2	2	30
3	3	44
4	4	56
5	5	73
6	6	96
7	7	121
8	8	151
9	9	196
10	10	276

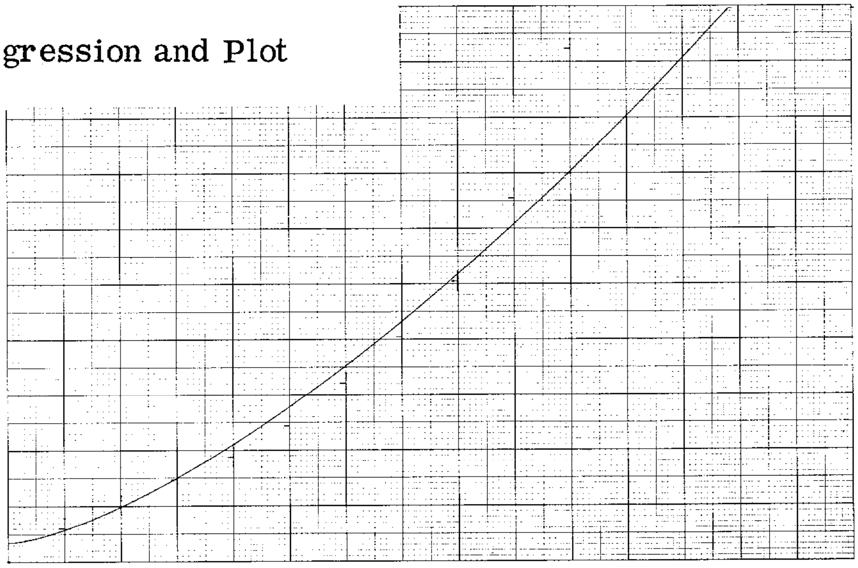
Results:

$r = .99$
$b = 1.45$
$a = 7.07$

STAT-PAC
X-9

General Power Regression and Plot

$Y = aX^b + C$



00	CLR	20	Plus Page	40	UP	27	80	0	00
01	STP	41	ENTRY	41	X	36	81	0	00
02	PNT	45		42	XEY	30	82	0	00
03	XTO	23		43	YE	24	83	0	00
04	a	13		44	8	10	84	0	00
05	DN	25		45	+	33	85	0	00
06	YTO	40		46	YE	24	86	0	00
07	-	34		47	8	10	87	0	00
08	f	15		48	DN	25	88	0	00
09	XEY	30		49	X	36	89	0	00
0a	-	34		4a	XEY	30	8a	0	00
0b	YTO	40		4b	YE	24	8b	0	00
0c	b	14		4c	7	07	8c	0	00
0d	CLR	20		4d	+	33	8d	0	00
10	STP	41	ENTRY	50	YE	24	90	0	00
11	PNT	45		51	7	07	91	0	00
12	PNT	45		52	DN	25	92	0	00
13	RDN	31		53	RDN	31	93	0	00
14	X	36		54	X	36	94	0	00
15	YTO	40		55	XEY	30	95	0	00
16	-	34		56	YE	24	96	0	00
17	e	12		57	9	11	97	0	00
18	RDN	31		58	+	33	98	0	00
19	5	05		59	YE	24	99	0	00
1a	0	00		5a	9	11	9a	0	00
1b	0	00		5b	DN	25	9b	0	00
1c	DIV	35		5c	EXP	74	9c	0	00
1d	YTO	40		5d	XEY	30	9d	0	00
20	c	16		60	EXP	74	a0	0	00
21	XEY	30		61	UP	27	a1	0	00
22	RDN	31		62	b	14	a2	0	00
23	DIV	35		63	-	34	a3	0	00
24	YTO	40		64	d	17	a4	0	00
25	d	17		65	DIV	35	a5	0	00
26	CLR	20		66	GTO	44	a6	0	00
27	1	01		67	-	34	a7	0	00
28	RUP	22		68	0	00	a8	0	00
29	YTO	40		69	0	00	a9	0	00
2a	0	00		6a	GTO	44	aa	0	00
2b	STP	41	ENTRY	6b	-	34	ab	0	00
2c	IFG	43		6c	3	03	ac	0	00
2d	6	06		6d	9	1	ad	0	00
30	a	13		70	0	00			
31	PNT	45		71	0	00			
32	PNT	45		72	0	00			
33	LN	65		73	0	00			
34	RUP	22		74	0	00			
35	XFR	67		75	0	00			
36	-	34		76	0	00			
37	f	15		77	0	00			
38	XEY	30		78	0	00			
39	RDN	31		79	0	00			
3a	-	34		7a	0	00			
3b	DN	25		7b	0	00			
3c	LN	65		7c	0	00			
3d	AC+	60		7d	0	00			

$\Sigma(\ln X_i)^2$

X_{shift}

Y_{shift}

X_{scale}

Y_{scale}

$\Sigma \ln Y$ X

$\Sigma(\ln Y_i \ln X_i)$ $\Sigma \ln X$ ΔX

			Plus Page
b0	0	00	
b1	0	00	
b2	0	00	
b3	0	00	
b4	0	00	
b5	0	00	
b6	0	00	
b7	0	00	
b8	0	00	
b9	0	00	
ba	0	00	
bb	0	00	
bc	0	00	
bd	0	00	
c0	0	00	
c1	0	00	
c2	0	00	
c3	0	00	
c4	0	00	
c5	0	00	
c6	0	00	
c7	0	00	
c8	0	00	
c9	0	00	
ca	0	00	
cb	0	00	
cc	0	00	
cd	0	00	
d0	0	00	
d1	0	00	
d2	0	00	
d3	0	00	
d4	0	00	
d5	0	00	
d6	0	00	
d7	0	00	
d8	0	00	
d9	0	00	
da	0	00	
db	0	00	
dc	0	00	
dd	0	00	

```
00   a     13    Minus
01   RUP   22    Page
02   XEY   30
03   -     34
04   c     16
05   DIV   35
06   XFR   67
07   -     34
08   e     12
09   XEY   30
0a   RDN   31
0b   +     33
0c   RDN   31
0d   XEY   30

10   FMT   42
11   DN    25
12   RDN   31
13   XEY   30
14   -     34
15   -     34
16   RDN   31
17   XEY   30
18   FMT   42
19   DN    25
1a   RDN   31
1b   +     33
1c   XEY   30
1d   RUP   22

20   FMT   42
21   DN    25
22   UP    27
23   5     05
24   0     00
25   -     34
26   CLX   37
27   RDN   31
28   FMT   42
29   DN    25
2a   FMT   42
2b   UP    27
2c   YE    24
2d   0     00

30   1     01
31   +     33
32   CLX   37
33   UP    27
34   RUP   22
35   GTO   44
36   +     33
37   2     02
38   8     10
39   1     01
3a   -     34
3b   e     12
3c   UP    27
3d   X     36

40   RUP   22
41   YE    24
42   9     11
43   X     36
44   RDN   31
45   -     34
46   YE    24
47   f     15
48   DN    25
49   UP    27
4a   X     36
4b   XEY   30
4c   YE    24
4d   8     10

50   RDN   31
51   XEY   30
52   X     36
53   RDN   31
54   -     34
55   f     15
56   X     36
57   DN    25
58   √     76
59   UP    27
5a   YE    24
5b   7     07
5c   DN    25
5d   X     36

60   RUP   22
61   XEY   30
62   YE    24
63   8     10
64   YTO   40
65   0     00
66   e     12
67   X     36
68   DN    25
69   -     34
6a   UP    27
6b   DN    25
6c   f     15
6d   CHS   32

70   DIV   35
71   DN    25
72   UP    27
73   YE    24
74   e     12
75   X     36
76   XEY   30
77   YE    24
78   0     00
79   -     34
7a   XEY   30
7b   YE    24
7c   8     10
7d   XEY   30

80   DIV   35
81   XEY   30
82   EXP   74
83   YE    24
84   7     07
85   RDN   31
86   DIV   35
87   e     12
88   RUP   22
89   XTO   23
8a   0     00
8b   YTO   40
8c   1     01
8d   PNT   45    S

90   CLR   20
91   c     16
92   UP    27
93   8     10
94   DIV   35
95   5     05
96   0     00
97   0     00
98   X     36
99   a     13
9a   XEY   30
9b   -     34
9c   AC+   60
9d   RCL   61

a0   AC-   63
a1   +     33
a2   AC+   60
a3   DN    25
a4   LN    65
a5   UP    27
a6   XFR   67
a7   1     01
a8   X     36
a9   DN    25
aa   EXP   74
ab   UP    27
ac   XFR   67
ad   0     00
```

△ Y

C

```
b0   X     36    Minus
b1   b     14    Page
b2   -     34
b3   d     17
b4   DIV   35
b5   e     12
b6   UP    27
b7   a     13
b8   -     34
b9   c     16
ba   DIV   35
bb   7     07
bc   5     05
bd   0     00

c0   0     00
c1   X<Y   52
c2   d     17
c3   5     05
c4   RUP   22
c5   XEY   30
c6   5     05
c7   EEX   26
c8   3     03
c9   X<Y   52
ca   9     11
cb   d     17
cc   DN    25
cd   XEY   30

d0   FMT   42
d1   DN    25
d2   GTO   44
d3   9     11
d4   d     17
d5   CLR   20
d6   FMT   42
d7   UP    27
d8   END   46
```

EXPONENTIAL REGRESSION AND PLOT

This program computes the least squares fit and a correlation coefficient of n pairs of data points for an exponential function of the form:

$$y = ae^{bx}$$

The program also plots each point and the least squares curve after the unknowns have been calculated.

The equation is linearized into

$$\ln y = \ln a + bx$$

or

$$Y = A + bx$$

Using a linear regression method,

$$b = \frac{n \sum x Y - \sum x \sum Y}{n \sum x^2 - (\sum x)^2}$$

$$A = \frac{\sum Y - b \sum x}{n}$$

$$a = e^A$$

the correlation coefficient is given by

$$r = \frac{n \sum x Y - \sum x \sum Y}{\sqrt{\left[n \sum x^2 - (\sum x)^2\right]\left[n \sum Y^2 - (\quad Y)^2\right]}}$$

Note: $Y_i > 0 \quad i = 1, \cdots, n$

Reference: Statistical Theory and Methodology in Science and Engineering
by K. A. Brownlee

John Wiley and Sons, 1965

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A
Using the origin controls, locate the pen in the lower left corner.

ENTER PROGRAM A: (Starting Address is (0)(0))

SET: Decimal wheel at 6 or less

PRESS: END

PRESS: CONTINUE

ENTER TRANSLATION CONSTANTS:

$Y_{shift} \rightarrow Y$

$X_{shift} \rightarrow X$

PRESS: CONTINUE

ENTER SCALING CONSTANTS:

$Y_{scale} \rightarrow Y$

$X_{scale} \rightarrow X$

PRESS: CONTINUE

DISPLAY

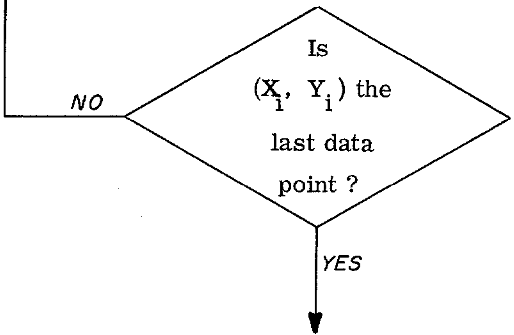
0 ——— Z
0 ——— Y
i ——— X

Each data point will be plotted as entered.

ENTER DATA: $Y_i \rightarrow Y$

$X_i \rightarrow X$

PRESS: PRINT



PRESS: SET FLAG

PRESS: CONTINUE

USER INSTRUCTIONS (con't)

ENTER PROGRAM B: (Starting Address is (0)(0))

PRESS: END

PRESS: CONTINUE

DISPLAY

a ——— Z
b ——— Y
r ——— X

PRESS: PRINT

PRESS: CONTINUE

The least squares curve $y = ae^{bx}$ will now be plotted. To interrupt the plot or stop it when finished, PRESS: STOP.

To re-run program return to beginning of USER INSTRUCTIONS, ENTER PROGRAM B and proceed as before.

EXAMPLE

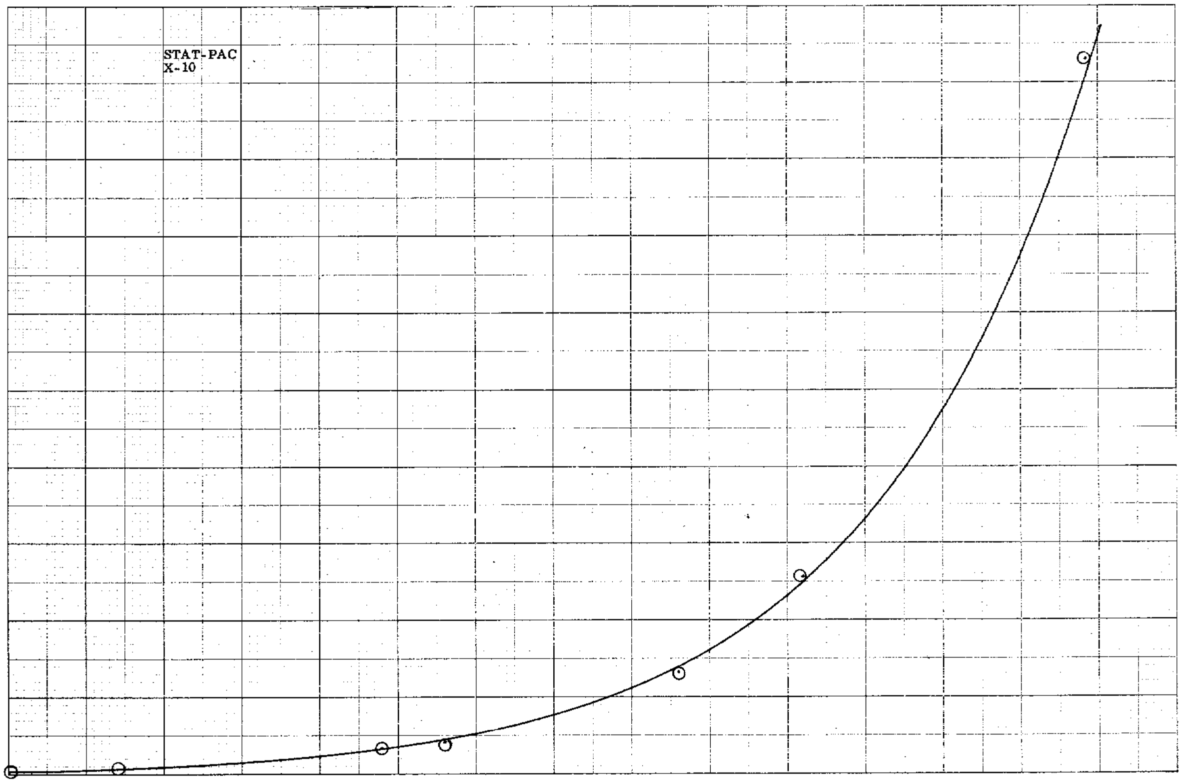
X	Y
.5	7.12
1.2	11.67
2.9	38.85
3.3	48.03
4.8	138.70
5.6	262.00
7.4	935.64

Let: $X_{shift} = .5$ $X_{scale} \geq \frac{7.4 - .5}{15} \approx .5 \frac{\text{units}}{\text{inch}}$

$Y_{shift} = 5$ $Y_{scale} \geq \frac{935.64-5}{10} \approx 100 \frac{\text{units}}{\text{inch}}$

Solution: a = 4.925
b = .705
r = .9998

$Y = 4.925e^{.705X}$


STAT-PAC
X-10

STAT-PAC X-10 Program A

00	CLR	20	
01	STP	41	ENTRY
02	XTO	23	
03	a	13	
04	YTO	40	
05	b	14	
06	CLR	20	
07	STP	41	ENTRY
08	XTO	23	
09	c	16	
0a	YTO	40	
0b	d	17	
0c	CLR	20	
0d	1	01	
10	XTO	23	
11	0	00	
12	STP	41	ENTRY
13	XEY	30	
14	LN	65	
15	AC+	60	
16	UP	27	
17	X	36	
18	XEY	30	
19	YEX	24	
1a	7	07	
1b	+	33	
1c	YEX	24	
1d	7	07	
20	DN	25	
21	X	36	
22	XEY	30	
23	YEX	24	
24	9	11	
25	+	33	
26	YEX	24	
27	9	11	
28	DN	25	
29	RDN	31	
2a	X	36	
2b	XEY	30	
2c	YEX	24	
2d	8	10	
30	+	33	
31	YEX	24	
32	8	10	
33	RUP	22	
34	EXP	74	
35	RDN	31	
36	a	13	
37	-	34	
38	c	16	
39	DIV	35	
3a	b	14	
3b	RUP	22	
3c	XEY	30	
3d	-	34	
40	d	17	
41	DIV	35	
42	5	05	
43	0	00	
44	0	00	
45	X	36	
46	RUP	22	
47	X	36	
48	CLX	37	
49	RDN	31	
4a	FMT	42	
4b	DN	25	
4c	FMT	42	
4d	UP	27	
50	YEX	24	
51	0	00	
52	IFG	43	
53	5	05	
54	b	14	
55	1	01	
56	+	33	
57	DN	25	
58	GTO	44	
59	1	01	
5a	0	00	
5b	DN	25	
5c	UP	27	
5d	YEX	24	
60	8	10	
61	X	36	
62	e	12	
63	UP	27	
64	X	36	
65	DN	25	
66	-	34	
67	UP	27	
68	YEX	24	
69	7	07	
6a	XEY	0	
6b	YEX	24	
6c	8	0	
6d	STP	41	
70	0	00	
71	0	00	
72	0	00	
73	0	00	
74	0	00	
75	0	00	
76	0	00	
77	0	00	
78	0	00	
79	0	00	
7a	0	00	
7b	0	00	
7c	0	00	
7d	0	00	
80	0	00	
81	0	00	
82	0	00	
83	0	00	
84	0	00	
85	0	00	
86	0	00	
87	0	00	
88	0	00	
89	0	00	
8a	0	00	
8b	0	00	
8c	0	00	
8d	0	00	
90	0	00	
91	0	00	
92	0	00	
93	0	00	
94	0	00	
95	0	00	
96	0	00	
97	0	00	
98	0	00	
99	0	00	
9a	0	00	
9b	0	00	
9c	0	00	
9d	END	46	

Program B

00	YTO	40
01	8	10
02	X	36
03	DN	25
04	YEX	24
05	f	15
06	XEY	30
07	UP	27
08	X	36
09	RDN	31
0a	-	34
0b	DN	25
0c	YEX	24
0d	f	15

X_{shift}

Y_{shift}

X_{scale}

Y_{scale}

x

Δx

STAT-PAC X-10 Program B

```
10  YTO  40
11  0    00
12  X    36
13  DN   25
14  √    76
15  YEX  24
16  8    10
17  YTO  40
18  8    10
19  XEY  30
1a  YEX  24
1b  9    11
1c  X    36
1d  UP   27

20  RCL  61
21  X    36
22  DN   25
23  -    34
24  UP   27
25  YEX  24
26  9    11
27  DN   25
28  DIV  35
29  YEX  24
2a  0    00
2b  DN   25
2c  DIV  35
2d  e    12

30  UP   27
31  DN   25
32  CHS  32
33  X    36
34  f    15
35  +    33
36  XEY  30
37  YEX  24
38  8    10
39  XEY  30
3a  DIV  35
3b  DN   25
3c  EXP  74
3d  YEX  24

40  0    00
41  XTO  23
42  1    01
43  RDN  31
44  STP  41
45  CLR  20
46  c    16
47  UP   27
48  6    06
49  DIV  35
4a  a    13
4b  XEY  30
4c  -    34
4d  AC+  60

50  RCL  61
51  AC-  63
52  +    33
53  AC+  60
54  DN   25
55  YEX  24
56  0    00
57  YTO  40
58  0    00
59  X    36
5a  DN   25
5b  EXP  74
5c  YEX  24
5d  1    01

60  YTO  40
61  1    01
62  X    36
63  b    14
64  -    34
65  d    17
66  DIV  35
67  e    12
68  UP   27
69  a    13
6a  -    34
6b  c    16
6c  DIV  35
6d  5    05

70  0    00
71  0    00
72  X    36
73  RUP  22
74  X    36
75  DN   25
76  XEY  30
77  FMT  42
78  DN   25
79  GTO  44
7a  5    05
7b  0    00
7c  END  46
```

CUBIC (THIRD DEGREE) REGRESSION PLOT

This program plots the data points and the cubic curve associated with a third degree regression curve fit. The raw data points may be plotted before or after program STAT-PAC IV-9 is run.

