HEWLETT-PACKARD

HP-67/HP-97

Stat Pac I

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Introduction

The 21 programs of Stat Pac I have been drawn from the fields of general statistics and related areas.

Each program in this pac is represented by one or more magnetic cards and a section in this manual. The manual provides a description of the program with relevant equations, a set of instructions for using the program, and one or more example problems, each of which includes a list of the actual keystrokes required for its solution. Program listings for all the programs in the pac appear at the back of this manual. Explanatory comments have been incorporated in the listings to facilitate your understanding of the actual working of each program. Thorough study of a commented listing can help you to expand your programming repertoire, since interesting techniques can often be found in this way.

On the face of each magnetic card are various mnemonic symbols which provide shorthand instructions to the use of the program. You should first familiarize yourself with a program by running it once or twice while following the complete User Instructions in the manual. Thereafter, the mnemonics on the cards themselves should provide the necessary instructions, including what variables are to be input, which user-definable keys are to be pressed, and what values will be output. A full explanation of the mnemonic symbols for magnetic cards may be found in appendix A.

If you have already worked through a few programs in the Standard Pac, you will understand how to load a program and how to interpret the User Instructions form. If these procedures are not clear to you, take a few minutes to review the sections, "Loading a Program" and "Format of User Instructions," in your Standard Pac.

We hope that Stat Pac I will assist you in the solution of numerous problems in your discipline. We would very much appreciate knowing your reactions to the programs in this pac, and to this end we have provided a questionnaire inside the front cover of this manual. Would you please take a few minutes to give us your comments on these programs? It is in the comments we receive from you that we learn how best to increase the usefulness of programs like these.

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A WORD ABOUT PROGRAM USAGE

This application pac has been designed for both the HP-97 Programmable Printing Calculator and the HP-67 Programmable Pocket Calculator. The most significant difference between the HP-67 and the HP-97 calculators is the printing capability of the HP-97. The two calculators also differ in a few minor ways. The purpose of this section is to discuss the ways that the programs in this pac are affected by the differences in the two machines, and to suggest how you can make optimal use of your machine, be it an HP-67 or an HP-97.

Some of the computed results in this pac are output by PRINTx statements. On the HP-97, these results will be output on the printer. On the HP-67, each PRINT command will be interpreted as a PAUSE: the program will halt, display the result for about five seconds, then continue execution. The term "PRINT/PAUSE" is used to describe this output condition.

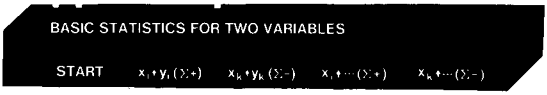
If you own an HP-67, you may want more time to copy down the number displayed by a PRINT/PAUSE. All you need to do is press down any key on the keyboard. If the command being executed is PRINTx (eight rapid blinks of the decimal point), pressing down a key will cause the program to halt. If the command being executed is PRINT STACK (two slow blinks of the decimal point), the number in the display will remain there until the depressed key is released; then the next register in the stack will be displayed, and so on. After display of all four registers, the program will halt execution if a key was pressed at any time during the display of the stack contents. In both cases, execution of the halted program may be re-initiated by pressing R/S.

HP-97 users may also want to keep a permanent record of the values input to a certain program. A convenient way to do this is to set the Print Mode switch to NORMAL before running the program. In this mode, all input values and their corresponding user-definable keys will be listed on the printer, thus providing a record of the entire operation of the program.

Another area that could reflect differences between the HP-67 and the HP-97 is in the keystroke solutions to example problems. It is sometimes necessary in these solutions to include operations that involve prefix keys, namely, f on the HP-97 and f, g, and h on the HP-67. For example, the operation 10^x is performed on the HP-97 as f 10^x and on the HP-67 as g 10^x. In such cases, the keystroke solution omits the prefix key and indicates only the operation (as here, [10^x]). As you work through the example problems, take care to press the appropriate prefix keys (if any) for your calculator.

Also in keystroke solutions, those values which are output by the command PRINTx will be followed by three asterisks (***).

BASIC STATISTICS FOR TWO VARIABLES



This program calculates means, standard deviations, covariance, correlation coefficient, coefficients of variation, sums of data points, sum of multiplication of data points, and sums of squares of data points derived from a set of ungrouped data points $\{(x_i, y_i), i = 1, 2, \ldots, n\}$, or grouped data points $\{(x_i, y_i, f_i), i = 1, 2, \ldots, n\}$. f_i denotes the frequency of repetition of (x_i, y_i).

$$\text{means } \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

$$\text{standard deviations } s_x = \sqrt{\frac{\Sigma x_i^2 - n\bar{x}^2}{n-1}}$$

$$\left(\text{or } s_x' = \sqrt{\frac{\Sigma x_i^2 - n\bar{x}^2}{n}}\right)$$

$$s_y = \sqrt{\frac{\Sigma y_i^2 - n\bar{y}^2}{n-1}}$$

$$\left(\text{or } s_y' = \sqrt{\frac{\Sigma y_i^2 - n\bar{y}^2}{n}}\right)$$

covariance

$$s_{xy} = \frac{1}{n-1}\left(\Sigma x_i y_i - \frac{1}{n}\Sigma x_i \Sigma y_i\right)$$

$$\left(\text{or } s_{xy}' = \frac{1}{n}\left[\Sigma x_i y_i - \frac{1}{n}\Sigma x_i \Sigma y_i\right]\right)$$

$$\text{correlation coefficient } \gamma_{xy} = \frac{s_{xy}}{s_x s_y}$$

$$\text{Coefficients of variation } V_x = \frac{s_x}{\bar{x}} \cdot 100 \quad , \; V_y = \frac{s_y}{\bar{y}} \cdot 100$$

Note: n is a positive integer and $n > 1$.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		f A	1.00
4	For grouped data points, go to 8			
5	For ungrouped data points,			
	do 6~7			
	for i=1, 2,..., n			
6	Input x_i	x_i	ENTER↑	
	y_i	y_i	B	i
7	If you made a mistake in			
	inputting x_k and y_k, then			
	correct by →	x_k	ENTER↑	
		y_k	C	i–1
8	For grouped data points			
	do 9~10			
	for i=1, 2,..., n			
9	Input x_i	x_i	ENTER↑	
	y_i	y_i	ENTER↑	
	f_i	f_i	D	Σf_i
10	If you made a mistake in			
	inputting x_k, y_k and f_k, then			
	correct by →	x_k	ENTER↑	
		y_k	ENTER↑	
		f_k	E	$\Sigma f_i - f_k$
11	Calculate means: $\bar{x}$		f B	$\bar{x}$
	$\bar{y}$		R/S	$\bar{y}$
12	Calculate coefficients of			
	variation: V_x		f B	V_x
	V_y		R/S	V_y

13	Calculate standard deviations:			
	s_x		f C	s_x
	s_y		R/S	s_y
	s_x'		f C	s_x'
	s_y'		R/S	s_y'
14	Calculate: covariance			
	s_{xy}		f D	s_{xy}
	s_{xy}'		R/S	s_{xy}'
15	Calculate correlation			
	coefficient γ_{xy}		f D	γ_{xy}
16	Calculate sums: Σx_i		f E	Σx_i
	Σy_i		R/S	Σy_i
	$\Sigma x_i y_i$		R/S	$\Sigma x_i y_i$
17	Calculate sums of squares			
	Σx_i^2		f E	Σx_i^2
	Σy_i^2		R/S	Σy_i^2
	For a new case, go to 2			
	*Note: to clear print mode			
	press →		CLF	
			0	

Example 1:

For the following set of data, find the means, standard deviations, covariance, correlation coefficient, coefficients of variation, and the sums.

x_i	26	30	44	50	62	68	74
y_i	92	85	78	81	54	51	40

Keystrokes: **Outputs:**

A ⟶ 0.00

f A ⟶ 1.00

26 ENTER↑ 92 B ⟶ 26.00 *** (x_1)

92.00 *** (y_1)

1.00 ***

100 ENTER↑ 100 B →	100.00 ***	(x_2)
	100.00 ***	(y_2) (error)
	2.00 ***	
100 ENTER↑ 100 C →	100.00 ***	(x_2)
	100.00 ***	(y_2) (correction)
	1.00 ***	
30 ENTER↑ 85 B →	30.00 ***	(x_2)
	85.00 ***	(y_2)
	2.00 ***	
44 ENTER↑ 78 B →	44.00 ***	(x_3)
	78.00 ***	(y_3)
	3.00 ***	
50 ENTER↑ 81 B →	50.00 ***	(x_4)
	81.00 ***	(y_4)
	4.00 ***	
62 ENTER↑ 54 B →	62.00 ***	(x_5)
	54.00 ***	(y_5)
	5.00 ***	
68 ENTER↑ 51 B →	68.00 ***	(x_6)
	51.00 ***	(y_6)
	6.00 ***	
74 ENTER↑ 40 B →	74.00 ***	(x_7)
	40.00 ***	(y_7)
	7.00 ***	
f B →	50.57 ***	($\bar{x}$)
R/S →	68.71 ***	($\bar{y}$)
f B →	36.58 ***	(V_x)
R/S →	29.10 ***	(V_y)
f C →	18.50 ***	(s_x)
R/S →	20.00 ***	(s_y)
f C →	17.13 ***	(s_x')
R/S →	18.51 ***	(s_y')

f D → -354.14 ***(s_{xy})
R/S → -303.55 ***(s_{xy}')
f D → -0.96 *** (γ_{xy})
f E → 354.00 *** (Σx_i)
R/S → 481.00 *** (Σy_i)
R/S → 22200.00***($\Sigma x_i y_i$)
f E → 19956.00***(Σx_i^2)
R/S → 35451.00***(Σy_i^2)

Example 2:

Apply the program to the following set of grouped data.

x_i	4.8	5.2	3.8	4.4	4.1
y_i	15.1	11.5	14.3	13.6	12.8
f_i	1	3	1	6	2

Keystrokes:	Outputs:	
A →	0.00	
f A →	1.00	
4.8 ENTER↑ 15.1 ENTER↑ 1 D →	4.80 ***	(x_1)
	15.10 ***	(y_1)
	1.00 ***	(f_1)
	1.00 ***	
5.2 ENTER↑ 11.5 ENTER↑ 3 D →	5.20 ***	(x_2)
	11.50 ***	(y_2)
	3.00 ***	(f_2)
	4.00 ***	(Σf_i)
10 ENTER↑ 10 ENTER↑ 4 D →	10.00 ***	
	10.00 ***	(error)
	4.00 ***	
	8.00 ***	
10 ENTER↑ 10 ENTER↑ 4 E →	10.00 ***	
	10.00 ***	(correction)
	4.00 ***	
	4.00 ***	
3.8 ENTER↑ 14.3 ENTER↑ 1 D →	3.80 ***	(x_3)
	14.30 ***	(y_3)
	1.00 ***	(f_3)
	5.00 ***	(Σf_i)

4.4 ENTER↑ 13.6 ENTER↑ 6 D →	4.40 ***	(x_4)
	13.60 ***	(y_4)
	6.00 ***	(f_4)
	11.00 ***	(Σf_i)
4.1 ENTER↑ 12.8 ENTER↑ 2 D →	4.10 ***	(x_5)
	12.80 ***	(y_5)
	2.00 ***	(f_5)
	13.00 ***	(Σf_i)
f B →	4.52 ***	($\bar{x}$)
R/S →	13.16 ***	($\bar{y}$)
f B →	9.93 ***	(V_x)
R/S →	8.42 ***	(V_y)
f C →	0.45 ***	(s_x)
R/S →	1.11 ***	(s_y)
f C →	0.43 ***	(s_x')
R/S →	1.07 ***	(s_y')
f D →	−0.31 ***	(s_{xy})
R/S →	−0.28 ***	(s_{xy}')
f D →	−0.62 ***	(γ_{xy})
f E →	58.80 ***	(Σx_i)
R/S →	171.10 ***	(Σy_i)
R/S →	770.22 ***	($\Sigma x_i y_i$)
f E →	268.38 ***	(Σx_i^2)
R/S →	2266.69 ***	(Σy_i^2)

FACTORIAL, PERMUTATION AND COMBINATION

FACTORIAL. PERMUTATION AND COMBINATION

START | P? | $n \to n!$ | $m \uparrow n \to {}_mP_n$ | $m \uparrow n \to {}_mC_n$

This program finds the extended range factorial (n can be greater than 69), permutation and combination. Permutation and combination are functions of the factorial, but this program will not use the factorial key, so that better accuracy and larger range can be obtained.

The equations are:

Factorial $$n! = n\,(n-1)\,(n-2) \cdot \cdot \cdot 2 \cdot 1$$

Permutation $${}_mP_n = \frac{m!}{(m-n)!} = m(m-1)\ldots(m-n+1)$$

Combination $${}_mC_n = \frac{m!}{(m-n)!\,n!} = \frac{m(m-1)\ldots(m-n+1)}{1 \cdot 2 \cdot \ldots \cdot n}$$

where m, n are integers and $0 \leqslant n \leqslant m$.

Notes: 1. ${}_mP_0 = 1$, ${}_mP_1 = m$, ${}_mP_m = m!$
therefore n! should be used for large m.

2. ${}_mC_0 = {}_mC_m = 1$

3. ${}_mC_1 = {}_mC_{m-1} = m$

4. ${}_mC_n = {}_mC_{m-n}$

5. In calculating n!, the accuracy will be reduced for n>69, since it is calculated by taking Log., ie
$$n! = \log^{-1}\left[\log(n) + \log\left[(n-1)!\right]\right]$$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	Go to 5 or 6 or 7			
5	Calculate n!			
	(i) $n \leqslant 69$	n	C	n!
	(ii) n>69**	n	C	n
			R/S	exp. of 10
			R/S	decimal No.
6	Calculate $_mP_n$	m	ENTER↑	
		n	D	$_mP_n$
7	Calculate $_mC_n$	m	ENTER↑	
		n	E	$_mC_n$
	For a new case, go to 2			
	*Note: to clear print mode			
	press →		CLF	
			0	
	**In Print Mode, the 3-number			
	result will be printed out			
	automatically.			

Examples:

1. $5! = 120$
2. $69! = 1.711224524 \times 10^{98}$
3. $70! = 1.197857069 \times 10^{100}$
4. $100! = 9.332622518 \times 10^{157}$
5. ${}_{27}P_5 = 9687600.00$
6. ${}_{73}C_4 = 1088430.00$

Keystrokes:	Output:	
A ⟶	0.00	
B ⟶	1.00	
5 C ⟶	5.00 ***	
	120.00 ***	(5!)
69 C ⟶	69.00 ***	
	1.711224524+98 ***	(69!)
70 C ⟶	70.00***	
	100.00 ***	(10^{100})
	1.197857069 ***	(decimal no.)
100 C ⟶	100.00 ***	
	157.00 ***	(10^{157})
	9.332622518 ***	(decimal no.)
27 ENTER↑ 5 D ⟶	27.00 ***	
	5.00 ***	
	9687600.00 ***	(${}_{27}P_5$)
73 ENTER↑ 4 E ⟶	73.00 ***	
	4.00 ***	
	1088430.00 ***	(${}_{73}C_4$)

MOMENTS, SKEWNESS AND KURTOSIS (FOR GROUPED OR UNGROUPED DATA)

For grouped or ungrouped data, moments are used to describe sets of data, skewness is used to measure the lack of symmetry in a distribution, and kurtosis is the relative peakness or flatness of a distribution. For a given set of data

$$\{x_1, x_2, \cdot\cdot\cdot, x_n\}:$$

1st moment $$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

2nd moment $$m_2 = \frac{1}{n} \Sigma x_i^2 - \bar{x}^2$$

3rd moment $$m_3 = \frac{1}{n} \Sigma x_i^3 - \frac{3}{n} \bar{x} \Sigma x_i^2 + 2\bar{x}^3$$

4th moment $$m_4 = \frac{1}{n} \Sigma x_i^4 - \frac{4}{n} \bar{x} \Sigma x_i^3 + \frac{6}{n} \bar{x}^2 \Sigma x_i^2 - 3\bar{x}^4$$

Moment coefficient of skewness

$$\gamma_1 = \frac{m_3}{m_2^{3/2}}$$

Moment coefficient of kurtosis

$$\gamma_2 = \frac{m_4}{m_2^2}$$

This program also provides the option for calculating those statistics for grouped data (using similar formulas as for ungrouped data):

data	y_1	y_2	...	y_m
frequency	f_1	f_2	...	f_m

Note that for this case, 1st moment

$$\bar{x} = \frac{\sum_{i=1}^{m} f_i x_i}{\sum_{i=1}^{m} f_i}$$

Reference: Theory and Problems of Statistics, M. R. Spiegel, Schaum's Outline, McGraw-Hill, 1961

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		f A	1.00
4	For grouped data, go to 12,			
5	Do 6~7 for i = 1, 2, ..., n for			
	ungrouped data			
6	Input x_i	x_i	B	i
7	If you made a mistake in			
	inputting x_k, then correct by →	x_k	C	i-1
8	Calculate: $\bar{x}$		f B	$\bar{x}$
9	Calculate: m_2		f C	m_2
	m_3		R/S	m_3
	m_4		R/S	m_4
10	Calculate: γ_1		f D	γ_1
	γ_2		R/S	γ_2
11	For a new case, go to 2			
12	Do 13~14 for j = 1, 2, ..., m			
	for grouped data			
13	Input	y_j	ENTER↑	
	and	f_j	D	j
14	If you made a mistake in			
	inputting y_h and f_h, then			
	correct by	y_h	ENTER↑	
		f_h	E	j–1
15	Go to 8			
	*Note: to clear print mode			
	press →		CLF	
			0	

Examples:

1. Ungrouped data

i	1	2	3	4	5	6	7	8	9
x_i	2.1	3.5	4.2	6.5	4.1	3.6	5.3	3.7	4.9

$\bar{x} = 4.21$, $m_2 = 1.39$, $m_3 = 0.39$, $m_4 = 5.49$

$\gamma_1 = 0.24$, $\gamma_2 = 2.84$

Keystrokes:	Outputs:	
A →	0.00	
f A →	1.00	
2.1 B →	2.10 ***	(x_1)
	1.00 ***	
4 B →	4.00 ***	(x_2) (error)
	2.00 ***	
4 C →	4.00 ***	(x_2) (correction)
	1.00 ***	
3.5 B →	3.50 ***	(x_2)
	2.00 ***	
4.2 B →	4.20 ***	(x_3)
	3.00 ***	
6.5 B →	6.50 ***	(x_4)
	4.00 ***	
4.1 B →	4.10 ***	(x_5)
	5.00 ***	
3.6 B →	3.60 ***	(x_6)
	6.00 ***	
5.3 B →	5.30 ***	(x_7)
	7.00 ***	

3.7 B → 3.70 *** (x_8)
8.00 ***

4.9 B → 4.90 *** (x_9)
9.00 ***

f B → 4.21 *** ($\bar{x}$)

f C → 1.39 *** (m_2)

R/S → .39 *** (m_3)

R/S → 5.49 *** (m_4)

f D → 0.24 *** (γ_1)

R/S → 2.84 *** (γ_2)

2. Grouped data

j	1	2	3	4	5
y_j	3	2	4	6	1
f_j	4	5	3	2	1

$\bar{x} = 3.13$, $m_2 = 1.98$, $m_3 = 2.14$, $m_4 = 11.05$
$\gamma_1 = 0.77$, $\gamma_2 = 2.81$

Keystrokes:	**Outputs:**	
A →	0.00	
f A →	1.00	
3 ENTER↑ 4 D →	3.00***	(y_1)
	4.00 ***	(f_1)
	1.00 ***	

2 ENTER↑ 5 D →	2.00 ***	(y_2)
	5.00 ***	(f_2)
	2.00 ***	
5 ENTER↑ 5 D →	5.00 ***	(y_3) (error)
	5.00 ***	(f_3)
	3.00 ***	
5 ENTER↑ 5 E →	5.00 ***	(y_3)
	5.00 ***	(f_3) (correction)
	2.00 ***	
4 ENTER↑ 3 D →	4.00 ***	(y_3)
	3.00 ***	(f_3)
	3.00 ***	
6 ENTER↑ 2 D →	6.00 ***	(y_4)
	2.00 ***	(f_4)
	4.00 ***	
1 ENTER↑ 1 D →	1.00 ***	(y_5)
	1.00 ***	(f_5)
	5.00 ***	
f B →	3.13 ***	($\bar{x}$)
f C →	1.98 ***	(m_2)
R/S →	2.14 ***	(m_3)
R/S →	11.05 ***	(m_4)
f D →	0.77 ***	(γ_1)
R/S →	2.81 ***	(γ_2)

RANDOM NUMBER GENERATOR

Random numbers are useful in a wide variety of applications, e.g., simulation, sampling, computer programming, numerical analysis and games. This program calculates (1) uniformly distributed numbers, (2) random integers, (3) normally distributed numbers, (4) exponentially distributed numbers, (5) mean, standard deviation and counter of the numbers generated.

This program calculates:

1. Uniformly distributed pseudo random numbers u_i in the range $a < u_i < b$: The multiplicative linear congruential method is used.

$$u_{i+1} = f_{i+1}(b - a) + a$$

where $i = 0, 1, 2, \ldots$ and

$$f_{i+1} = \text{fractional part of } (997\, f_i)$$

$$f_o = 0.5284163.$$

The period has length 500000, i.e., 500000 different numbers can be generated before repeating. The least significant digits (the righthand digits) of u_i are not as random as the most significant digits (the left-hand digits). Thus random digits, if needed, should be taken from the most significant end of the numbers. This generator passes the chi-square frequency test for uniformity, serial test and run tests for randomness.

If a different sequence of numbers is desired, a different starting value f_o ($0 < f_o < 1$) can be used. Some program steps (the starting value stored under LBL [0]) must be changed accordingly. Note that if $10^7 \times f_o$ is not divisible by 2 or 5, then the period of the generator has length 500000. All the tests mentioned above should be applied to the new generator before using it.

2. Pseudo random integers d_i such that $1 \leq d_i \leq k$:

Suppose u_i ($i = 1, 2, \ldots$) is a sequence of uniformly distributed pseudo random numbers between 0 and 1.

$$d_i = 1 + \text{integer part of } (ku_i)$$

3. Normally distributed pseudo random numbers n_i if the mean m and the standard deviation σ are given:

Suppose u_i (i = 1, 2, ...) is a sequence of uniformly distributed pseudo random numbers between 0 and 1.

Let

$$v_1 = (2u_i - 1) \quad , \quad v_2 = (2u_{i+1} - 1)$$
$$S = v_1^2 + v_2^2 \qquad (i = 1, 2, \ldots)$$

If $S \geqslant 1$, discard the two uniform numbers u_i, u_{i+1} and generate the next two numbers in the sequence until $S < 1$. Then compute the normally distributed numbers according to the following equations

$$n_i = \sigma v_1 \sqrt{\frac{-2 \ln S}{S}} + m$$

$$n_{i+1} = \sigma v_2 \sqrt{\frac{-2 \ln S}{S}} + m$$

4. Exponentially distributed pseudo random numbers e_i with mean μ:

Suppose u_i (i = 1, 2, ...) is a sequence of uniformly distributed pseudo random numbers between 0 and 1.

$$e_i = -\mu \ln u_i$$

5. The mean $\bar{x}$, standard deviation s and counter n of the random numbers computed:

$$\bar{x} = \sum_{i=1}^{n} x_i/n$$

$$s = \sqrt{\frac{\Sigma x_i^2 - n\bar{x}^2}{n-1}}$$

where x_i can be u_i, d_i, n_i or e_i.

Reference:

Donald E. Knuth, *The Art of Computer Programming*, Vol. 2, Addison-Wesley, 1971.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	For random integers, go to 6			
	For normal numbers, go to 9			
	For exponential numbers,			
	go to 12			
3	Input interval range for			
	uniform numbers	a	ENTER↑	
		b	f A	b
4	Perform 4 for i=1, 2, . . .		A	u_i
5	For $\bar{x}$ and s, go to 14			
6	Input maximum integer			
	desired	k	f B	k
7	Perform 7 for i=1, 2, . . .		B	d_i
8	For $\bar{x}$ and s, go to 14			
9	For normal numbers, input			
	mean	m	ENTER↑	
	and standard deviation	σ	f C	σ
10	Perform 10 for i=1, 2, . . .		C	n_i
11	For $\bar{x}$ and s, go to 14			
12	Input mean for exponential			
	numbers	μ	f D	μ
13	Perform 13 for i=1, 2, . . .		D	e_i
14	Optional: Calculate the mean		E	$\bar{x}$
	the standard deviation		R/S	s
	the counter		R/S	n
15	To continue the calculation,			
	go back to 4, 7, 10, or 13			
16	For a new case, go to 2			

Example 1:

Generate a sequence of uniform pseudo random numbers between 0 and 1.