The curve being plotted is given by:

$$f(X) = Y = a_0 + a_1X + a_2X^2 + a_3X^3$$

where the a_i are determined by program STAT-PAC IV-9. The user specifies the following:

X_{max} = Largest Value of X_i, to Be Plotted at X = 10 in; or 25.4 cm.
X_{min} = Smallest Value of X_i, to Be Plotted at X = 0 in; or 0 cm.
Y_{max} = Largest Value of Y_i, to Be Plotted at Y = 5 in; or 12.7 cm.
Y_{min} = Smallest Value of Y_i, to Be Plotted at Y = 0 in; or 0 cm.
X_{final} = The Point to Which f(X) is Desired for Prediction
$X_{initial}$ = The Point from Which f(X) is Desired to Be Plotted. $(X_{initial} \geq X_{min})$

The point $\{ X_{max}, Y \}$ is plotted at X = 10 in., or 25.4 cm. in order to leave 5 in., or 12.7 cm. of plotting surface remaining so that f(X) may be plotted beyond the limits of known (X_i, Y_i) for prediction purposes. The plot position of the points X_{max}, and Y_{max} may be changed by altering program steps (+)(0)(6) through (+)(0)(8) for X_{max}, and (+)(1)(7) through (+)(1)(a) for Y_{max}.

USER INSTRUCTIONS

PRESS: STOP

Using the origin controls, locate the pen in the lower left corner of the paper.

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

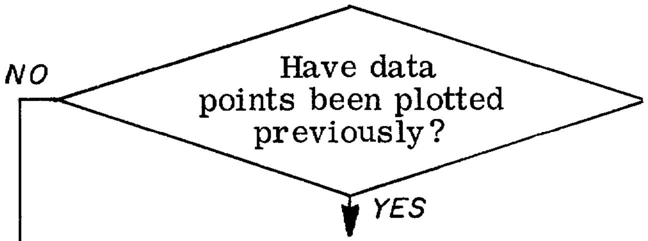
ENTER DATA:

$X_{max} \longrightarrow Y$
$X_{min} \longrightarrow X$

PRESS: CONTINUE

ENTER DATA

$Y_{max} \longrightarrow Y$
$Y_{min} \longrightarrow X$



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

0 ———— Z
1 ———— Y
0 ———— X

PRESS: CONTINUE

DISPLAY

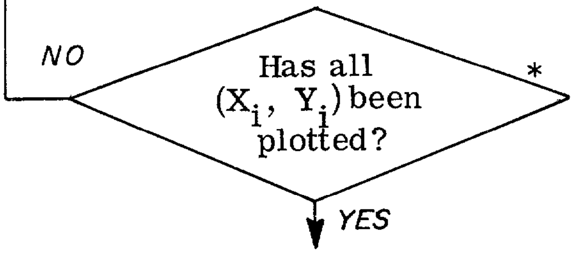
0 ———— Z
i ———— Y
0 ———— X

USER INSTRUCTIONS (Con't)

ENTER DATA:

$Y_i \longrightarrow Y$
$X_i \longrightarrow X$

PRESS: CONTINUE



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

0 ———— Z
i + 1 ———— Y
0 ———— X

ENTER DATA:

$a_3 \longrightarrow Z$
$a_2 \longrightarrow Y$
$a_1 \longrightarrow X$

PRESS: CONTINUE

ENTER DATA:

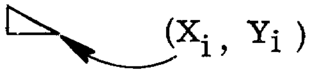
$X_{final} \longrightarrow Z$
$X_{initial} \longrightarrow Y$
$a_0 \longrightarrow X$

PRESS: CONTINUE

The cubic curve is now plotted. The plot will automatically stop when X reaches X_{final} .

To run another case

*The point (X_i, Y_i) is plotted as



EXAMPLE

Using the data from the example of program STAT-PAC IV-9.

X	Y
-2	-23
-1	2
0	5
1	4
2	17

Let:

$$\left.\begin{array}{l} X_{max} = 2 \\ X_{min} = -2 \end{array}\right\}$$

$$\left.\begin{array}{l} Y_{max} = 17 \\ Y_{min} = -23 \end{array}\right\}$$

$$\left.\begin{array}{l} a_3 = 3 \\ a_2 = -2 \\ a_1 = -2 \end{array}\right\}$$

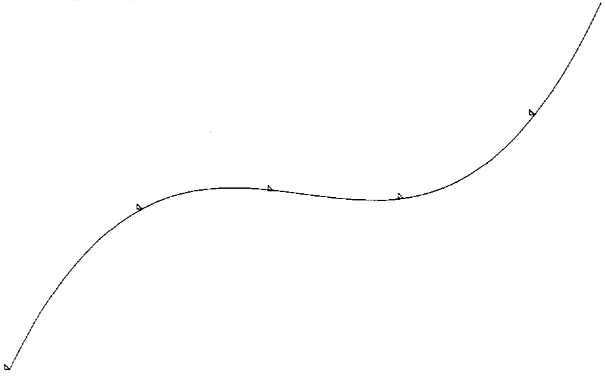
$$\left.\begin{array}{l} X_{final} = 2.5 \\ X_{initial} = -2 \\ a_0 = 5 \end{array}\right\}$$

The resulting plot is shown

STAT-PAC
X-11

Cubic (Third Degree) Regression Plot
$Y = 5 - 2X - 2X^2 + 3X^3$

Cubic coefficients determined by program STAT-PAC IV-9, points and fitted curve generated by program STAT-PAC X-11.



STAT-PAC X-11 9100B ONLY

00	CLR	20	Plus Page	40	-	34		80	CNT	47
01	STP	41	ENTRY	41	d	17		81	CNT	47
02	PNT	45		42	X	36		82	CNT	47
03	PNT	45		43	DN	25		83	CNT	47
04	-	34		44	XEY	30		84	CNT	47
05	UP	27		45	FMT	42		85	CNT	47
06	5	05		46	DN	25		86	CNT	47
07	EEX	26		47	UP	27		87	CNT	47
08	3	03		48	5	05		88	CNT	47
09	RUP	22		49	0	00		89	CNT	47
0a	DIV	35		4a	-	34		8a	CNT	47
0b	DN	25		4b	RDN	31		8b	CNT	47
0c	AC+	60		4c	FMT	42		8c	CNT	47
0d	0	00		4d	DN	25		8d	CNT	47
10	UP	27		50	XEY	30		90	CNT	47
11	UP	27		51	RUP	22		91	CNT	47
12	STP	41	ENTRY	52	+	33		92	CNT	47
13	PNT	45		53	RDN	31		93	CNT	47
14	PNT	45		54	XEY	30		94	CNT	47
15	-	34		55	FMT	42		95	CNT	47
16	UP	27		56	DN	25		96	CNT	47
17	2	02		57	RUP	22		97	CNT	47
18	5	05		58	+	33		98	CNT	47
19	0	00		59	XEY	30		99	CNT	47
1a	0	00		5a	RDN	31		9a	CNT	47
1b	RUP	22		5b	-	34		9b	CNT	47
1c	DIV	35		5c	DN	25		9c	CNT	47
1d	DN	25		5d	XEY	30		9d	CNT	47
20	YTO	40		60	FMT	42		a0	CNT	47
21	c	16		61	DN	25		a1	CNT	47
22	XTO	23		62	FMT	42		a2	CNT	47
23	d	17		63	UP	27		a3	CNT	47
24	0	00		64	0	00		a4	CNT	47
25	UP	27		65	UP	27		a5	CNT	47
26	1	01		66	b	14		a6	CNT	47
27	UP	27		67	UP	27		a7	CNT	47
28	0	00		68	1	01		a8	CNT	47
29	IFG	43		69	+	33		a9	CNT	47
2a	7	07		6a	0	00		aa	CNT	47
2b	0	00		6b	GTO	44		ab	CNT	47
2c	STP	41	ENTRY	6c	2	02		ac	CNT	47
2d	IFG	43		6d	c	16		ad	CNT	47
30	7	07		70	STP	41	ENTRY			
31	0	00		71	PNT	45				
32	PNT	45		72	PNT	45				
33	PNT	45		73	XTO	23				
34	RUP	22		74	b	14				
35	XTO	23		75	YTO	40				
36	b	14		76	a	13				
37	e	12		77	0	00				
38	-	34		78	RDN	31				
39	f	15		79	DN	25				
3a	X	36		7a	GTO	44				
3b	c	16		7b	-	34				
3c	XEY	30		7c	0	00				
3d	RDN	31		7d	0	00				

a_0

a_3

a_2/X

n/a_1

Y_{min}

Y_f

X_{min}

X_f

```
b0  CNT  47    Plus
b1  CNT  47    Page
b2  CNT  47
b3  CNT  47
b4  CNT  47
b5  CNT  47
b6  CNT  47
b7  CNT  47
b8  CNT  47
b9  CNT  47
ba  CNT  47
bb  CNT  47
bc  CNT  47
bd  CNT  47

c0  CNT  47
c1  CNT  47
c2  CNT  47
c3  CNT  47
c4  CNT  47
c5  CNT  47
c6  CNT  47
c7  CNT  47
c8  CNT  47
c9  CNT  47
ca  CNT  47
cb  CNT  47
cc  CNT  47
cd  CNT  47

d0  CNT  47
d1  CNT  47
d2  CNT  47
d3  CNT  47
d4  CNT  47
d5  CNT  47
d6  CNT  47
d7  CNT  47
d8  CNT  47
d9  CNT  47
da  CNT  47
db  CNT  47
dc  CNT  47
dd  CNT  47
```

STAT-PAC X-11

00	XTO	23	Minus Page
01	9	11	
02	DN	25	
03	STP	41	ENTRY
04	PNT	45	
05	PNT	45	
06	XTO	23	
07	8	10	
08	YTO	40	
09	-	34	
0a	f	15	
0b	DN	25	
0c	YTO	40	
0d	-	34	
10	e	12	
11	-	34	
12	EEX	26	
13	2	02	
14	DIV	35	
15	YTO	40	
16	-	34	
17	d	17	
18	XFR	67	
19	-	34	
1a	f	15	
1b	UP	27	
1c	b	14	
1d	XEY	30	
20	X	36	
21	UP	27	
22	X	36	
23	XEY	30	
24	YE	24	
25	a	13	
26	XEY	30	
27	X	36	
28	XEY	30	
29	YE	24	
2a	a	13	
2b	RUP	22	
2c	+	33	
2d	XEY	30	
30	RUP	22	
31	UP	27	
32	X	36	
33	X	36	
34	XFR	67	
35	9	11	
36	X	36	
37	DN	25	
38	+	33	
39	XFR	67	
3a	8	10	
3b	+	33	
3c	c	16	
3d	-	34	
40	d	17	
41	X	36	
42	XFR	67	
43	-	34	
44	f	15	
45	UP	27	
46	e	12	
47	-	34	
48	f	15	
49	X	36	
4a	DN	25	
4b	FMT	42	
4c	DN	25	
4d	XFR	67	
50	-	34	
51	f	15	
52	UP	27	
53	XFR	67	
54	-	34	
55	d	17	
56	+	33	
57	XFR	67	
58	-	34	
59	e	12	
5a	X<Y	52	
5b	6	06	
5c	5	05	
5d	YTO	40	
60	-	34	
61	f	15	
62	GTO	44	
63	1	01	
64	8	10	
65	CLR	20	
66	FMT	42	
67	UP	27	
68	GTO	44	
69	+	33	
6a	0	00	
6b	0	00	
6c	END	46	

x_0

x_{final}

Δx

LEAST SQUARES REGRESSION
(Linear, Parabolic, Power, Exponential)

This program allows the user to calculate and plot the equations of four different regression curves with a single entry of his discrete data.

This regression package is divided into six individual programs as described below and has an error correcting option.

I. MAIN PROGRAM

All data and scaling factors for the four regression programs are entered through this program. Also entered is a desired tolerance on the regression equations. As each data point is entered, a vertical line is plotted above and below this point to allow the user to see if the particular regression curves are within this tolerance.

The main section of the program calculates and accumulates the sums which are needed to generate the least squares coefficients.

II. MAIN PROGRAM CORRECTOR

This is an optional program which may be run if while running the MAIN PROGRAM the user has entered incorrect data points. The program simply deletes these incorrect points from the series of accumulated sums and plots an "X" over such points.

III. LINEAR REGRESSION & PLOT

This program calculates a & b for the equation: $Y = bx + a$.
The defining equations are:

$$b = \frac{\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^{n} (X_i - \overline{X})^2}$$

$$a = \overline{Y} - b\overline{X}$$

$$\text{where: } \overline{X} = \frac{1}{n} \sum_{i=1}^{n} X_i \; ; \; \overline{Y} = \frac{1}{n} \sum_{i=1}^{n} Y_i$$

IV. PARABOLIC REGRESSION & PLOT

This program calculates a_0, a_1, a_2 for the equation $Y = a_0 + a_1X + a_2X^2$.
The coefficients are found by simultaneously solving the following normal equations:

$$\Sigma Y = a_0 n + a_1 \Sigma X + a_2 \Sigma X^2$$
$$\Sigma XY = a_0 \Sigma X + a_1 \Sigma X^2 + a_2 \Sigma X^3$$
$$\Sigma X^2 Y = a_0 \Sigma X^2 + a_1 \Sigma X^3 + a_2 \Sigma X^4$$

V. POWER CURVE REGRESSION & PLOT

This program calculates the correlation coefficient and a and b for a power curve of the form $Y = aX^b$.

The equation is linearized into $\ln Y = b\ln X + \ln a$

where $$b = \frac{n\,\Sigma\,(\ln X\ \ln Y) - \Sigma\ln X\ \Sigma\ln Y}{n\Sigma(\ln X)^2 - (\Sigma\ln X)^2}$$

and $$r = \frac{n\Sigma\ln X\ \ln Y - (\Sigma\ln X)(\Sigma\ln Y)}{\sqrt{[n\Sigma(\ln X)^2 - (\Sigma\ln X)^2][n\Sigma(\ln Y)^2 - (\Sigma\ln Y)^2]}}$$

$$\ln a = \frac{\Sigma\ln Y}{n} - \frac{\Sigma\ln X}{n}\ b$$

VI. EXPONENTIAL REGRESSION & PLOT

This program claculates the correlation coefficient and a and b for an exponential function of the form $Y = ae^{bX}$.

The equation is linearized into $\ln Y = \ln a + bX$
or $Y = A + bX$

Using a linear regression method,

$$b = \frac{n\Sigma XY - \Sigma X\Sigma Y}{n\Sigma X^2 - (\Sigma X)^2}$$

$$A = \frac{\Sigma Y - b\Sigma X}{n}$$

$$a = e^A$$

the correlation coefficient is given by

$$r = \frac{n\Sigma XY - \Sigma X\Sigma Y}{\sqrt{[n\Sigma X^2 - (\Sigma X)^2]\ [n\Sigma Y^2 - (\Sigma Y)^2]}}$$

To convert the program for plotting in centimeters, make the following changes:

Program	Step	Change to
I	(-)(0)(8)	1
	(-)(0)(9)	9
	(-)(0)(a)	7
	(-)(c)(5)	2
	(-)(c)(6)	0
II	(-)(9)(0)	5
	(-)(9)(1)	0
	(-)(9)(2)	.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: STOP

Using the origin controls, locate the pen in the lower left corner of the paper.

SET: Decimal Wheel at 6 or less

PRESS: END

ENTER MAIN PROGRAM:
Side A followed by Side B

PRESS: CONTINUE

ENTER SHIFT FACTORS:

$$Y_{shift} \longrightarrow Y$$
$$X_{shift} \longrightarrow X$$

PRESS: CONTINUE

ENTER SCALE FACTORS:

$$Y_{scale} \longrightarrow Y$$
$$X_{scale} \longrightarrow X$$

PRESS: CONTINUE

ENTER DESIRED TOLERANCE ON Y:

$$\Delta Y \longrightarrow X$$

PRESS: CONTINUE

→DISPLAY

0 ——— Z
i ——— Y
0 ——— X

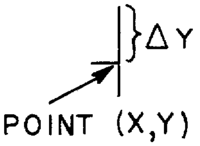
ENTER COORDINATES:

$$Y_i \longrightarrow Y$$
$$X_i \longrightarrow X$$

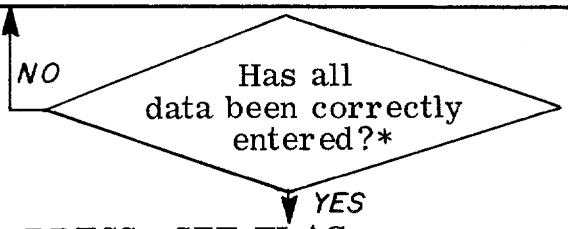
NOTE: $X_i > 0$, $Y_i > 0$

PRESS: CONTINUE

[Point is plotted as



USER INSTRUCTIONS (Con't)



PRESS: SET FLAG
PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
n ——— X

(n = total number of data points)

*Check the printer output to determine whether any incorrect data points have been entered in the program. If an error has been made, enter the correct values until all correct (X_i, Y_i) sets have been entered.

If all entered data is correct, proceed to the regression programs.

Otherwise:

PRESS: GO TO (-)(0)(0)

ENTER MAIN PROGRAM CORRECTOR

PRESS: GO TO (-)(0)(0)

PRESS: CONTINUE

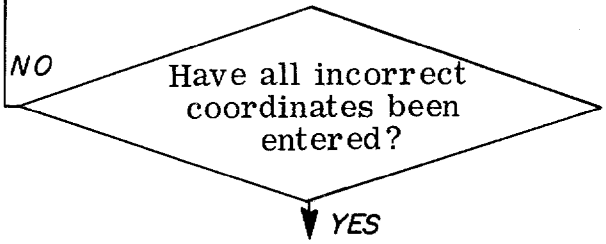
→DISPLAY

0 ——— Z
10i ——— Y
0 ——— X

ENTER i^th SET OF COORDINATES TO BE DELETED:

$$Y_i \longrightarrow Y$$
$$X_i \longrightarrow X$$

PRESS: CONTINUE



PRESS: SET FLAG

USER INSTRUCTIONS (Con't)

PRESS: CONTINUE

DISPLAY

0 ———— Z
0 ———— Y
10K———— X

(K = number of data points deleted)

Any of the four regression programs may now be run in any order any number of times. Individual instructions are given below. All curves are self-terminating.

POWER CURVE: $Y = aX^b$

PRESS: GO TO (-)(0)(0)

ENTER PROGRAM

PRESS: GO TO (-)(0)(0)

PRESS: CONTINUE

DISPLAY

r ———— Z
b ———— Y
a ———— X

PRESS: CONTINUE

The curve is now plotted.

EXPONENTIAL CURVE: $Y = ae^{bX}$
(Same as POWER CURVE)

LINEAR CURVE: $Y = bX + a$

(Same as POWER CURVE except without r)

PARABOLIC CURVE: $Y=a_0+a_1X+a_2X^2$

PRESS: GO TO (-)(0)(0)

ENTER PROGRAM A

PRESS: GO TO (-)(0)(0)

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

a_0 ———— Z
a_1 ———— Y
a_2 ———— X

PRESS: GO TO (-)(0)(0)

ENTER PROGRAM B

PRESS: GO TO (-)(0)(0)

PRESS: CONTINUE

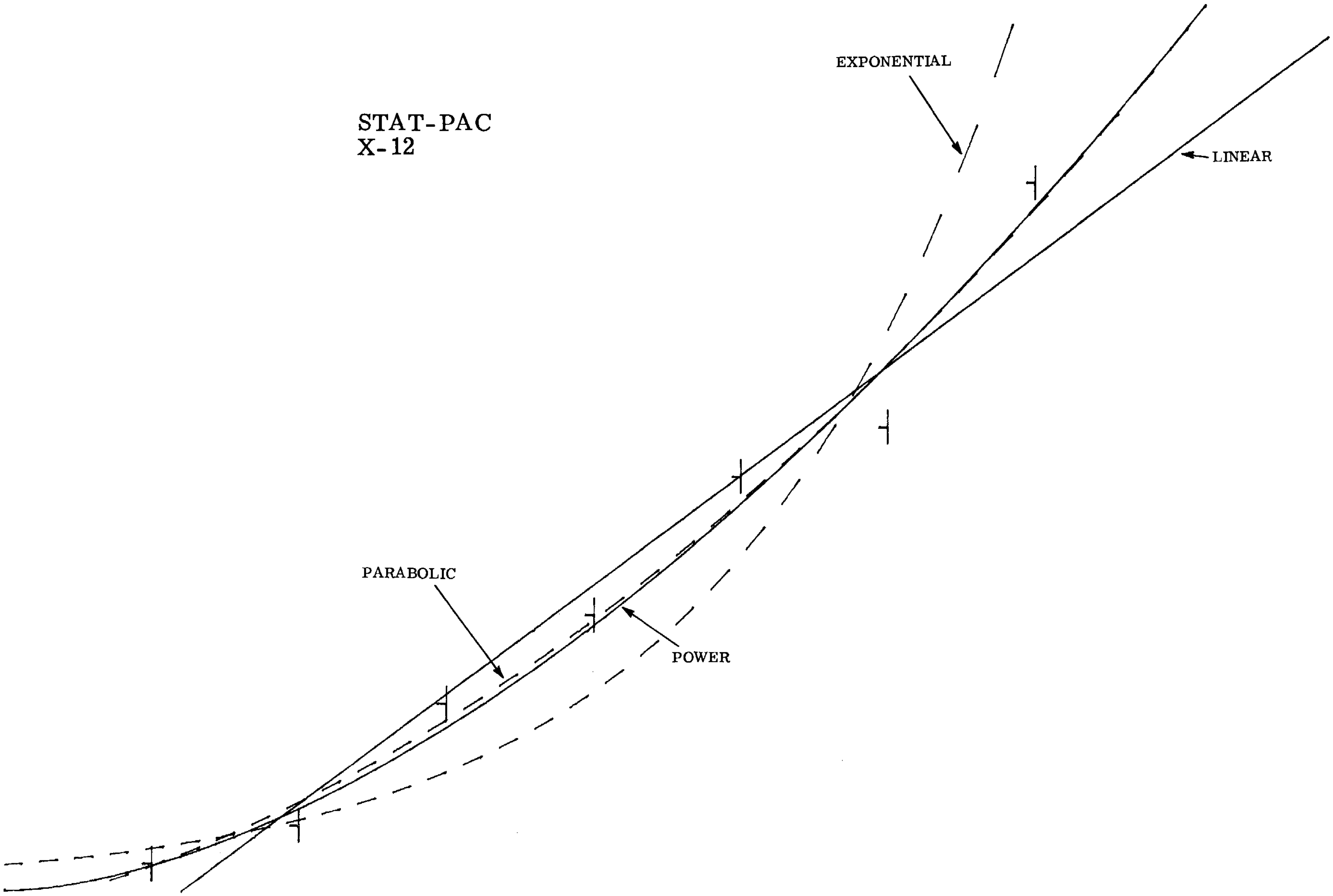
The curve is now plotted.

EXAMPLE

$X_{shift} = Y_{shift} = 0$
$X_{scale} = .6$
$Y_{scale} = 16$
$\Delta Y = 3$

X	Y
1	5
2	12
3	34
4	50
5	75
6	84
7	128

PARABOLA: $1.64X^2 + 6.64X - 4.00$
EXPONENTIAL: $4.63e^{.51}$, r = .96
POWER: $4.66X^{1.68}$, r = .994
LINEAR: $19.79X - 23.7$


STAT-PAC
X-12
EXPONENTIAL
LINEAR
PARABOLIC
POWER

STAT-PAC X-12 General Loader 9100B ONLY

00	0	00	Plus Page	40	0	00	80	0	00
01	0	00		41	0	00	81	0	00
02	0	00		42	0	00	82	0	00
03	0	00		43	0	00	83	0	00
04	0	00		44	0	00	84	0	00
05	0	00		45	0	00	85	0	00
06	0	00		46	0	00	86	0	00
07	0	00		47	0	00	87	0	00
08	0	00		48	0	00	88	0	00
09	0	00		49	0	00	89	0	00
0a	0	00		4a	0	00	8a	0	00
0b	0	00		4b	0	00	8b	0	00
0c	0	00		4c	0	00	8c	0	00
0d	0	00		4d	0	00	8d	0	00
10	0	00		50	0	00	90	0	00
11	0	00		51	0	00	91	0	00
12	0	00		52	0	00	92	0	00
13	0	00		53	0	00	93	0	00
14	0	00		54	0	00	94	0	00
15	0	00		55	0	00	95	0	00
16	0	00		56	0	00	96	0	00
17	0	00		57	0	00	97	0	00
18	0	00		58	0	00	98	0	00
19	0	00		59	0	00	99	0	00
1a	0	00		5a	0	00	9a	0	00
1b	0	00		5b	0	00	9b	0	00
1c	0	00		5c	0	00	9c	0	00
1d	0	00		5d	0	00	9d	0	00
20	0	00		60	0	00	a0	0	00
21	0	00		61	0	00	a1	0	00
22	0	00		62	0	00	a2	0	00
23	0	00		63	0	00	a3	0	00
24	0	00		64	0	00	a4	0	00
25	0	00		65	0	00	a5	0	00
26	0	00		66	0	00	a6	0	00
27	0	00		67	0	00	a7	0	00
28	0	00		68	0	00	a8	0	00
29	0	00		69	0	00	a9	0	00
2a	0	00		6a	0	00	aa	0	00
2b	0	00		6b	0	00	ab	0	00
2c	0	00		6c	0	00	ac	0	00
2d	0	00		6d	0	00	ad	0	00
30	0	00		70	0	00			
31	0	00		71	0	00			
32	0	00		72	0	00			
33	0	00		73	0	00			
34	0	00		74	0	00			
35	0	00		75	0	00			
36	0	00		76	0	00			
37	0	00		77	0	00			
38	0	00		78	0	00			
39	0	00		79	0	00			
3a	0	00		7a	0	00			
3b	0	00		7b	0	00			
3c	0	00		7c	0	00			
3d	0	00		7d	0	00			

b0	0	00	Plus Page
b1	0	00	
b2	0	00	
b3	0	00	
b4	0	00	
b5	0	00	
b6	0	00	
b7	0	00	
b8	0	00	
b9	0	00	
ba	0	00	
bb	0	00	
bc	0	00	
bd	0	00	
c0	0	00	
c1	0	00	
c2	0	00	
c3	0	00	
c4	0	00	
c5	0	00	
c6	0	00	
c7	0	00	
c8	0	00	
c9	0	00	
ca	0	00	
cb	0	00	
cc	0	00	
cd	0	00	
d0	0	00	
d1	0	00	
d2	0	00	
d3	0	00	
d4	0	00	
d5	0	00	
d6	0	00	
d7	0	00	
d8	0	00	
d9	0	00	
da	0	00	
db	0	00	
dc	0	00	
dd	CLR	20	

STAT-PAC X-12 General Loader 9100B ONLY

```
00  STP  41   ENTRY Minus Page
01  XTO  23
02  d    17
03  YTO  40
04  c    16
05  CLR  20
06  STP  41   ENTRY
07  UP   27
08  5    05
09  0    00
0a  0    00
0b  DIV  35
0c  YTO  40
0d  b    14

10  RUP  22
11  DIV  35
12  YTO  40
13  a    13
14  CLR  20
15  STP  41   ENTRY
16  UP   27
17  a    13
18  X    36
19  YTO  40
1a  -    34
1b  0    00
1c  CLR  20
1d  XTO  23

20  -    34
21  e    12
22  XTO  23
23  -    34
24  f    15
25  1    01
26  XTO  23
27  0    00
28  RUP  22
29  STP  41   ENTRY
2a  IFG  43
2b  d    17
2c  8    10
2d  PNT  45

30  PNT  45
31  AC+  60
32  UP   27
33  LN   65
34  YE   24
35  3    03
36  +    33
37  YE   24
38  3    03
39  RUP  22
3a  LN   65
3b  YE   24
3c  4    04
3d  +    33

40  YE   24
41  4    04
42  X    36
43  XEY  30
44  YE   24
45  1    01
46  +    33
47  YE   24
48  1    01
49  DN   25
4a  X    36
4b  XEY  30
4c  YE   24
4d  -    34

50  f    15
51  +    33
52  YE   24
53  -    34
54  f    15
55  DN   25
56  UP   27
57  X    36
58  XEY  30
59  YE   24
5a  -    34
5b  e    12
5c  +    33
5d  YE   24

60  -    34
61  e    12
62  DN   25
63  EXP  74
64  XEY  30
65  LN   65
66  UP   27
67  X    36
68  XEY  30
69  YE   24
6a  2    02
6b  +    33
6c  YE   24
6d  2    02

70  DN   25
71  EXP  74
72  X    36
73  XEY  30
74  YE   24
75  6    06
76  +    33
77  YE   24
78  6    06
79  XEY  30
7a  X    36
7b  XEY  30
7c  YE   24
7d  5    05

80  +    33
81  YE   24
82  5    05
83  DN   25
84  UP   27
85  X    36
86  XEY  30
87  YE   24
88  9    11
89  +    33
8a  YE   24
8b  9    11
8c  XEY  30
8d  X    36

90  XEY  30
91  YE   24
92  8    10
93  +    33
94  YE   24
95  8    10
96  XEY  30
97  X    36
98  XEY  30
99  YE   24
9a  7    07
9b  +    33
9c  YE   24
9d  7    07

a0  d    17
a1  -    34
a2  b    14
a3  DIV  35
a4  c    16
a5  RUP  22
a6  XEY  30
a7  -    34
a8  a    13
a9  X    36
aa  XFR  67
ab  -    34
ac  0    00
ad  +    33
```

```
b0  RDN  31       Minus
b1  XEY  30       Page
b2  FMT  42
b3  DN   25
b4  RDN  31
b5  XEY  30
b6   -   34
b7   -   34
b8  RDN  31
b9  XEY  30
ba  FMT  42
bb  DN   25
bc  RDN  31
bd   +   33

c0  XEY  30
c1  RUP  22
c2  FMT  42
c3  DN   25
c4  UP   27
c5   5   05
c6   0   00
c7   -   34
c8  CLX  37
c9  RDN  31
ca  FMT  42
cb  DN   25
cc  FMT  42
cd  UP   27

d0  YE   24
d1   0   00
d2   1   01
d3   +   33
d4  DN   25
d5  GTO  44
d6   2   02
d7   6   06
d8   1   01
d9   -   34
da  YTO  40
db   0   00
dc  DN   25
dd  END  46
```

STAT-PAC X-12 Corrector 9100B ONLY

00	1	01
01	0	00
02	1	01
03	XTO	23
04	-	34
05	d	17
06	RUP	22
07	STP	41
08	IFG	43
09	c	16
0a	a	13
0b	PNT	45
0c	PNT	45
0d	AC-	63
10	UP	27
11	LN	65
12	YE	24
13	3	03
14	-	34
15	YE	24
16	3	03
17	RUP	22
18	LN	65
19	YE	24
1a	4	04
1b	-	34
1c	YE	24
1d	4	04
20	X	36
21	XEY	30
22	YE	24
23	1	01
24	-	34
25	YE	24
26	1	01
27	DN	25
28	X	36
29	XEY	30
2a	YE	24
2b	-	34
2c	f	15
2d	-	34
30	YE	24
31	-	34
32	f	15
33	DN	25
34	UP	27
35	X	36
36	XEY	30
37	YE	24
38	-	34
39	e	12
3a	-	34
3b	YE	24
3c	-	34
3d	e	12
40	DN	25
41	EXP	74
42	XEY	30
43	LN	65
44	UP	27
45	X	36
46	XEY	30
47	YE	24
48	2	02
49	-	34
4a	YE	24
4b	2	02
4c	DN	25
4d	EXP	74
50	X	36
51	XEY	30
52	YE	24
53	6	06
54	-	34
55	YE	24
56	6	06
57	XEY	30
58	X	36
59	XEY	30
5a	YE	24
5b	5	05
5c	-	34
5d	YE	24
60	5	05
61	DN	25
62	UP	27
63	X	36
64	XEY	30
65	YE	24
66	9	11
67	-	34
68	YE	24
69	9	11
6a	XEY	30
6b	X	36
6c	XEY	30
6d	YE	24
70	8	10
71	-	34
72	YE	24
73	8	10
74	XEY	30
75	X	36
76	XEY	30
77	YE	24
78	7	07
79	-	34
7a	YE	24
7b	7	07
7c	YE	24
7d	0	00
80	1	01
81	-	34
82	YE	24
83	0	00
84	d	17
85	-	34
86	b	14
87	DIV	35
88	c	16
89	RUP	22
8a	XEY	30
8b	-	34
8c	a	13
8d	X	36
90	1	01
91	0	00
92	0	00
93	-	34
94	RUP	22
95	XEY	30
96	+	33
97	RDN	31
98	FMT	42
99	DN	25
9a	RUP	22
9b	-	34
9c	-	34
9d	XEY	30
a0	RDN	31
a1	+	33
a2	+	33
a3	XEY	30
a4	RUP	22
a5	FMT	42
a6	DN	25
a7	RUP	22
a8	+	33
a9	+	33
aa	RDN	31
ab	FMT	42
ac	UP	27
ad	FMT	42

STAT-PAC X-12 Corrector 9100B ONLY

```
b0   DN    25
b1   RUP   22
b2    -    34
b3    -    34
b4   RUP   22
b5   XEY   30
b6    -    34
b7    -    34
b8   CLX   37
b9   RDN   31
ba   XEY   30
bb   FMT   42
bc   DN    25
bd   FMT   42

c0   UP    27
c1   YE    24
c2    -    34
c3    d    17
c4    1    01
c5    +    33
c6   DN    25
c7   GTO   44
c8    0    00
c9    3    03
ca    1    01
cb    -    34
cc   DN    25
cd   END   46
```

```
00  RCL  61          40  -    34          80  X<Y  52
01  XFR  67          41  CLX  37          81  9    11
02  0    00          42  RDN  31          82  c    16
03  DIV  35          43  XTO  23          83  XEY  30
04  YTO  40          44  -    34          84  RDN  31
05  -    34          45  c    16          85  c    16
06  d    17          46  YTO  40          86  -    34
07  UP   27          47  -    34          87  a    13
08  f    15          48  d    17          88  X    36
09  XEY  30          49  PNT  45          89  CLX  37
0a  DIV  35          4a  PNT  45          8a  X>Y  53
0b  YTO  40          4b  STP  41          8b  5    05
0c  -    34          4c  b    14          8c  c    16
0d  c    16          4d  UP   27          8d  5    05

10  XEY  30          50  EEX  26          90  EEX  26
11  X    36          51  2    02          91  3    03
12  RDN  31          52  X    36          92  X<Y  52
13  X    36          53  d    17          93  5    05
14  e    12          54  XEY  30          94  c    16
15  RUP  22          55  -    34          95  DN   25
16  X    36          56  XTO  23          96  XEY  30
17  DN   25          57  -    34          97  FMT  42
18  -    34          58  b    14          98  DN   25
19  XFR  67          59  YTO  40          99  GTO  44
1a  -    34          5a  -    34          9a  5    05
1b  d    17          5b  0    00          9b  c    16
1c  UP   27          5c  YE   24          9c  CLX  37
1d  f    15          5d  -    34          9d  UP   27

20  X    36          60  0    00          a0  UP   27
21  DN   25          61  XFR  67          a1  FMT  42
22  -    34          62  -    34          a2  UP   27
23  XFR  67          63  b    14          a3  END  46
24  6    06          64  +    33
25  +    33          65  YTO  40
26  f    15          66  -    34
27  UP   27          67  0    00
28  XFR  67          68  XFR  67
29  -    34          69  -    34
2a  c    16          6a  d    17
2b  X    36          6b  X    36
2c  XFR  67          6c  XFR  67
2d  9    11          6d  -    34

30  XEY  30          70  c    16
31  -    34          71  +    33
32  DN   25          72  XFR  67
33  DIV  35          73  -    34
34  UP   27          74  0    00
35  DN   25          75  UP   27
36  XFR  67          76  d    17
37  -    34          77  -    34
38  c    16          78  b    14
39  X    36          79  DIV  35
3a  XFR  67          7a  7    07
3b  -    34          7b  5    05
3c  d    17          7c  0    00
3d  XEY  30          7d  0    00
```

STAT-PAC X-12 Power 9100B ONLY

```
00  XFR  67
01   3   03
02  UP   27
03   X   36
04  XFR  67
05   2   02
06  UP   27
07  XFR  67
08   0   00
09   X   36
0a  RDN  31
0b  XEY  30
0c   -   34
0d  YTO  40

10   -   34
11   0   00
12  XFR  67
13   -   34
14   e   12
15  RUP  22
16   X   36
17  XFR  67
18   4   04
19  UP   27
1a   X   36
1b  DN   25
1c   -   34
1d  YTO  40

20   -   34
21   1   01
22  XFR  67
23   4   04
24  UP   27
25  XFR  67
26   3   03
27   X   36
28  XFR  67
29   1   01
2a  UP   27
2b  XFR  67
2c   0   00
2d   X   36

30  RDN  31
31  XEY  30
32   -   34
33  XFR  67
34   -   34
35   0   00
36  DIV  35
37  YTO  40
38   -   34
39   0   00
3a   √   76
3b   X   36
3c  XFR  67
3d   -   34

40   1   01
41   √   76
42  DIV  35
43  XFR  67
44   -   34
45   0   00
46  UP   27
47  XFR  67
48   3   03
49   X   36
4a  XFR  67
4b   4   04
4c  XEY  30
4d   -   34

50  XFR  67
51   0   00
52  DIV  35
53  XFR  67
54   -   34
55   0   00
56  XEY  30
57  EXP  74
58  XTO  23
59   -   34
5a   0   00
5b  YTO  40
5c   -   34
5d   1   01

60  PNT  45
61  PNT  45
62  STP  41
63   b   14
64  UP   27
65  EEX  26
66   2   02
67   X   36
68   d   17
69  XEY  30
6a   -   34
6b  XTO  23
6c   -   34
6d   c   16

70  YTO  40
71   -   34
72   d   17
73  YE   24
74   -   34
75   d   17
76  XFR  67
77   -   34
78   c   16
79   +   33
7a  YTO  40
7b   -   34
7c   d   17
7d  XFR  67

80   -   34
81   1   01
82  XEY  30
83  LN   65
84   X   36
85  XFR  67
86   -   34
87   0   00
88  XEY  30
89  EXP  74
8a   X   36
8b  XFR  67
8c   -   34
8d   d   17

90  UP   27
91   d   17
92   -   34
93   b   14
94  DIV  35
95   7   07
96   5   05
97   0   00
98   0   00
99  X<Y  52
9a   b   14
9b   7   07
9c  XEY  30
9d  RDN  31

a0   c   16
a1   -   34
a2   a   13
a3   X   36
a4  CLX  37
a5  X>Y  53
a6   7   07
a7   3   03
a8   5   05
a9  EEX  26
aa   3   03
ab  X<Y  52
ac   7   07
ad   3   03
```

b0	DN	25
b1	XEY	30
b2	FMT	42
b3	DN	25
b4	GTO	44
b5	7	07
b6	3	03
b7	CLX	37
b8	UP	27
b9	UP	27
ba	FMT	42
bb	UP	27
bc	END	46