Keystrokes:	Outputs:	
0 ENTER↑ 1 f A	0.00 ***	(a)
	1.00 ***	(b)
A	0.83 ***	
A	0.56 ***	
A	0.27 ***	
A	0.04 ***	
A	0.20 ***	
A	0.75 ***	
A	0.83 ***	
A	0.95 ***	
E	0.55 ***	(mean)
R/S	0.34 ***	(s)
R/S	8.00 ***	(counter)
A	0.68 ***	
A	0.63 ***	
A	0.22 ***	
	etc.	

Example 2:

Use the random number generator to simulate the successive tosses of a die.

Keystrokes:	Outputs:	
6 f B	6.00 ***	(k)
B	5.00 ***	
B	4.00 ***	
B	2.00 ***	
B	1.00 ***	
B	2.00 ***	
B	5.00 ***	
	etc.	

Example 3:

A professor decides to assign grades randomly and without bias to the students. The grades should have a normal distribution with average grade being 75 and standard deviation being 10. How can the random number generator be used for this purpose?

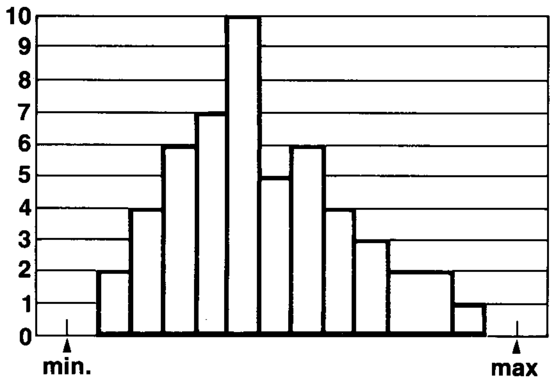
Keystrokes:	Outputs:
75 ENTER↑ 10 f C →	75.00 *** (m)
	10.00 *** (σ)
C →	87.42 ***
C →	77.17 ***
C →	67.44 ***
C →	81.23 ***
C →	89.91 ***
C →	85.32 ***
	etc.

Example 4:

Suppose a radioactive substance emits alpha particles at a rate such that on the average, one particle is emitted every 5 seconds. Note that the amount of time between two successive emissions has the exponential distribution with mean 5. Generate a sequence of random numbers so that each of them can be used as the amount of time between two emissions.

Keystrokes:	Outputs:
5 f D →	5.00 *** (μ)
D →	0.93 ***
D →	2.92 ***
D →	6.49 ***
D →	15.93 ***
D →	8.14 ***
D →	1.44 ***
	etc.

HISTOGRAM



A histogram or bar chart can provide a meaningful way of representing tabular data or the output of an algorithm. By viewing a histogram, trends and biases can be spotted easily.

This program sorts input data into 24 intervals or bins of equal width between specified upper and lower limits.

One is added to the bin whose number is calculated. This procedure is repeated for all x values in the data set. After all data has been input, pressing [f] [B], [R/S], [R/S] will cause the printout of the total number of inputs, the mean of the inputs, and the standard deviation of the inputs. Pressing [f] [A] gives the number of inputs in each bin and a representation of the histogram. The bins are arranged in maximum x value to minimum x value order.

The 24 intervals are stored three at a time in registers $R_1 \sim R_8$.

Incorrect values may be deleted at any time by keying them in and pressing [E]. However, if the value is out of bounds, then "Error" will be displayed. Press [CL x] and continue.

To start the program you must specify the minimum expected value x_{min} and the maximum expected value x_{max}.

Equations:

For the histogram:

$$\text{mean} = \frac{\Sigma x}{n}$$

$$\text{standard deviation} = \sqrt{\frac{\Sigma x^2 - n\,\bar{x}^2}{n-1}}$$

$$y_i = 1 + \text{INT}\left[24\,\frac{x_i - x_{min}}{x_{max} - x_{min}}\right]$$

where

y_i = interval number
x_i = input data
x_{min} = lower limit of histogram
x_{max} = upper limit of histogram
INT = integer part of

Remark:

Because each interval is represented by only three digits, overflow from one interval to the next lower interval will occur for most of the intervals if there are more than 999 counts in the interval.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	Input minimum value	x_{min}	ENTER↑	
	maximum value	x_{max}	C	x_{min}, x_{max}
5	Do 6~7 for i=1, 2, . . . , n			
6	Input x_i	x_i	D	i
7	If you made a mistake in			
	inputting x_k, then correct by →	x_k	E	i−1
8	List histogram		f A	LIST

9	Print n, $\bar{x}$, s		f B	n
			R/S	$\bar{x}$
			R/S	s
10	For a new case, go to 2			
	*Note: to clear print mode			
	press→		CLF	
			0	

Example 1:

Compute a histogram of the following data with $x_{min}=0$, $x_{max}=24$.(18.1, 14.3, 8.4, 0.7, 20.2, 14, 17.2, 24, 8.8, 5.7, 13.2, 22.1, 15.7, 18.9, 23).

Keystrokes:	Outputs:	
A ——→	0.00	
B ——→	1.00	
0 ENTER↑ 24 C ——→	0.00 ***	(x_{min})
	24.00 ***	(x_{max})
18.1 D ——→	18.10 ***	
	1. ***	
14.3 D ——→	14.30 ***	
	2. ***	
8.4 D ——→	8.40 ***	
	3. ***	
0.7 D ——→	0.70 ***	
	4. ***	
9.9 D ——→	9.90 ***	(error)
	5. ***	
9.9 E ——→	9.90 ***	(correction)
	4. ***	
20.2 D ——→	20.20 ***	
	5. ***	
14.0 D ——→	14.00 ***	
	6. ***	
17.2 D ——→	17.20 ***	
	7. ***	

24 D →	24.00 ***	
	8. ***	
8.8 D →	8.80 ***	
	9. ***	
5.7 D →	5.70 ***	
	10. ***	
13.2 D →	13.20 ***	
	11. ***	
22.1 D →	22.10 ***	
	12. ***	
15.7 D →	15.70 ***	
	13. ***	
18.9 D →	18.90 ***	
	14. ***	
23 D →	23.00 ***	
	15. ***	
f B →	15.00 ***	(n)
R/S →	14.95 ***	($\bar{x}$)
R/S →	6.71 ***	(s)
f A →	0.00 ***	
	1.00 ***	
	1. ***	
	1.00 ***	
	2.00 ***	
	0. ***	
	2.00 ***	
	3.00 ***	
	0. ***	
	3.00 ***	
	4.00 ***	
	0. ***	
	4.00 ***	
	5.00 ***	
	0. ***	

5.00 ***
6.00 ***
1. ***

6.00 ***
7.00 ***
0. ***

7.00 ***
8.00 ***
0. ***

8.00 ***
9.00 ***
2. ***

9.00 ***
10.00 ***
0. ***

10.00 ***
11.00 ***
0. ***

11.00 ***
12.00 ***
0. ***

12.00 ***
13.00 ***
0. ***

13.00 ***
14.00 ***
1. ***

14.00 ***
15.00 ***
2. ***

15.00 ***
16.00 ***
1. ***

16.00 ***
17.00 ***
0. ***

17.00 ***
18.00 ***
1. ***

18.00 ***
19.00 ***
2. ***

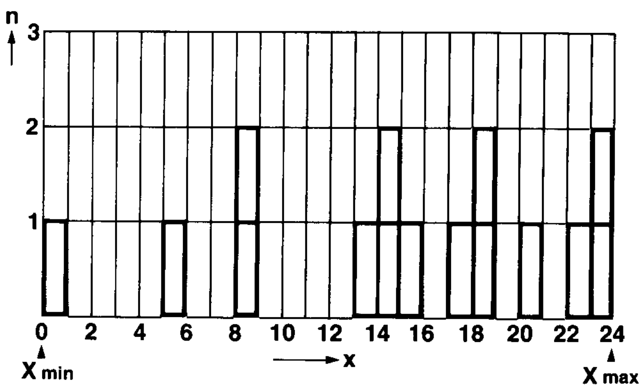
19.00 ***
20.00 ***
0. ***

20.00 ***
21.00 ***
1. ***

21.00 ***
22.00 ***
0. ***

22.00 ***
23.00 ***
1. ***

23.00 ***
24.00 ***
2. ***



ANALYSIS OF VARIANCE (ONE WAY)

The one-way analysis of variance is used to test if observed differences among k sample means can be attributed to chance or whether they are indicative of actual differences among the corresponding population means. Suppose the i^{th} sample has n_i observations (samples may have equal or unequal number of observations). The null hypothesis we want to test is that the k population means are all equal. This program generates the complete ANOVA table.

1. Mean of observations in the i^{th} sample (i=1, 2, - - -, k)

$$\bar{x}_i = \frac{1}{n_i} \sum_{j=1}^{n_i} x_{ij}$$

2. Standard deviation of observations in the i^{th} sample

$$s_i = \left[\left(\sum_{j=1}^{n_i} x_{ij}^2 - n_i \bar{x}_i^2 \right) \Big/ (n_i - 1) \right]^{1/2}$$

3. Sum of observations in the i^{th} sample

$$Sum_i = \sum_{j=1}^{n_i} x_{ij}$$

4. Total sum of squares

$$TSS = \sum_{i=1}^{k} \sum_{j=1}^{n_i} x_{ij}^2 - \frac{\left(\sum_{i=1}^{k} \sum_{j=1}^{n_i} x_{ij} \right)^2}{\sum_{i=1}^{k} n_i}$$

5. Treatment sum of squares

$$\text{TrSS} = \sum_{i=1}^{k} \frac{\left(\sum_{j=1}^{n_i} x_{ij} \right)^2}{n_i} - \frac{\left(\sum_{i=1}^{k} \sum_{j=1}^{n_i} x_{ij} \right)^2}{\sum_{i=1}^{k} n_i}$$

6. Error sum of squares

$$\text{ESS} = \text{TSS} - \text{TrSS}$$

7. Treatment degrees of freedom

$$df_1 = k - 1$$

8. Error degrees of freedom

$$df_2 = \sum_{i=1}^{k} n_i - k$$

9. Total degrees of freedom

$$df_3 = df_1 + df_2 = \sum_{i=1}^{k} n_i - 1$$

10. Treatment mean square

$$\text{TrMS} = \frac{\text{TrSS}}{df_1}$$

11. Error mean square

$$\text{EMS} = \frac{\text{ESS}}{df_2}$$

12. The F ratio

$$F = \frac{\text{TrMS}}{\text{EMS}} \quad \text{(with degrees of freedom } df_1, df_2\text{)}$$

Reference:

J. E. Freund, *Mathematical Statistics*, Prentice Hall, 1962.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	Do 4 ~ 8 for i=1, 2, ..., k			
5	Do 6 for j=1, 2, ..., n_i			
6	Input x_{ij}	x_{ij}	C	j
7	If you made a mistake in			
	inputting x_{im}, then correct by	x_{im}	D	j–1
8	Calculate: mean $\bar{x}_i$		E	$\bar{x}_i$
	standard deviation s_i		R/S	s_i
	sum Sum_i		R/S	Sum_i
9	Calculate: total sum of squares		f A	TSS
	treatment sum of squares		R/S	TrSS
	error sum of squares		R/S	ESS
10	Calculate degrees of freedom			
	df_1		f B	df_1
	df_2		R/S	df_2
	df_3		R/S	df_3
11	Calculate:			
	treatment mean square		f C	TrMS
	error mean square		R/S	EMS
	F ratio		R/S	F
12	For a new case, go to 2			
	*Note: to clear print mode			
	press →		CLF	
			0	

Example:

The following are the scores obtained in an achievement test by random samples of students from four different schools:

i \ j	1	2	3	4	5	6	7
School 1	88	99	96	68	85		
School 2	78	62	98	83	61	88	
School 3	80	61	74	92	78	54	77
School 4	71	65	90	46			

Calculate the ANOVA table and test the null hypothesis that the differences among the sample means can be attributed to chance. Use significance level $\alpha = 0.01$.

Keystrokes:	Outputs:	
A	0.00	
B	1.00	
88 **C**	88.00 ***	
	1.00 ***	
99 **C**	99.00 ***	
	2.00 ***	
96 **C**	96.00 ***	
	3.00 ***	
68 **C**	68.00 ***	
	4.00 ***	
85 **C**	85.00 ***	
	5.00 ***	
E	87.20 ***	($\bar{x}_1$)
R/S	12.15 ***	(s_1)
R/S	436.00 ***	(Sum_1)
78 **C**	78.00 ***	
	1.00 ***	

62 **C** →	62.00 ***	
	2.00 ***	
98 **C** →	98.00 ***	
	3.00 ***	
83 **C** →	83.00 ***	
	4.00 ***	
61 **C** →	61.00 ***	
	5.00 ***	
88 **C** →	88.00 ***	
	6.00 ***	
E →	78.33 ***	($\bar{x}_2$)
R/S →	14.62 ***	(s_2)
R/S →	470.00 ***	(Sum_2)
80 **C** →	80.00 ***	
	1.00 ***	
61 **C** →	61.00 ***	
	2.00 ***	
74 **C** →	74.00 ***	
	3.00 ***	
92 **C** →	92.00 ***	
	4.00 ***	
78 **C** →	78.00 ***	
	5.00 ***	
54 **C** →	54.00 ***	
	6.00 ***	
77 **C** →	77.00 ***	
	7.00 ***	
E →	73.71 ***	($\bar{x}_3$)
R/S →	12.61 ***	(s_3)
R/S →	516.00 ***	(Sum_3)

71 C	71.00 ***	
	1.00 ***	
66 C	66.00 ***	(error)
	2.00 ***	
66 D	66.00 ***	(correction)
	1.00 ***	
65 C	65.00 ***	
	2.00 ***	
90 C	90.00 ***	
	3.00 ***	
46 C	46.00 ***	
	4.00 ***	
E	68.00 ***	($\bar{x}_4$)
R/S	18.13 ***	(s_4)
R/S	272.00 ***	(Sum_4)
f A	4530.00 ***	(TSS)
R/S	930.44 ***	(TrSS)
R/S	3599.56 ***	(ESS)
f B	3.00 ***	(df_1)
R/S	18.00 ***	(df_2)
R/S	21.00 ***	(df_3)
f C	310.15 ***	(TrMS)
R/S	199.98 ***	(EMS)
R/S	1.55 ***	(F)

ANOVA Table

	SS	df	MS	F
Treatments	930.44	3	310.15	1.55
Error	3599.56	18	199.98	
Total	4530.00	21		

Since $F = 1.55$ does not exceed $F_{.01,3,18} = 5.09$, the null hypothesis can not be rejected. We conclude that the means of the scores for the four schools are not significantly different.

TWO WAY ANALYSIS OF VARIANCE (NO REPLICATIONS)

The analysis of variance is the analysis of the total variability of a set of data (measured by their total sum of squares) into components which can be attributed to different sources of variation.

The two way analysis of variance tests the row effects and the column effects independently. This program will generate the ANOVA table for the case such that (1) each cell only has one observation and (2) the row and column effects do not interact.

Equations:

1. Sums

$$\text{Row } RS_i = \sum_j x_{ij} \qquad i = 1, 2, \ldots, r$$

$$\text{Column } CS_j = \sum_i x_{ij} \qquad j = 1, 2, \ldots, c$$

2. Sums of squares

$$\text{Total TSS} = \Sigma\Sigma x_{ij}^2 - (\Sigma\Sigma x_{ij})^2/rc$$

$$\text{Row RSS} = \sum_i \left(\sum_j x_{ij} \right)^2 / c - (\Sigma\Sigma x_{ij})^2/rc$$

$$\text{Column CSS} = \sum_j \left(\sum_i x_{ij} \right)^2 / r - (\Sigma\Sigma x_{ij})^2/rc$$

$$\text{Error ESS} = \text{TSS} - \text{RSS} - \text{CSS}$$

3. Degrees of freedom

$$\text{Row } df_1 = r - 1$$

$$\text{Column } df_2 = c - 1$$

$$\text{Error } df_3 = (r - 1)(c - 1)$$

4. F ratios

$$\text{Row } F_1 = \frac{RSS}{df_1} \Big/ \frac{ESS}{df_3}$$

$$\text{Column } F_2 = \frac{CSS}{df_2} \Big/ \frac{ESS}{df_3}$$

Reference:

Dixon and Massey, *Introduction to Statistical Analysis,* McGraw-Hill, 1969.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	Input: number of rows r	r	ENTER↑	
	number of columns c	c	C	c
5	Do 5~9 for i=1, 2, . . . , r			
	Do 6 for j=1, 2, . . . , c			
6	Input x_{ij}	x_{ij}	D	j
7	If you made a mistake in			
	inputting x_{im}, then correct by	x_{im}	E	j–1
8	Calculate row sums RS_i		f A	RS_i
9	Re-initialize for columns		f B	0.00
10	Do 11~14 for j=1, 2, . . . , c			
11	Do 12 for i=1, 2, ..., r			
12	Input x_{ij}	x_{ij}	D	i
13	If you made a mistake in			
	inputting x_{hj}, then correct by	x_{hj}	E	i–1
14	Calculate column sums CS_i		f C	CS_i
15	Calculate F ratios: Row F_1		f D	F_1
	Column F_2		R/S	F_2

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
16	Calculate degrees of freedom:			
	row df_1		f D	df_1
	column df_2		R/S	df_2
	error df_3		R/S	df_3
17	Calculate sums of squares:			
	row RSS		f E	RSS
	column CSS		R/S	CSS
	error ESS		R/S	ESS
	total TSS		R/S	TSS
18	For a new case, go to 2			
	*Note: to clear print mode			
	press →		CLF	
			0	

Example:

Apply this program to analyze the following set of data.

i \ j	Column 1	2	3	4
Row 1	7	6	8	7
2	2	4	4	4
3	4	6	5	3

Keystrokes:	**Outputs:**	
A ——→	0.00	
B ——→	1.00	
3 ENTER↑ 4 C ——→	3.00 ***	(r)
	4.00 ***	(c)
7 D ——→	7.00 ***	
	1.00 ***	
6 D ——→	6.00 ***	
	2.00 ***	
8 D ——→	8.00 ***	
	3.00 ***	

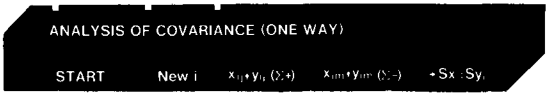
Keystrokes	Display	
7 [D] →	7.00 ***	
	4.00 ***	
[f] [A] →	28.00 ***	(RS_1)
2 [D] →	2.00 ***	
	1.00 ***	
4 [D] →	4.00 ***	
	2.00 ***	
4 [D] →	4.00 ***	
	3.00 ***	
4 [D] →	4.00 ***	
	4.00 ***	
[f] [A] →	14.00 ***	(RS_2)
4 [D] →	4.00 ***	
	1.00 ***	
7 [D] →	7.00 ***	(error)
	2.00 ***	
7 [E] →	7.00 ***	(correction)
	1.00 ***	
6 [D] →	6.00 ***	
	2.00 ***	
5 [D] →	5.00 ***	
	3.00 ***	
3 [D] →	3.00 ***	
	4.00 ***	
[f] [A] →	18.00 ***	(RS_3)
[f] [B] →	0.00 ***	
7 [D] →	7.00 ***	
	1.00 ***	
2 [D] →	2.00 ***	
	2.00 ***	
4 [D] →	4.00 ***	
	3.00 ***	
[f] [C] →	13.00 ***	(CS_1)
6 [D] →	6.00 ***	
	1.00 ***	
4 [D] →	4.00 ***	
	2.00 ***	

6 [D] →	6.00 ***	
	3.00 ***	
[f] [C] →	16.00 ***	(CS_2)
8 [D] →	8.00 ***	
	1.00 ***	
4 [D] →	4.00 ***	
	2.00 ***	
5 [D] →	5.00 ***	
	3.00 ***	
[f] [C] →	17.00 ***	(CS_3)
7 [D] →	7.00 ***	
	1.00 ***	
4 [D] →	4.00 ***	
	2.00 ***	
3 [D] →	3.00 ***	
	3.00 ***	
[f] [C] →	14.00 ***	(CS_4)
[f] [D] →	11.70 ***	(F_1)
[R/S] →	1.00 ***	(F_2)
[f] [D] →	2.00 ***	(df_1)
[R/S] →	3.00 ***	(df_2)
[R/S] →	6.00 ***	(df_3)
[f] [E] →	26.00 ***	(RSS)
[R/S] →	3.33 ***	(CSS)
[R/S] →	6.67 ***	(ESS)
[R/S] →	36.00 ***	(TSS)

ANOVA

	SS	df	F ratio
Row	26.00	2	11.70
Column	3.33	3	1.00
Error	6.67	6	
Total	36.00		

ANALYSIS OF COVARIANCE (ONE WAY)



The one way analysis of covariance program tests the effect of one variable separately from the effect of a second variable, if the second variable represents an actual measurement for each individual (rather than a category).

Suppose (x_{ij}, y_{ij}) represents the j^{th} observation from the i^{th} population ($i = 1,2, \ldots, k$, $j = 1,2, \ldots, n_i$). Note that samples may have equal or unequal number of observations. The analysis of covariance tests for a difference in means of residuals. The residuals are the differences of the observations and a regression quantity based on the associated second variable. The analysis of covariance procedure is based on the separations of the sums of squares and the sums of products into several portions. This program will generate the complete ANOCOV table.

Equations:

1. Sums and sums of squares

$$Sx_i = \sum_j x_{ij} \quad (i = 1,2,\ldots,k)$$

$$TSSx = \Sigma\,\Sigma x_{ij}^2 - \frac{(\Sigma\Sigma x_{ij})^2}{\sum_i n_i}$$

$$ASSx = \sum_i \frac{\left(\sum_j x_{ij}\right)^2}{n_i} - \frac{(\Sigma\Sigma x_{ij})^2}{\sum_i n_i}$$

$$WSSx = TSSx - ASSx$$

2. Degrees of freedom

$$df_1 = k - 1$$

$$df_2 = \sum_i n_i - k$$

3. Mean squares and F statistic

$$AMSx = \frac{ASSx}{df_1}$$

$$WMSx = \frac{WSSx}{df_2}$$

$$F_x = \frac{AMSx}{WMSx} \text{ with degrees of freedom } df_1, df_2$$

By changing x_{ij} to y_{ij}, similar formulas for y_{ij} can be obtained.

4. Sums of products

$$TSP = \Sigma\Sigma x_{ij}\, y_{ij} - \frac{(\Sigma\Sigma x_{ij})\ (\Sigma\Sigma y_{ij})}{\sum_i n_i}$$

$$ASP = \sum_i \frac{\left(\sum_j x_{ij}\right)\left(\sum_j y_{ij}\right)}{n_i} - \frac{(\Sigma\Sigma x_{ij})(\Sigma\Sigma y_{ij})}{\sum_i n_i}$$

$$WSP = TSP - ASP$$

5. Residual sums of squares

$$TSS\hat{y} = TSSy - \frac{(TSP)^2}{TSSx}$$

$$WSS\hat{y} = WSSy - \frac{(WSP)^2}{WSSx}$$

$$ASS\hat{y} = TSS\hat{y} - WSS\hat{y}$$

6. Residual degrees of freedom

$$df_3 = k - 1$$

$$df_4 = \sum_i n_i - k - 1$$

7. Residual mean squares and F statistic

$$AMS\hat{y} = \frac{ASS\hat{y}}{df_3}$$

$$WMS\hat{y} = \frac{WSS\hat{y}}{df_4}$$

$$F = \frac{AMS\hat{y}}{WMS\hat{y}} \text{ with degrees of freedom } df_3, df_4$$

ANOCOV Table

					Residuals			
	degrees of freedom	SSx	SP	SSy	degrees of freedom	$SS\hat{y}$	$MS\hat{y}$	F statistic
Among means	df_1	ASSx	ASP	ASSy	df_3	$ASS\hat{y}$	$AMS\hat{y}$	F
Within groups	df_2	WSSx	WSP	WSSy	df_4	$WSS\hat{y}$	$WMS\hat{y}$	
Total		TSSx	TSP	TSSy		$TSS\hat{y}$		

Remarks:

1. F_x can be used to test if the X means are equal (ANOVA for X).
2. F_y can be used to test if the Y means (not making use of the X values) are equal (ANOVA for unadjusted Y).