STAT-PAC X-12 Exponential 9100B ONLY

```
00  XFR  67
01  0    00
02  UP   27
03  XFR  67
04  -    34
05  f    15
06  X    36
07  f    15
08  UP   27
09  XFR  67
0a  4    04
0b  X    36
0c  DN   25
0d  -    34

10  YTO  40
11  -    34
12  0    00
13  XFR  67
14  0    00
15  UP   27
16  XFR  67
17  9    11
18  X    36
19  f    15
1a  UP   27
1b  X    36
1c  DN   25
1d  -    34

20  YTO  40
21  -    34
22  1    01
23  XFR  67
24  0    00
25  UP   27
26  XFR  67
27  -    34
28  e    12
29  X    36
2a  XFR  67
2b  4    04
2c  UP   27
2d  X    36

30  DN   25
31  -    34
32  XFR  67
33  -    34
34  1    01
35  X    36
36  DN   25
37  √    76
38  UP   27
39  XFR  67
3a  -    34
3b  0    00
3c  XEY  30
3d  DIV  35

40  YTO  40
41  -    34
42  2    02
43  XFR  67
44  -    34
45  0    00
46  UP   27
47  XFR  67
48  -    34
49  1    01
4a  DIV  35
4b  f    15
4c  UP   27
4d  DN   25

50  X    36
51  XFR  67
52  4    04
53  XEY  30
54  -    34
55  XFR  67
56  0    00
57  DIV  35
58  XFR  67
59  -    34
5a  2    02
5b  RDN  31
5c  EXP  74
5d  XTO  23

60  -    34
61  0    00
62  YTO  40
63  -    34
64  1    01
65  PNT  45
66  PNT  45
67  STP  41
68  b    14
69  UP   27
6a  EEX  26
6b  2    02
6c  X    36
6d  d    17

70  XEY  30
71  -    34
72  XTO  23
73  -    34
74  2    02
75  YTO  40
76  -    34
77  3    03
78  YE   24
79  -    34
7a  3    03
7b  XFR  67
7c  -    34
7d  2    02

80  +    33
81  YTO  40
82  -    34
83  3    03
84  XFR  67
85  -    34
86  1    01
87  X    36
88  XFR  67
89  -    34
8a  0    00
8b  XEY  30
8c  EXP  74
8d  X    36

90  XFR  67
91  -    34
92  3    03
93  UP   27
94  d    17
95  -    34
96  b    14
97  DIV  35
98  7    07
99  5    05
9a  0    00
9b  0    00
9c  X<Y  52
9d  c    16

a0  2    02
a1  XEY  30
a2  RDN  31
a3  c    16
a4  -    34
a5  a    13
a6  X    36
a7  CLX  37
a8  X>Y  53
a9  7    07
aa  8    10
ab  5    05
ac  EEX  26
ad  3    03
```

```
b0   X<Y   52
b1    7    07
b2    8    10
b3   DN    25
b4   XEY   30
b5   IFG   43
b6    b    14
b7    b    14
b8   FMT   42
b9   UP    27
ba   SFL   54
bb   FMT   42
bc   DN    25
bd   GTO   44

c0    7    07
c1    8    10
c2   CLX   37
c3   UP    27
c4   UP    27
c5   FMT   42
c6   UP    27
c7   END   46
```

STAT-PAC X-12 Parabolic Program 1 9100B ONLY

```
00  XFR  67        40   f    15        80  DIV  35
01   8   10        41   X    36        81  UP   27
02  UP   27        42  YTO   40        82  DN   25
03   f   15        43   -    34        83   e   12
04   X   36        44   1    01        84  XEY  30
05   2   02        45  UP    27        85  XTO  23
06   X   36        46  XFR   67        86   -   34
07  XFR  67        47   8    10        87   4   04
08   9   11        48   X    36        88  XFR  67
09   X   36        49  XFR   67        89   9   11
0a  UP   27        4a   9    11        8a  RUP  22
0b   X   36        4b  UP    27        8b   X   36
0c   X   36        4c   X    36        8c  RDN  31
0d  DN   25        4d  DN    25        8d   -   34

10   -   34        50   -    34        90  YTO  40
11   f   15        51   e    12        91   -   34
12  UP   27        52   X    36        92   2   02
13   X   36        53  DN    25        93  XFR  67
14  XFR  67        54  YE    24        94   6   06
15   7   07        55   -    34        95  XEY  30
16   X   36        56   1    01        96  XFR  67
17  RDN  31        57   -    34        97   8   10
18   -   34        58  YTO   40        98  RUP  22
19  XFR  67        59   -    34        99   X   36
1a   9   11        5a   2    02        9a  DN   25
1b  RUP  22        5b  XFR   67        9b   -   34
1c   X   36        5c   5    05        9c  XFR  67
1d  XFR  67        5d  XEY   30        9d   -   34

20   0   00        60  XFR   67        a0   0   00
21   X   36        61   9    11        a1   X   36
22  RDN  31        62   X    36        a2  XFR  67
23   +   33        63  XFR   67        a3   -   34
24  XFR  67        64   8    10        a4   2   02
25   8   10        65  UP    27        a5  UP   27
26  RUP  22        66  XFR   67        a6   f   15
27  XEY  30        67   6    06        a7   X   36
28   X   36        68   X    36        a8  RDN  31
29   X   36        69  DN    25        a9   -   34
2a  RDN  31        6a   -    34        aa  YTO  40
2b   -   34        6b  XFR   67        ab   -   34
2c  YTO  40        6c   -    34        ac   3   03
2d   -   34        6d   2    02        ad  XFR  67

30   0   00        70  XEY   30
31  XFR  67        71  UP    27
32   5   05        72  XFR   67
33  UP   27        73   0    00
34   f   15        74   X    36
35   X   36        75  RUP   22
36  XFR  67        76  YE    24
37   9   11        77   -    34
38  UP   27        78   0    00
39  XFR  67        79  RUP   22
3a   6   06        7a  XEY   30
3b   X   36        7b   -    34
3c  DN   25        7c  DN    25
3d   -   34        7d  XEY   30
```

STAT-PAC X-12 Parabolic Program 1 9100B ONLY

```
b0   9    11
b1   XEY  30
b2   XFR  67
b3   -    34
b4   0    00
b5   X    36
b6   DN   25
b7   RDN  31
b8   X    36
b9   RDN  31
ba   -    34
bb   XFR  67
bc   -    34
bd   3    03

c0   XEY  30
c1   DIV  35
c2   YTO  40
c3   -    34
c4   3    03
c5   DN   25
c6   X    36
c7   XFR  67
c8   -    34
c9   2    02
ca   XEY  30
cb   -    34
cc   XFR  67
cd   -    34

d0   0    00
d1   DIV  35
d2   YTO  40
d3   -    34
d4   0    00
d5   XFR  67
d6   -    34
d7   3    03
d8   UP   27
d9   XFR  67
da   -    34
db   4    04
dc   PNT  45
dd   END  46
```

STAT-PAC X-12 Parabolic Program 2 9100B ONLY

```
00   XTO   23
01   -     34
02   a     13
03   YTO   40
04   -     34
05   b     14
06   DN    25
07   YTO   40
08   -     34
09   c     16
0a   b     14
0b   UP    27
0c   EEX   26
0d   2     02

10   X     36
11   d     17
12   XEY   30
13   -     34
14   XTO   23
15   -     34
16   9     11
17   YTO   40
18   -     34
19   8     10
1a   YE    24
1b   -     34
1c   8     10
1d   XFR   67

20   -     34
21   9     11
22   +     33
23   YTO   40
24   -     34
25   8     10
26   DN    25
27   UP    27
28   UP    27
29   X     36
2a   XFR   67
2b   -     34
2c   a     13
2d   X     36

30   XFR   67
31   -     34
32   b     14
33   RUP   22
34   X     36
35   XFR   67
36   -     34
37   c     16
38   +     33
39   DN    25
3a   +     33
3b   XFR   67
3c   -     34
3d   8     10

40   UP    27
41   d     17
42   -     34
43   b     14
44   DIV   35
45   7     07
46   5     05
47   0     00
48   0     00
49   X<Y   52
4a   6     06
4b   d     17
4c   XEY   30
4d   RDN   31

50   c     16
51   -     34
52   a     13
53   X     36
54   CLX   37
55   X>Y   53
56   1     01
57   a     13
58   5     05
59   EEX   26
5a   3     03
5b   X<Y   52
5c   1     01
5d   a     13

60   DN    25
61   XEY   30
62   IFG   43
63   6     06
64   8     10
65   FMT   42
66   UP    27
67   SFL   54
68   FMT   42
69   DN    25
6a   GTO   44
6b   1     01
6c   a     13
6d   CLX   37

70   UP    27
71   UP    27
72   FMT   42
73   UP    27
74   END   46
```

LOG NORMAL DISTRIBUTION WITH PLOT

STAT-PAC
X-13

This program computes the value of the log-normal distribution for a given set of parameters, a, μ , σ , over a specified range of X.

Definitions:

$$f(X) = \frac{1}{(X - a)\, \sigma \sqrt{2\pi}} \quad \exp\left[- \frac{(\ln [X - a] - \mu)^2}{2\sigma^2}\right]$$

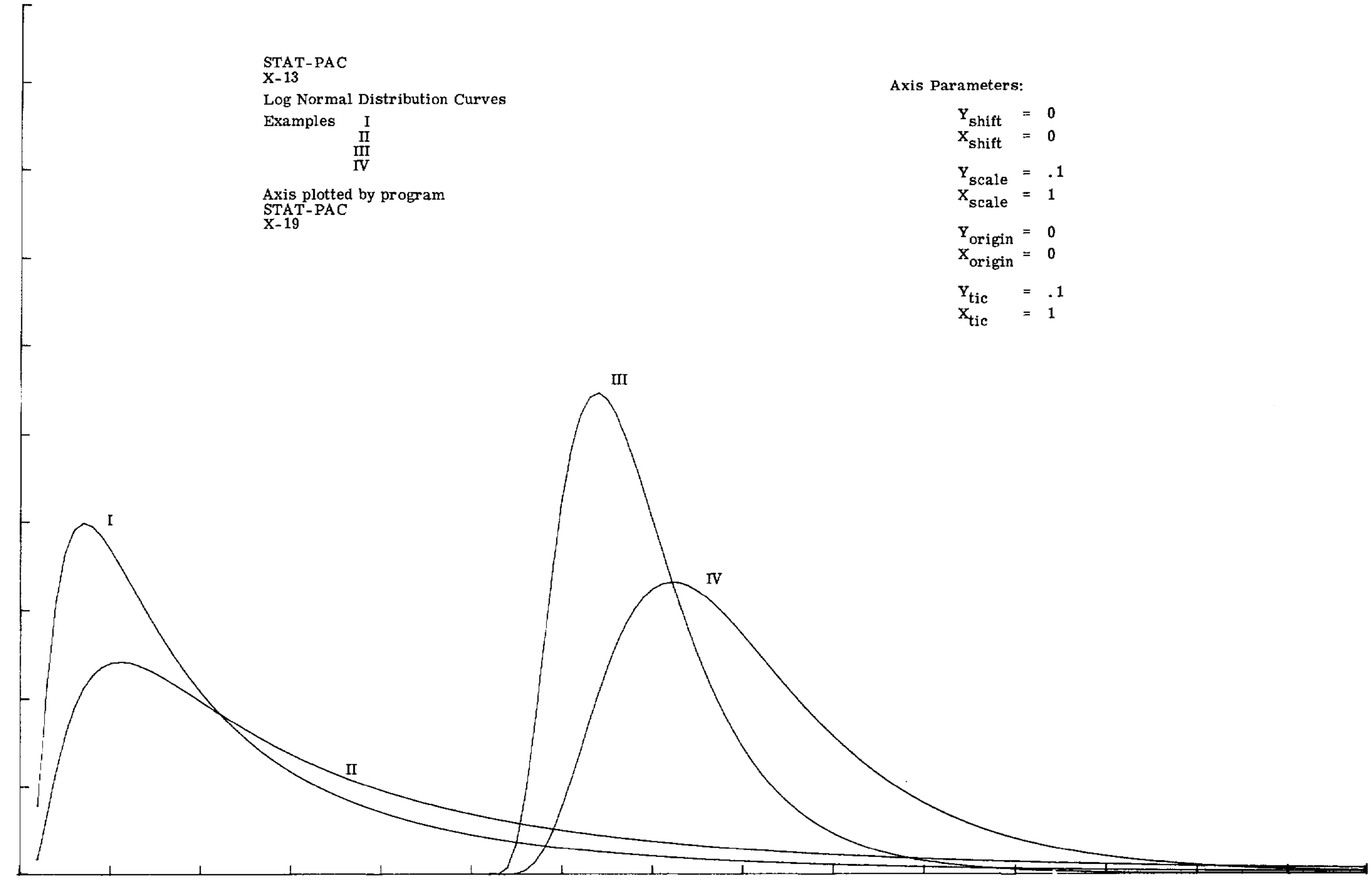
where

$$X > a$$

The program determines f(X) over a range

$$X_i \leq X \leq X_f$$

in increments of ΔX.



USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: END

ENTER PROGRAM

→ PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
1 ——— X

ENTER DATA:

a → Z
σ → Y
μ → X

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
2 ——— X

ENTER DATA:

ΔX → Z
X_f → Y
X_i → X

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
3 ——— X

ENTER DATA:

Y_{plot} → Y
X_{plot} → X *

→ PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

f(X) ——— Z
X ——— Y
ΔX ——— X

f(X) is now plotted.

When $X = X_f$

$*Y_{plot} = 500/Y_{scale}$

$X_{plot} = 500/X_{scale}$

EXAMPLES

I.

a = 0
σ = 1
μ = .5

ΔX = .1
X_f = 15
X_i = .1

II.

a = 0
σ = 1
μ = 1

ΔX = .1
X_f = 15
X_i = .1

III.

a = 5
σ = .5
μ = .5

ΔX = .1
X_f = 15
X_i = 5.1

IV.

a = 5
σ = .5
μ = 1

ΔX = .1
X_f = 15
X_i = 5.1

For all cases

Y_{plot} = 5000
X_{plot} = 500

STAT-PAC X-13

00	CLR	20	
01	1	01	
02	STP	41	ENTRY
03	PNT	45	
04	PNT	45	
05	AC+	60	
06	DN	25	
07	YTO	40	
08	d	17	
09	0	00	
0a	UP	27	
0b	UP	27	
0c	2	02	
0d	STP	41	ENTRY
10	PNT	45	
11	PNT	45	
12	XTO	23	
13	c	16	
14	YTO	40	
15	b	14	
16	DN	25	
17	YTO	40	
18	a	13	
19	0	00	
1a	UP	27	
1b	UP	27	
1c	3	03	
1d	STP	41	ENTRY
20	PNT	45	
21	PNT	45	
22	XTO	23	
23	9	11	
24	YTO	40	
25	8	10	
26	c	16	
27	UP	27	
28	d	17	
29	-	34	
2a	DN	25	
2b	LN	65	
2c	UP	27	
2d	f	15	
30	-	34	
31	DN	25	
32	UP	27	
33	X	36	
34	2	02	
35	DIV	35	
36	e	12	
37	UP	27	
38	X	36	
39	DN	25	
3a	DIV	35	
3b	DN	25	
3c	CHS	32	
3d	EXP	74	
40	UP	27	
41	e	12	
42	DIV	35	
43	c	16	
44	UP	27	
45	d	17	
46	-	34	
47	DN	25	
48	DIV	35	
49	π	56	
4a	√	76	
4b	DIV	35	
4c	2	02	
4d	√	76	
50	DIV	35	
51	c	16	
52	UP	27	
53	a	13	
54	PNT	45	
55	PNT	45	
56	+	33	
57	YTO	40	
58	c	16	
59	XEY	30	
5a	YEX	24	
5b	9	11	
5c	YTO	40	
5d	9	11	
60	X	36	
61	RUP	22	
62	YEX	24	
63	8	10	
64	YTO	40	
65	8	10	
66	X	36	
67	XEY	30	
68	RUP	22	
69	FMT	42	
6a	DN	25	
6b	c	16	
6c	UP	27	
6d	b	14	
70	X>Y	53	
71	2	02	
72	8	10	
73	X=Y	50	
74	2	02	
75	8	10	
76	FMT	42	
77	UP	27	
78	CLR	20	
79	FMT	42	
7a	UP	27	
7b	END	46	

$Y_p = 500/Y_{scale}$

$X_p = 500/X_{scale}$

ΔX

X_f

X_i

a

σ

μ

GOMPERTZ CURVE PLOT

This program plots the data values used in calculating a Gompertz Curve (STAT-PAC IV-12) and plots the Gompertz curve best fitting these data points.

The Gompertz curve is given by:

$$Y = Ka^{(b^X)}$$

or by:

$$\log Y = \log K + b^X \log a.$$

The parameters log K, b, and log a are determined by program STAT-PAC IV-12.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: STOP

Using the origin controls, locate the pen in the lower left corner of the paper.

SET: Decimal Wheel at 6 or less

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA:

X_{max} → Z
Y_{max} → Y *
Y_{min} → X

PRESS: CONTINUE

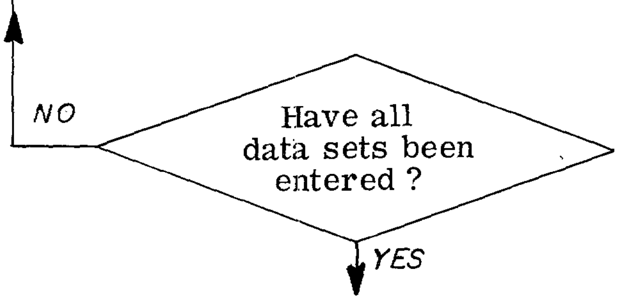
DISPLAY

0 ——— Z
i ——— Y
0 ——— X

ENTER DATA:

Y_i → Y
X_i → X

PRESS: CONTINUE



PRESS: SET FLAG

USER INSTRUCTIONS (Con't)

PRESS: CONTINUE

DISPLAY

0 ——— Z
n ——— Y (n = number of data points entered)
0 ——— X

ENTER DATA:

log a → Z
b → Y
log K → X

PRESS: CONTINUE

The Gompertz curve will be plotted.

To terminate: PRESS: STOP

To run another case:

PRESS: END

* The point X_{max} will be plotted at X = 8 inches (20.3 cm.)