Reference:

Dixon and Massey, *Introduction to Statistical Analysis,* McGraw-Hill, 1969.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2 of card 1			
2	Initialize		A	0.00
3	To set print mode*		f E	1.00
4	Do 5~9 for i=1, 2, . . . , k			
5	Initialize for new i		B	i
6	Do 6 for j=1, 2, . . . , n_i			
7	Input x_{ij} and y_{ij}	x_{ij}	ENTER↑	
		y_{ij}	C	j
8	If you made a mistake in			
	inputting x_{im} and y_{im}, then			
	correct by	x_{im}	ENTER↑	
		y_{im}	D	j−1
9	Calculate the i^{th} sums Sx_i		E	Sx_i
	Sy_i		R/S	Sy_i
10	Calculate: the total sum of			
	squares for x		f A	TSSx
	Among means sum of squares			
	for x		R/S	ASSx
	Within groups sum of squares			
	for x		R/S	WSSx
11	Calculate: the total sum of			
	squares for y		f A	TSSy
	Among means sum of squares			
	for y		R/S	ASSy
	Within groups sum of squares			
	for y		R/S	WSSy
12	Calculate: F_x		f B	F_x
	F_y		R/S	F_y
	degrees of freedom df_1		R/S	df_1
	df_2		R/S	df_2

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
13	Load side 1 of card 2			
14	Calculate: the total sum of			
	products (SP)		f C	TSP
	among means SP		R/S	ASP
	within group SP		R/S	WSP
15	Calculate: TSSŷ		f D	TSSŷ
	WSSŷ		R/S	WSSŷ
	ASSŷ		R/S	ASSŷ
16	Calculate: residual mean			
	squares		f E	AMSŷ
	WMSŷ		R/S	WMSŷ
	the F statistic		R/S	F
	the degrees of freedom df_3		R/S	df_3
	df_4		R/S	df_4
17	For a new case, go to 1			
	*Note: to clear print mode			
	press →		CLF	
			0	

Example:

			j			
			1	2	3	4
i	1	x	3	2	1	2
		y	10	8	8	11
	2	x	4	3	3	5
		y	12	12	10	13
	3	x	1	2	3	1
		y	6	5	8	7

($k = 3$, $n_1 = n_2 = n_3 = 4$)

Keystrokes:

Load side 1 and side 2 of card 1

Keystrokes:	Outputs:	
A	0.00	
f E	1.00	
B	1.00 ***	
3 ENTER↑ 10 C	3.00 ***	
	10.00 ***	
	1.00 ***	
2 ENTER↑ 8 C	2.00 ***	
	8.00 ***	
	2.00 ***	
5 ENTER↑ 5 C	5.00 ***	
	5.00 ***	(error)
	3.00 ***	
5 ENTER↑ 5 D	5.00 ***	
	5.00 ***	(correction)
	2.00 ***	
1 ENTER↑ 8 C	1.00 ***	
	8.00 ***	
	3.00 ***	
2 ENTER↑ 11 C	2.00 ***	
	11.00 ***	
	4.00 ***	
E	8.00 ***	(Sx_1)
R/S	37.00 ***	(Sy_1)
B	2.00 ***	
4 ENTER↑ 12 C	4.00 ***	
	12.00 ***	
	1.00 ***	
3 ENTER↑ 12 C	3.00 ***	
	12.00 ***	
	2.00 ***	
3 ENTER↑ 10 C	3.00 ***	
	10.00 ***	
	3.00 ***	

5 ENTER↑ 13 C →	5.00 ***
	13.00 ***
	4.00 ***
E →	15.00 *** (Sx_2)
R/S →	47.00 *** (Sy_2)
B →	3.00 ***
1 ENTER↑ 6 C →	1.00 ***
	6.00 ***
	1.00 ***
2 ENTER↑ 5 C →	2.00 ***
	5.00 ***
	2.00 ***
3 ENTER↑ 8 C →	3.00 ***
	8.00 ***
	3.00 ***
1 ENTER↑ 7 C →	1.00 ***
	7.00 ***
	4.00 ***
E →	7.00 *** (Sx_3)
R/S →	26.00 *** (Sy_3)
f A →	17.00 *** (TSSx)
R/S →	9.50 *** (ASSx)
R/S →	7.50 *** (WSSx)
f A →	71.67 *** (TSSy)
R/S →	55.17 *** (ASSy)
R/S →	16.50 *** (WSSy)
f B →	5.70 *** (Fx)
R/S →	15.05 *** (Fy)
R/S →	2.00 *** (df_1)
R/S →	9.00 *** (df_2)

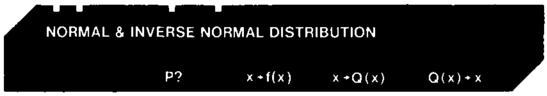
Load side 1 of card 2

f C	27.00 ***	(TSP)
R/S	20.75 ***	(ASP)
R/S	6.25 ***	(WSP)
f D	28.78 ***	(TSSŷ)
R/S	11.29 ***	(WSSŷ)
R/S	17.49 ***	(ASSŷ)
f E	8.75 ***	(AMSŷ)
R/S	1.41 ***	(WMSŷ)
R/S	6.20 ***	(F)
R/S	2.00 ***	(df_3)
R/S	8.00 ***	(df_4)

ANOCOV Table

					Residuals			
	df	SSx	SP	SSy	df	SSŷ	MSŷ	F
Among means	2	9.50	20.75	55.17	2	17.49	8.75	6.20
Within groups	9	7.50	6.25	16.50	8	11.29	1.41	
Total		17.00	27.00	71.67		28.78		

NORMAL AND INVERSE NORMAL DISTRIBUTION



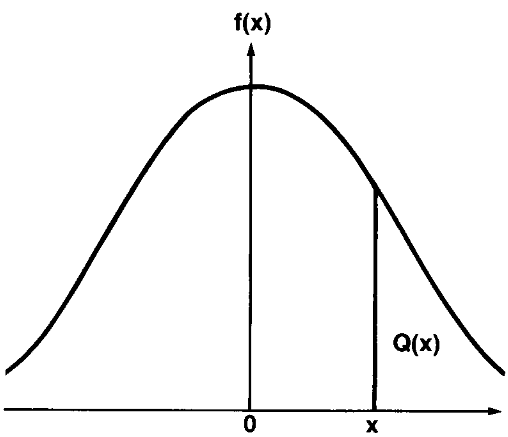
This program evaluates the standard normal density function f(x) and the normal integral Q(x) for given x. If Q is given, x can also be found.

The standard normal distribution has mean 0 and standard deviation 1.

Equations:

1. Standard normal density

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$



2. Normal integral

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt$$

Polynomial approximation is used to compute Q(x) for given x.

Define $R = f(x)\,(b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5) + \epsilon(x)$

where $|\epsilon(x)| < 7.5 \times 10^{-8}$

$$t = \frac{1}{1 + r|x|}\ , \quad r = 0.2316419$$

$b_1 = .319381530, \quad b_2 = -.356563782$

$b_3 = 1.781477937, \quad b_4 = -1.821255978$

$b_5 = 1.330274429$

$$\text{Then } Q(x) = \begin{cases} R & \text{if } x \geqslant 0 \\ 1-R & \text{if } x < 0 \end{cases}$$

3. Inverse normal

For a given $Q > 0$, x can be found such that

$$Q = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}}\, dt.$$

The following rational approximation is used:

$$\text{Define } y = t - \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3} + \epsilon(Q)$$

where $|\epsilon(Q)| < 4.5 \times 10^{-4}$

$$t = \begin{cases} \sqrt{\ln \dfrac{1}{Q^2}} & \text{if } 0 < Q \leqslant 0.5 \\ \sqrt{\ln \dfrac{1}{(1-Q)^2}} & \text{if } 0.5 < Q < 1 \end{cases}$$

$c_0 = 2.515517$ $\quad$ $d_1 = 1.432788$
$c_1 = 0.802853$ $\quad$ $d_2 = 0.189269$
$c_2 = 0.010328$ $\quad$ $d_3 = 0.001308$

Then $$x = \begin{cases} y & \text{if } 0 < Q \leq 0.5 \\ -y & \text{if } 0.5 < Q < 1 \end{cases}$$

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions,* National Bureau of Standards, 1970.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2 of			
	card 1			
2	Initialize		A	0.00
3	Load side 1 and side 2 of			
	card 2			
4	To set print mode*		B	1.00
	Optional: Step 5			
5	Input x to compute f(x)	x	C	f(x)
6	Input x to compute Q(x)	x	D	Q(x)
	For a new case of x, go to			
	5 or 6			
7	Input Q(x) to compute x	Q(x)	E	x
	For a new case of Q(x), go			
	to 7			
	*Note: to clear print mode			
	press →		0	
			STO	
			A	
			STO	
			B	

Example 1:

Find f(x) and Q(x) for x = 1.18 and x = −2.28.

Keystrokes:	Outputs:	
Load side 1 and side 2 of card 1		
A ———→	0.00	
Load side 1 and side 2 of card 2		
B ———→	1.00	
1.18 C ———→	1.18 ***	
	0.20 ***	(f(1.18))
1.18 D ———→	1.18 ***	
	0.12 ***	(Q(1.18))
2.28 CHS D ———→	−2.28 ***	
	0.99 ***	(Q(−2.28))
2.28 CHS C ———→	−2.28 ***	
	0.03 ***	(f(−2.28))

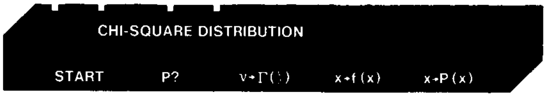
Example 2:

Given Q = 0.12 and Q =0.95, find x.

(If you have run through Example 1, then you can proceed; otherwise you have to load programs as described in Example 1).

Keystrokes:	Outputs:	
0.12 E ———→	0.12 ***	
	1.18 ***	(x)
0.95 E ———→	0.95 ***	
	−1.65 ***	(x)

CHI-SQUARE DISTRIBUTION

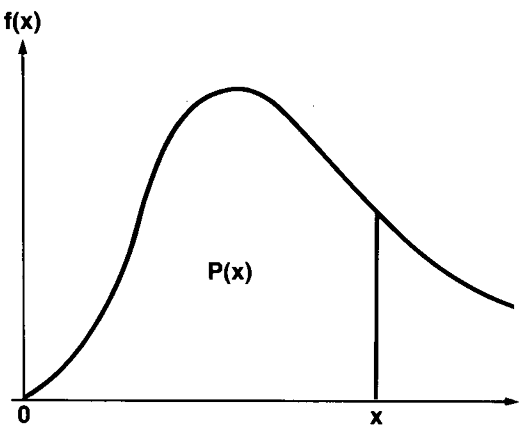


This program evaluates the chi-square density

$$f(x) = \frac{1}{2^{\frac{\nu}{2}}\,\Gamma\left(\frac{\nu}{2}\right)}\, x^{\frac{\nu}{2}-1}\, e^{-\frac{x}{2}}$$

where $x \geqslant 0$

ν is the degrees of freedom.



Series approximation is used to evaluate the cumulative distribution

$$P(x) = \int_0^x f(t)\, dt$$

$$= \left(\frac{x}{2}\right)^{\frac{\nu}{2}} \frac{e^{-\frac{x}{2}}}{\Gamma\left(\frac{\nu+2}{2}\right)} \left[1 + \sum_{k=1}^{\infty} \frac{x^k}{(\nu+2)(\nu+4)\ldots(\nu+2k)}\right]$$

The program computes successive partial sums of the above series. When two consecutive partial sums are equal, the value is used as the sum of the series.

Notes: 1. Program requires $\nu < 141$. If $\nu > 141$, erroneous overflow will result.

2. If both x and ν are large, f(x) may overflow the machine.

3. If ν is even,

$$\Gamma\left(\frac{\nu}{2}\right) = \left(\frac{\nu}{2} - 1\right)!$$

If ν is odd,

$$\Gamma\left(\frac{\nu}{2}\right) = \left(\frac{\nu}{2} - 1\right)\left(\frac{\nu}{2} - 2\right) \cdots \left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right)$$

4. $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions,* National Bureau of Standards, 1970.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	Input degrees of freedom ν	ν	C	$\Gamma(\nu/2)$
	Optional: Step 5			
5	Input x to compute f(x)	x	D	f(x)
6	Input x to compute P(x)	x	E	P(x)
	(i) For a new case with the			
	same ν go to 5 or 6			
	(ii) For a new case with			
	different ν, go to 2			
	*Note: to clear print mode			
	press →		CLF	
			0	

Example 1:

If degrees of freedom $\nu = 20$, find f(x), P(x) for x = 9.6 and x = 15.

Keystrokes:	Outputs:	
A ⟶	0.00	
B ⟶	1.00	
20 C ⟶	20.00 ***	
	362880.00 ***	(Γ(20/2))
9.6 D ⟶	9.60 ***	
	0.02 ***	(f(9.6))
9.6 E ⟶	9.60 ***	
	0.03 ***	(P(9.6))
15 E ⟶	15.00 ***	
	0.22 ***	(P(15))
15 D ⟶	15.00 ***	
	0.06 ***	(f(15))

Example 2:

If $\nu = 3$, find f(x) and P(x) for x =7.82.

Keystrokes:	Outputs:	
A ⟶	0.00	
B ⟶	1.00	
3 C ⟶	3.00 ***	
	0.89 ***	(Γ(3/2))
7.82 D ⟶	7.82 ***	
	0.02 ***	(f(7.82))
7.82 E ⟶	7.82 ***	
	0.95 ***	(P(7.82))

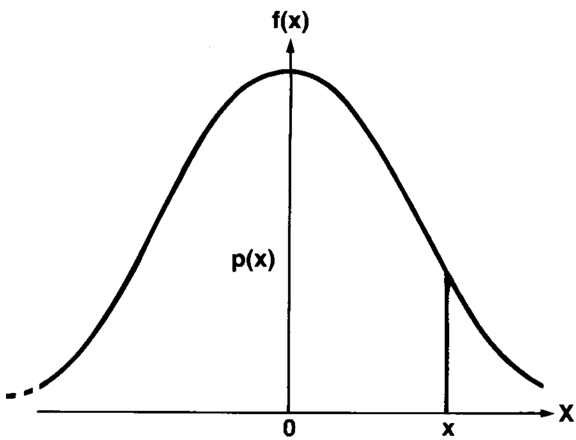
t DISTRIBUTION

This program evaluates the t density function f(x) and the cumulative distribution P(x) for a given x and degrees of freedom ν.

Equations:

1. Density function

$$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi\nu}\,\Gamma\left(\frac{\nu}{2}\right)}\left(1 + \frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$



2. Cumulative distribution function

$$P(x) = \int_{-\infty}^{x} f(y)dy$$

$$\text{Let} \quad \theta = \tan^{-1}\left(\frac{|x|}{\sqrt{\nu}}\right)$$

(a) ν even

$$\text{Let} \quad R = \sin\theta \left\{ 1 + \frac{1}{2}\cos^2\theta + \frac{1\cdot 3}{2\cdot 4}\cos^4\theta + \ldots + \frac{1\cdot 3\cdot 5 \ \ldots \ (\nu\text{-}3)}{2\cdot 4\cdot 6 \ \ldots \ (\nu\text{-}2)}\cos^{\nu-2}\theta \right\}$$

(b) ν odd

$$\text{Let} \quad R = \begin{cases} \dfrac{2\theta}{\pi} & \text{if } \nu = 1 \\ \dfrac{2\theta}{\pi} + \dfrac{2}{\pi}\cos\theta \left\{ \sin\theta \left[1 + \dfrac{2}{3}\cos^2\theta + \ldots \ \dfrac{2\cdot 4 \ldots (\nu\text{-}3)}{1\cdot 3 \ldots (\nu\text{-}2)} \cos^{\nu-3}\theta \right] \right\} & \text{if } \nu>1 \end{cases}$$

$$\text{Then} \quad P(x) = \begin{cases} \dfrac{1+R}{2} & \text{if } x>0 \\ \dfrac{1-R}{2} & \text{if } x\leq 0 \end{cases}$$

Remark:

The program requires $\nu<141$ for f(x), otherwise erroneous overflow will result.

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, 1970.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	Input degrees of freedom ν	ν	C	ν
	Optional: Step 5			
5	Input x to compute f(x)	x	D	f(x)
6	Input x to compute P(x)	x	E	P(x)
7	(i) For a new case with the			
	same ν go to 5 or 6			
	(ii) For a new case with a			
	different ν, go to 2			
	*Note: to clear print mode			
	press →		CLF	
			0	

Example 1:

Find f(x) and P(x) for x = 2.2, ν = 11.

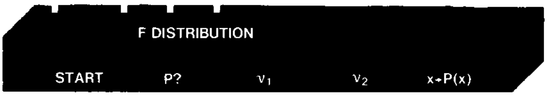
Keystrokes:	Outputs:	
A →	0.00	
B →	1.00	
11 C →	11.00 ***	(ν)
2.2 E →	2.20 ***	(x)
	0.97***	(P(2.2))
2.2 D →	2.20 ***	(x)
	0.04 ***	(f(2.2))

Example 2:

Find f(x) and P(x) for x = −1.75, ν = 30.

Keystrokes:	Outputs:	
A	0.00	
B	1.00	
30 C	30.00 ***	(ν)
1.75 CHS D	−1.75 ***	(x)
	0.09 ***	(f(−1.75))
1.75 CHS E	−1.75 ***	(x)
	0.05 ***	(P(−1.75))

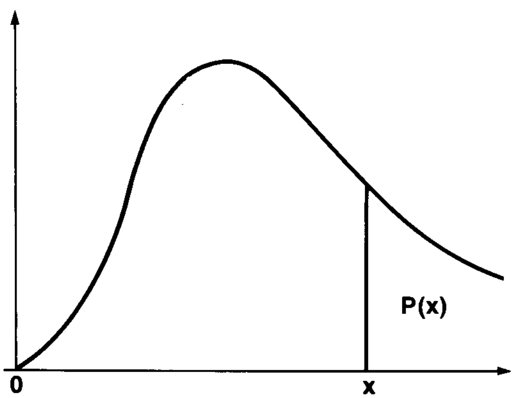
F DISTRIBUTION



This program evaluates the integral of the F distribution

$$P(x) = \int_x^{\infty} \frac{\Gamma\left(\frac{\nu_1 + \nu_2}{2}\right) y^{\frac{\nu_1}{2} - 1} \left(\frac{\nu_1}{\nu_2}\right)^{\frac{\nu_1}{2}}}{\Gamma\left(\frac{\nu_1}{2}\right) \Gamma\left(\frac{\nu_2}{2}\right) \left(1 + \frac{\nu_1}{\nu_2} y\right)^{\frac{\nu_1 + \nu_2}{2}}} \, dy$$

for given values of x (>0), degrees of freedom ν_1, ν_2, provided either ν_1 or ν_2 is even.



The integral is evaluated by means of the following series:

(1) ν_1 even

$$P(x) = t^{\frac{\nu_2}{2}} \left[1 + \frac{\nu_2}{2}(1 - t) + \ldots + \frac{\nu_2(\nu_2 + 2) \ldots (\nu_2 + \nu_1 - 4)}{2 \cdot 4 \ldots (\nu_1 - 2)} (1-t)^{\frac{\nu_1 - 2}{2}} \right]$$

(2) ν_2 even

$$P(x) = 1 - (1 - t)^{\frac{\nu_1}{2}} \left[1 + \frac{\nu_1}{2} t + \ldots + \frac{\nu_1(\nu_1 + 2) \ldots (\nu_2 + \nu_1 - 4)}{2 \cdot 4 \ldots (\nu_2 - 2)} t^{\frac{\nu_2 - 2}{2}} \right]$$

where $t = \dfrac{\nu_2}{\nu_2 + \nu_1 x}$

Note: Usually ν_1 is identified as the degree of freedom for numerator, and ν_2 is identified as the degree of freedom for denominator.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	Input ν_1	ν_1	C	ν_1
5	Input ν_2	ν_2	D	ν_2
6	Input x to calculate P(x)	x	E	P(x)
7	For a new case go to 2			
	*Note: to clear print mode			
	press →		CLF	
			0	

Examples:

1. $\nu_1 = 7,\ \nu_2 = 6$

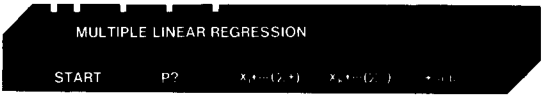
   P(4.21) = 0.05

2. $\nu_1 = 4,\ \nu_2 = 20$

   P(2.25) = 0.10

Keystrokes:	Outputs:	
A ⟶	0.00	
B ⟶	1.00	
7 C ⟶	7.00 ***	(ν_1)
6 D ⟶	6.00 ***	(ν_2)
4.21 E ⟶	4.21 ***	(x)
	0.05 ***	(P(x))
4 C ⟶	4.00 ***	(ν_1)
20 D ⟶	20.00 ***	(ν_2)
2.25 E ⟶	2.25 ***	(x)
	0.10 ***	(P(x))

MULTIPLE LINEAR REGRESSION



For a set of data points $\{(x_i, y_i, z_i), i = 1, 2, \ldots, n\}$ this program fits a linear equation of the form

$$z = a + bx + cy$$

by the least squares method.
Regression coefficients a, b, c can be found by solving the normal equations:

$$\left.\begin{aligned} \Sigma z_i &= an + b\,\Sigma x_i + c\,\Sigma y_i \\ \Sigma x_i z_i &= a\,\Sigma x_i + b\,\Sigma x_i^2 + c\,\Sigma x_i y_i \\ \Sigma y_i z_i &= a\,\Sigma y_i + b\,\Sigma x_i y_i + c\,\Sigma y_i^2 \end{aligned}\right\} \quad i = 1, 2, \ldots, n$$

$$c = \frac{A - B}{\left[n\Sigma x_i^2 - (\Sigma x_i)^2\right]\left[n\Sigma y_i^2 - (\Sigma y_i)^2\right] - \left[n\Sigma x_i y_i - (\Sigma x_i)(\Sigma y_i)\right]^2}$$

$$\text{where} \quad A = \left[n\Sigma x_i^2 - (\Sigma x_i)^2\right]\left[n\Sigma y_i z_i - (\Sigma y_i)(\Sigma z_i)\right]$$

$$B = \left[n\Sigma x_i y_i - (\Sigma x_i)(\Sigma y_i)\right]\left[n\Sigma x_i z_i - (\Sigma x_i)(\Sigma z_i)\right]$$

$$b = \frac{\left[n\Sigma x_i z_i - (\Sigma x_i)(\Sigma z_i)\right] - c\left[n\Sigma x_i y_i - (\Sigma x_i)(\Sigma y_i)\right]}{n\Sigma x_i^2 - (\Sigma x_i)^2}$$

$$a = \frac{1}{n}(\Sigma z_i - c\,\Sigma y_i - b\,\Sigma x_i)$$

$$R^2 = \frac{a\,\Sigma z_i + b\,\Sigma x_i z_i + c\,\Sigma y_i z_i - \frac{1}{n}(\Sigma z_i)^2}{(\Sigma z_i^2) - \frac{(\Sigma z_i)^2}{n}}$$

Reference: Introduction to the Theory of Statistics, Mood and Graybill, McGraw-Hill, 1963

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	Do 5~6 for i = 1,2, . . . ,n			
5	Input x_i	x_i	ENTER↑	
	y_i	y_i	ENTER↑	
	z_i	z_i	C	i
6	If you made a mistake in			
	inputting x_k, y_k, and z_k,			
	then correct by →	x_k	ENTER↑	
		y_k	ENTER↑	
		z_k	D	i–1
7	Calculate coefficients: a		E	a
	b		R/S	b
	c		R/S	c
8	Calculate the square of			
	multiple correlation			
	coefficient R^2		f A	R^2
9	Calculate estimated z from			
	regression, input: x	x	ENTER↑	
	y	y	f B	$\hat{z}$
10	Repeat step 9 for different			
	(x, y)'s			
11	Recall sums: Σx_i		f C	Σx_i
	Σy_i		R/S	Σy_i
	Σz_i		R/S	Σz_i
12	Recall sums of squares: Σx_i^2		f D	Σx_i^2
	Σy_i^2		R/S	Σy_i^2
	Σz_i^2		R/S	Σz_i^2

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
13	Recall sums of cross			
	products: $\Sigma x_i y_i$		f E	$\Sigma x_i y_i$
	$\Sigma x_i z_i$		R/S	$\Sigma x_i z_i$
	$\Sigma y_i z_i$		R/S	$\Sigma y_i z_i$
14	For a new case, go to 2			
	*Note: to clear print mode			
	press →		CLF	
			0	

Example:

A set of data points are given as the following

i	1	2	3	4
x_i	1.5	0.45	1.8	2.8
y_i	0.7	2.3	1.6	4.5
z_i	2.1	4.0	4.1	9.4

Find the regression line, coefficients a, b, c, R^2, $\hat{z}$, sums, sums of squares, and sums of products.

Keystrokes:	**Outputs:**	
A ——→	0.00	
B ——→	1.00	
1.5 ENTER↑ 0.7 ENTER↑ 2.1 C ——→	1.50 ***	
	0.70 ***	
	2.10 ***	
	1.00 ***	
9 ENTER↑ 9 ENTER↑ 9 C ——→	9.00 ***	
	9.00 ***	(error)
	9.00 ***	
	2.00 ***	

9 ENTER↑ 9 ENTER↑ 9 D ⟶ 9.00 ***
9.00 *** (correction)
9.00 ***
1.00 ***

0.45 ENTER↑ 2.3 ENTER↑ 4 C ⟶ 0.45 ***
2.30 ***
4.00 ***
2.00 ***

1.8 ENTER↑ 1.6 ENTER↑ 4.1 C ⟶ 1.80 ***
1.60 ***
4.10 ***
3.00 ***

2.8 ENTER↑ 4.5 ENTER↑ 9.4 C ⟶ 2.80 ***
4.50 ***
9.40 ***
4.00 ***

E ⟶ -0.10 *** (a)
R/S ⟶ 0.79 *** (b)
R/S ⟶ 1.63 *** (c)

f A ⟶ 1.00 *** (R^2)
DSP 9 PRINTx ⟶ 0.998411259 ***
DSP 2
2 ENTER↑ 3 f B ⟶ 2.00 ***
3.00 ***
6.37 *** ($\hat{z}$)

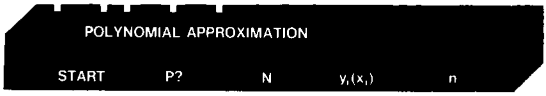
f C ⟶ 6.55 *** (Σx_i)
R/S ⟶ 9.10 *** (Σy_i)
R/S ⟶ 19.60 *** (Σz_i)
f D ⟶ 13.53 *** (Σx_i^2)
R/S ⟶ 28.59 *** (Σy_i^2)
R/S ⟶ 125.58 *** (Σz_i^2)
f E ⟶ 17.57 *** ($\Sigma x_i y_i$)
R/S ⟶ 38.65 *** ($\Sigma x_i z_i$)
R/S ⟶ 59.53 *** ($\Sigma y_i z_i$)

Regression line is

$$z = -0.10 + 0.79x + 1.63y$$

For x =2 and y = 3, $\hat{z} = 6.37$

POLYNOMIAL APPROXIMATION



Suppose $x_0, x_1, \ldots, x_N$ are equally spaced points ($x_0 < x_N$) at which the corresponding values $f(x_0), f(x_1), \ldots, f(x_N)$ of a function $f(x)$ are known.

This program approximates in the least squares sense the function $f(x)$ by a polynomial of degree m, where $2 \leqslant m \leqslant 4$. The special Chebyshev polynomials for discrete intervals are used.

Equations:

Let $f_n(x)$ be the orthogonal polynomials ($x = 0, 1, 2, \ldots, N$) such that

$$f_0(x) = 1$$

$$f_1(x) = 1 - \frac{2x}{N} \text{ and}$$

$$(n+1)(N-n)\, f_{n+1}(x) = (2n+1)(N-2x)\, f_n(x) - n(N+n+1)\, f_{n-1}(x)$$

where

$$n = 1, 2, \ldots, m - 1.$$

Then let

$$(f_n, f_n) = \frac{(N+n+1)!\,(N-n)!}{(2n+1)(N!)^2}$$

$$(f, f_n) = \sum_{j=0}^{n} f_n(j)\, f(x_j)$$

and

$$a_n = \frac{(f, f_n)}{(f_n, f_n)}.$$

This program computes all values of (f, f_n) for $n = 0, 1, 2, 3, 4$. If the degree $m = 4$, all terms are used; if $m = 3$, (f, f_4) is replaced by zero in later calculations; and if $m = 2$, (f, f_4) and (f, f_3) are both replaced by zero.

Let $g_n(u)$ be the symmetrical form of the orthogonal polynomial in the domain $-1 < u < 1$ such that

$$g_0(u) = 1 \qquad g_1(u) = u$$

and

$$g_{n+1}(u) = \frac{(2 + 1)\,N}{(n + 1)\,(N - n)}\, u g_n(u) - \frac{n(N + n + 1)}{(n + 1)\,(N - n)}\, g_{n-1}(u)$$

where

$$n = 1, 2, \ldots, m - 1.$$

The program computes the coefficients of the polynomial

$$\sum_{n=0}^{N} a_n\, g_n(u) = b_0 + b_1 u + b_2 u^2 + b_3 u^3 + b_4 u^4. \qquad (1)$$

Then $g_n(u)$ is shifted to a proper interval between x_0 and x_N by letting

$$u = \beta + \alpha x$$

where

$$\alpha = \frac{-2}{x_N - x_0}$$

$$\beta = \frac{x_N + x_0}{x_N - x_0}$$

The transformation is done in two steps. First, let $z = u - \beta$, thus (1) becomes

$$c_0 + c_1 z + c_2 z^2 + c_3 z^3 + c_4 z^4 \qquad (2)$$

where

$$c_0 = b_0 + b_1 \beta + b_2 \beta^2 + b_3 \beta^3 + b_4 \beta^4$$

$$c_1 = b_1 + 2b_2 \beta + 3b_3 \beta^2 + 4b_4 \beta^3$$

$$c_2 = b_2 + 3b_3 \beta + 6b_4 \beta^2$$

$$c_3 = b_3 + 4 b_4 \beta$$

$$c_4 = b_4.$$

Then set $z = \alpha x$ and (2) becomes

$$d_0 + d_1 x + d_2 x^2 + d_3 x^3 + d_4 x^4 \tag{3}$$

where

$$d_i = \alpha^i c_i \ (i = 0, 1, 2, 3, 4).$$

(3) is the polynomial approximation for the function f(x).

Note: $N \geqslant 4$ has to be satisfied in order to make the program work.

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions,* National Bureau of Standards, 1970.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2 of card 1			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	Input N**	N	C	N
5	Do 6 for i = 0,1,2, . . . , N			
6	Input y_i (x_i)	$y_i (x_i)$	D	i
7	Input n for nth order fit	n	E	0.00
8	Load side 1 and side 2 of card 2			
9	To continue execution of			
	program		f A	1.00

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
10	Input x_N	x_N	ENTER↑	
	and x_0	x_0	f B	
11	To obtain the coefficients: d_i		f C	d_0
			R/S	d_1
			R/S	d_2
			R/S	d_3
			R/S	d_4
12	To evaluate y from the	x	f D	$\hat{y}$
	polynomial			
13	For a new case, go to 1			
	**N = No. of data −1			
	*Note: to clear print mode			
	press →		CLF	
			0	

Example:

Find a third order polynomial approximation for the following data.

x	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3
f(x)	2.72	3.49	4.48	5.75	7.39	9.49	12.18	15.64	20.09

(Note: $f(x) = e^x$)

Keystrokes: **Outputs:**

Load side 1 and side 2 of Card 1

A → 0.00
B → 1.00
8 C → 8.00 *** (N)

2.72 D → 2.72 ***
1.00 ***

3.49 D → 3.49 ***
2.00 ***

Keys	Output	
4.48 **D** →	4.48***	
	3.00 ***	
5.75 **D** →	5.75 ***	
	4.00 ***	
7.39 **D** →	7.39 ***	
	5.00 ***	
9.49 **D** →	9.49 ***	
	6.00 ***	
12.18 **D** →	12.18 ***	
	7.00 ***	
15.64 **D** →	15.64 ***	
	8.00 ***	
20.09 **D** →	20.09 ***	
	9.00 ***	
3 **E** →	3.00 ***	(n)

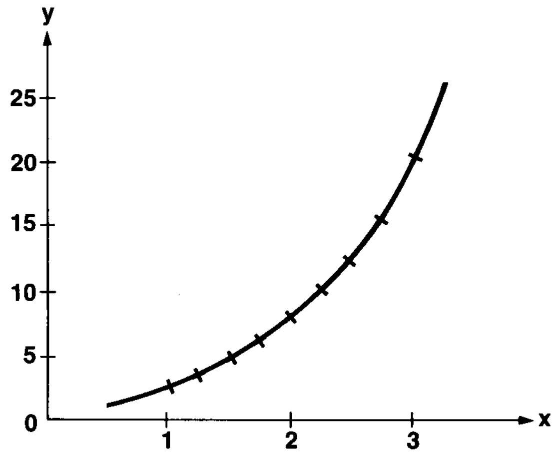
Load side 1 and side 2 of Card 2

Keys	Output	
f **A** →	1.00	
3 **ENTER↑** 1 **f** **B** →	3.00 ***	(x_N)
	1.00 ***	(x_o)
f **C** →	-1.79 ***	(d_0)
R/S →	7.03 ***	(d_1)
R/S →	-3.85 ***	(d_2)
R/S →	1.31 ***	(d_3)
R/S →	0.00 ***	(d_4)
2 **f** **D** →	2.00 ***	
	7.35 ***	($\hat{y}$)

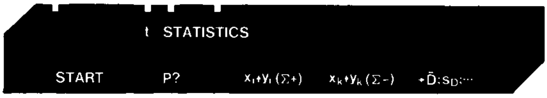
3 [f] [D] ——→ 3.00 ***
20.06 *** ($\hat{y}$)

1 [f] [D] ——→ 1.00 ***
2.69 *** ($\hat{y}$)

The polynomial is $-1.79 + 7.03\,x - 3.85\,x^2 + 1.31\,x^3$.



t STATISTICS



I. Paired t Statistic

Given a set of paired observations from two normal populations with means μ_1, μ_2 (unknown)

x_i	x_1	x_2	...	x_n
y_i	y_1	y_2	...	y_n

let

$$D_i = x_i - y_i$$

$$\bar{D} = \frac{1}{n} \sum_{i=1}^{n} D_i$$

$$s_D = \sqrt{\frac{\Sigma D_i^2 - \frac{1}{n} (\Sigma D_i)^2}{n - 1}}$$

$$s_{\bar{D}} = \frac{s_D}{\sqrt{n}}$$

The test statistic

$$t = \frac{\bar{D}}{s_{\bar{D}}}$$

which has n – 1 degrees of freedom (df) can be used to test the null hypothesis

$$H_0: \mu_1 = \mu_2$$

Reference:

Statistics in Research, B. Ostle, Iowa State University Press, 1963.

II. t Statistic For Two Means

Suppose $\{x_1, x_2, \ldots, x_{n1}\}$ and $\{y_1, y_2, \ldots, y_{n2}\}$ are independent random samples from two normal populations having means μ_1, μ_2 (unknown) and the same unknown variance σ^2.

We want to test the null hypothesis

H_0: $\mu_1 - \mu_2 = d$

Define

$$\bar{x} = \frac{1}{n_1} \sum_{i=1}^{n_1} x_i$$

$$\bar{y} = \frac{1}{n_2} \sum_{i=1}^{n_2} y_i$$

$$t = \frac{\bar{x} - \bar{y} - d}{\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \sqrt{\frac{\Sigma x_i^2 - n_1\bar{x}^2 + \Sigma y_i^2 - n_2\bar{y}^2}{n_1 + n_2 - 2}}}$$

We can use this t statistic which has the t distribution with $n_1 + n_2 - 2$ degrees of freedom (df) to test the null hypothesis H_0.

Note: n_2, Σy_i, Σy_i^2, n_1, Σx_i, Σx_i^2 are in registers R_1 through R_6.

Reference:

Statistical Theory and Methodology in Science and Engineering, K. A. Brownlee, John Wiley & Sons, 1965.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	For paired t statistic, go to 6			
5	For t statistic for two means,			
	go to 11			

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	Paired t statistic:			
6	Do 7~8 for i = 1,2, . . . , n			
7	Input: x_i	x_i	ENTER↑	
	y_i	y_i	C	i
8	If you made a mistake in			
	inputting x_k, y_k then correct by	x_k	ENTER↑	
		y_k	D	i−1
9	Calculate: $\bar{D}$		E	$\bar{D}$
	s_D		R/S	s_D
	test statistic:		R/S	t
	degrees of freedom		R/S	df
10	For a new case, go to 2			
	t statistic for two means:			
11	Do 12 ~ 13 for i = 1,2, . . ., n_1			
12	Input x_i	x_i	f A	i
13	If you made a mistake in			
	inputting x_k, then correct by →	x_k	f B	i−1
14	Null hypothsis test	d	f C	d
15	Do 16~17 for j = 1,2, . . . , n_2	y_j	f A	j
16	If you made a mistake in			
	inputting y_k, then correct by →	y_k	f B	j−1
17	Calculate: t		f D	t
	df		R/S	df
18	For a different value of d	d	f C	d
	Calculate: t		f D	t
	df		R/S	df
19	For a new case, go to 2			
	*Note: to clear print mode			
	press →		CLF	
			0	

Example 1:

x_1	14	17.5	17	17.5	15.4
y_1	17	20.7	21.6	20.9	17.2

$\bar{D}$ = −3.20
s_D = 1.00
t = −7.16
df = 4.00

Keystrokes:	Outputs:	
A	0.00	
B	1.00	
14 ENTER↑ 17 C	14.00 ***	
	17.00 ***	
	1.00 ***	
17 ENTER↑ 15 C	17.00 ***	
	15.00 ***	(error)
	2.00 ***	
17 ENTER↑ 15 D	17.00 ***	
	15.00 ***	(correction)
	1.00 ***	
17.5 ENTER↑ 20.7 C	17.50 ***	
	20.70 ***	
	2.00 ***	
17 ENTER↑ 21.6 C	17.00 ***	
	21.60 ***	
	3.00 ***	
17.5 ENTER↑ 20.9 C	17.50 ***	
	20.90 ***	
	4.00 ***	
15.4 ENTER↑ 17.2 C	15.40 ***	
	17.20 ***	
	5.00 ***	

E ⟶	−3.20 ***	($\bar{D}$)
R/S ⟶	1.00 ***	(s_D)
R/S ⟶	−7.16 ***	(t)
R/S ⟶	4.00 ***	(df)

Example 2:

x: 79, 84, 108, 114, 120, 103, 122, 120
y: 91, 103, 90, 113, 108, 87, 100, 80, 99, 54
$n_1 = 8$
$n_2 = 10$
If $d = 0$ (i.e., H_0: $\mu_1 = \mu_2$)
then $t = 1.73$, $df = 16.00$

Keystrokes:	Outputs:	
A ⟶	0.00	
B ⟶	1.00	
79 f A ⟶	79.00 ***	
	1.00 ***	
84 f A ⟶	84.00 ***	
	2.00 ***	
99 f A ⟶	99.00 ***	
	3.00 ***	(error)
99 f B ⟶	99.00 ***	
	2.00 ***	(correction)
108 f A ⟶	108.00 ***	
	3.00 ***	
114 f A ⟶	114.00 ***	
	4.00 ***	
120 f A ⟶	120.00 ***	
	5.00 ***	
103 f A ⟶	103.00 ***	
	6.00 ***	
122 f A ⟶	122.00 ***	
	7.00 ***	

120 f A ⟶ 120.00 ***
8.00 ***

0 f C ⟶ 0.00 *** (d)

91 f A ⟶ 91.00 ***
1.00 ***

103 f A ⟶ 103.00 ***
2.00 ***

90 f A ⟶ 90.00 ***
3.00 ***

113 f A ⟶ 113.00 ***
4.00 ***

108 f A ⟶ 108.00 ***
5.00 ***

87 f A ⟶ 87.00 ***
6.00 ***

100 f A ⟶ 100.00 ***
7.00 ***

80 f A ⟶ 80.00 ***
8.00 ***

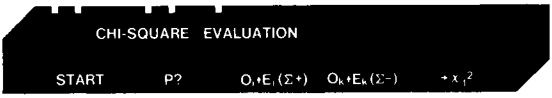
99 f A ⟶ 99.00 ***
9.00 ***

54 f A ⟶ 54.00 ***
10.00 ***

f D ⟶ 1.73 *** (t)

R/S ⟶ 16.00 *** (df)

CHI-SQUARE EVALUATION



This program calculates the value of the χ^2 statistic for the goodness of fit test by the equation

$$\chi_1^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

where O_i = observed frequency
E_i = expected frequency

If the expected values are equal

$$\left(E = E_i = \frac{\Sigma O_i}{n} \text{ for all i} \right)$$

then

$$\chi_2^2 = \frac{n \Sigma O_i^2}{\Sigma O_i} - \Sigma O_i$$

Note: In order to apply the goodness of fit test to a set of given data, combining some classes may be necessary to make sure that each expected frequency is not too small (say, not less than 5).

Reference: Mathematical Statistics, J. E. Freund, Prentice Hall, 1962

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	For equal expected values,			
	go to 10			
5	Do 6~7 for i = 1,2, ... , n			
6	Input: O_i	O_i	ENTER↑	
	E_i	E_i	C	i

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
7	If you made a mistake in			
	inputting O_k and E_k, then			
	correct by →	O_k	ENTER↑	
		E_k	D	i−1
8	Calculate χ_1^2		E	χ_1^2
9	For a new case, go to 2			
10	Do 11~12 for i = 1,2, . . . , n			
	for equal expected values			
11	Input: O_i	O_i	f A	i
12	If you made a mistake in			
	inputting O_h, then correct by →	O_h	f B	i−1
13	Calculate: χ_2^2		f C	χ_2^2
	E		f D	E
14	For a new case, go to 2			
	*Note: to clear print mode			
	press →		CLF	
			0	

Examples 1:

Find the value of χ^2 statistic for the goodness of fit for the following data set:

O_i	8	50	47	56	5	14
E_i	9.6	46.75	51.85	54.4	8.25	9.15

$\chi_1^2 = 4.84$

Keystrokes: **Outputs:**

A ——————→ 0.00
B ——————→ 1.00
8 ENTER↑ 9.6 C ——————→ 8.00 ***
9.60 ***
1.00 ***

50 ENTER↑ 46.75 C →	50.00 *** 46.75 *** 2.00 ***	
47 ENTER↑ 51.85 C →	47.00 *** 51.85 *** 3.00 ***	
56 ENTER↑ 54.4 C →	56.00 *** 54.40 *** 4.00 ***	
5 ENTER↑ 8.25 C →	5.00 *** 8.25 *** 5.00 ***	
100 ENTER↑ 100 C →	100.00 *** 100.00 *** 6.00 ***	(error)
100 ENTER↑ 100 D →	100.00 *** 100.00 *** 5.00 ***	(correction)
14 ENTER↑ 9.15 C →	14.00 *** 9.15 *** 6.00 ***	
E →	4.84 ***	(χ_1^2)

Example 2:

The following table shows the observed frequencies in tossing a die 120 times. χ^2 can be used to test if the die is fair.

Note: Assume that the expected frequencies are equal.

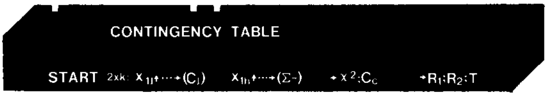
number	1	2	3	4	5	6
frequency O_i	25	17	15	23	24	16

$\chi_2^2 = 5.00$

E = 20.00

Keystrokes:	Outputs:	
A →	0.00	
B →	1.00	
25 f A →	25.00 ***	
	1.00 ***	
17 f A →	17.00 ***	
	2.00 ***	
19 f A →	19.00 ***	(error)
	3.00 ***	
19 f B →	19.00 ***	(correction)
	2.00 ***	
15 f A →	15.00 ***	
	3.00 ***	
23 f A →	23.00 ***	
	4.00 ***	
24 f A →	24.00 ***	
	5.00 ***	
16 f A →	16.00 ***	
	6.00 ***	
f C →	5.00 ***	(χ_2^2)
f D →	20.00 ***	(E)

CONTINGENCY TABLE



Contingency tables can be used to test the null hypothesis that two variables are independent.

I. 2 x k CONTINGENCY TABLE

i \ j	1	2	...	k	Totals
1	x_{11}	x_{12}	...	x_{1k}	R_1
2	x_{21}	x_{22}	...	x_{2k}	R_2
Totals	C_1	C_2	...	C_k	T

Test statistic

$$\chi^2 = \frac{T}{R_1} \sum_{i=1}^{k} \frac{x_{1i}^2}{C_i} + \frac{T}{R_2} \sum_{i=1}^{k} \frac{x_{2i}^2}{C_i} - T$$

Degrees of freedom df = k − 1

Pearson's coefficient of contingency C_c measures the degree of association between the two variables

$$C_c = \sqrt{\frac{\chi^2}{T + \chi^2}}$$

II. 3 x k CONTINGENCY TABLE

i \ j	1	2	...	k	Totals
1	x_{11}	x_{12}	...	x_{1k}	R_1
2	x_{21}	x_{22}	...	x_{2k}	R_2
3	x_{31}	x_{32}	...	x_{3k}	R_3
Totals	C_1	C_2	...	C_k	T

This program computes the χ^2 statistic (with 2(k - 1) degrees of freedom) for testing the independence of the two variables. Also Pearson's coefficient of contingency C_c, which measures the degree of association between the two variables, is calculated.

Equations:

$$\text{Row sum } R_i = \sum_{j=1}^{k} x_{ij} \quad i = 1, 2, 3$$

$$\text{Column sum } C_j = \sum_{i=1}^{3} x_{ij} \quad j = 1, 2, \ldots, k$$

$$\text{Total } T = \sum_{i=1}^{3} \sum_{j=1}^{k} x_{ij}$$

Chi-square statistic

$$\chi^2 = \sum_{i=1}^{3} \sum_{j=1}^{k} \frac{(x_{ij} - E_{ij})^2}{E_{ij}}$$

$$= T \left(\sum_{i=1}^{3} \sum_{j=1}^{k} \frac{x_{ij}^2}{R_i C_j} \right) - T$$

Where the expected frequency

$$E_{ij} = \frac{R_i C_j}{T}$$

Contingency coefficient

$$C_c = \sqrt{\frac{\chi^2}{T + \chi^2}}$$

Reference:

B. Ostle, *Statistics in Research,* Iowa State University Press, 1972.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		f A	1.00
4	For 2 x k go to 5			
	For 3 x k go to 11			
5	Do 6~7 for j = 1,2, . . . , k			
	for 2 × k			
6	Input: x_{1j}	x_{1j}	ENTER↑	
	x_{2j}	x_{2j}	B	j
	(Optional) Calculate column			
	sum C_j		R/S	C_j
7	If you made a mistake in			
	inputting x_{1h} and x_{2h}, then			
	correct by →	x_{1h}	ENTER↑	
		x_{2h}	C	j-1
	(Optional) Calculate column			
	sum C_h (correction)		R/S	$-C_h$
8	Calculate: χ^2		D	χ^2
	C_c		R/S	C_c
9	Calculate: row sums R_1		E	R_1
	R_2		R/S	R_2
	total T		R/S	T
10	For a new case go to 2			
11	Do 12~13 for j =1,2, . . . , k			
	for 3 X k			
12	Input x_{1j}	x_{1j}	ENTER↑	
	x_{2j}	x_{2j}	ENTER↑	
	x_{3j}	x_{3j}	f B	j
	(Optional) Calculate column			
	sum C_j		R/S	C_j

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
13	If you made a mistake in			
	inputting X_{1h} , X_{2h} and X_{3h},			
	then correct by →	x_{1h}	ENTER↑	
		x_{2h}	ENTER↑	
		x_{3h}	f C	j-1
	(Optional) Calculate column			
	sum C_h (correction)		R/S	$-C_h$
14	Calculate: χ^2		f D	χ^2
	C_c		R/S	C_c
15	Calculate: row sums R_1		f E	R_1
	R_2		R/S	R_2
	R_3		R/S	R_3
	total T		R/S	T
16	For a new case go to 2			
	*Note: to clear print mode			
	press →		CLF	
			0	

Example 1:

A random sample of 250 men and 250 women were polled as to their desires concerning the ownership of television sets. The data in the following table resulted. Apply the program to analyze the result of the poll, i.e., can the hypothesis that the desire to own a television set is independent of sex be rejected?