The point Y_{min} will be plotted at Y = 0 inches.

The point $Y_{max} - Y_{min}$ will be plotted at Y = 5 inches (12.7 cm.)

EXAMPLE

See next page.

EXAMPLE

The table below lists the sales (100 lb. bags) of commercial fertilizer at Farson's General Feed Store, Wray, Colorado. The store's management would like to predict its fertilizer volume to determine if a branch feed store should be opened in nearby Haxton.

Year	Qtr.	X	Qtr'ly Sales (Y) (100 lb. bags)
1966 -	1	0	124
	2	1	160
	3	2	163
	4	3	144
1967 -	1	4	195
	2	5	222
	3	6	241
	4	7	313
1968 -	1	8	305
	2	9	356
	3	10	363
	4	11	458
1969 -	1	12	406
	2	13	441
	3	14	550

Results from program
STAT-PAC IV-12

log a = -1.1
b = .94
log K = 3.16

N = 5

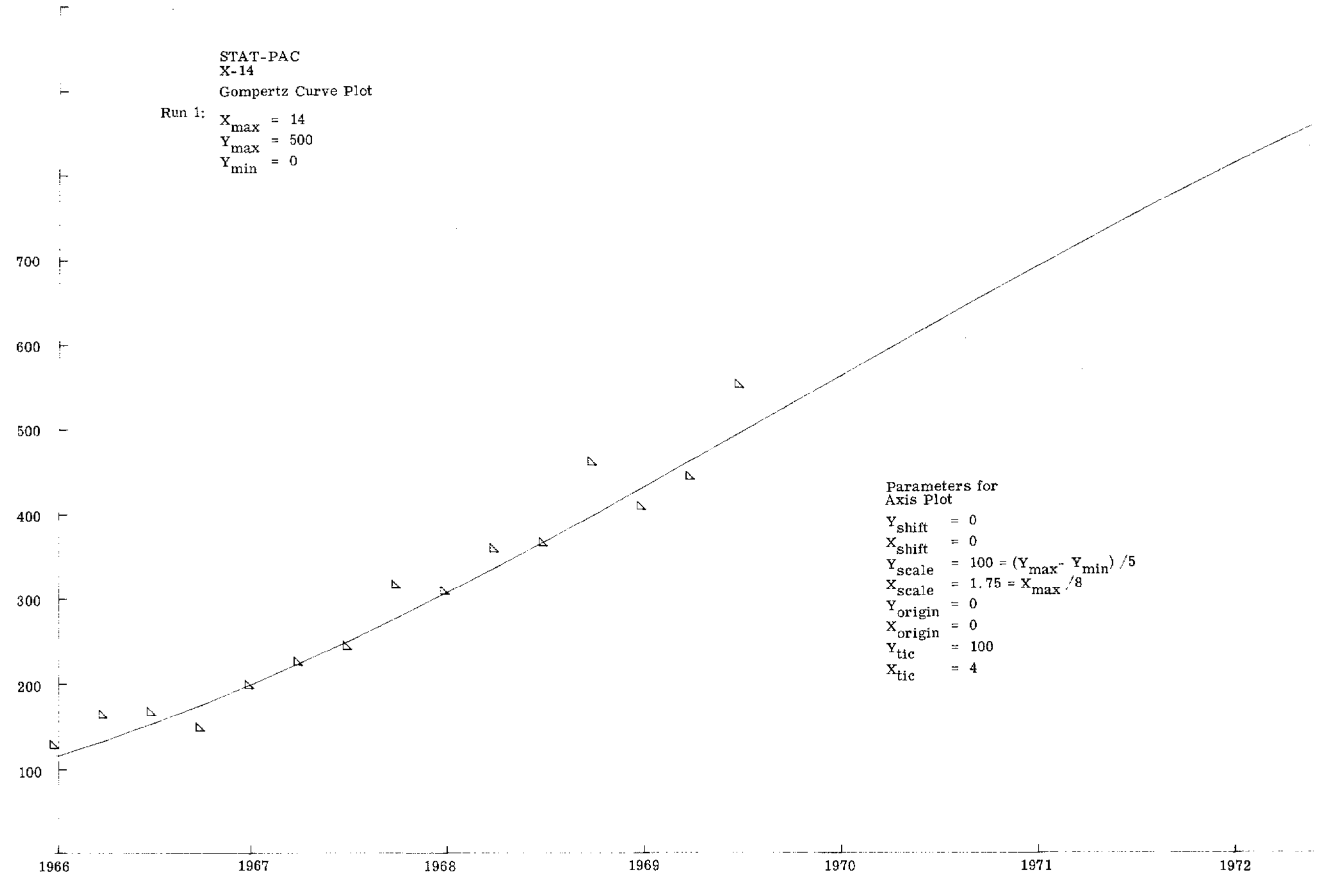
Run No. 1

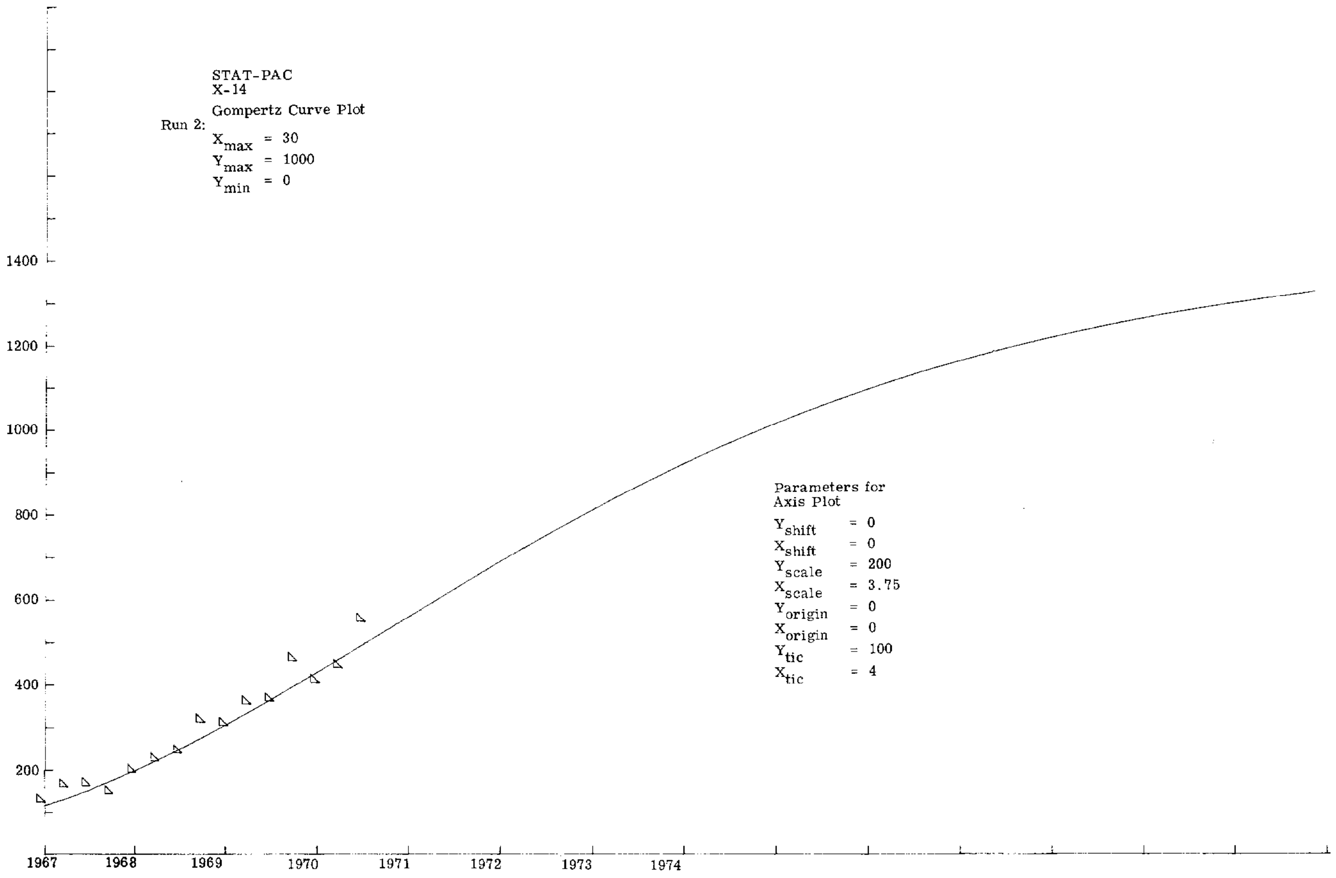
X_{max} = 14
Y_{max} = 500
Y_{min} = 0

Run No. 2

X_{max} = 30
Y_{max} = 1000
Y_{min} = 0

Notice that by changing X_{max} from 14 to 30, a better estimate of future fertilizer sales is obtained.


STAT-PAC
X-14
Gompertz Curve Plot
Run 1:
X max = 14
Y max = 500
Y min = 0
Parameters for Axis Plot
Y shift = 0
X shift = 0
Y scale = 100 = (Y max - Y min) /5
X scale = 1.75 = X max /8
Y origin = 0
X origin = 0
Y tic = 100
X tic = 4
700
600
500
400
300
200
100
1966
1967
1968
1969
1970
1971
1972


STAT-PAC
X-14
Gompertz Curve Plot
Run 2:
X max = 30
Y max = 1000
Y min = 0
Parameters for Axis Plot
Y shift = 0
X shift = 0
Y scale = 200
X scale = 3.75
Y origin = 0
X origin = 0
Y tic = 100
X tic = 4
1400
1200
1000
800
600
400
200
1967
1968
1969
1970
1971
1972
1973
1974

STAT-PAC X-14

```
00  CLR  20              40  RDN  31              80  DN   25
01  STP  41   ENTRY      41  +    33              81  EXP  74
02  PNT  45              42  XEY  30              82  UP   27
03  PNT  45              43  RUP  22              83  a    13
04  XTO  23              44  FMT  42              84  -    34
05  a    13              45  DN   25              85  e    12
06  -    34              46  XEY  30              86  X    36
07  2    02              47  RUP  22              87  DN   25
08  5    05              48  -    34              88  XEY  30
09  0    00              49  RDN  31              89  UP   27
0a  0    00              4a  +    33              8a  f    15
0b  XEY  30              4b  DN   25              8b  X    36
0c  DIV  35              4c  FMT  42              8c  RDN  31
0d  4    04              4d  DN   25              8d  FMT  42

10  EEX  26              50  FMT  42              90  DN   25
11  3    03              51  UP   27              91  RUP  22
12  RUP  22              52  0    00              92  DIV  35
13  DIV  35              53  UP   27              93  1    01
14  DN   25              54  d    17              94  .    21
15  AC+  60              55  UP   27              95  +    33
16  0    00              56  1    01              96  DN   25
17  UP   27              57  +    33              97  GTO  44
18  1    01              58  0    00              98  6    06
19  UP   27              59  GTO  44              99  c    16
1a  0    00              5a  2    02              9a  END  46
1b  CNT  47              5b  0    00
1c  CNT  47              5c  1    01
1d  CNT  47              5d  -    34

20  STP  41   ENTRY      60  0    00
21  IFG  43              61  STP  41   ENTRY
22  5    05              62  PNT  45
23  c    16              63  PNT  45
24  PNT  45              64  XTO  23
25  PNT  45              65  d    17
26  RUP  22              66  YTO  40
27  XTO  23              67  c    16
28  d    17              68  DN   25
29  f    15              69  YTO  40
2a  X    36              6a  b    14
2b  RUP  22              6b  0    00
2c  XEY  30              6c  UP   27
2d  a    13              6d  UP   27

30  -    34              70  c    16
31  e    12              71  LN   65
32  X    36              72  X    36
33  DN   25              73  DN   25
34  XEY  30              74  EXP  74
35  FMT  42              75  UP   27
36  DN   25              76  b    14
37  UP   27              77  X    36
38  5    05              78  d    17
39  0    00              79  +    33
3a  -    34              7a  1    01
3b  RDN  31              7b  EXP  74
3c  FMT  42              7c  LOG  75
3d  DN   25              7d  DIV  35
```

Y_{min}

log a

b

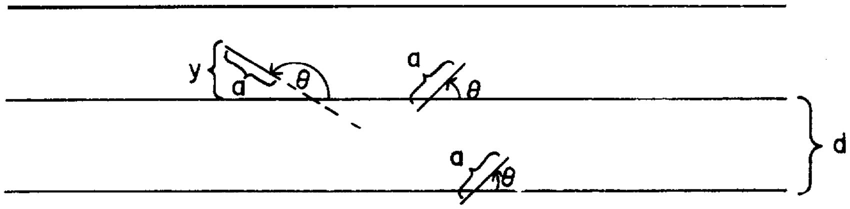
n log K

Y_f

X_f

BUFFON NEEDLE EXPERIMENT WITH PLOT

This program simulates the well known Buffon Needle Experiment described in the reference. The problem is to determine the probability that a needle of length a will intersect a line on a grid of lines separated by distance d . The geometry of the experiment is seen below:



The needle of length $a \le d$ is randomly tossed on the grid. By considering the angle θ between the needle and a line of the grid, and the distance Y between the upper tip of the needle and the line it intersects or the nearest line below it, the possible relations can be derived:

$$0 \le \theta \le \pi$$

$$0 \le Y \le d$$

The needle intersects a line when:

$$0 \le Y \le a \sin \theta \ .$$

It can be shown that the probability of an intersection is theoretically given by:

$$P_T = \frac{2a}{\pi d}$$

This formula involving π has intrigued mathematicians since it allows one to experimentally determine π.

This program randomly chooses a θ from a uniform distribution such that:

$$0 \le \theta \le \pi$$

then independently chooses a Y from a uniform distribution such that:

$$0 \le Y \le 1$$

The grid distance d is defined to be 1.

Thus $P_T = \frac{2a}{\pi}$.

The value of Y is then compared against

$$Y \le a \sin \theta.$$

If Y is less than or equal to $\underline{a \sin \theta}$, an intersection has occurred. The number of failing throws are then used to evaluate the experimental probability:

$$P_E = \frac{\# \text{ successful}}{\text{successful} + \text{failed}}$$

The program displays:

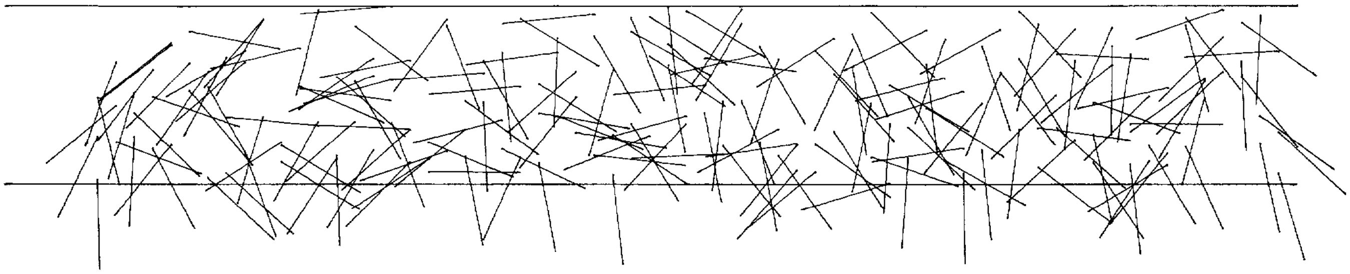
\# trials

P_E

P_T

and proceeds to toss another needle.

STAT-PAC
X-15
Buffon Needle Experiment (With Plot)
Monte-Carlo simulation of throwing needles onto a grid.



Reference: Theory of Probability, M. E. Munroe, McGraw-Hill, 1951.

USER INSTRUCTIONS

SET: RADIANS

SET: Decimal Wheel at 6 or less

PRESS: STOP

Using Origin Controls, set Origin at X = 0 in., Y = 4 in., or (X = 0 cm., Y = 10 cm.).

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

DISPLAY

0 ———— Z
0 ———— Y
1 ———— X

ENTER DATA:

$R_Y \longrightarrow Z$ *
$R_\theta \longrightarrow Y$
$a \longrightarrow X$

PRESS: CONTINUE

DISPLAY

Trials —— Z
P_E ———— Y
P_T ———— X

The program continues indefinitely.

*R_Y is an initializing value for the routine which generates the random values of Y. R_Y should be $0 \leq R_Y \leq 1$.

R_θ is a similar initializer for generating R_θ. R_θ should not equal R_Y. R_θ should be $0 \leq R_\theta \leq \pi$.

EXAMPLE

R_Y = .75

R_θ = .50

a = .50

Results after 2100 trials:

P_E = .312

P_T = .318

STAT-PAC X-15 9100B ONLY

```
00  CLR  20   Plus   40   2   02           80  CNT  47
01  EEX  26   Page   41  XTO  23           81  CNT  47
02   3   03          42   9   11           82  CNT  47
03  XEY  30          43   1   01           83  CNT  47
04  FMT  42          44  STP  41   ENTRY   84  CNT  47
05  DN   25          45  PNT  45           85  CNT  47
06  CNT  47          46  PNT  45           86  CNT  47
07  EEX  26          47  XTO  23           87  CNT  47
08   3   03          48   d   17           88  CNT  47
09  FMT  42          49  RUP  22           89  CNT  47
0a  DN   25          4a  XTO  23           8a  CNT  47
0b   2   02          4b   a   13           8b  CNT  47
0c  EEX  26          4c   2   02           8c  CNT  47
0d   3   03          4d   X   36           8d  CNT  47

10  FMT  42          50   π   56           90  CNT  47
11  DN   25          51  DIV  35           91  CNT  47
12   3   03          52  YTO  40           92  CNT  47
13  EEX  26          53   c   16           93  CNT  47
14   3   03          54   π   56           94  CNT  47
15  FMT  42          55  RUP  22           95  CNT  47
16  DN   25          56   +   33           96  CNT  47
17   4   04          57  DN   25           97  CNT  47
18  EEX  26          58  UP   27           98  CNT  47
19   3   03          59   X   36           99  CNT  47
1a  FMT  42          5a  DN   25           9a  CNT  47
1b  DN   25          5b  UP   27           9b  CNT  47
1c   5   05          5c   X   36           9c  CNT  47
1d  EEX  26          5d  DN   25           9d  CNT  47

20   3   03          60  UP   27           a0  CNT  47
21  FMT  42          61   X   36           a1  CNT  47
22  DN   25          62  DN   25           a2  CNT  47
23   6   06          63  UP   27           a3  CNT  47
24  EEX  26          64  INT  64           a4  CNT  47
25   3   03          65   -   34           a5  CNT  47
26  FMT  42          66  YTO  40           a6  CNT  47
27  DN   25          67   b   14           a7  CNT  47
28   7   07          68   π   56           a8  CNT  47
29  EEX  26          69   X   36           a9  CNT  47
2a   3   03          6a  GTO  44           aa  CNT  47
2b  FMT  42          6b   -   34           ab  CNT  47
2c  DN   25          6c   0   00           ac  CNT  47
2d  FMT  42          6d   0   00           ad  CNT  47

30  UP   27          70  CNT  47                        θ
31  IFG  43          71  CNT  47
32   3   03          72  CNT  47                        x plot
33   9   11          73  CNT  47
34  CLR  20          74  CNT  47
35  SFL  54          75  CNT  47
36  GTO  44          76  CNT  47
37   0   00          77  CNT  47
38   4   04          78  CNT  47
39  CLR  20          79  CNT  47
3a  FMT  42          7a  CNT  47
3b  UP   27          7b  CNT  47
3c   5   05          7c  CNT  47
3d  EEX  26          7d  CNT  47               a sin θ
```

```
b0   CNT  47   Plus
b1   CNT  47   Page
b2   CNT  47
b3   CNT  47
b4   CNT  47
b5   CNT  47
b6   CNT  47
b7   CNT  47
b8   CNT  47
b9   CNT  47
ba   CNT  47
bb   CNT  47
bc   CNT  47
bd   CNT  47

c0   CNT  47
c1   CNT  47
c2   CNT  47
c3   CNT  47
c4   CNT  47
c5   CNT  47
c6   CNT  47
c7   CNT  47
c8   CNT  47
c9   CNT  47
ca   CNT  47
cb   CNT  47
cc   CNT  47
cd   CNT  47

d0   CNT  47
d1   CNT  47
d2   CNT  47
d3   CNT  47
d4   CNT  47
d5   CNT  47
d6   CNT  47
d7   CNT  47
d8   CNT  47
d9   CNT  47
da   CNT  47
db   CNT  47
dc   CNT  47
dd   CNT  47
```