Results of Sample Poll on Television Ownership

Classification	Men	Women	Total
Want television	80	120	200
Don't want television	170	130	300
Total	250	250	500

Keystrokes:	**Outputs:**	
A	0.00	
f A	1.00	
80 ENTER↑ 170 B	80.00 ***	
	170.00 ***	
	1.00 ***	
120 ENTER↑ 130 B	120.00 ***	
	130.00 ***	
	2.00 ***	
D	13.33 ***	(χ^2)

$$\chi^2 = 13.33 > \chi^2_{.99(1)} = 6.63$$

Thus, the hypothesis that desire to own a television set is independent of sex is rejected.

Example 2:

Find test statistic χ^2 and coefficient of contingency C_c for the following set of data.

	1	2	3
A	2	5	4
B	3	8	7

Keystrokes:	**Outputs:**	
A	0.00	
f A	1.00	
2 ENTER↑ 3 B	2.00 ***	
	3.00 ***	
	1.00 ***	
R/S	5.00 ***	(C_1)
5 ENTER↑ 8 B	5.00 ***	
	8.00 ***	
	2.00 ***	

R/S	13.00 ***	(C_2)
6 ENTER↑ 9 B	6.00 ***	(error)
	9.00 ***	
	3.00 ***	
R/S	15.00 ***	(C_3)
6 ENTER↑ 9 C	6.00 ***	(correction)
	9.00 ***	
	2.00 ***	
R/S	–15.00 ***	($-C_3$)
4 ENTER↑ 7 B	4.00 ***	
	7.00 ***	
	3.00 ***	
R/S	11.00 ***	(C_3)
D	0.02 ***	(χ^2)
R/S	0.03 ***	(C_c)
E	11.00 ***	(R_1)
R/S	18.00 ***	(R_2)
R/S	29.00 ***	(T)

Example 3:

Find test statistic χ^2 and coefficient of contingency C_c for the following set of data.

i \ j	1	2	3	4
1	36	67	49	58
2	31	60	49	54
3	58	87	80	68

Keystrokes:	**Outputs:**	
A	0.00	
f A	1.00	
36 ENTER↑ 31 ENTER↑ 58 f B	36.00 ***	
	31.00 ***	
	58.00 ***	
	1.00 ***	
R/S	125.00 ***	(C_1)
67 ENTER↑ 60 ENTER↑ 87 f B	67.00 ***	
	60.00 ***	
	87.00 ***	
	2.00 ***	
R/S	214.00 ***	(C_2)
4 ENTER↑ 49 ENTER↑ 80 f B	4.00 ***	(error)
	49.00 ***	
	80.00 ***	
	3.00 ***	
R/S	133.00 ***	(C_3)
49 ENTER↑ 49 ENTER↑ 80 f B	49.00 ***	
	49.00 ***	
	80.00 ***	
	4.00 ***	
R/S	178.00 ***	(C_4)
4 ENTER↑ 49 ENTER↑ 80 f C	4.00 ***	(correction)
	49.00 ***	
	80.00 ***	
	3.00 ***	
R/S	–133.00 ***	($-C_3$)

58 ENTER↑ 54 ENTER↑ 68 f B →	58.00 ***	
	54.00 ***	
	68.00 ***	
	4.00 ***	
R/S →	180.00 ***	(C_4)
f D →	3.36 ***	(χ^2)
R/S →	0.07 ***	(C_c)
f E →	210.00 ***	(R_1)
R/S →	194.00 ***	(R_2)
R/S →	293.00 ***	(R_3)
R/S →	697.00 ***	(T)

SPEARMAN'S RANK CORRELATION COEFFICIENT

Spearman's rank correlation coefficient is a measure of rank correlation under the following circumstance: n individuals are ranked from 1 to n according to some specified characteristic by 2 observers, and we wish to know if the 2 rankings are substantially in agreement with one another.

Spearman's rank correlation coefficient is defined by

$$r_s = 1 - \frac{6\sum_{i=1}^{n} D_i^2}{n(n^2 - 1)}$$

where n = number of paired observations (x_i, y_i)
D_i = rank (x_i) - rank (y_i) = $R_i - S_i$

If the X and Y random variables from which these n pairs of observations are derived are independent, then r_s has zero mean and a variance equals to

$$\frac{1}{n - 1}$$

A test for the null hypothesis

$$H_0\text{: X, Y are independent}$$

is using

$$z = r_s\sqrt{n - 1}$$

which is approximately a standardized normal variable (for large n, say $n \geqslant 10$).

If the null hypothesis of independence is not rejected, we can infer that the population correlation coefficient $\rho(x, y) = 0$, but dependence between the variables does not necessarily imply that $\rho(x, y) \neq 0$.

Note: $-1 \leqslant r_s \leqslant 1$

$r_s = 1$ indicates complete agreement in order of the ranks and $r_s = -1$ indicates complete agreement in the opposite order of the ranks.

Reference: Nonparametric Statistical Inference, J. D. Gibbons, McGraw Hill, 1971

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	Do 5~6 for i=1, 2, - - - - , n			
5	Input R_i	R_i	ENTER↑	
	S_i	S_i	C	i
6	If you made a mistake in			
	inputting R_k and S_k, then			
	correct by →	R_k	ENTER↑	
		S_k	D	i−1
7	Calculate: r_s		E	r_s
	z		R/S	z
8	For a new case, go to 2			
	*Note: to clear print mode			
	press →		CLF	
			0	

Example:

The following data set is the result of two tests in a class; find r_s and z.

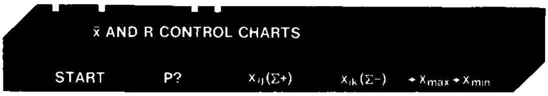
Student	x_i Math Grade	y_i Stat Grade	R_i Rank of x_i	S_i Rank of y_i
1	82	81	6	7
2	67	75	14	11
3	91	85	3	4
4	98	90	1	2
5	74	80	11	8
6	52	60	15	15
7	86	94	4	1
8	95	78	2	9
9	79	83	9	6
10	78	76	10	10
11	84	84	5	5
12	80	69	8	13
13	69	72	13	12
14	81	88	7	3
15	73	61	12	14

Keystrokes:	Outputs:
A	0.00
B	1.00
6 ENTER↑ 7 C	6.00 ***
	7.00 ***
	1.00 ***
14 ENTER↑ 11 C	14.00 ***
	11.00 ***
	2.00 ***
3 ENTER↑ 4 C	3.00 ***
	4.00 ***
	3.00 ***
1 ENTER↑ 2 C	1.00 ***
	2.00 ***
	4.00 ***

11 ENTER↑ 8 C →	11.00 *** 8.00 *** 5.00 ***	
5 ENTER↑ 5 C →	5.00 *** 5.00 *** 6.00 ***	(errors)
5 ENTER↑ 5 D →	5.00 *** 5.00 *** 5.00 ***	(correction)
15 ENTER↑ 15 C →	15.00 *** 15.00 *** 6.00 ***	
4 ENTER↑ 1 C →	4.00 *** 1.00 *** 7.00 ***	
2 ENTER↑ 9 C →	2.00 *** 9.00 *** 8.00 ***	
9 ENTER↑ 6 C →	9.00 *** 6.00 *** 9.00 ***	
10 ENTER↑ 10 C →	10.00 *** 10.00 *** 10.00 ***	
5 ENTER↑ 5 C →	5.00 *** 5.00 *** 11.00 ***	
8 ENTER↑ 13 C →	8.00 *** 13.00 *** 12.00 ***	

13 ENTER↑ 12 C	→	13.00 ***
		12.00 ***
		13.00 ***
7 ENTER↑ 3 C	→	7.00 ***
		3.00 ***
		14.00 ***
12 ENTER↑ 14 C	→	12.00 ***
		14.00 ***
		15.00 ***
E	→	0.76 *** (r_s)
R/S	→	2.85 *** (z)

$\bar{x}$ AND R CONTROL CHARTS



In quality control, a chart is used to decide periodically whether a process is in statistical control. The use of such a chart facilitates the detection and elimination of assignable causes of process variation, thereby reducing rejects and rework, improving product quality, and lowering inspection cost. The x chart and R chart are two of the most frequently encountered, they deal with measurement data.

Suppose x_{ij} represents the j^{th} data point from the i^{th} sample, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. This program computes (1) the sample mean $\bar{x}$ and the sample range R_i, (2) the over-all mean $\bar{\bar{x}}$ and the average range $\bar{R}$, (3) the upper control limit $U_{\bar{x}}$ and the lower control limit $L_{\bar{x}}$ for $\bar{x}$, and (4) the upper control limit U_R and the lower control limit L_R for R.

Equations:

1.

$$\bar{x}_i = \sum_{j=1}^{n} x_{ij}/n$$

$$R_i = x_{max} - x_{min}$$

where x_{max} is the maximum of the x values and x_{min} is the minimum of the x values in the i^{th} sample.

2.

$$\bar{\bar{x}} = \sum_{i=1}^{m} \bar{x}_i/m$$

$$\bar{R} = \sum_{i=1}^{m} R_i/m$$

3.

$$L_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R}$$

$$U_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R}$$

where A_2 is the factor for the $\bar{x}$ chart, which can be found in the following table.

4.

$$L_R = D_3\bar{R}$$

$$U_R = D_4\bar{R}$$

D_3 and D_4 are factors for the R chart, which can be found in the table.

Factors for determining from R the 3-sigma control limits for x and R charts.

Number of observations in subgroup n	Factor for x chart A_2	Factors for R chart	
		Lower limit D_3	Upper limit D_4
2	1.88	0	3.27
3	1.02	0	2.57
4	0.73	0	2.28
5	0.58	0	2.11
6	0.48	0	2.00
7	0.42	0.08	1.92
8	0.37	0.14	1.86
9	0.34	0.18	1.82
10	0.31	0.22	1.78
11	0.29	0.26	1.74
12	0.27	0.28	1.72
13	0.25	0.31	1.69
14	0.24	0.33	1.67
15	0.22	0.35	1.65
16	0.21	0.36	1.64
17	0.20	0.38	1.62
18	0.19	0.39	1.61
19	0.19	0.40	1.60
20	0.18	0.41	1.59

All factors are based on the normal distribution.

The table is reproduced from *Statistical Quality Control,* by Grant and Leavenworth, 1972, with permission of McGraw-Hill Book Company.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	Do 5~ 9 for i=1, 2, . . . , m			
5	Do 6~7 for j=1, 2, . . . , n			
6	Input x_{ij}	x_{ij}	C	j
7	If you made a mistake in			
	inputting x_{ik}, then correct			
	by ** →	x_{ik}	D	j–1
8	Calculate: x_{max}		E	x_{imax}
	x_{min}		E	x_{imin}
9	Calculate: the mean $\bar{x}_i$		f A	$\bar{x}_i$
	the range R_i		f A	R_i
10	Calculate: $\bar{\bar{x}}$		f B	$\bar{\bar{x}}$
	$\bar{R}$		f B	$\bar{R}$
11	Calculate the $\bar{x}$ limits:			
	the lower	A_2	f C	$L_{\bar{x}}$
	the upper		f C	$U_{\bar{x}}$
12	Calculate L_R	D_3	f D	L_R
13	Calculate U_R	D_4	f E	U_R
14	For a new case, go to 2			
	*Note: to clear print mode			
	press →		0	
			STO	
			E	
	**Note: If there are two or more			
	x_{ik}'s entered incorrectly (one			
	follows the other), then do not			
	try to correct them, go to step 2.			

Example:

For the following set of data, find the lower and upper control limits for $\bar{x}$ and R.

	i \ j	1	2	3	4	5
	1	10.04	10.00	10.02	10.01	10.02
Sample	2	10.00	10.01	10.03	10.02	10.01
	3	10.02	10.02	10.02	10.04	10.01

(Note: $n = 5, A_2 = 0.58, D_3 = 0, D_4 = 2.11$)

Keystrokes:	**Outputs:**	
A	0.00	
B	1.00	
10.04 C	10.04 ***	
	1.00 ***	
10 C	10.00 ***	
	2.00 ***	
10.02 C	10.02 ***	
	3.00 ***	
11.11 C	11.11 ***	(error)
	4.00 ***	
11.11 D	11.11 ***	(correction)
	3.00 ***	
10.01 C	10.01 ***	
	4.00 ***	
10.02 C	10.02 ***	
	5.00 ***	
E	10.04 ***	($x_{1\ max}$)
E	10.00 ***	($x_{1\ min}$)
f A	10.02 ***	($\bar{x}_1$)
f A	0.04 ***	(R_1)
10 C	10.00 ***	
	1.00 ***	
10.01 C	10.01 ***	
	2.00 ***	
10.03 C	10.03 ***	
	3.00 ***	
10.02 C	10.02 ***	
	4.00 ***	

Keystrokes	Output	
10.01 [C]	10.01 ***	
	5.00 ***	
[E]	10.03 ***	($x_{2\ max}$)
[E]	10.00 ***	($x_{2\ min}$)
[A]	10.01 ***	($\bar{x}_2$)
[A]	0.03 ***	(R_2)
10.02 [C]	10.02 ***	
	1.00 ***	
10.02 [C]	10.02 ***	
	2.00 ***	
10.04 [C]	10.04 ***	(error)
	3.00 ***	
10.04 [D]	10.04 ***	(correction)
	2.00 ***	
10.02 [C]	10.02 ***	
	3.00 ***	
10.04 [C]	10.04 ***	
	4.00 ***	
10.01 [C]	10.01 ***	
	5.00 ***	
[E]	10.04 ***	($x_{3\ max}$)
[E]	10.01 ***	($x_{3\ min}$)
[f] [A]	10.02 ***	($\bar{x}_3$)
[f] [A]	0.03 ***	(R_3)
[f] [B]	10.02 ***	($\bar{\bar{x}}$)
[f] [B]	0.03 ***	($\bar{R}$)
0.58 [f] [C]	10.00 ***	($L_{\bar{x}}$)
[f] [C]	10.04 ***	($U_{\bar{x}}$)
0 [f] [D]	0.00 ***	(L_R)
2.11 [f] [E]	0.07 ***	(U_R)

Reference:

Grant and Leavenworth, *Statistical Quality Control,* McGraw-Hill, 1972

OPERATING CHARACTERISTIC CURVES

This program evaluates the probability P_a of acceptance for a single sampling plan with finite or infinite lot size.

Equations:

1. Finite lot size

The hypergeometric distribution is used to evaluate the probability P_a. The lot size N, sample size n and the acceptance number c (maximum allowable number of defectives in the sample) should be given. The probability P_a, which is the ordinate of the type A operating characteristic curve, can be computed for the different values of the fraction defective p in the lot.

$$P_a = \sum_{x=0}^{c} f(x)$$

$$f(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$

where f(x) is the hypergeometric density function, M is the number of defectives in a lot which is calculated as the integer part of Np.

The recursive relation

$$f(x+1) = \frac{(x-M)(x-n)}{(x+1)(N-M-n+x+1)} f(x)$$

$$(x = 0, 1, 2, \ldots, n-1)$$

is used to find the probability

$$P_a = \sum_{x=0}^{c} f(x)$$

with starting value

$$f(0) = \frac{\binom{N-M}{n}}{\binom{N}{n}}$$

The binomial coefficient $\binom{N}{n}$ is computed by the formula

$$\binom{N}{n} = \frac{N(N-1)\ \dots\ (N-n+1)}{1 \cdot 2 \cdot \dots \cdot n}$$

2. Infinite lot size

The binomial distribution is used to evaluate the probability P_a. The sample size n and the acceptance number c should be given. The probability P_a, which is the ordinate of the type B operating characteristic curve, can be computed for different values of the fraction defective p.

$$P_a = \sum_{x=0}^{c} f(x)$$

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

where $0 \leq p < 1$.

The recursive relation

$$f(x+1) = \frac{p(n-x)}{(x+1)(1-p)} f(x)$$

$$(x = 0, 1, 2, \dots, n-1)$$

is used to find the probability

$$P_a = \sum_{x=0}^{c} f(x)$$

with starting value

$$f(0) = (1-p)^n$$

Remarks:

1. The program requires that $0 \leq p < 1$.
2. For the type A curve (finite lot size), if $c = 0$, $P_a = f(0)$.

3. For certain combinations of N, n ,and c (usually when they are large), an overflow condition will occur. In that case, the program halts and the display shows all 9's.
4. If the lot size is finite (type A), the execution time mainly depends on the sample size n and the acceptance number c; the larger they are, the longer it takes.
5. The type A OC curve for finite lot sizes is really a set of discrete points, since defectives can occur only as whole numbers. For very large lot sizes, these points come very close together, giving a practically continuous curve.

   Type B OC curves can be considered as suitable approximations to type A OC curves, provided the sample size n is small compared with the lot size N (in general, if $n/N \leqslant 0.1$).
6. The lot size N has a relatively small effect on the type A OC curve as long as n/N is not large. The absolute sample size n is a much more controlling factor in determining the type A OC curve.

7. The acceptance number *c* affects drastically the probability of acceptance for the type *B OC* curve for any given fraction defective *p*.

References:

1. Dodge and Romig, *Sampling Inspection Tables,* John Wiley and Sons, 1959.
2. Grant and Leavenworth, *Statistical Quality Control,* McGraw-Hill, 1972.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	Initialize		A	0.00
3	To set print mode*		B	1.00
4	For infinite lot size (type B),			
	go to 11			
5	For finite lot size (type A), do 6~ 9			
6	Input lot size	N	C	N
7	Input: sample size	n	ENTER↑	n
	acceptance number	c	D	c
8	Calculate probability P_a	p	E	P_a

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
9	For a different p, go to 8			
10	For a new case, go to 2			
11	Input: sample size	n	ENTER↑	n
	acceptance number	c	f D	c
12	Calculate probability P_a	p	f E	P_a
13	For a different p, go to 12			
14	For a new case, go to 2			
	*Note: to clear print mode			
	press →		CLF	
			0	

Example 1:

Find the type A OC curve for the sampling plan with N = 200, n = 20 and c = 0 (compute P_a for p = 0, 0.02, 0.04, 0.06, 0.08, 0.1, 0.12, 0.14).

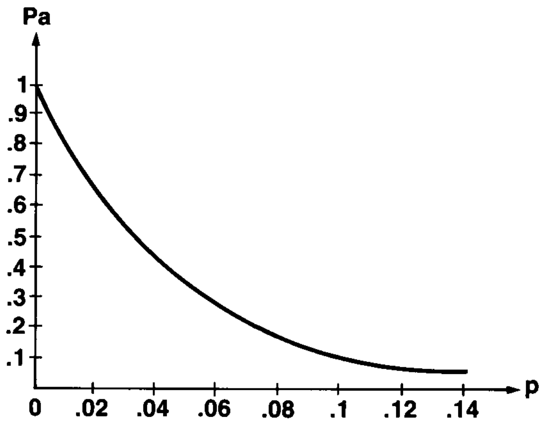
Keystrokes:	Outputs:	
A →	0.00	
B →	1.00	
200 C →	200.00 ***	(N)
20 ENTER↑ 0 D →	20.00 ***	(n)
	0.00 ***	(c)
0 E →	0.00 ***	
	1.00 ***	
0.02 E →	0.02 ***	
	0.65 ***	
0.04 E →	0.04 ***	
	0.42 ***	
0.06 E →	0.06 ***	
	0.27 ***	

0.08 **E** ⟶ 0.08 ***
0.17 ***

0.1 **E** ⟶ 0.10 ***
0.11 ***

0.12 **E** ⟶ 0.12 ***
0.07 ***

0.14 **E** ⟶ 0.14 ***
0.04 ***



Example 2:

Find the type B OC curve for the sampling plan with n = 200, c = 1 (compute P_a for p = 0, 0.01, 0.02, 0.03 and 0.04).

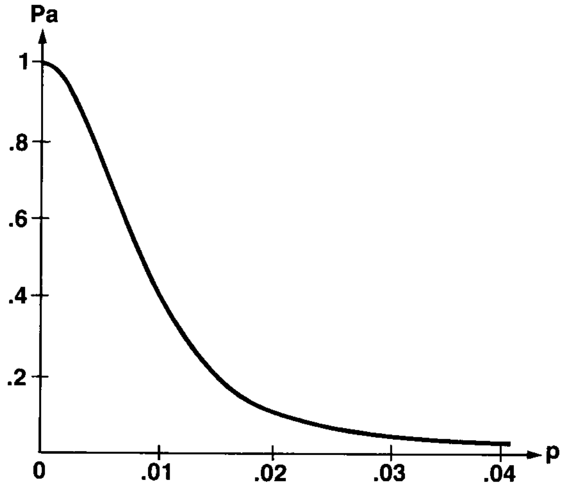
Keystrokes:	Outputs:	
A ⟶	0.00	
B ⟶	1.00	
200 **ENTER↑** 1 **f** **D** ⟶	200.00 ***	(n)
	1.00 ***	(c)
0 **f** **E** ⟶	0.00 ***	
	1.00 ***	

0.01 [f] [E] → 0.01 ***
0.40 ***

0.02 [f] [E] → 0.02 ***
0.09 ***

0.03 [f] [E] → 0.03 ***
0.02 ***

0.04 [f] [E] → 0.04 ***
2.656338303–03 ***



SINGLE- AND MULTI-SERVER QUEUES

I. Infinite Customers

Suppose there are n ($n \geq 1$) identical stations available to service calls from an infinite number of customers. Let λ be the arrival rate of customers (Poisson input), μ be the service rate of each server (exponential service), and let the service discipline be first-come, first-served. Assume all customers wait in a single line and are directed to whichever station is available. Assume further that, no customers are lost from the queue.

This program computes the following values for given n, λ and μ.

Equations:

1. The intensity factor

$$\rho = \frac{\lambda}{\mu}$$

(ρ must be less than n)

2. The probability that all servers are idle

$$P_0 = \left[\sum_{k=0}^{n-1} \frac{\rho^k}{k!} + \frac{\rho^n}{n!\left(1 - \frac{\rho}{n}\right)} \right]^{-1}$$

3. The probability that all servers are busy

$$P_b = \frac{\rho^n P_0}{n!\left(1 - \frac{\rho}{n}\right)}$$

4. The average number of customers in the queue

$$L_q = \frac{\rho P_b}{n-\rho}$$

5. The average number of customers in the system (waiting or being served)

$$L = L_q + \rho$$

6. The average waiting time in the queue

$$T_q = \frac{L_q}{\lambda}$$

7. The average flow time through the system

$$T = \frac{L}{\lambda}$$

8. The probability of waiting longer than time t

$$P(t) = P_b \, e^{-(n\mu - \lambda)\, t}$$

Remarks:

1. n must be an integer greater than or equal to 1.
2. $\rho < n$, otherwise the queue increases without bound.
3. λ and μ are rates, that is, numbers per unit time.

II. Finite Customers

Suppose there are n ($n \geqslant 1$) identical stations available to service calls. This program handles the case in which demand arises from a finite rather than an infinite population of customers.

Let the number of customers m be fixed; let a be the mean time between service calls; and s be the mean time to serve one customer. Given m, n, s and a, this program computes the following values.

Equations:

1. The average number of customers in the system (waiting or being served)

$$L = \frac{\sum_{k=0}^{m} k\, Q_k}{\sum_{k=0}^{m} Q_k}$$

where

$$Q_0 = 1$$

$$(m - k + 1)\,\rho\, Q_{k-1} = \begin{cases} kQ_k & \text{if } 1 \leq k \leq n \\ nQ_k & \text{if } n < k \leq m \end{cases}$$

and

$$\rho = \frac{s}{a}$$

2. The average flow time through the system

$$T = aL$$

3. The average number of customers in the queue

$$L_q = m\left[(\rho + 1)\left(\frac{L}{M} - 1\right) + 1\right]$$

4. The average waiting time in the queue

$$T_q = aL_q$$

5. The over-all efficiency factor of the system

$$F = -(\rho + 1)\left(\frac{L}{m} - 1\right)$$

Remarks:

1. For large values of m and/or small values of ρ, the calculation of Q_k in the routine under f C may underflow. To avoid this, the program tests to see if $Q_k < 10^{-90}$. If it does, the program will halt its recursive solution for Q_k and go directly to the calculation of L. This should not affect the calculated value of L.