Minus Page

00	DN	25
01	XTO	23
02	8	10
03	SIN	70
04	UP	27
05	d	17
06	X	36
07	YTO	40
08	7	07
09	a	13
0a	UP	27
0b	π	56
0c	+	33
0d	DN	25
10	UP	27
11	X	36
12	DN	25
13	UP	27
14	X	36
15	DN	25
16	UP	27
17	X	36
18	DN	25
19	UP	27
1a	INT	64
1b	-	34
1c	YTO	40
1d	a	13
20	DN	25
21	X<Y	52
22	3	03
23	3	03
24	X=Y	50
25	3	03
26	3	03
27	e	12
28	UP	27
29	1	01
2a	+	33
2b	YTO	40
2c	e	12
2d	f	15
30	GTO	44
31	3	03
32	b	14
33	f	15
34	UP	27
35	1	01
36	+	33
37	YTO	40
38	f	15
39	e	12
3a	XEY	30
3b	+	33
3c	UP	27
3d	DN	25
40	XEY	30
41	DIV	35
42	c	16
43	PSE	57
44	CNT	47
45	a	13
46	UP	27
47	EEX	26
48	3	03
49	X	36
4a	XFR	67
4b	9	11
4c	FMT	42
4d	DN	25
50	XFR	67
51	7	07
52	UP	27
53	EEX	26
54	3	03
55	X	36
56	DN	25
57	-	34
58	XFR	67
59	8	10
5a	COS	73
5b	UP	27
5c	d	17
5d	X	36
60	EEX	26
61	3	03
62	X	36
63	XFR	67
64	9	11
65	XEY	30
66	-	34
67	DN	25
68	FMT	42
69	DN	25
6a	XFR	67
6b	9	11
6c	UP	27
6d	EEX	26
70	2	02
71	+	33
72	7	07
73	EEX	26
74	3	03
75	X>Y	53
76	7	07
77	c	16
78	5	05
79	EEX	26
7a	2	02
7b	UP	27
7c	0	00
7d	YTO	40
80	9	11
81	XEY	30
82	FMT	42
83	UP	27
84	b	14
85	RDN	31
86	GTO	44
87	+	33
88	5	05
89	4	04
8a	END	46

CONTROL ELLIPSE

Individual or separate control of related variables will result in errors of "over" and "under" control. These errors become more pronounced if the variables are correlated. Control of individual observations, assuming that they come from a bivariate normal distribution, may be done by means of a control ellipse or by means of Hotelling's T^2 (1) control chart. The results obtained by each method are identical.

This program accepts the sample variances and the covariance, sample size, and F_α obtained from the F tables with a significance level of α, and outputs the approximation to χ^2 denoted T^2, the analytical calculations for the ellipse, and the plot. The plot indicates pictorially the nature of the out-of-control conditions.

The following equations are used:

$$T^2 = \frac{2(N-1)}{(n-2)} \quad F_\alpha$$

$$\lambda = \frac{(S_X{}^2 + S_Y{}^2) \pm \sqrt{(S_X{}^2 + S_Y{}^2)^2 - 4[S_X{}^2 S_Y{}^2 - (S_{XY})^2]}}{2}$$

λ_1 (+)

λ_2 (-)

ϕ – The angle of rotation = arc tan $\left[\frac{\lambda_1 - S_X{}^2}{S_{XY}}\right]$

b = Length of the semi-minor axis = $\sqrt{\lambda_2 T^2}$

a = Length of the semi-major axis = $\sqrt{\lambda_1 T^2}$

The plot is generated by calculating r, the radius from the following equation:

$$r = \frac{ep}{1 - e\cos\theta}$$

where:

e – eccentricity
p – magnification factor
θ – angle through which r is rotated to plot the complete ellipse (360°)

References: (1) Hotelling, Harold, "Multivariate Quality Control, "Techniques of Statistical Analysis, Ed. by Eisenhart, Hastay, and Wallis, McGraw-Hill, New York, 1947.

(2) "Quality Control Methods", J. Edward Jackson, *Industrial Quality Control*, Vol. 12 (7), January, 1956.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

SET: DEGREES ☐

SET: Decimal Wheel at 6 or less

Using the origin controls, locate the pen in the middle of the paper. This point corresponds to $\overline{X}$, $\overline{Y}$.

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA:

X, Y Scale → Z
F_α → Y
N → X

Problem units/inch (cm.)

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
T^2 ——— X

PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA:

S_{XY} → Z
S_Y^2 → Y
S_X^2 → X

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

λ_1 ——— Z
λ_2 ——— Y
ϕ ——— X

PRESS: CONTINUE

DISPLAY

b ——— Z
a ——— Y
0 ——— X

PRESS: CONTINUE

The calculator returns to the second data entry point to allow the user to input new values for S_{XY}, S_Y^2 and S_X^2 and plot the new corresponding ellipse.

To rerun the program from the beginning:

PRESS: END

NOTE: To run the program and plot on metric paper, GO TO location (+)(2)(2) and enter a 2.

The plot is incrementing by 3°. To speed-up the plot, GO TO location (+)(7)(6) and enter the appropriate increment.

EXAMPLE

See next page.

EXAMPLE

The Cumminsky Chemical Company of Williams, Michigan has two chemicals, A and B, that they are trying to control simultaneously since the quality of the final product, C, depends on the joint effect of these two chemicals.

The following sample data has been determined:

$X_{scale} = Y_{scale} = 10$

$F_\alpha = 3.12$ (@ $\alpha = .05$)

$N = 75$

$S_{XY} = 68.87$ CASE 1

$S_Y{}^2 = 108.00$

$S_X{}^2 = 102.68$

The calculator computes and prints out:

$T^2 = 6.325479$

$\lambda_1 = 174.261350$

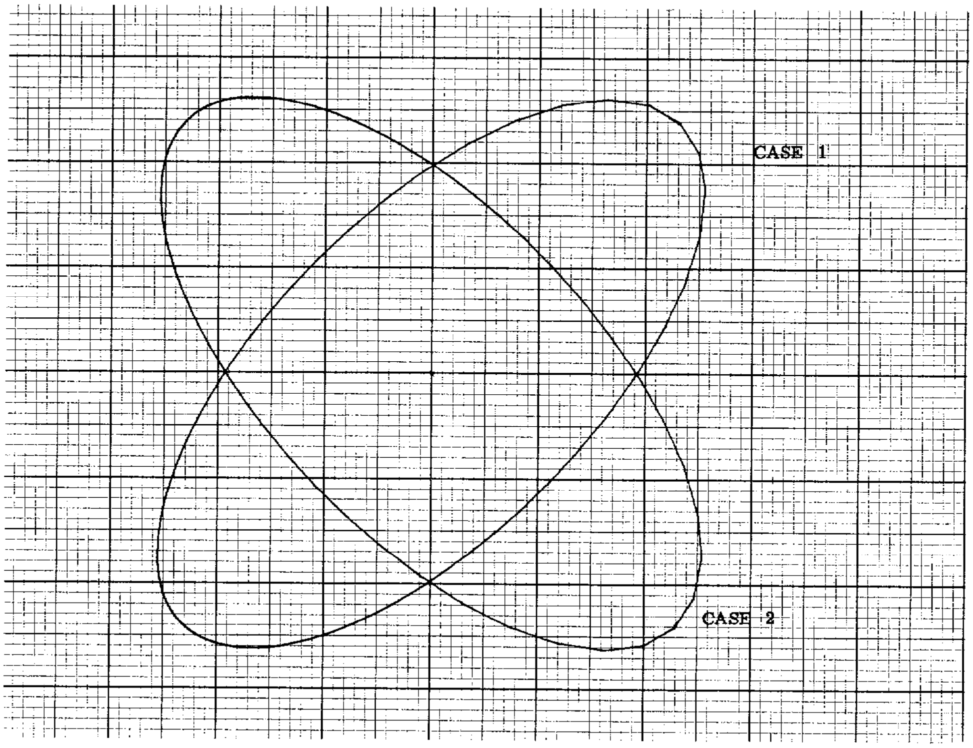
$\lambda_2 = 36.418650$

angle of rotation = 46.105932^o

semi-minor axis length = 15.177794

semi-major axis length = 33.200702

CASE 2 is obtained by using $S_{XY} = -68.87$.



STAT-PAC X-16 9100B ONLY

00	CLR	20	Plus Page
01	STP	41	ENTRY
02	PNT	45	
03	PNT	45	
04	RDN	31	
05	YTO	40	
06	c	16	
07	RDN	31	
08	1	01	
09	-	34	
0a	YTO	40	
0b	d	17	
0c	2	02	
0d	X	36	
10	DN	25	
11	X	36	
12	d	17	
13	UP	27	
14	1	01	
15	-	34	
16	DN	25	
17	DIV	35	
18	YTO	40	
19	b	14	
1a	CLR	20	
1b	b	14	
1c	PNT	45	
1d	PNT	45	
20	c	16	
21	UP	27	
22	5	05	
23	0	00	
24	0	00	
25	DIV	35	
26	YTO	40	
27	c	16	
28	CLR	20	
29	STP	41	ENTRY
2a	PNT	45	
2b	PNT	45	
2c	AC+	60	
2d	X	36	
30	DN	25	
31	YTO	40	
32	d	17	
33	RDN	31	
34	X	36	
35	DN	25	
36	-	34	
37	4	04	
38	X	36	
39	f	15	
3a	UP	27	
3b	e	12	
3c	+	33	
3d	DN	25	
40	UP	27	
41	X	36	
42	RDN	31	
43	XEY	30	
44	-	34	
45	DN	25	
46	√	76	
47	-	34	
48	YTO	40	
49	e	12	
4a	+	33	
4b	+	33	
4c	2	02	
4d	DIV	35	
50	f	15	
51	UP	27	
52	DN	25	
53	-	34	
54	d	17	
55	DIV	35	
56	DN	25	
57	ARC	72	
58	TAN	71	
59	YTO	40	
5a	f	15	
5b	UP	27	
5c	YE	24	
5d	e	12	
60	2	02	
61	DIV	35	
62	e	12	
63	PNT	45	
64	PNT	45	
65	b	14	
66	X	36	
67	RDN	31	
68	√	76	
69	RDN	31	
6a	X	36	
6b	DN	25	
6c	√	76	
6d	XTO	23	
70	a	13	
71	UP	27	
72	CLX	37	
73	PNT	45	s
74	PNT	45	
75	DN	25	
76	RDN	31	
77	X	36	
78	DN	25	
79	RDN	31	
7a	GTO	44	
7b	-	34	
7c	0	00	
7d	0	00	
80	CNT	47	
81	CNT	47	
82	CNT	47	
83	CNT	47	
84	CNT	47	
85	CNT	47	
86	CNT	47	
87	CNT	47	
88	CNT	47	
89	CNT	47	
8a	CNT	47	
8b	CNT	47	
8c	CNT	47	
8d	CNT	47	
90	CNT	47	
91	CNT	47	
92	CNT	47	
93	CNT	47	
94	CNT	47	
95	CNT	47	
96	CNT	47	
97	CNT	47	
98	CNT	47	
99	CNT	47	
9a	CNT	47	
9b	CNT	47	
9c	CNT	47	
9d	CNT	47	
a0	CNT	47	
a1	CNT	47	
a2	CNT	47	
a3	CNT	47	
a4	CNT	47	
a5	CNT	47	
a6	CNT	47	
a7	CNT	47	
a8	CNT	47	
a9	CNT	47	
aa	CNT	47	
ab	CNT	47	
ac	CNT	47	
ad	CNT	47	

	c	
	p	
a	r	
	T^2	
	Scale	
S_{XY}	θ	
S_Y^2	2λ2	Φ
S_X^2	λ_1	e

```
b0   CNT  47      Plus
b1   CNT  47      Page
b2   CNT  47
b3   CNT  47
b4   CNT  47
b5   CNT  47
b6   CNT  47
b7   CNT  47
b8   CNT  47
b9   CNT  47
ba   CNT  47
bb   CNT  47
bc   CNT  47
bd   CNT  47

c0   CNT  47
c1   CNT  47
c2   CNT  47
c3   CNT  47
c4   CNT  47
c5   CNT  47
c6   CNT  47
c7   CNT  47
c8   CNT  47
c9   CNT  47
ca   CNT  47
cb   CNT  47
cc   CNT  47
cd   CNT  47

d0   CNT  47
d1   CNT  47
d2   CNT  47
d3   CNT  47
d4   CNT  47
d5   CNT  47
d6   CNT  47
d7   CNT  47
d8   CNT  47
d9   CNT  47
da   CNT  47
db   CNT  47
dc   CNT  47
dd   CNT  47
```

STAT-PAC X-16 9100B ONLY

Minus Page

```
00   X    36
01   DN   25
02   XEY  30
03   -    34
04   DN   25
05   √    76
06   XTO  23
07   8    10
08   UP   27
09   a    13
0a   DIV  35
0b   YTO  40
0c   f    15
0d   X    36

10   +    33
11   UP   27
12   CLX  37
13   XTO  23
14   d    17
15   COS  73
16   RUP  22
17   XTO  23
18   a    13
19   f    15
1a   X    36
1b   1    01
1c   XEY  30
1d   -    34

20   f    15
21   DIV  35
22   a    13
23   X    36
24   YTO  40
25   9    11
26   UP   27
27   d    17
28   XEY  30
29   RCT  66
2a   YTO  40
2b   -    34
2c   f    15
2d   XTO  23

30   -    34
31   e    12
32   UP   27
33   e    12
34   COS  73
35   X    36
36   e    12
37   SIN  70
38   RUP  22
39   X    36
3a   DN   25
3b   -    34
3c   e    12
3d   COS  73

40   UP   27
41   XFR  67
42   8    10
43   X    36
44   DN   25
45   -    34
46   e    12
47   COS  73
48   UP   27
49   XFR  67
4a   -    34
4b   f    15
4c   X    36
4d   XFR  67

50   -    34
51   e    12
52   RUP  22
53   XTO  23
54   -    34
55   e    12
56   e    12
57   SIN  70
58   X    36
59   DN   25
5a   +    33
5b   e    12
5c   SIN  70
5d   UP   27

60   XFR  67
61   8    10
62   X    36
63   DN   25
64   -    34
65   XFR  67
66   -    34
67   e    12
68   UP   27
69   c    16
6a   DIV  35
6b   XEY  30
6c   RDN  31
6d   DIV  35

70   DN   25
71   XEY  30
72   FMT  42
73   DN   25
74   d    17
75   UP   27
76   3    03
77   +    33
78   3    03
79   6    06
7a   2    02
7b   X<Y  52
7c   9    11
7d   7    07

80   DN   25
81   XTO  23
82   d    17
83   COS  73
84   UP   27
85   f    15
86   X    36
87   1    01
88   XEY  30
89   -    34
8a   f    15
8b   XEY  30
8c   DIV  35
8d   XFR  67

90   9    11
91   X    36
92   d    17
93   XEY  30
94   GTO  44
95   2    02
96   9    11
97   CLR  20
98   FMT  42
99   UP   27
9a   GTO  44
9b   +    33
9c   2    02
9d   8    10

a0   END  46
```

X	X'
Y	Y'

GENERAL PARAMETRIC PLOT (SELF - SCALING)

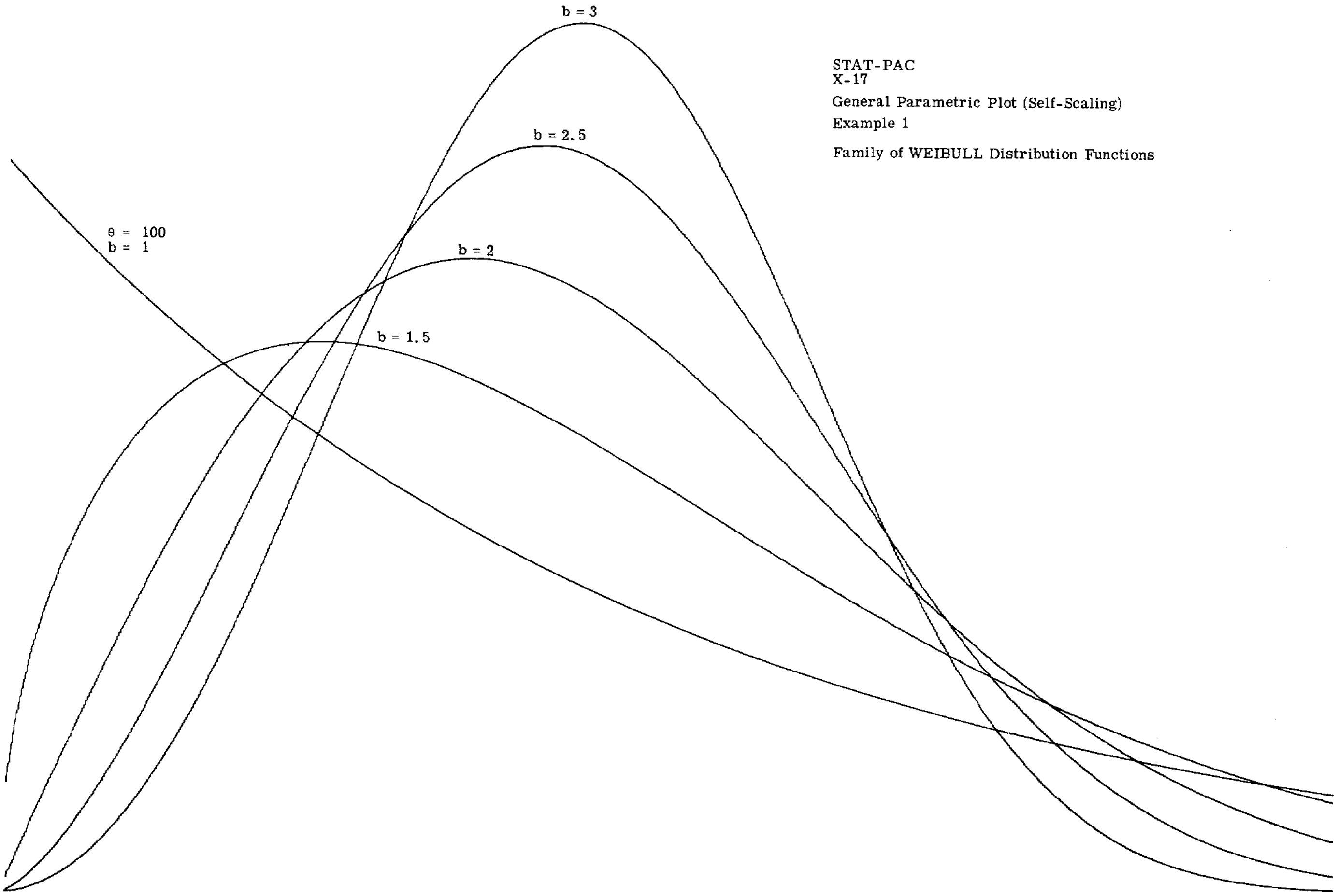
This program permits the plotting of a family of curves F(X, a, b) where a and b are parametrically varied. The program scales itself and outputs the plotter scaling constants so that axes may be placed on the plot.

The entire family may be plotted using the same set of scaling constants, or each may be individually scaled to plot full range.

The function to be plotted f(X) = F(X, a, b) is placed into the calculator by the user as a subroutine. The subroutine then evaluates F(X, a, b) where X is varied from

$$X_{min} \rightarrow X_{max} \quad .$$

The parameters a and b remain constant until the curve is completed. Then the user may parametrically vary a and b to generate a family of curves.



STAT-PAC
X-17
General Parametric Plot (Self-Scaling)
Example 1
Family of WEIBULL Distribution Functions

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: STOP

SET: Decimal Wheel at 6 or less

Using the origin controls, locate the pen in the lower left corner of the paper.

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: GO TO (-)(3)(0)

SET: | PROGRAM | |

Enter the subroutine which generates F(X, a, b) leaving F(X, a, b) in the Y register. The last step should be RETURN. Registers (-)(3) thru (-)(d) are available. The flag is not available. The quantities X, a, and b are found:

X —— registers X and e
a —— register (-) f
b —— register (-) e

Do not alter registers e, (-)f, or (-) e.

SET: | | RUN |

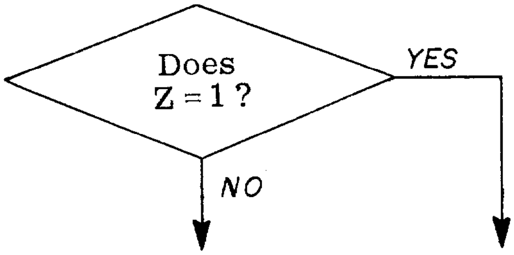
PRESS: END

PRESS: CONTINUE

ENTER DATA:

0 or 1 → Z *
b → Y
a → X

PRESS: CONTINUE



USER INSTRUCTIONS (Con't)

ENTER DATA:

$X_{max} \rightarrow Y$
$X_{min} \rightarrow X$

PRESS: CONTINUE

The calculator now evaluates F(X, a, b) over the range $X_{min} \rightarrow X_{max}$ searching for F_{max} and F_{min}.

F(X, a, b) is now plotted.

DISPLAY

1 —— Z
Y_{shift} —— Y
X_{shift} —— X

PRESS: CONTINUE

DISPLAY

2 —— Z
Y_{scale} —— Y
X_{scale} —— X

*If 1 is placed in the Z register F(X, a, b) is plotted using scaling constants of the previous run.

EXAMPLE

Generate a family of Weibull Distribution Functions given by:

$$F(X, b, \theta) = \frac{bX^{(b-1)}}{\theta^b} \quad e^{-\left(\frac{X}{\theta}\right)^b}$$

for the cases: (b) (a)

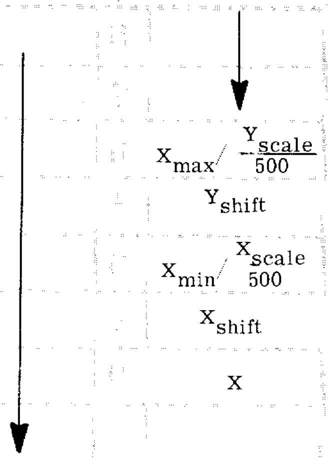
1. θ = 100 , b = 3.0
2. θ = 100 , b = 2.5
3. θ = 100 , b = 2.0
4. θ = 100 , b = 1.5
5. θ = 100 , b = 1.0

$X_{min} = 1$, $X_{max} = 200$

For cases 2, 3, 4, and 5 use the same scaling constants as for case 1. (i.e., place a 1 in the Z register.)

STAT-PAC X-17

```
00  CLR  20    Plus   Page
01  STP  41    ENTRY
02  PNT  45
03  PNT  45
04  XTO  23
05   -   34
06   f   15
07  YTO  40
08   -   34
09   e   12
0a  DN   25
0b   0   00
0c  X=Y  50
0d   1   01

10   b   14
11   d   17
12  UP   27
13   f   15
14   -   34
15  YTO  40
16   e   12
17  GTO  44
18   -   34
19   0   00
1a   3   03
1b   1   01
1c  STP  41         ENTRY
1d  PNT  45

20  PNT  45
21  XTO  23
22   d   17
23  YTO  40
24   c   16
25   -   34
26   5   05
27   0   00
28  CNT  47
29  DIV  35
2a   d   17
2b  XEY  30
2c   -   34
2d  AC+  60

30  EEX  26
31   9   11
32   9   11
33  XTO  23
34   b   14
35  CHS  32
36  XTO  23
37   a   13
38  RCL  61
39   +   33
3a  YTO  40
3b   e   12
3c   c   16
3d  X<Y  52

40   8   10
41   0   00
42  DN   25
43  SFL  54
44  GTO  44
45  SUB  77
46   -   34
47   3   03
48   0   00
49  IFG  43
4a   7   07
4b   3   03
4c   b   14
4d   -   34

50   a   13
51  DIV  35
52   e   12
53  UP   27
54   d   17
55   -   34
56   c   16
57  DIV  35
58   7   07
59   6   06
5a   0   00
5b   0   00
5c  X<Y  52
5d   6   06

60   8   10
61  DN   25
62  FMT  42
63  DN   25
64  GTO  44
65   -   34
66   0   00
67   3   03
68   0   00
69  UP   27
6a  UP   27
6b  FMT  42
6c  UP   27
6d  GTO  44

70   -   34
71   0   00
72   c   16
73   b   14
74  X>Y  53
75  YTO  40
76   b   14
77   a   13
78  X<Y  52
79  YTO  40
7a   a   13
7b  GTO  44
7c   3   03
7d   7   07

80  RCL  61
81  AC-  63
82  UP   27
83   3   03
84  CNT  47
85  DIV  35
86   d   17
87  XEY  30
88   -   34
89  AC+  60
8a  UP   27
8b   5   05
8c   0   00
8d  CNT  47

90  DIV  35
91  YTO  40
92   c   16
93   a   13
94  UP   27
95   b   14
96   -   34
97   5   05
98  EEX  26
99   3   03
9a  GTO  44
9b   -   34
9c   0   00
9d   0   00

a0  CNT  47
a1  CNT  47
a2  CNT  47
a3  CNT  47
a4  CNT  47
a5  CNT  47
a6  CNT  47
a7  CNT  47
a8  CNT  47
a9  CNT  47
aa  CNT  47
ab  CNT  47
ac  CNT  47
ad  CNT  47
     +
```

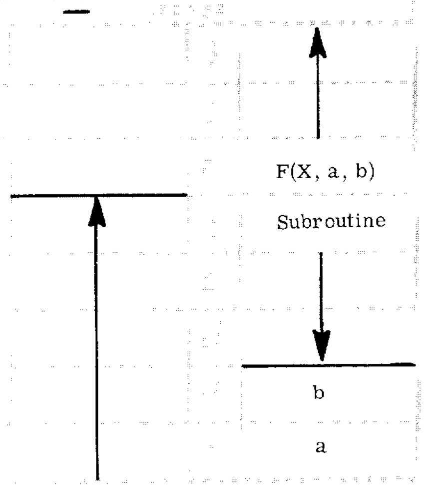


```
b0   CNT  47     Plus
b1   CNT  47     Page
b2   CNT  47
b3   CNT  47
b4   CNT  47
b5   CNT  47
b6   CNT  47
b7   CNT  47
b8   CNT  47
b9   CNT  47
ba   CNT  47
bb   CNT  47
bc   CNT  47
bd   CNT  47

c0   CNT  47
c1   CNT  47
c2   CNT  47
c3   CNT  47
c4   CNT  47
c5   CNT  47
c6   CNT  47
c7   CNT  47
c8   CNT  47
c9   CNT  47
ca   CNT  47
cb   CNT  47
cc   CNT  47
cd   CNT  47

d0   CNT  47
d1   CNT  47
d2   CNT  47
d3   CNT  47
d4   CNT  47
d5   CNT  47
d6   CNT  47
d7   CNT  47
d8   CNT  47
d9   CNT  47
da   CNT  47
db   CNT  47
dc   CNT  47
dd   CNT  47
```

```
00   DIV  35     Minus
01   YTO  40     Page
02    a   13
03   RCL  61
04   AC-  63
05    +   33
06   AC+  60
07   DN   25
08   GTO  44
09    +   33
0a    4   04
0b    4   04
0c    1   01
0d   UP   27
```

```
10    b   14
11   UP   27
12    d   17
13   PNT  45     s
14   PNT  45
15    5   05
16   EEX  26
17    2   02
18   UP   27
19    a   13
1a   XEY  30
1b    X   36
1c   UP   27
1d    c   16

20    X   36
21    2   02
22   RDN  31
23   PNT  45     s
24   PNT  45
25    0   00
26   UP   27
27   UP   27
28   GTO  44
29    +   33
2a    0   00
2b    1   01
2c   CNT  47
2d   CNT  47

30   UP   27
31   XFR  67
32    -   34
33    f   15
34   UP   27
35    1   01
36    -   34
37   DN   25
38   XEY  30
39   LN   65
3a    X   36
3b   DN   25
3c   EXP  74
3d   UP   27

40   XFR  67
41    -   34
42    f   15
43    X   36
44   UP   27
45   XFR  67
46    -   34
47    e   12
48   LN   65
49    X   36
4a   DN   25
4b   EXP  74
4c   DIV  35
4d    e   12
```

```
50   UP   27
51   XFR  67
52    -   34
53    e   12
54   DIV  35
55   XFR  67
56    -   34
57    f   15
58   XEY  30
59   LN   65
5a    X   36
5b   DN   25
5c   EXP  74
5d   CHS  32

60   EXP  74
61    X   36
62   RTN  77
```



Example #1

Weibull Family

```
30  UP   27
31  XFR  67
32  -    34
33  f    15
34  UP   27
35  1    01
36  -    34
37  DN   25
38  XEY  30
39  LN   65
3a  X    36
3b  DN   25
3c  EXP  74
3d  UP   27

40  XFR  67
41  -    34
42  f    15
43  X    36
44  UP   27
45  XFR  67
46  -    34
47  e    12
48  LN   65
49  X    36
4a  DN   25
4b  EXP  74
4c  DIV  35
4d  e    12

50  UP   27
51  XFR  67
52  -    34
53  e    12
54  DIV  35
55  XFR  67
56  -    34
57  f    15
58  XEY  30
59  LN   65
5a  X    36
5b  DN   25
5c  EXP  74
5d  CHS  32

60  EXP  74
61  X    36
62  RTN  77
63  END  46
```

Example #2

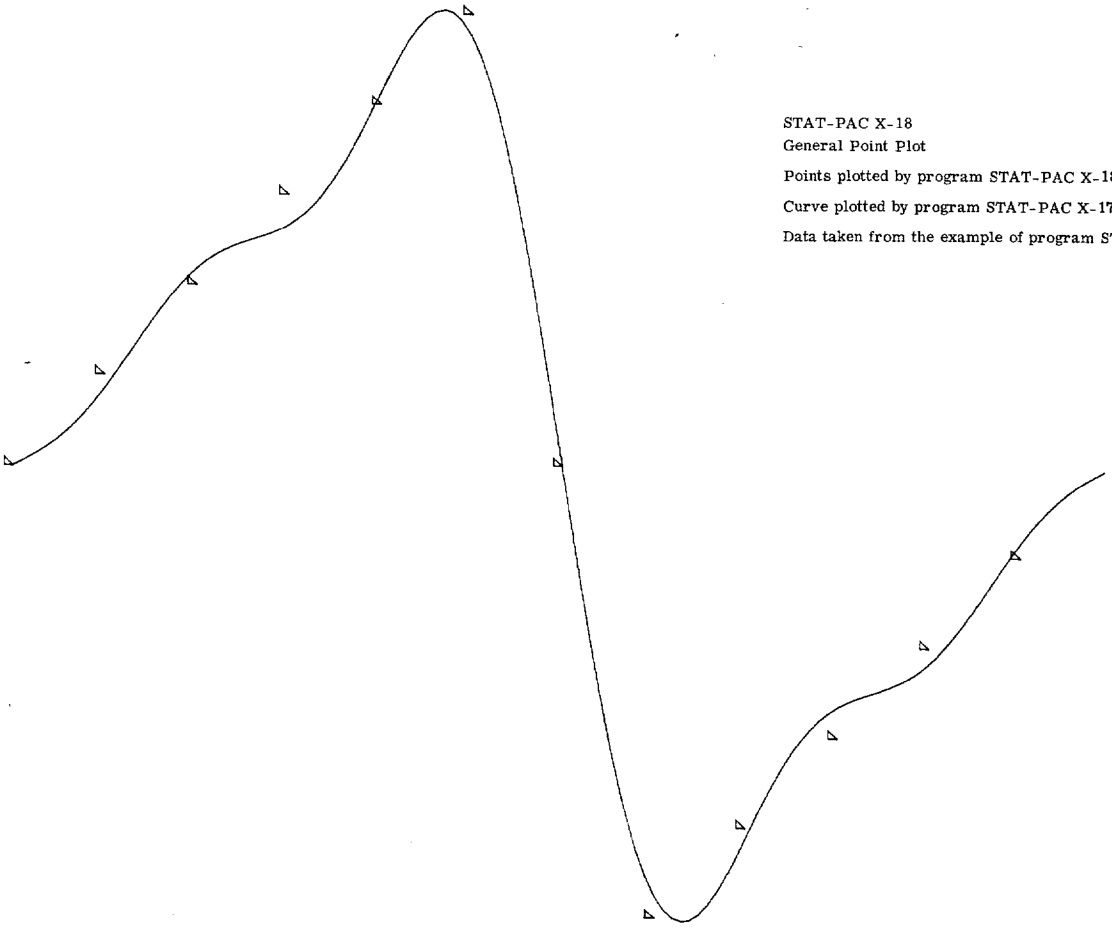
```
30  SIN  70
31  UP   27
32  .    21
33  9    11
34  7    07
35  7    07
36  X    36
37  e    12
38  UP   27
39  +    33
3a  DN   25
3b  SIN  70
3c  UP   27
3d  .    21

40  4    04
41  5    05
42  3    03
43  X    36
44  DN   25
45  -    34
46  e    12
47  UP   27
48  3    03
49  X    36
4a  DN   25
4b  SIN  70
4c  UP   27
4d  .    21

50  2    02
51  6    06
52  2    02
53  X    36
54  DN   25
55  +    33
56  e    12
57  UP   27
58  4    04
59  X    36
5a  DN   25
5b  SIN  70
5c  UP   27
5d  .    21

60  1    01
61  5    05
62  1    01
63  X    36
64  DN   25
65  -    34
66  RTN  77
67  END  46
```

Fourier Series used in Example from STAT-PAC X-18

GENERAL POINT PLOT

This program plots a series of points (X_i, Y_i). The program is written to plot $(X_{max} - X_{min})$ at 15 inches (38.1 cm.). To convert the X axis to units of centimeters, place 3000 in locations (0)(6) through (0)(9). This will cause $(X_{max} - X_{min})$ to be plotted at 15 cm.

The Y axis is programmed to plot $(Y_{max} - Y_{min})$ at 5 inches. To convert to units of centimeters, place 2000 in locations (1)(8) through (1)(b).



STAT-PAC X-18
General Point Plot

Points plotted by program STAT-PAC X-18.

Curve plotted by program STAT-PAC X-17.

Data taken from the example of program STAT-PAC V-2.

USER INSTRUCTIONS

DEPRESS: X Y Z on the 9120A

PRESS: STOP

SET: Decimal Wheel at 6 or less

Using the origin controls, locate the pen in the lower left corner of the paper.

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

ENTER DATA:

$X_{max} \rightarrow Y$
$X_{min} \rightarrow X$

PRESS: CONTINUE

ENTER DATA:

$Y_{max} \rightarrow Y$
$Y_{min} \rightarrow X$

PRESS: CONTINUE

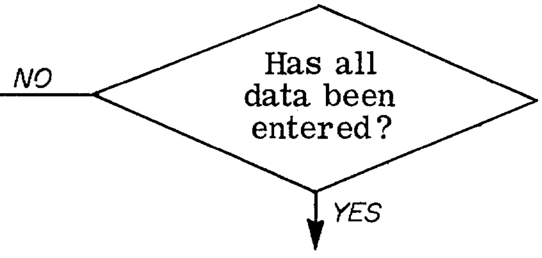
DISPLAY

0 —— Z
i —— Y
0 —— X

ENTER DATA:

$Y_i \rightarrow Y$
$X_i \rightarrow X$

PRESS: CONTINUE



PRESS: SET FLAG

USER INSTRUCTIONS (Con't)

PRESS: CONTINUE

To run another case

EXAMPLE

Plot the data from the example of program STAT-PAC V-2.

X	Y	i
0	0	1
$\frac{\pi}{6}$	.262	2
$2\frac{\pi}{6}$	.524	3
$3\frac{\pi}{6}$	.786	4
$4\frac{\pi}{6}$	1.047	5
$5\frac{\pi}{6}$	1.309	6
$6\frac{\pi}{6}$	0	7
$7\frac{\pi}{6}$	-1.309	8
$8\frac{\pi}{6}$	-1.047	9
$9\frac{\pi}{6}$	-.786	10
$10\frac{\pi}{6}$	-.524	11
$11\frac{\pi}{6}$	-.262	12

$X_{max} = 15(\frac{\pi}{6})$

$X_{min} = 0$

$Y_{max} = 1.309$

$Y_{min} = -1.309$

The curve fitted thru these data points is obtained by the use of Program STAT-PAC X-17 (General Parametric Plot).

EXAMPLE (Con't)

Plot the Fourier Series obtained from the example of program STAT-PAC V-2 .

$f(X)=.977 \sin X-.453 \sin 2X+.262 \sin 3X-.151 \sin 4X$

$X_{max} = 15 (\frac{\pi}{6})$

$X_{min} = 0$

$a = 0$

$b = 0$

STAT-PAC X-18

```
00  CLR   20
01  STP   41    ENTRY
02  PNT   45
03  PNT   45
04   -    34
05  UP    27
06   7    07
07   5    05
08   0    00
09   0    00
0a  RUP   22
0b  DIV   35
0c  DN    25
0d  AC+   60

10   0    00
11  UP    27
12  UP    27
13  STP   41    ENTRY
14  PNT   45
15  PNT   45
16   -    34
17  UP    27
18   5    05
19   0    00
1a   0    00
1b   0    00
1c  RUP   22
1d  DIV   35

20  DN    25
21  YTO   40
22   c    16
23  XTO   23
24   d    17
25   0    00
26  UP    27
27   1    01
28  UP    27
29   0    00
2a  STP   41    ENTRY
2b  IFG   43
2c   6    06
2d   c    16

30  PNT   45
31  PNT   45
32  RUP   22
33  XTO   23
34   b    14
35   e    12
36   -    34
37   f    15
38   X    36
39   c    16
3a  XEY   30
3b  RDN   31
3c   -    34
3d   d    17

40   X    36
41  DN    25
42  XEY   30
43  FMT   42
44  DN    25
45  UP    27
46   5    05
47   0    00
48   -    34
49  RDN   31
4a  FMT   42
4b  DN    25
4c  XEY   30
4d  RUP   22

50   +    33
51  RDN   31
52  XEY   30
53  FMT   42
54  DN    25
55  RUP   22
56   +    33
57  XEY   30
58  RDN   31
59   -    34
5a  DN    25
5b  XEY   30
5c  FMT   42
5d  DN    25

60  FMT   42
61  UP    27
62   0    00
63  UP    27
64   b    14
65  UP    27
66   1    01
67   +    33
68   0    00
69  GTO   44
6a   2    02
6b   a    13
6c  CLR   20
6d  FMT   42

70  UP    27
71  GTO   44
72   0    00
73   0    00
74  END   46
```

STAT-PAC
X-19

AXES PLOT

This program will plot the abscissa and ordinate axes. It was written to be used in conjunction with the other plotter programs to facilitate interpretation and interpolation of plotted data.