2. For certain combinations of m, n, s and a, an overflow condition will occur. In that case, the program halts and the display shows all 9's.

3. The execution time for L depends on m; the larger m is, the longer it takes. A rough estimate of the time for this routine (f C) is given by m/30 minutes.

4. Suppose instead of knowing s and a, the service rate μ of each server and the arrival rate λ are given. Then the following formulas can be used to compute s and a in order to run this program.

$$s = \frac{1}{\mu}$$

$$a = \frac{1}{\lambda}$$

Note that $$\rho = \frac{\lambda}{\mu}$$

References:

1. H. M. Wagner, *Principles of Operations Research with Applications to Managerial Decisions,* Prentice-Hall, 1969.

2. James Martin, *Systems Analysis for Data Transmission,* Prentice-Hall, 1972.

3. Hillier and Lieberman, *Introduction to Operations Research,* Holden-Day, 1970.

4. Peck and Hazelwood, *Finite Queuing Tables,* John Wiley and Sons, 1958.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2			
2	For finite customers go to 11			
3	Do 4 ~ 9 for infinite customers			
4	Input μ	μ	ENTER↑	
	λ	λ	ENTER↑	
	n	n	A	ρ
5	Calculate P_o		B	P_o
	P_b		B	P_b
6	Calculate: L_q		C	L_q
	L		C	L

7	Calculate: T_q		D	T_q
	T		D	T
8	Input t to calculate P(t)	t	E	P(t)
9	For a different t, go to 8			
10	For a new case, go to 2			
11	Do 12 ~ 16 for finite customers			
12	Input: number of customers	m	ENTER↑	
	number of servers	n	f A	m
13	Input: service time	s	ENTER↑	
	arrival time	a	f B	ρ
14	Calculate: customers in system		f C	L
	time through system		f C	T
15	Calculate: queue length		f D	L_q
	waiting time in queue		f D	T_q
16	Calculate efficiency factor F		f E	F
17	For a new case, go to 2			

Example 1:

Bank customers arrive at a bank on the average of 1.2 customers per minute. They join a common queue for 3 tellers, each teller serves at a rate of 30 customers per hour. Find ρ, P_0, P_b, L_q, L, T_q, T and the probability P(2) that a customer will have to wait for more than 2 minutes.

(Note: Service rate $\mu = \frac{30}{60} =$ 0.5 customers per minute
Arrival rate λ = 1.2 customers per minute)

Keystrokes: .5 ENTER↑ 1.2 ENTER↑ 3 A ⟶

Outputs:

0.5 *** (μ)
1.20 *** (λ)
3.00 *** (n)
2.40 *** (ρ)

Keystrokes		Outputs	
B	→	0.06 ***	(P_0)
B	→	0.65 ***	(P_b)
C	→	2.59 ***	(L_q)
C	→	4.99 ***	(L)
D	→	2.16 ***	(T_q)
D	→	4.16 ***	(T)
2 E	→	2.00 ***	(t)
		0.36 ***	(P(t))

Example 2:

A laundromat has 12 washers which require an average of 4 hours of service after every 60 hours of operation. If there is only one service person in the laundromat, find ρ, L, T, L_q, T_q and F.

Keystrokes:		**Outputs:**	
12 ENTER↑ 1 f A	→	12.00 ***	(m)
		1.00 ***	(n)
4 ENTER↑ 60 f B	→	4.00 ***	(s)
		60.00 ***	(a)
		0.07 ***	(ρ)
f C	→	1.64 ***	(L)
f C	→	98.66 ***	(T)
f D	→	0.95 ***	(L_q)
f D	→	57.24 ***	(T_q)
f E	→	0.92 ***	(F)

PROGRAM LISTINGS

The following listings are included for your reference. A table of keycodes and keystrokes corresponding to the symbols used in the listings can be found in Appendix E of your Owners Handbook.

Program **Page**

1. Basic Statistics for Two Variables **L01-01**
2. Factorial, Permutation and Combination **L02-01**
3. Moments, Skewness and Kurtosis (For Grouped or Ungrouped Data) **L03-01**
4. Random Number Generator **L04-01**
5. Histogram **L05-01**
6. Analysis of Variance (One Way) **L06-01**
7. Two Way Analysis of Variance **L07-01**
8. Analysis of Covariance (One Way) **L08-01**
   Card 1
   Card 2
9. Normal and Inverse Normal Distribution **L09-01**
   Card 1
   Card 2
10. Chi-Square Distribution **L10-01**
11. t Distribution **L11-01**
12. F Distribution **L12-01**
13. Multiple Linear Regression **L13-01**
14. Polynomial Approximation **L14-01**
    Card 1
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15. t Statistics **L15-01**
16. Chi-Square Evaluation **L16-01**
17. Contingency Table **L17-01**
18. Spearman's Rank Correlation Coefficient **L18-01**
19. $\bar{x}$ and R Control Chart **L19-01**
20. Operating Characteristic Curves **L20-01**
21. Single- and Multi-Server Queues **L21-01**

Basic Statistics for Two Variables

001	*LBLA		057	STO9	
002	CLRG		058	P⇄S	
003	CF0	Initialize	059	GSB9	
004	CF1		060	GSB8	
005	CF2		061	RTN	
006	P⇄S		062	*LBLE	
007	CLRG		063	SF1	Correction for x_k, y_k, f_k.
008	P⇄S		064	GSBD	
009	0		065	CF1	
010	RTN		066	RTN	
011	*LBLa	Set flag 0 for print.	067	*LBL3	
012	SF0		068	F1?	
013	1		069	GTO4	
014	RTN		070	Σ+	
015	*LBLC	Correction for x_k, y_k.	071	RTN	
016	F0?		072	*LBL4	
017	GSB0		073	Σ-	
018	SF1		074	RTN	
019	X⇄Y		075	*LBLb	
020	Σ-		076	x̄	$\bar{x}$, $\bar{y}$
021	GSB9		077	GSB9	
022	GSB8		078	R/S	
023	CF1		079	X⇄Y	
024	RTN		080	GSB9	
025	*LBLB		081	GSB8	
026	F0?	Input x_i, y_i.	082	RTN	
027	GSB0		083	*LBLc	
028	X⇄Y		084	x̄	
029	Σ+		085	STO0	
030	GSB9		086	X⇄Y	
031	GSB8		087	P⇄S	
032	RTN		088	STO0	
033	*LBLD	Input x_i, y_i, f_i.	089	P⇄S	V_x, V_y
034	STOC		090	S	
035	F1?		091	EEX	
036	CHS		092	2	
037	ST+9		093	[illegible]	
038	R↓		094	X⇄Y	
039	STOB		095	LSTX	
040	R↓		096	×	
041	STOA		097	X⇄Y	
042	R↑		098	RCL0	
043	F0?		099	÷	
044	GSB0		100	GSB9	
045	R↑		101	R/S	
046	ABS		102	X⇄Y	
047	GSB9		103	P⇄S	
048	STOI		104	RCL0	
049	*LBL2		105	P⇄S	
050	RCLB		106	÷	
051	RCLA		107	GSB9	
052	GSB3		108	GSB8	
053	DSZI		109	RTN	
054	GTO2		110	*LBLd	
055	RCL9		111	S	
056	P⇄S		112	GSB9	

REGISTERS

0 $\bar{x}$	1	2	3	4	5	6	7	8	9 Σf_i
S0 $\bar{y}$	S1	S2	S3	S4 Σx_i	S5 Σx_i^2	S6 Σy_i	S7 Σy_i^2	S8 $\Sigma x_i y_i$	S9 n

A x_i	B y_i	C f_i	D	E	I

Step	Key	Comment
113	R/S	
114	X⇄Y	s_x, s_y
115	GSB9	
116	GSB8	
117	RTN	
118	*LBLc	
119	S	
120	*LBL1	
121	P⇄S	s_x', s_y'
122	RCL9	
123	P⇄S	
124	ENT↑	
125	X⇄Y	
126	1	
127	-	
128	÷	
129	√X	
130	÷	
131	GSB9	
132	F2?	
133	GSB8	
134	CF2	
135	R/S	
136	LSTX	
137	S	
138	X⇄Y	
139	SF2	
140	GTO1	
141	RTN	
142	*LBLd	
143	x̄	
144	X⇄Y	
145	P⇄S	
146	STO0	
147	RCL8	
148	RCL4	
149	RCL0	
150	×	s_{xy}, s_{xy}'
151	-	
152	RCL9	
153	1	
154	-	
155	÷	
156	P⇄S	
157	STOE	
158	GSB9	
159	R/S	
160	P⇄S	
161	RCL9	
162	P⇄S	
163	ENT↑	
164	X⇄Y	
165	1	
166	-	
167	÷	
168	÷	
169	GSB9	
170	RTN	
171	*LBLd	
172	S	
173	RCLE	
174	÷	
175	×	γ_{xy}
176	1/X	
177	GSB9	
178	GSB8	
179	RTN	
180	*LBLe	
181	RCLΣ	
182	GSB9	$\Sigma x_i, \Sigma y_i$
183	R/S	
184	X⇄Y	$\Sigma x_i y_i$
185	GSB9	
186	R/S	
187	P⇄S	
188	RCL8	
189	P⇄S	
190	GSB9	
191	GSB8	
192	RTN	
193	*LBLe	
194	P⇄S	
195	RCL7	
196	RCL5	$\Sigma x_i^2, \Sigma y_i^2$
197	P⇄S	
198	GSB9	
199	R/S	
200	X⇄Y	
201	GSB9	
202	GSB8	
203	RTN	
204	*LBL0	
205	X⇄Y	
206	PRTX	Print x_i, y_i.
207	X⇄Y	
208	PRTX	
209	RTN	
210	*LBL9	
211	F0?	Subroutine for print.
212	PRTX	
213	RTN	
214	*LBL8	
215	F0?	Subroutine for space.
216	SPC	
217	RTN	

LABELS

A Start	B $x_i \uparrow y_i$ (Σ+)	C $x_k \uparrow y_k$ (Σ−)	D x_i, y_i, f_i	E x_k, y_k, f_k
a Print	b Used	c Used	d Used	e Σx_i. . . .
0 Print x_i, y_i	1 Useo	2 Used	3 Used	4 Used
5	6	7	8 Space	9 Print

FLAGS

0	Print
1	Σ− (Correct.)
2	S_y'
3	

SET STATUS

FLAGS	ON	OFF
0	☐	☒
1	☐	☒
2	☐	☒
3	☐	☒

TRIG	
DEG	☒
GRAD	☐
RAD	☐

DISP	
FIX	☒
SCI	☐
ENG	☐
n	2

Factorial, Permutation and Combination

001	*LBLA		057	LSTX	
002	CLRG	Initialize	058	X≤Y?	
003	CF0		059	GSB6	
004	0		060	STO7	
005	RTN		061	1	
006	*LBLB	Set flag 0 for print.	062	STOI	
007	SF0		063	+	
008	1		064	STO6	
009	RTN		065	CLX	
010	*LBLD		066	X=Y?	
011	X⇄Y		067	GTO3	
012	GSBa		068	*LBL0	
013	X⇄Y	Input m, n for $_mP_n$	069	R↓	
014	GSBa		070	1	
015	X>Y?		071	RCLI	
016	GTO2		072	+	
017	ENT↑		073	STOI	
018	0		074	X>Y?	
019	X=Y?		075	GTO5	
020	GTO3		076	RCL7	
021	CLX		077	X⇄Y	
022	1		078	+	
023	X=Y?		079	LSTX	
024	GTO4		080	÷	
025	-		081	RCL6	
026	STOI		082	×	
027	R↓		083	STO6	
028	STO7		084	GTO0	
029	*LBL1		085	*LBL4	
030	RCL7		086	R↓	
031	1		087	R↓	$_mP_1 = m$
032	-		088	GSBa	
033	STO7		089	GSBb	
034	×		090	RTN	
035	DSZI		091	*LBL6	
036	GTO1		092	STO6	
037	GSBa		093	X⇄Y	
038	GSBb		094	RTN	
039	RTN		095	*LBL5	
040	*LBL2		096	RCL6	
041	0		097	GSBa	
042	÷		098	GSBb	
043	*LBL3	Error	099	RTN	
044	ENT↑		100	*LBLC	
045	1		101	GSBa	
046	GSBa		102	STO1	
047	GSBb		103	STO3	Input n for n!
048	RTN		104	6	
049	*LBLE		105	9	
050	X⇄Y		106	X⇄Y	
051	GSBa		107	X≤Y?	
052	X⇄Y	Input m, n for $_mC_n$	108	GTO9	
053	GSBa		109	X⇄Y	
054	X>Y?		110	-	
055	GTO2		111	STOI	
056	-		112	LSTX	

REGISTERS

0	1	2	3	4	5	6	7	8	9
	m, n	Log(69!)+....	(n – i)			Used	m	n – 1	

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
					m – 69

Step	Key Entry	Comments
113	N!	
114	LOG	
115	STO2	
116	RCL1	
117	ENT↑	
118	LOG	
119	ST+2	
120	*LBL7	
121	DSZI	
122	GTO8	
123	GTO7	
124	*LBL7	
125	RCL2	
126	INT	
127	GSBa	
128	RCL2	
129	FRC	
130	10^x	
131	DSP9	
132	GSBa	
133	DSP2	
134	RCL1	
135	GSBb	
136	R/S	
137	R↓	
138	R↓	
139	R/S	
140	R↑	
141	RTN	
142	*LBL8	
143	RCL3	Take log (n − 1).
144	1	
145	-	
146	STO3	
147	LOG	
148	ST+2	
149	GTO7	
150	*LBL9	For n ≤ 69.
151	N!	
152	GSBa	
153	GSBb	
154	RTN	
155	*LBLa	Subroutine for print.
156	F0?	
157	PRTX	
158	RTN	
159	*LBLb	Subroutine for space.
160	F0?	
161	SPC	
162	RTN	

LABELS

A Start	B Print?	C n→n!	D m↑n→$_mP_n$	E m↑n→$_mC_n$
a Print	b Space	c	d	e
0 $_mC_n$	1 m!	2 Error	3 $_mP_0$, $_mC_0$	4 $_mP_1$, $_mC_1$
5 Output	6 x⇄y	7 n > 69	8 n > 69	9 n ≤ 69

FLAGS

0	Print
1	
2	
3	

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☐
3	☐	☒			n	2

Moments, Skewness and Kurtosis (For Grouped or Ungrouped Data)

Step	Key	Comment
001	*LBLA	
002	CLRG	
003	P⇄S	Initialize
004	CLRG	
005	P⇄S	
006	CF1	
007	CF0	
008	0	
009	RTN	
010	*LBLa	Set flag 0 for print.
011	SF0	
012	1	
013	RTN	
014	*LBLB	
015	STOA	
016	GSB9	Input x_i for ungrouped data.
017	Σ+	
018	*LBL1	
019	P⇄S	
020	RCL4	
021	RCL5	
022	RCL9	
023	P⇄S	
024	STO1	
025	R↓	
026	STO3	
027	R↓	
028	STO2	
029	RCLA	
030	3	
031	Y^x	
032	F1?	
033	CHS	
034	ST+4	
035	RCLA	
036	4	
037	Y^x	
038	F1?	
039	CHS	
040	ST+5	
041	RCL1	
042	GSB9	
043	GSB8	
044	RTN	
045	*LBLC	
046	GSB9	Correction for x_k.
047	SF1	
048	Σ-	
049	GSB1	
050	CF1	
051	RTN	
052	*LBLD	
053	X⇄Y	Input y_i, f_i for grouped data.
054	GSB9	
055	X⇄Y	
056	GSB9	

Step	Key	Comment
057	F1?	
058	CHS	
059	ST+1	
060	X⇄Y	
061	x	
062	ST+2	
063	LSTX	
064	x	
065	ST+3	
066	LSTX	
067	x	
068	ST+4	
069	LSTX	
070	x	
071	ST+5	
072	RCLC	
073	1	
074	F1?	
075	CHS	
076	+	
077	STOC	
078	GSB9	
079	GSB8	
080	RTN	
081	*LBLE	
082	SF1	Correction for y_h, f_h.
083	GSBD	
084	CF1	
085	RTN	
086	*LBLb	
087	RCL2	
088	RCL1	
089	÷	$\overline{x}$
090	STO6	
091	GSB9	
092	GSB8	
093	RTN	
094	*LBLc	
095	RCL3	
096	RCL1	
097	÷	
098	RCL6	
099	X²	
100	STO8	
101	-	
102	STO7	m_2
103	GSB9	
104	R/S	
105	RCL4	
106	RCL3	
107	RCL6	m_3
108	x	
109	3	
110	x	
111	-	
112	RCL1	

REGISTERS

0	1	2	3	4	5	6	7	8	9
	n or Σf_i	Σx_i or $\Sigma f_i y_i$	Σx_i^2 or $\Sigma f_i y_i^2$	Σx_i^3 or $\Sigma f_i y_i^3$	Σx_i^4 or $\Sigma f_i y_i^4$	$\overline{x}$, m_4	m_2	$\overline{x}^2$	m_3

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
				Σx_i	Σx_i^2	Σy_i	Σy_i^2	Σ_{xy}	n

A	B	C	D	E	I
x_i		n			

Key entry	Comment
+	
RCL6	
RCL8	
x	
2	
x	
+	
ST09	
GSB9	
R/S	
RCL5	
RCL6	
RCL4	
x	m_4
4	
x	
-	
RCL8	
RCL3	
x	
6	
x	
+	
RCL1	
÷	
RCL8	
X²	
3	
x	
-	
ST06	
GSB9	
GSB8	
RTN	
*LBLd	
RCL9	γ_1
RCL7	
1	
.	
5	
Yˣ	
÷	
GSB9	
R/S	
RCL6	
RCL7	
X²	
÷	γ_2
GSB9	
GSB8	
RTN	
*LBL9	Subroutine for print.
F0?	
PRTX	
RTN	
*LBL8	
F0?	
SPC	Subroutine for space.
RTN	

LABELS					FLAGS
A Start	B x_i (Σ+)	C x_k (Σ−)	D $y_i \uparrow f_i$ (Σ+)	E $y_h \uparrow f_h$ (Σ−)	0 Print
a Print?	b $\rightarrow \bar{x}$	c $\rightarrow m_2, m_3, m_4$	d $\rightarrow \gamma_1 ; \gamma_2$	e	1 Correction
0	1 Used	2	3	4	2
5	6	7	8 Space	9 Print	3

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☐
3	☐	☒			n	2

Random Number Generator

Step	Key	Comment
001	*LBLa	Input a, b, clear secondary registers.
002	P⇄S	
003	CLRG	
004	P⇄S	
005	GTOc	
006	*LBLA	
007	GSB7	
008	RCLC	
009	RCLD	
010	-	
011	x	u_i
012	RCLD	
013	+	
014	*LBL2	
015	PRTX	
016	STO9	
017	Σ+	
018	RCLA	
019	+	
020	STOA	
021	RCL9	
022	X^2	
023	RCLB	
024	+	
025	STOB	
026	1	
027	RCLI	
028	+	
029	STOI	
030	RCL9	
031	RTN	
032	*LBLb	
033	STOD	
034	GSB0	Input k.
035	RCLD	
036	*LBL3	
037	PRTX	
038	SPC	
039	RTN	
040	*LBLB	
041	GSB7	
042	RCLD	Random digit generator.
043	x	
044	INT	
045	1	
046	+	
047	GTO2	
048	*LBLc	
049	STOC	
050	X⇄Y	Input m, σ.
051	STOD	
052	GSB0	
053	RCLD	
054	GSB4	
055	RCLC	
056	GTO3	
057	*LBL5	
058	*LBLC	
059	GSB7	
060	STO7	
061	GSB7	
062	2	Normal number generator.
063	x	
064	1	
065	-	
066	STO2	
067	RCL7	
068	2	
069	x	
070	1	
071	-	
072	STO1	
073	→P	
074	X^2	
075	1	
076	X≤Y?	
077	GTO5	
078	R↓	
079	ENT↑	
080	LN	
081	2	
082	x	
083	CHS	
084	X⇄Y	
085	÷	
086	√X	
087	STO8	
088	RCL1	
089	GSB6	
090	RTN	
091	*LBLC	
092	RCL8	
093	RCL2	
094	*LBL6	
095	x	
096	RCLC	
097	x	
098	RCLD	
099	+	
100	GTO2	
101	*LBLd	
102	GTOb	
103	*LBLD	
104	GSB7	
105	LN	
106	CHS	Input μ, exponential random number.
107	RCLD	
108	x	
109	GTO2	
110	*LBLE	
111	SPC	$\bar{x}$
112	$\bar{x}$	

REGISTERS

0	1	2	3	4	5	6	7	8	9
	V_1	V_2					u_i	$\sqrt{-2 \ln S/S}$	

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
				Σx_i	Σx_i^2				n

A	B	C	D	E	I
Used	Used	b or σ	a or k or m or μ	f_{i+1}	index n

Step	Key	Comments
113	GSB4	
114	R/S	
115	S	s
116	GSB4	
117	R/S	
118	P⇄S	
119	RCL9	
120	P⇄S	
121	GSB4	n
122	SPC	
123	RTN	
124	*LBL4	Subroutine to print.
125	PRTX	
126	RTN	
127	*LBL0	
128	.	
129	5	
130	2	
131	8	
132	4	u_0
133	1	
134	6	
135	3	
136	STOE	
137	0	
138	STOA	
139	STOB	
140	STOI	
141	SPC	
142	RTN	
143	*LBL7	
144	RCLE	
145	9	
146	9	
147	7	Random number generator.
148	×	
149	FRC	
150	STOE	
151	RTN	

LABELS					FLAGS
A →u_i	B →d_i	C →n_i	D →e_i	E →$\overline{x}$; s; n	0
a stb→	b k→	c m↑σ→	d μ→	e	1
0 u_0 to R_E	1 Used	2	3 Print, space	4 Print	2
5 Used	6 n_i, n_{i+1}	7 997 x u_i	8	9	3

SET STATUS				
FLAGS	ON	OFF	TRIG	DISP
0	☐	☒	DEG ☒	FIX ☒
1	☐	☒	GRAD ☐	SCI ☐
2	☐	☒	RAD ☐	ENG ☐
3	☐	☒		n 2

Histogram

```
*LBLA        Initialize
CLRG
P⇄S
CLRG
P⇄S
CF0
CF1
0
RTN
*LBLC
R↓
GSB8         Input xmin, xmax.
STOC
R↑
STOD
GSB8
GSB7
GSB7
X⇄Y
-
2
4
STOE
÷
STOA
RTN
*LBLE
0
STOI
X⇄Y
GSB8         Correction
Σ-
SF1
GSBc
CF1
RTN
*LBLD
STO0
0
STOI         Input xi.
R↓
GSB8
RCLC
X>Y?
GTO0
R↓
RCLD
X⇄Y
X>Y?
GTO0
0
X⇄Y
Σ+
*LBLc
STO9
RCL0
RCLD
X⇄Y
X=Y?
GSB2
RCLC
-
RCLA
1/X
×
INT
1
X⇄Y
+
LSTX
3
÷
INT
1
+
STOI
1
-
3
×
-
GTO1
*LBL9
RCL9
DSP0
GSB8
GSB7
DSP2
CF1
R/S
RTN
*LBL1
GSBd
F1?
CHS
ST+i
GTO9
*LBLd
3
CHS
×
10ˣ
RTN
*LBLa
SPC
0            Listing
STOI
RCLC
STOB
*LBL5
ISZI
2
```

REGISTERS

0 x_i	1 1, 2, 3	2 4, 5, 6	3 7, 8, 9	4 10, 11, 12	5 13, 14, 15	6 16, 17, 18	7 19, 20, 21	8 22, 23, 24	9 n
S0	S1	S2	S3	S4 Σx_i	S5 Σx_i^2	S6 Σy_i	S7 Σy_i^2	S8 $\Sigma x_i y_i$	S9 n

A $(x_{max}-x_{min})/24$	B x_{min}, last range	C x_{min}	D x_{max}	E 24	I Counter 1–8