The required inputs are:

X_{shift}, Y_{shift} — These should be the same as used in the previously run problem.

X_{scale}, Y_{scale} — These should also be the same as used in the previously run problem.

X_{origin}, Y_{origin} — Simply the value you wish the axes cross-point to take on. Must be expressed in terms of the scaling constants.

X_{tic}, Y_{tic} — The desired distance between tic marks. Must also be expressed in terms of the scaling constants.

The axes will then cross at scaled point

$$\left(\frac{500}{X_{scale}} \left[X_{origin} - X_{shift} \right], \frac{500}{Y_{scale}} \left[Y_{origin} - Y_{shift} \right] \right)$$

and the scaled distances between tic marks on the X and Y axes will be

$$\left[\frac{500}{X_{scale}} \right] [X_{tic}] \text{ and } \left[\frac{500}{Y_{scale}} \right] [Y_{tic}] \text{ respectively.}$$

NOTE: All entries should be made in terms of the problem variables.

To convert this program to recognize centimeters rather than inches as the basic unit of scaling, program a 2 in location 0-9 and a 6 in location 5-9.

USER INSTRUCTIONS

PRESS: STOP

Using the origin controls, locate the pen in the lower left corner of the paper, or if plotting axes on a graph which has just been plotted, leave the pen at the location dictated by pressing the STOP key.

SET: Decimal Wheel at 6 or less

PRESS: END

ENTER PROGRAM

PRESS: CONTINUE

ENTER TRANSLATION CONSTANTS:

$$Y_{shift} \longrightarrow Y$$
$$X_{shift} \longrightarrow X$$

PRESS: CONTINUE

ENTER SCALING CONSTANTS:

$$Y_{scale} \longrightarrow Y$$
$$X_{scale} \longrightarrow X$$

PRESS: CONTINUE

ENTER DESIRED ORIGIN*:

$$Y_{origin} \longrightarrow Y$$
$$X_{origin} \longrightarrow X$$

NOTE: $Y_{origin} \geq Y_{shift}$
$X_{origin} \geq X_{shift}$

PRESS: CONTINUE

ENTER TIC MARK INCREMENTS:

$$Y_{tic} \longrightarrow Y$$
$$X_{tic} \longrightarrow X$$

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

*NOTE: Y_{origin}, X_{origin}, Y_{tic} and X_{tic} are expressed in terms of the scaling constants (i.e., the problem variables).

The axes will now be plotted.

To rerun the program, re-enter the magnetic card and proceed as before.

EXAMPLE

See next page.

EXAMPLE

Let:

Y_{shift} = -500 units

X_{shift} = -500 units

Y_{origin} = 1500 units

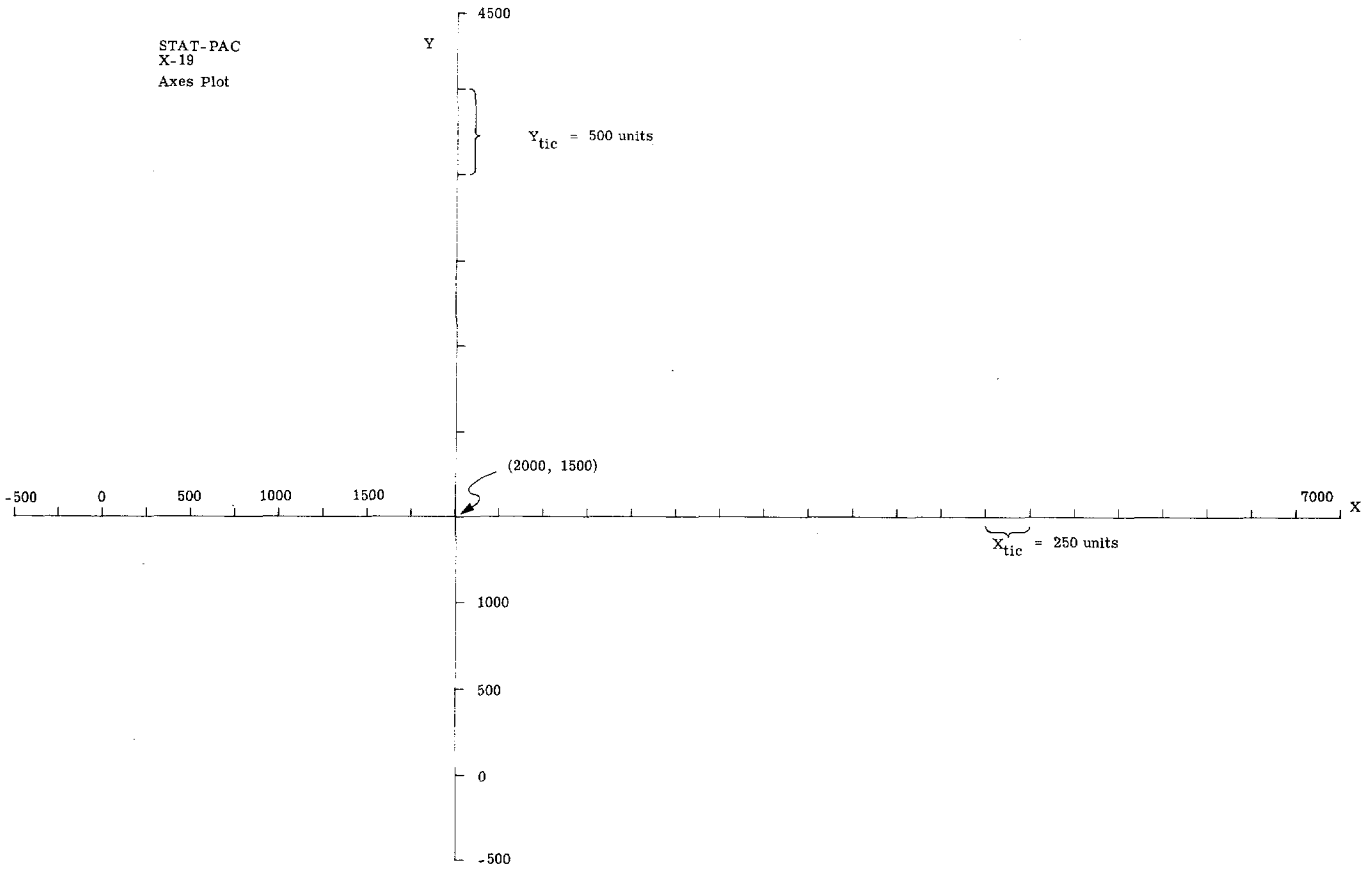
X_{origin} = 2000 units

$Y_{scale} = 500 \frac{\text{units}}{\text{inch}}$

$X_{scale} = 500 \frac{\text{units}}{\text{inch}}$

$Y_{tic} = 500 \frac{\text{units}}{\text{tic mark}}$

$X_{tic} = \overset{2500}{\cancel{250}} \frac{\text{units}}{\text{tic mark}}$



STAT-PAC X-19

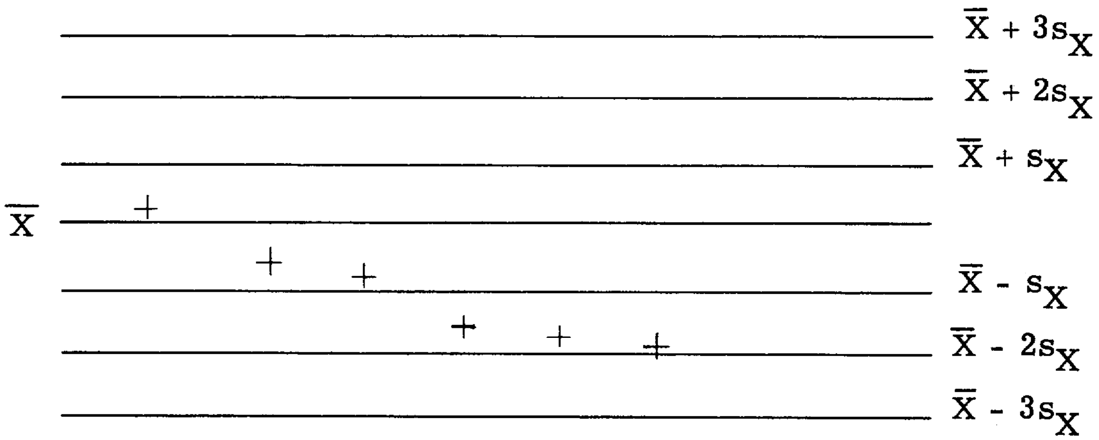
```
00  CLR  20
01  STP  41  ENTRY
02  XTO  23
03  c    16
04  YTO  40
05  d    17
06  CLR  20
07  STP  41  ENTRY
08  UP   27
09  5    05
0a  0    00
0b  0    00
0c  DIV  35
0d  YTO  40

10  b    14
11  RUP  22
12  DIV  35
13  YTO  40
14  0    00
15  CLR  20
16  STP  41  ENTRY
17  UP   27
18  c    16
19  -    34
1a  b    14
1b  DIV  35
1c  YEX  24
1d  d    17

20  DN   25
21  -    34
22  XEY  30
23  YEX  24
24  0    00
25  YTO  40
26  0    00
27  X    36
28  YTO  40
29  c    16
2a  CLR  20
2b  STP  41  ENTRY
2c  UP   27
2d  YEX  24

30  0    00
31  RDN  31
32  X    36
33  YEX  24
34  b    14
35  c    16
36  RDN  31
37  DIV  35
38  d    17
39  XEY  30
3a  X<Y  52
3b  ARC  72
3c  -    34
3d  X<Y  52

40  3    03
41  c    16
42  X=Y  50
43  ARC  72
44  -    34
45  -    34
46  AC+  60
47  DN   25
48  CLX  37
49  IFG  43
4a  SFL  54
4b  XEY  30
4c  FMT  42
4d  DN   25

50  UP   27
51  IFG  43
52  SFL  54
53  UP   27
54  RCL  61
55  AC-  63
56  +    33
57  AC+  60
58  7    07
59  5    05
5a  IFG  43
5b  SFL  54
5c  INT  64
5d  IFG  43

60  SFL  54
61  5    05
62  EEX  26
63  3    03
64  X<Y  52
65  9    11
66  0    00
67  DN   25
68  IFG  43
69  SFL  54
6a  XEY  30
6b  FMT  42
6c  DN   25
6d  XEY  30

70  IFG  43
71  SFL  54
72  XEY  30
73  UP   27
74  5    05
75  0    00
76  +    33
77  RDN  31
78  XEY  30
79  IFG  43
7a  SFL  54
7b  XEY  30
7c  FMT  42
7d  DN   25

80  XEY  30
81  IFG  43
82  SFL  54
83  XEY  30
84  RUP  22
85  -    34
86  DN   25
87  XEY  30
88  IFG  43
89  SFL  54
8a  XEY  30
8b  GTO  44
8c  4    04
8d  c    16

90  RUP  22
91  XEY  30
92  IFG  43
93  SFL  54
94  XEY  30
95  FMT  42
96  DN   25
97  FMT  42
98  UP   27
99  IFG  43
9a  a    13
9b  8    10
9c  UP   27
9d  RCL  61

a0  AC-  63
a1  d    17
a2  RDN  31
a3  b    14
a4  SFL  54
a5  GTO  44
a6  3    03
a7  a    13
a8  CLR  20
a9  FMT  42
aa  UP   27
ab  END  46
```

$\overline{X}$ CONTROL CHART PLOT

This program constructs an $\overline{X}$ Control Chart and plots observed samples on the chart. The control chart consists of lines at $\overline{X}$, $\overline{X} \pm s_X$, $\overline{X} \pm 2s_X$, and $\overline{X} \pm 3s_X$.

In general the levels $\overline{X} \pm 2s_X$ are warning levels, and $\overline{X} \pm 3s_X$ are termed control limits. The user determines.

$\overline{X}$ – Expected Value of Population
s_X – Expected Standard Deviation of Population
Δt – t Axis Increment



$\overline{X}$ CONTROL CHART

This program has an option of updating the control chart as observations are available.

USER INSTRUCTIONS

SET: Decimal Wheel at 6 or less

PRESS: STOP

Using the origin controls, locate the pen in the lower left corner of the paper.

DEPRESS: X on the 9120A

PRESS: END

ENTER PROGRAM

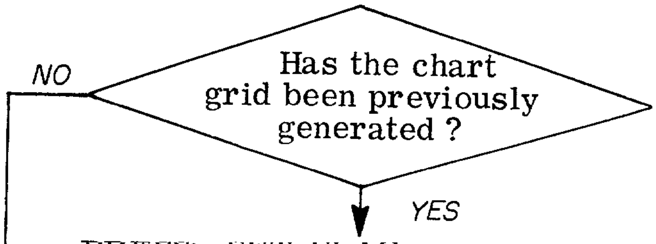
PRESS: CONTINUE

DISPLAY

0 ———— Z
0 ———— Y
0 ———— X

ENTER DATA:

$\Delta t \longrightarrow Z$
$s_X \longrightarrow Y$
$\bar{X} \longrightarrow X$



PRESS: SET FLAG

→ PRESS: CONTINUE

→ DISPLAY

0 ———— Z
n ———— Y
0 ———— X

(n represents the entry to be input; if this is not the first pass at the chart, place the correct n $\longrightarrow$ Y).

ENTER DATA: $X_n \longrightarrow X$

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

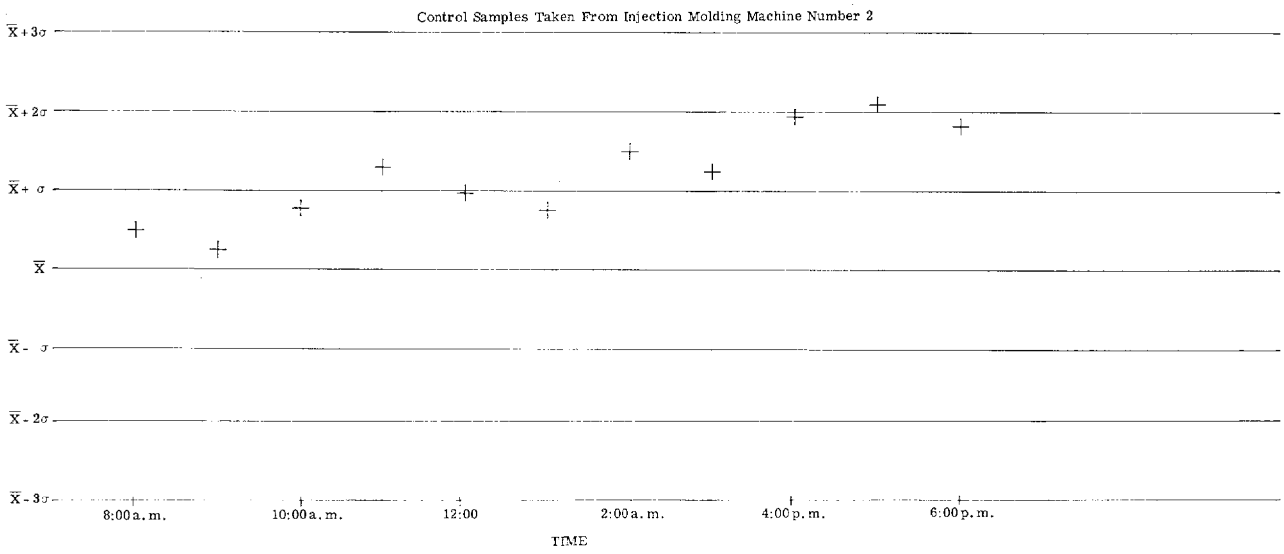
If program Axis Plot STAT-PAC X-19 is to be run, use:

$$Y_{shift} = -5s_X - \bar{X}$$
$$Y_{scale} = s_X$$

$$X_{shift} = 0$$
$$X_{scale} = \Delta t$$

STAT-PAC
X-20

X Control Chart Plot



EXAMPLE

$\Delta t = 1$

$s_X = 1$

$\overline{X} = 0$

$X_1 = .50$

$X_2 = .25$

$X_3 = .78$

$X_4 = 1.30$

$X_5 = .97$

$X_6 = .76$

$X_7 = 1.50$

$X_8 = 1.25$

$X_9 = 1.94$

$X_{10} = 2.10$

$X_{11} = 1.82$

STAT-PAC X-20

```
00  CLR   20
01  STP   41    ENTRY
02  AC+   60
03  RUP   22
04  XTO   23
05  d     17
06  PNT   45
07  0     00
08  RUP   22
09  PNT   45
0a  RUP   22
0b  PNT   45
0c  PNT   45
0d  PNT   45

10  IFG   43
11  4     04
12  4     04
13  EEX   26
14  3     03
15  RUP   22
16  +     33
17  4     04
18  EEX   26
19  3     03
1a  X<Y   52
1b  4     04
1c  4     04
1d  b     14

20  FMT   42
21  DN    25
22  2     02
23  EEX   26
24  3     03
25  FMT   42
26  DN    25
27  UP    27
28  +     33
29  DN    25
2a  FMT   42
2b  DN    25
2c  6     06
2d  EEX   26

30  3     03
31  FMT   42
32  DN    25
33  7     07
34  .     21
35  5     05
36  EEX   26
37  3     03
38  FMT   42
39  DN    25
3a  FMT   42
3b  UP    27
3c  5     05
3d  EEX   26

40  2     02
41  GTO   44
42  1     01
43  6     06
44  0     00
45  UP    27
46  FMT   42
47  UP    27
48  UP    27
49  1     01
4a  XEY   30
4b  STP   41    ENTRY
4c  XEY   30
4d  PNT   45

50  XEY   30
51  PNT   45
52  PNT   45
53  YTO   40
54  c     16
55  UP    27
56  e     12
57  UP    27
58  5     05
59  X     36
5a  DN    25
5b  +     33
5c  f     15
5d  -     34

60  e     12
61  DIV   35
62  5     05
63  EEX   26
64  2     02
65  X     36
66  c     16
67  UP    27
68  d     17
69  DIV   35
6a  5     05
6b  EEX   26
6c  2     02
6d  X     36

70  DN    25
71  FMT   42
72  DN    25
73  UP    27
74  5     05
75  0     00
76  -     34
77  RDN   31
78  FMT   42
79  DN    25
7a  RUP   22
7b  +     33
7c  +     33
7d  RDN   31

80  FMT   42
81  DN    25
82  RUP   22
83  -     34
84  RUP   22
85  XEY   30
86  +     33
87  RDN   31
88  XEY   30
89  FMT   42
8a  UP    27
8b  FMT   42
8c  DN    25
8d  RDN   31

90  XEY   30
91  -     34
92  -     34
93  DN    25
94  XEY   30
95  FMT   42
96  DN    25
97  FMT   42
98  UP    27
99  0     00
9a  UP    27
9b  c     16
9c  UP    27
9d  1     01

a0  +     33
a1  0     00
a2  GTO   44
a3  4     04
a4  b     14
a5  END   46
```

n

Δt

S_x

$\bar{X}$

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