Step	Key	Comment
113	ST09	
114	GSBe	
115	RCLi	
116	EEX	
117	3	
118	×	
119	INT	
120	DSP0	
121	PRTX	
122	SPC	
123	DSP2	
124	1	
125	ST09	
126	GSBe	
127	RCLi	
128	EEX	
129	3	
130	×	
131	FRC	
132	EEX	
133	3	
134	×	
135	INT	
136	DSP0	
137	PRTX	Listing
138	SPC	
139	DSP2	
140	0	
141	ST09	
142	GSBe	
143	RCLi	
144	EEX	
145	6	
146	×	
147	FRC	
148	EEX	
149	3	
150	×	
151	INT	
152	DSP0	
153	PRTX	
154	SPC	
155	DSP2	
156	RCLI	
157	8	
158	X>Y?	
159	GTO5	
160	RTN	
161	*LBLe	
162	RCLB	
163	PRTX	
164	RCLC	
165	RCLA	
166	RCLI	
167	3	
168	×	

Step	Key	Comment
169	RCL9	
170	-	
171	×	
172	+	
173	PRTX	
174	STOB	
175	RTN	
176	*LBLB	Set flag 0 for print.
177	SF0	
178	1	
179	RTN	
180	*LBLb	
181	GSB7	
182	GSB7	For n, $\bar{x}$, s
183	P⇄S	
184	RCL9	
185	P⇄S	
186	GSB8	
187	R/S	
188	$\bar{x}$	
189	GSB8	
190	R/S	
191	S	
192	GSB8	
193	GSB7	
194	RTN	
195	*LBL8	Subroutine for print.
196	F0?	
197	PRTX	
198	RTN	
199	*LBL7	Subroutine for space.
200	F0?	
201	SPC	
202	RTN	
203	*LBL2	Correction for input equals to x_{max}.
204	RCLA	
205	2	
206	÷	
207	-	
208	RTN	

LABELS

A Start	B Print	C x_{max}, x_{min}	D Input	E Correct
a List	b n; $\bar{x}$; s	c y	d 10^x	e Used
0 Error	1 Sorting	2 Cor. for x_{max}	3	4
5 Listing	6	7 Space	8 Print	9 Print index

FLAGS

0	Print
1	Correction
2	
3	

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☐
3	☐	☒			n	2

Analysis of Variance (One Way)

001	*LBLA	
002	CLRG	
003	P⇄S	
004	CLRG	Initialize
005	P⇄S	
006	CF0	
007	CF1	
008	CF2	
009	0	
010	RTN	
011	*LBLC	Input x_{ij}.
012	F2?	
013	GSB9	
014	GSB3	
015	Σ+	
016	*LBL8	
017	P⇄S	
018	RCL4	
019	RCL5	
020	P⇄S	
021	STOB	
022	R↓	
023	STOA	
024	R↓	
025	GSB3	
026	GSB0	
027	RTN	
028	*LBLE	
029	1	
030	ST+9	
031	SF2	
032	RCLA	
033	ST+7	
034	RCLB	
035	ST+4	
036	P⇄S	
037	RCL9	$\bar{x}_i$
038	P⇄S	
039	ST+6	
040	RCLA	
041	X²	
042	P⇄S	
043	RCL9	
044	P⇄S	
045	÷	
046	ST+5	
047	x̄	
048	GSB3	
049	R/S	
050	S	s_i
051	GSB3	
052	R/S	
053	RCLA	
054	GSB3	
055	GSB0	Sum_i
056	GSB0	

057	RTN	
058	*LBL9	
059	P⇄S	Clear registers for new i.
060	CLRG	
061	P⇄S	
062	RTN	
063	*LBLD	
064	GSB3	Correction
065	Σ-	
066	GSB8	
067	RTN	
068	*LBLB	Set flag 0 for print.
069	SF0	
070	1	
071	RTN	
072	*LBLa	
073	RCL4	
074	RCL7	
075	X²	
076	RCL6	
077	÷	TSS
078	STO8	
079	-	
080	STO0	
081	GSB3	TrSS
082	R/S	
083	RCL5	
084	RCL8	
085	-	
086	STO1	
087	GSB3	
088	R/S	
089	RCL0	ESS
090	RCL1	
091	-	
092	STO2	
093	GSB3	
094	GSB0	
095	RTN	
096	*LBL0	Subroutine for space.
097	F0?	
098	SPC	
099	RTN	
100	*LBL3	Subroutine to print.
101	F0?	
102	PRTX	
103	RTN	
104	*LBLb	
105	RCL9	
106	1	df_1
107	-	
108	STO3	
109	GSB3	
110	R/S	
111	RCL6	df_2
112	RCL9	

REGISTERS

0 TSS	1 T_rSS	2 ESS	3 df_1	4 $\Sigma\Sigma x_{ij}^2$	5 $\Sigma(\Sigma x_{ij})^2/n_i$	6 Σn_i	7 $\Sigma\Sigma x_{ij}$	8 Σx_{ij}	9 k
S0	S1	S2	S3	S4 Σx_i	S5 Σx_i^2	S6 Σy_i	S7 Σy_i^2	S8 $\Sigma x_i y_i$	S9 n

A Σx_{ij}, df_2	B Σx_{ij}^2, F	C	D	E	I

Step	Key	Comments
113	-	
114	STOA	
115	GSB3	
116	R/S	
117	RCLA	
118	RCL3	df_3
119	+	
120	GSB3	
121	GSB0	
122	RTN	
123	*LBLc	
124	RCL1	T_rMS
125	RCL3	
126	÷	
127	GSB3	
128	R/S	
129	RCL2	EMS
130	RCLA	
131	÷	
132	GSB3	
133	R/S	
134	÷	
135	GSB3	
136	GSB0	F
137	STOB	
138	RTN	

LABELS

A Start	B Print?	C x_{ij} (Σ+)→	D x_{im} (Σ−)→	E $\overline{x}$, s_i, sum_i
a TSS, ...	b df_1, ...	c T_rMS ...	d	e
0 Space	1	2	3 Print	4
5	6	7	8 Σx_i, Σx_i^2	9 CLREG

FLAGS

0	Print
1	Correction
2	New data
3	

SET STATUS

FLAGS	ON	OFF
0	☐	☒
1	☐	☒
2	☐	☒
3	☐	☒

TRIG	
DEG	☒
GRAD	☐
RAD	☐

DISP	
FIX	☒
SCI	☐
ENG	☐
n	2

Two Way Analysis of Variance

Step	Key Entry	Comments
001	*LBLA	
002	CLRG	Initialize
003	CF0	
004	0	
005	RTN	
006	*LBLC	
007	X⇄Y	Input r, c.
008	STO5	
009	GSB9	
010	R↓	
011	STO6	
012	GSB9	
013	GSB8	
014	RTN	
015	*LBLD	
016	ST+7	Input x_{ij}.
017	GSB9	
018	X²	
019	ST+2	
020	RCLA	
021	1	
022	+	
023	STOA	
024	GSB9	
025	RTN	
026	*LBLa	
027	RCL7	
028	ST+1	
029	X²	Calculate RS_i.
030	ST+3	
031	*LBL0	
032	RCL7	
033	0	
034	STOA	
035	STO7	
036	X⇄Y	
037	GSB9	
038	GSB8	
039	RTN	
040	*LBLb	
041	RCLA	Re-initialize for column.
042	STOB	
043	0	
044	STO2	
045	STOA	
046	GSB8	
047	RTN	
048	*LBLc	
049	RCL7	CS_i
050	X²	
051	ST+4	
052	GTO0	
053	*LBLd	
054	RCL1	F_1, F_2
055	X²	
056	RCL5	
057	RCL6	
058	×	
059	÷	
060	STO7	
061	CHS	
062	RCL2	
063	+	
064	STO1	
065	RCL3	
066	RCL6	
067	÷	
068	RCL7	
069	-	
070	STO2	
071	RCL4	
072	RCL5	
073	÷	
074	RCL7	
075	-	F_1, F_2
076	STO3	
077	RCL2	
078	+	
079	CHS	
080	RCL1	
081	+	
082	STO4	
083	RCL5	
084	1	
085	-	
086	STO5	
087	RCL6	
088	1	
089	-	
090	STO6	
091	×	
092	STO7	
093	÷	
094	STO8	
095	RCL2	
096	RCL5	
097	÷	
098	RCL8	
099	÷	
100	GSB8	
101	GSB9	
102	R/S	
103	RCL3	
104	RCL6	
105	÷	
106	RCL8	
107	÷	
108	GSB9	
109	GSB8	
110	RTN	
111	*LBLd	
112	RCL5	

REGISTERS

0	1	2	3	4	5	6	7	8	9
	$\Sigma\Sigma x_{ij}$, TSS	$\Sigma\Sigma x_{ij}^2$, RSS	Used	Used	r, r − 1	c, c − 1	Used	Used	0
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
i, j	r				

113	CSB9	
114	R/S	df_1, df_2, df_3
115	RCL6	
116	CSB9	
117	R/S	
118	RCL7	
119	CSB9	
120	CSB8	
121	RTN	
122	*LBLe	RSS, CSS, ESS, TSS
123	RCL2	
124	CSB9	
125	R/S	
126	RCL3	
127	CSB9	
128	R/S	
129	RCL4	
130	CSB9	
131	R/S	
132	RCL1	
133	CSB9	
134	CSB8	
135	RTN	
136	*LBLE	For correction of x_{im} or x_{hj}.
137	CSB9	
138	CHS	
139	ST+7	
140	X²	
141	ST-2	
142	RCLA	
143	1	
144	-	
145	STOA	
146	CSB9	
147	RTN	Set flag 0 for print.
148	*LBLB	
149	SF0	
150	1	
151	RTN	
152	*LBL9	Subroutine for print.
153	F0?	
154	PRTX	
155	RTN	
156	*LBL8	Subroutine for space.
157	F0?	
158	SPC	
159	RTN	

LABELS					FLAGS
A Start	B Print?	C r↑c→	D x_{ij} (Σ+)→	E x_{im} (Σ−)→	0 Print
a →RS_i	b Re-int.	c →CS_i	d→F_1, F_2...	e→RSS;...	1
0 0 to R_A,R_7	1	2	3	4	2
5	6	7	8 Space	9 Print	3

SET STATUS				
FLAGS	ON	OFF	TRIG	DISP
0	☐	☒	DEG ☒	FIX ☒
1	☐	☒	GRAD ☐	SCI ☐
2	☐	☒	RAD ☐	ENG ☐
3	☐	☒		n 2

Analysis of Covariance (One Way) (Card 1)

Step	Key	Comment
001	*LBLA	
002	CLRG	
003	P⇄S	Initialize
004	CLRG	
005	P⇄S	
006	CF0	
007	0	
008	STO8	
009	RTN	
010	*LBLB	
011	P⇄S	
012	0	For new i.
013	STO4	
014	STO5	
015	STO6	
016	STO7	
017	STO8	
018	STO9	
019	P⇄S	
020	ISZI	
021	RCLI	
022	GSB0	
023	GSB1	
024	RTN	
025	*LBL9	Subroutine to print x_{ij}, y_{ij}.
026	R↓	
027	PRTX	
028	R↑	
029	PRTX	
030	RTN	
031	*LBLe	Set flag 0 for print.
032	SF0	
033	1	
034	STO8	
035	RTN	
036	*LBLD	Correction for x_{im}, y_{im}.
037	F0?	
038	GSB9	
039	X⇄Y	
040	Σ-	
041	STO9	
042	GSB0	
043	GSB1	
044	RTN	
045	*LBLC	Input for x_{ij}, y_{ij}.
046	F0?	
047	GSB9	
048	X⇄Y	
049	Σ+	
050	STO9	
051	GSB0	
052	GSB1	
053	RTN	
054	*LBLE	
055	P⇄S	
056	RCL4	
057	RCLA	
058	+	
059	STOA	
060	RCL5	
061	RCLB	
062	+	
063	STOB	
064	RCL6	
065	RCLC	
066	+	
067	STOC	
068	RCL7	
069	RCLD	
070	+	
071	STOD	
072	RCL8	
073	RCLE	
074	+	
075	STOE	
076	P⇄S	
077	RCL9	Σn_i
078	ST+0	
079	P⇄S	
080	ST+0	
081	RCL4	$\Sigma \frac{(\Sigma x_{ij})^2}{n_i}$
082	P⇄S	
083	GSB8	
084	P⇄S	
085	RCL6	$\Sigma \frac{(\Sigma y_{ij})^2}{n_i}$
086	GSB8	
087	RCL4	
088	RCL6	
089	×	
090	RCL9	
091	÷	
092	P⇄S	
093	ST+5	
094	P⇄S	
095	RCL6	
096	RCL4	
097	P⇄S	
098	GSB0	s_{xi}
099	R/S	
100	R↓	
101	GSB0	
102	GSB1	s_{yi}
103	R/S	
104	*LBL8	
105	X²	
106	RCL9	
107	÷	
108	ST+2	
109	RTN	
110	*LBLa	TSS_x
111	RCLB	
112	RCLA	

REGISTERS

0 Σn_i	1 TSS_x	2 Used	3 WSS_x	4 TSP_i	5 Used	6 WSP	7 WSS_Q	8 1 or 0	9 j
S0 Σn_i	S1 TSS_y	S2 Used	S3 WSS_y	S4 Σx_{ij}	S5 Σx_{ij}^2	S6 Σy_{ij}	S7 Σy_{ij}^2	S8 $\Sigma x_{ij}y_{ij}$	S9 $n_{i \neq j}$

A $\Sigma\Sigma x_{ij}$	B $\Sigma\Sigma x_{ij}^2$	C $\Sigma\Sigma y_{ij}$	D $\Sigma\Sigma y_{ij}^2$	E $\Sigma\Sigma x_{ij}y_{ij}$	I i = 1, 2, ... k

Step	Key	Comment
113	GSB7	
114	R/S	ASS_x
115	RCL2	
116	RCLA	
117	GSB6	
118	R/S	WSS_x
119	-	
120	STO3	
121	GSB0	
122	GSB1	
123	R/S	
124	*LBLa	TSS_y
125	RCLD	
126	RCLC	
127	P⇄S	
128	GSB7	
129	P⇄S	
130	R/S	ASS_y
131	P⇄S	
132	RCL2	
133	RCLC	
134	GSB6	
135	P⇄S	
136	R/S	
137	-	WSS_y
138	P⇄S	
139	STO3	
140	P⇄S	
141	GSB0	
142	GSB1	
143	R/S	
144	*LBL7	For $(TSS)_{x \text{ or } y}$
145	X²	
146	RCL0	
147	÷	
148	-	
149	STO1	
150	GSB0	
151	RTN	
152	*LBL6	For $(ASS)_{x \text{ or } y}$
153	X²	
154	RCL0	
155	÷	
156	-	
157	STO2	
158	GSB0	
159	RTN	
160	*LBLb	
161	RCL2	
162	RCLI	
163	1	
164	-	
165	÷	
166	RCL3	
167	RCL0	F_x
168	RCLI	
169	-	
170	÷	
171	÷	
172	GSB0	
173	R/S	
174	P⇄S	
175	RCL2	
176	RCLI	
177	1	
178	-	
179	÷	
180	RCL3	
181	P⇄S	
182	RCL0	
183	RCLI	
184	-	
185	÷	
186	÷	F_y
187	GSB0	
188	R/S	
189	RCLI	
190	1	
191	-	df_1
192	GSB0	
193	R/S	
194	RCL0	
195	RCLI	
196	-	df_2
197	GSB0	
198	GSB1	
199	RTN	
200	*LBL0	Subroutine for print.
201	F0?	
202	PRTX	
203	RTN	
204	*LBL1	Subroutine for space.
205	F0?	
206	SPC	
207	RTN	

LABELS

A Start	B New i	C $x_{ij}\uparrow y_{ij}$ (Σ+)	D $x_{im}\uparrow y_{im}$ (Σ−)	E s_{xi}; s_{yi}
a TSS_x; ...	b F_x; ...	c	d	e Print
0 Print	1 Space	2	3	4
5	6 ASS_x	7 TSS_x	8 $\Sigma\Sigma x_{ij}/n_i$	9 Print x_{ij}, y_{ij}

FLAGS

0	Print
1	
2	
3	

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☐
3	☐	☒			n	2

(Card 2)

Step	Key	Comments
001	*LBLc	
002	GSB2	
003	RCLE	
004	RCLA	
005	RCLC	
006	×	
007	RCL0	
008	÷	
009	-	TSP
010	STO4	
011	GSB0	
012	R/S	
013	RCL5	
014	RCLA	
015	RCLC	
016	×	
017	RCL0	
018	÷	ASP
019	-	
020	GSB0	
021	R/S	
022	-	WSP
023	STO6	
024	GSB0	
025	GSB1	
026	RTN	
027	*LBLd	
028	P⇄S	
029	RCL1	
030	P⇄S	
031	RCL4	
032	X²	
033	RCL1	
034	÷	$TSS_{\hat{y}}$
035	-	
036	GSB0	
037	R/S	
038	P⇄S	
039	RCL3	
040	P⇄S	
041	RCL6	
042	X²	
043	RCL3	
044	÷	
045	-	$WSS_{\hat{y}}$
046	STO7	
047	GSB0	
048	R/S	
049	-	$ASS_{\hat{y}}$
050	GSB0	
051	GSB1	
052	R/S	
053	*LBLe	
054	RCLI	
055	1	
056	-	
057	÷	
058	GSB0	$AMS_{\hat{y}}$
059	R/S	
060	RCL7	
061	RCL0	
062	RCLI	$WMS_{\hat{y}}$
063	-	
064	1	
065	-	
066	÷	
067	GSB0	
068	R/S	F
069	÷	
070	GSB0	
071	R/S	
072	RCLI	
073	1	df_3
074	-	
075	GSB0	
076	R/S	
077	RCL0	
078	RCLI	
079	-	df_4
080	1	
081	-	
082	GSB0	
083	GSB1	
084	RTN	
085	*LBL0	Subroutine for print.
086	F0?	
087	PRTX	
088	RTN	
089	*LBL1	Subroutine for space.
090	F0?	
091	SPC	
092	RTN	
093	*LBL2	Subroutine for setting
094	CF0	flag 0 for print.
095	RCL8	
096	1	
097	X=Y?	
098	SF0	
099	RTN	

REGISTERS

0	1	2	3	4	5	6	7	8	9
Σn_i	TSS_x	Used	WSS_x	TSP_i	Used	WSP	$WSS_{\hat{y}}$	1 or 0	j
S0 Σn_i	S1 TSS_y	S2 Used	S3 WSS_y	S4 Σx_{ij}	S5 Σx_{ij}^2	S6 Σy_{ij}	S7 Σy_{ij}^2	S8 $\Sigma x_{ij} y_{ij}$	S9 $n_{i \neq j}$

A	B	C	D	E	I
$\Sigma\Sigma x_{ij}$	$\Sigma\Sigma x_{ij}^2$	$\Sigma\Sigma y_{ij}$	$\Sigma\Sigma y_{ij}^2$	$\Sigma\Sigma x_{ij} y_{ij}$	i = 1, 2, ... k

LABELS					FLAGS
A	B	C	D	E	0 Print
a	b	c TSP; ...	d TSS¢; ...	e AMS¢; ...	1
0 Print	1 Space	2 Used	3	4	2
5	6	7	8	9	3

SET STATUS		
FLAGS	TRIG	DISP
ON OFF		
0 ☐ ☒	DEG ☒	FIX ☒
1 ☐ ☒	GRAD ☐	SCI ☐
2 ☐ ☒	RAD ☐	ENG ☐
3 ☐ ☒		n 2

Normal and Inverse Normal Distribution (Card 1)

001	*LBLA	
002	.	
003	2	
004	3	
005	1	
006	6	
007	4	
008	1	
009	9	
010	STO3	
011	1	
012	.	
013	3	
014	3	
015	0	
016	2	
017	7	
018	4	
019	4	
020	2	Store constants for normal distribution.
021	9	
022	STO4	
023	1	
024	.	
025	8	
026	2	
027	1	
028	2	
029	5	
030	5	
031	9	
032	7	
033	8	
034	CHS	
035	STO5	
036	1	
037	.	
038	7	
039	8	
040	1	
041	4	
042	7	
043	7	
044	9	
045	3	
046	7	
047	STO6	
048	.	
049	3	
050	5	
051	6	
052	5	
053	6	
054	3	
055	7	
056	8	

057	2	
058	CHS	
059	STO7	
060	.	
061	3	
062	1	
063	9	
064	3	
065	8	
066	1	
067	5	
068	3	
069	STO8	
070	P⇄S	
071	2	
072	.	
073	5	
074	1	
075	5	
076	5	
077	1	
078	7	
079	STO1	
080	.	
081	8	
082	0	
083	2	
084	8	
085	5	
086	3	
087	STO2	
088	.	
089	0	
090	1	
091	0	
092	3	
093	2	
094	8	Store constants for inverse N.D.
095	STO3	
096	1	
097	.	
098	4	
099	3	
100	2	
101	7	
102	8	
103	8	
104	STO4	
105	.	
106	1	
107	8	
108	9	
109	2	
110	6	
111	9	
112	STO5	

REGISTERS

0	1	2	3	4	5	6	7	8	9
			r	b_5	b_4	b_3	b_2	b_1	
S0	S1 C_0	S2 C_1	S3 C_2	S4 d_1	S5 d_2	S6 d_3	S7	S8	S9

A	B	C	D	E	I

```
113    .
114    0
115    0
116    1
117    3
118    0
119    8
120    STO6
121    P⇄S
122    0
123    STOA
124    STOB
125    RTN
```

LABELS					FLAGS
A Start	B	C	D	E	0
a	b	c	d	e	1
0	1	2	3	4	2
5	6	7	8	9	3

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☐
3	☐	☒			n	2

(Card 2)

Step	Key	Comment	Step	Key	Comment
001	*LBLB		057	×	
002	1		058	RCL2	
003	STOA	Store 1 in R_A for print.	059	×	
004	RTN		060	F0?	
005	*LBLC		061	GSB9	
006	GSB9		062	F0?	
007	STO1		063	GSB6	
008	ENT↑		064	RTN	
009	×	Input x and compute f(x).	065	*LBL1	
010	2		066	CF0	
011	÷		067	RCL1	
012	CHS		068	CHS	
013	e^x		069	STO1	
014	Pi		070	GSBc	
015	2		071	1	
016	×		072	X⇄Y	
017	√X		073	-	
018	÷		074	STO9	
019	STO2		075	GSB9	
020	GSB9		076	GSB6	
021	GSB6		077	RCL9	
022	GSB3		078	RTN	
023	RCL2		079	*LBLE	
024	RTN		080	GSB9	
025	*LBLD		081	X<0?	Input Q(x) and calculate x.
026	GSB9		082	GTO0	
027	STO1		083	1	
028	GSB5		084	X≤Y?	
029	GSBC		085	GTO0	
030	RCL1		086	R↓	
031	X<0?		087	.	
032	GTO1	Input x and calculate Q(x).	088	5	
033	SF0		089	X⇄Y	
034	*LBLc		090	X>Y?	
035	1		091	GSB8	
036	RCL1		092	ENT↑	
037	RCL3		093	×	
038	×		094	1/X	
039	+		095	LN	
040	1/X		096	√X	
041	ENT↑		097	P⇄S	
042	ENT↑		098	STO7	
043	ENT↑		099	RCL3	
044	RCL4		100	×	
045	×		101	RCL2	
046	RCL5		102	+	
047	+		103	RCL7	
048	×		104	×	
049	RCL6		105	RCL1	
050	+		106	+	
051	×		107	RCL7	
052	RCL7		108	RCL6	
053	+		109	×	
054	×		110	RCL5	
055	RCL8		111	+	
056	+		112	RCL7	

REGISTERS

0	1	2	3	4	5	6	7	8	9
	x	f(x)	r	b_5	b_4	b_3	b_2	b_1	

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
	C_0	C_1	C_2	d_1	d_2	d_3	t		

A	B	C	D	E	I
1 or 0	1 or 0				

Step	Key	Comment
113	×	
114	RCL4	
115	+	
116	RCL7	
117	×	
118	1	
119	+	
120	÷	
121	RCL7	
122	X⇄Y	
123	-	
124	P⇄S	
125	F1?	
126	CHS	
127	GSB9	
128	GSB6	
129	CF1	
130	RTN	
131	*LBL8	For (1 − Q)
132	SF1	
133	1	
134	-	
135	CHS	
136	RTN	
137	*LBL9	Subroutine to print.
138	RCLA	
139	X>0?	
140	GSB7	
141	R↓	
142	RTN	
143	*LBL7	
144	R↓	
145	PRTX	
146	R↑	
147	RTN	
148	*LBL6	Subroutine for space.
149	RCLA	
150	X>0?	
151	SPC	
152	R↓	
153	RTN	
154	*LBL5	Clear R_A for calculating Q(x).
155	RCLA	
156	X>0?	
157	GSB4	
158	R↓	
159	RTN	
160	*LBL4	
161	STOB	
162	CLX	
163	STOA	
164	RTN	
165	*LBL3	
166	RCLB	
167	X>0?	
168	GSB2	
169	RTN	
170	*LBL2	Restore 1 to R_A.
171	1	
172	STOA	
173	RTN	

LABELS

A	B Print?	C x→f(x)	D x→Q(x)	E Q(x)→x
a	b	c Q(x)	d	e
0 Error	1 x<0	2 1→R_A	3 R_B > 0?	4 0→R_A
5 R_A→R_B	6 Space	7 Print	8	9 Print

FLAGS

0	x positive
1	Q(x) > .5
2	
3	

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☐
3	☐	☒			n	2

Chi-Square Distribution

Step	Key	Comment
001	*LBLA	
002	CLRG	
003	CF0	Initialize
004	CF1	
005	0	
006	RTN	
007	*LBLB	
008	SF0	Set flag for print.
009	1	
010	RTN	
011	*LBLC	
012	GSB9	Input ν.
013	1	
014	STO3	
015	X⇄Y	Calculate $\Gamma\left(\frac{\nu}{2}\right)$
016	2	
017	÷	
018	STO1	
019	INT	
020	LSTX	
021	X≠Y?	
022	GTO1	
023	1	
024	-	
025	N!	
026	GSB9	
027	GSB8	
028	STO3	
029	R/S	
030	*LBL1	
031	.	
032	5	
033	X=Y?	
034	GTO2	
035	X⇄Y	
036	1	
037	-	
038	ST×3	
039	GTO1	
040	*LBL2	
041	Pi	
042	√X	
043	RCL3	
044	×	
045	STO3	
046	GSB9	
047	GSB8	
048	R/S	
049	*LBLD	
050	SF1	
051	*LBLe	Input x, calculate f(x).
052	GSB9	
053	STO2	
054	RCL1	
055	1	
056		

Step	Key	Comment
057	Y^x	
058	RCL2	
059	2	
060	÷	
061	CHS	
062	e^x	
063	×	
064	2	
065	RCL1	
066	Y^x	
067	÷	
068	RCL3	
069	÷	
070	STO5	
071	F1?	
072	GSB9	
073	F1?	
074	GSB8	
075	CF1	
076	RTN	
077	*LBLE	
078	GSBe	Input x, calculate P(x).
079	RCL2	
080	RCL1	
081	÷	
082	ST×5	
083	2	
084	RCL1	
085	×	
086	STO6	
087	1	
088	STO4	
089	*LBL3	
090	RCL2	
091	RCL6	
092	2	
093	+	
094	STO6	
095	÷	
096	RCL4	
097	×	
098	STO4	
099	+	
100	X≠Y?	
101	GTO3	
102	RCL5	
103	×	
104	GSB9	
105	GSB8	
106	RTN	
107	*LBL9	Subroutine for print.
108	F0?	
109	PRTX	
110	RTN	
111	*LBLc	
112	F0?	

REGISTERS

0	1	2	3	4	5	6	7	8	9
	$\nu/2$	x	1, $\Gamma(\nu/2)$	Used	f(x)	Used			

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I

113 SPC 114 RTN	Subroutine for space.		

LABELS					FLAGS
A Start	B Print?	C $\nu \rightarrow \Gamma(\nu/2)$	D $x \rightarrow f(x)$	E $x \rightarrow P(x)$	0 Print
a	b	c	d	e f(x)	1 P(x)
0	1 $\nu = ½$?	2 $\Gamma(½)$	3 P(x)	4	2
5	6	7	8 Space	9 Print	3

SET STATUS					
FLAGS	ON	OFF	TRIG		DISP
0	☐	☒	DEG ☒		FIX ☒
1	☐	☒	GRAD ☐		SCI ☐
2	☐	☒	RAD ☐		ENG ☐
3	☐	☒			n 2

t Distribution

Step	Key	Comment	Step	Key	Comment
001	*LBLA		057	X⇄Y	
002	CLRG		058	2	
003	CF0		059	÷	
004	CF1	Initialize	060	STO1	
005	0		061	INT	
006	RTN		062	LSTX	
007	*LBLB	Set flag 0 for print.	063	X≠Y?	
008	SF0		064	GTO1	
009	1		065	1	
010	RTN		066	-	Compute Γ(ν/2).
011	*LBLC		067	N!	
012	STO0	Input ν.	068	STO3	
013	GSB7		069	RTN	
014	GSB5		070	*LBL1	
015	RTN		071	.	
016	*LBLD	Input x, compute f(x).	072	5	
017	GSB7		073	X=Y?	
018	STOA		074	GTO2	
019	RCL0		075	X⇄Y	
020	GSBa		076	1	
021	STOB		077	-	
022	RCL0		078	ST×3	
023	1		079	GTO1	
024	+		080	*LBL2	Compute R(x).
025	GSBa		081	Pi	
026	STOC		082	√X	
027	RCLA		083	RCL3	
028	RCLC		084	×	
029	RCLB		085	STO3	
030	÷		086	RTN	
031	Pi		087	*LBLE	
032	RCL0		088	STOA	
033	×		089	GSB7	
034	√X		090	ABS	
035	÷		091	RCL0	
036	1		092	RAD	
037	RCLA		093	√X	
038	X²		094	÷	
039	RCL0		095	TAN⁻¹	
040	÷		096	STO2	
041	+		097	RCL0	
042	RCL0		098	2	
043	1		099	÷	
044	+		100	INT	
045	2		101	LSTX	
046	÷		102	X≠Y?	
047	CHS		103	GTO4	
048	Yˣ		104	0	
049	×		105	STO5	
050	STOD		106	*LBLb	
051	GSB7		107	RCL2	For even ν.
052	GSB5		108	COS	
053	RTN		109	X²	
054	*LBLa		110	STO3	
055	1		111	RCL2	
056	STO3		112	SIN	

REGISTERS

0	1	2	3	4	5	6	7	8	9
ν		θ	Used	sin θ	Used	Used	2θ/π, R	Used	

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
x	Γ(ν/2)	Γ((ν+1)/2)			

```
STO4
RCL0
2
X=Y?
GTO8
÷
1
-
STOI
1
STO6
*LBL3
RCL3
x
RCL5
1
+
x
LSTX
1
+
STO5
÷
ST+6
DSZI
GTO3
RCL6
RCL4
x
F1?
GTOe
RCLA
GTO6
*LBLe
RTN
*LBL4
RCL2
2
x
Pi
÷
STO7          For odd ν.
RCL0
1
STO5
ST-0
X=Y?
GTO9
SF1
GSBb
CF1
RCL2
COS
x
2
x
Pi
÷
RCL7
+
RCLA
GTO6
RTN
*LBL9
RCL7
RCLA
GTO6
RTN
*LBL8
RCL4
RCLA
GTO6
RTN
*LBL6         Compute P(x) from R(x) for x ≤ 0.
X>0?
GTO0
X⇄Y
1
-
CHS
2
÷
GSB7
GSB5
RTN
*LBL0         Compute P(x) for x > 0.
X⇄Y
1
+
2
÷
GSB7
GSB5
RTN
*LBL7         Subroutine to print.
F0?
PRTX
RTN
*LBL5
F0?           Subroutine to space
SPC
RTN
```

LABELS					FLAGS
A Start	B Print	C ν→	D x→f(x)	E x→P(x)	0 Print
a ν/2 integer	b even ν	c	d	e Used	1 odd ν
0 P(x), x > 0	1 ν/2 no integer	2 $\sqrt{\pi}$ x	3 even ν	4 odd ν	2
5 Space	6 P(x), x ≤ 0	7 Print	8 For even ν	9 For odd ν	3

SET STATUS		
FLAGS (ON / OFF)	TRIG	DISP
0 ☐ ☒	DEG ☒	FIX ☒
1 ☐ ☒	GRAD ☐	SCI ☐
2 ☐ ☒	RAD ☐	ENG ☐
3 ☐ ☒		n 2

F Distribution

Step	Key	Comment
001	*LBLA	
002	CLRG	
003	CF0	Initialize
004	CF1	
005	0	
006	RTN	
007	*LBLB	
008	SF0	Set flag 0 for print.
009	1	
010	RTN	
011	*LBLC	
012	STO1	Input ν_1.
013	GSB9	
014	RTN	
015	*LBLD	
016	STO2	Input ν_2.
017	GSB9	
018	RTN	
019	*LBLE	
020	GSB9	Input x.
021	STO6	
022	ENT↑	
023	RCL1	
024	×	
025	RCL2	
026	+	
027	RCL2	
028	X⇄Y	
029	÷	
030	STO3	
031	RCL1	
032	2	
033	÷	If ν_1 even then go to LBL d else go to LBL e.
034	FRC	
035	0	
036	X=Y?	
037	GTOd	
038	GTOe	
039	RTN	
040	*LBLd	
041	RCL3	
042	RCL2	
043	2	
044	STO7	
045	÷	
046	Y^x	For even ν_1.
047	STO4	
048	RCL1	
049	2	
050	-	
051	2	
052	÷	
053	STOI	
054	0	
055	X≠Y?	
056	GTO5	
057	RCL4	
058	GSB9	
059	RTN	
060	*LBL5	
061	1	
062	STO5	
063	RCL3	Even ν_1.
064	-	
065	STO3	
066	RCL2	
067	2	
068	÷	
069	×	
070	ST+5	
071	DSZI	
072	GTO3	
073	GTO2	
074	*LBL3	
075	RCL2	
076	2	
077	+	
078	STO2	
079	RCL7	
080	2	
081	+	
082	STO7	
083	÷	
084	RCL3	
085	×	
086	×	
087	ST+5	
088	DSZI	
089	GTO3	Even ν_1.
090	*LBL2	
091	RCL5	
092	RCL4	
093	×	
094	F1?	
095	GSB9	
096	F1?	
097	GSB8	
098	RTN	
099	*LBLe	
100	CF1	
101	RCL1	
102	RCL2	
103	STO1	For odd ν_1.
104	X⇄Y	
105	STO2	
106	1	
107	RCL3	
108	-	
109	STO3	
110	GSBd	
111	SF1	
112	1	

REGISTERS

0	1	2	3	4	5	6	7	8	9
	ν_1 or ν_2	ν_2 or ν_1	t, 1 − t	$t^{\nu_2/2}$ or $t^{\nu_1/2}$	Used	x	Used	Used	Used

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
					Used

Step	Key Entry	Comments
113	X⇄Y	
114	-	
115	GSB9	
116	GSB8	
117	R/S	
118	*LBL9	Subroutine for print.
119	F0?	
120	PRTX	
121	RTN	
122	*LBL8	Subroutine for space.
123	F0?	
124	SPC	
125	RTN	

LABELS

A Start	B Print?	C $\nu_1 \rightarrow$	D $\nu_2 \rightarrow$	E $x \rightarrow P(x)$
a	b	c	d even ν_1	e odd ν_1
0	1	2 output	3 for P(x)	4
5 for P(x)	6	7	8 Space	9 Print

FLAGS

0	Print
1	ν_1 even
2	
3	

SET STATUS

FLAGS	ON	OFF
0	☐	☒
1	☐	☒
2	☐	☒
3	☐	☒

TRIG	
DEG	☒
GRAD	☐
RAD	☐

DISP	
FIX	☒
SCI	☐
ENG	☐
n	2

Multiple Linear Regression

Step	Key	Comment
001	*LBLA	
002	CLRG	
003	CF0	Initialize
004	CF1	
005	0	
006	RTN	
007	*LBLC	
008	STOC	
009	R↓	
010	STOB	
011	R↓	
012	STOA	
013	GSB7	
014	7	
015	STOI	
016	R↓	Input x_i, y_i, z_i.
017	GSB1	
018	8	
019	STOI	
020	RCLB	Compute Σx_i, Σy_i, Σz_i; Σx_i^2, Σy_i^2, Σz_i^2; $\Sigma x_i y_i$, $\Sigma y_i z_i$, $\Sigma z_i x_i$.
021	GSB9	
022	GSB1	
023	9	
024	STOI	
025	RCLC	
026	GSB9	
027	GSB1	
028	RCLA	
029	RCLB	
030	x	
031	GSB2	
032	ST+1	
033	RCLA	
034	RCLC	
035	x	
036	GSB2	
037	ST+2	
038	RCLB	
039	RCLC	
040	x	
041	GSB2	
042	ST+3	
043	1	
044	GSB2	
045	ST+0	
046	RCL0	
047	GSB9	
048	RTN	
049	*LBL1	
050	GSB2	Subroutine for Σx_i, Σx_i^2 ...
051	ST+i	
052	RCLI	
053	3	
054	-	
055	STOI	
056	R↓	
057	X²	
058	GSB2	
059	ST+i	
060	RTN	
061	*LBLE	
062	RCL0	
063	RCL4	
064	x	
065	RCL7	
066	X²	
067	-	
068	STOD	
069	RCL0	
070	RCL3	
071	x	
072	RCL8	
073	RCL9	
074	x	
075	-	
076	x	
077	STOC	
078	RCL0	Calculating a, b, c.
079	RCL1	
080	x	
081	RCL7	
082	RCL8	
083	x	
084	-	
085	STOA	
086	RCL0	
087	RCL2	
088	x	
089	RCL7	
090	RCL9	
091	x	
092	-	
093	STOB	
094	x	
095	RCLC	
096	X⇄Y	
097	-	
098	RCLD	
099	RCL0	
100	RCL5	
101	x	
102	RCL8	
103	X²	
104	-	
105	x	
106	RCLA	
107	X²	
108	-	
109	÷	
110	STOC	
111	RCLB	
112	RCLA	

REGISTERS

0	1	2	3	4	5	6	7	8	9
n	$\Sigma x_i y_i$	$\Sigma x_i z_i$	$\Sigma y_i z_i$	Σx_i^2	Σy_i^2	Σz_i^2	Σx_i	Σy_i	Σz_i
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
Used, a	Used, b	z_i, B in b, c	$[n\Sigma x_j^2 - (\Sigma x_j)^2]$	$n\Sigma x_i^2 - (\Sigma x_{ij})^2$	

Step	Key Entry	Comments	Step	Key Entry	Comments
113	RCLC		169	GSB9	
114	×		170	ENT↑	Calculating approximated z value for given x, y.
115	−		171	RCLC	
116	RCLD		172	×	
117	÷		173	X⇄Y	
118	STOB		174	RCLB	
119	RCL9		175	×	
120	RCLC		176	+	
121	RCL8		177	RCLA	
122	×		178	+	
123	−		179	GSB9	
124	RCLB		180	RTN	
125	RCL7		181	*LBLc	
126	×		182	F0?	
127	−		183	SPC	Σx_i, Σy_i, Σz_i
128	RCL0		184	6	
129	÷	a	185	STOI	
130	STOA		186	GTO8	
131	GSB7		187	RTN	
132	R/S	b	188	*LBLd	Σx_i^2, Σy_i^2, Σz_i^2
133	RCLB		189	3	
134	GSB9		190	STOI	
135	R/S	c	191	GSB8	
136	RCLC		192	RTN	
137	GSB9		193	*LBLe	Σx_iy_i, Σx_iz_i, Σy_iz_i
138	R/S		194	0	
139	*LBLa		195	STOI	
140	RCLA		196	GSB8	
141	RCL9		197	RTN	
142	×		198	*LBL8	
143	RCLB		199	ISZI	
144	RCL2		200	RCLi	Print for LBL a, LBL b, LBL c
145	×		201	GSB9	
146	+		202	R/S	
147	RCLC		203	GTO8	
148	RCL3	Calculating R^2	204	RTN	
149	×		205	*LBLD	
150	+		206	SF1	
151	RCL9		207	GSBC	
152	X²		208	CF1	For correction of x_k, y_k, z_k.
153	RCL0		209	RTN	
154	÷		210	*LBLB	
155	−		211	SF0	
156	RCL6		212	1	
157	RCL9		213	RTN	
158	X²		214	*LBL7	Set flag 0 for print.
159	RCL0		215	F0?	
160	÷		216	SPC	
161	−		217	*LBL9	Print x_i, y_i, z_i.
162	÷		218	F0?	
163	GSB7		219	PRTX	
164	RTN		220	RTN	
165	*LBLb		221	*LBL2	Change sign for correction.
166	X⇄Y		222	F1?	
167	GSB7		223	CHS	
168	X⇄Y		224	RTN	

LABELS

A Start	B Print?	C Input	D Correction	E →a; b; c
a →R^2	b x↑y→$\hat{z}$	c →Σx_i, ...	d →Σx_i^2, ...	e Σx_iy_i, ...
0	1 Σx_i, Σx_i^2	2 CHS	3	4
5	6	7 Space	8 Print	9 Print

FLAGS

0	Print
1	Correction
2	
3	

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☐
3	☐	☒			n	2

Polynomial Approximation (Card 1)

```
*LBLA
CLRG        Initialize
CF0
CF1
0
RTN
*LBLC       Input N.
F0?
PRTX
STO7
F0?
SPC
RTN
*LBLD       Input yi.
F1?
GTO7
SF1
F0?
PRTX
STO1
STO2
STO3
STO4
STO5
2
STO6
*LBL0
1
STO9
RCL6
2
÷
F0?
PRTX
F0?
SPC
R/S
*LBL7
F0?
PRTX
ST+1
1
RCL6
RCL7
÷
-
x
ST+2
STO9
GSBe
ST+3
GSBe
ST+4
GSBe
ST+5
2
ST+6
GTO0
*LBLe
RCL8
ENT↑
+
1           Calculate fn(i) f(xi).
+
x
RCL7
RCL6
-
x
X⇄Y
RCL7
RCL8
+
1
+
x
RCL8
x
-
RCL7
RCL8
-
÷
RCL8
1
+
STO8
÷
RCL9
X⇄Y
STO9
RTN
*LBLE       Input order of polynomial.
F0?
PRTX
ENT↑
2
X=Y?
GTO2
CLX
3
X=Y?
GTO3
CLX
4
X=Y?
GTO4
CLX
0
÷
*LBL2
CLX
```

REGISTERS

0	1 (f, f_0), a_0	2 (f, f_1), a_1	3 (f, f_2), a_2	4 (f, f_3), a_3	5 (f, f_4), a_4	6 2j	7 N	8 n	9 $f_n(i)\ f(x_i)$
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A 1 for print	B	C	D	E	I

```
113  ST04
114  *LBL3
115  CLX
116  ST05
117  *LBL4
118  RCL1
119  RCL7
120  1
121  ST08
122  +
123  ÷
124  ST01
125  GSBd
126  ST÷2
127  GSBd
128  ST÷3
129  GSBd
130  ST÷4
131  GSBd
132  ST÷5
133  RCL7
134  RCL7
135  RCL7
136  1
137  -
138  2
139  x
140  ST06
141  ÷
142  3
143  x
144  ST08
145  R↓
146  2
147  +
148  RCL6
149  ÷
150  ST06
151  RCL3
152  x
153  ST-1
154  RCL8
155  RCL3
156  x
157  ST03
158  RCL7
159  RCL7
160  RCL7
161  2
162  -
163  ÷
164  5
165  x
166  3
167  ÷
168  RCL8
169  X⇄Y
170  x
171  ST09
172  LSTX
173  RCL6
174  x
175  ST07
176  CLX
177  R/S
178  *LBLd          (f, fi)
179  RCL7
180  RCL8
181  1
182  +
183  +
184  N!
185  RCL7
186  RCL8
187  -
188  N!
189  x
190  RCL8
191  RCL8
192  1
193  +
194  ST08
195  +
196  ÷
197  RCL7
198  N!
199  ENT↑
200  x
201  ÷
202  RTN
203  *LBLB          Set flag 0 for print.
204  SF0
205  1
206  ST0A
207  RTN
```

LABELS					FLAGS
A Start	B Print?	C N→	D y_i→	E n→	0 Print
a	b	c	d	e (f_n, f_n)	1 y_i, i > 1
0 Print i	1	2 Used	3 Used	4 Used	2
5	6	7 i > 1	8	9 $f_n(i)$ $f(x_i)$	3

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☐
3	☐	☒			n	2

(Card 2)

Step	Key	Comment
001	*LBLa	
002	R↑	
003	R↑	
004	3	
005	+	
006	ENT↑	
007	ENT↑	
008	5	
009	-	
010	÷	
011	2	
012	×	
013	3	
014	÷	
015	RCL7	Continue
016	+	
017	STO7	
018	RCL4	
019	×	
020	ST-2	
021	LSTX	
022	RCL9	
023	×	
024	STO4	
025	R↑	
026	R↑	
027	4	
028	+	
029	ENT↑	
030	ENT↑	
031	7	
032	-	
033	÷	
034	3	
035	×	
036	4	
037	÷	
038	RCL6	
039	X⇄Y	
040	×	
041	STO6	
042	LSTX	
043	RCL8	
044	×	
045	STO8	
046	R↑	
047	ENT↑	
048	ENT↑	
049	ENT↑	
050	3	
051	-	
052	÷	
053	7	
054	×	
055	4	
056	÷	
057	RCL7	
058	X⇄Y	
059	×	
060	ST+8	
061	LSTX	
062	RCL9	
063	×	
064	RCL5	
065	×	
066	STO5	
067	LSTX	
068	ST×8	
069	RCL6	
070	×	
071	ST+1	
072	RCL8	
073	ST-3	
074	*LBLB	
075	RCLA	
076	X>0?	
077	SF0	
078	1	
079	R/S	
080	*LBLb	
081	X⇄Y	
082	GSB9	Input x_N, x_0.
083	X⇄Y	
084	GSB8	
085	-	
086	STO8	
087	LSTX	
088	2	
089	×	
090	+	
091	RCL8	
092	÷	
093	STO6	
094	2	
095	RCL8	
096	÷	
097	STO8	
098	RCL1	
099	RCL6	
100	RCL2	
101	×	
102	+	
103	RCL6	
104	ENT↑	
105	×	
106	RCL3	
107	×	
108	+	
109	STO1	
110	RCL2	
111	RCL6	
112	RCL3	

REGISTERS

0	1	2	3	4	5	6	7	8	9
	, b_0, c_0, d_0	, b_1, c_1, d_1	b_2, c_2, d_2	b_3, c_3, d_3	b_4, c_4, d_4	Used	Used	Used	
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
1 for print					

```
x
2
x
+
ST02
RCL4
RCL6
x
3
x
ST+3
RCL6
x
ST+2
3
÷
RCL6
x
ST+1
RCL5
RCL6
x
4
x
ST+4
RCL6
x
1
.
5
x
ST+3
LSTX
÷
RCL6
x
ST+2
4
÷
RCL6
x
ST+1
RCL8
CHS
ENT↑
ENT↑
ENT↑
ST×2
x
ST×3
x
ST×4
x
ST×5
R/S
*LBLc
RCL1
GSB9        Output d0, d1, d2, d3, d4.
R/S
RCL2
GSB9
R/S
RCL3
GSB9
R/S
RCL4
GSB9
R/S
RCL5
GSB9
R/S
RTN
*LBLd
GSB9        Input x, calculate ŷ.
ST07
RCL2
x
RCL7
X²
RCL3
x
+
RCL7
3
Y^x
RCL4
x
+
RCL7
X²
X²
RCL5
x
+
RCL1
+
GSB8
RTN
*LBL9       Subroutine for space and print.
F0?
SPC
*LBL8
F0?
PRTX
RTN
```

LABELS					FLAGS
A	B Set flag 0	C	D	E	0 Print
a (continue)	b $x_N \uparrow x_0$	c $\to d_0, d_1,$	d $x \to \hat{y}$	e	1
0	1	2	3	4	2
5	6	7	8 Print	9 Used	3

SET STATUS		
FLAGS (ON / OFF)	TRIG	DISP
0 ☐ ☒	DEG ☒	FIX ☒
1 ☐ ☒	GRAD ☐	SCI ☐
2 ☐ ☒	RAD ☐	ENG ☐
3 ☐ ☒		n 2

t Statistics

Step	Key	Comment
001	*LBLA	
002	CF0	
003	0	Initialize
004	STO1	
005	STO2	
006	STO3	
007	RTN	
008	*LBLC	
009	F0?	Input x_i, y_i for paired t statistic.
010	GSB9	
011	-	
012	ST+2	
013	X²	
014	ST+3	
015	RCL1	
016	1	
017	+	
018	STO1	
019	GSB0	
020	RTN	
021	*LBLE	
022	RCL2	$\bar{D}$, S_D, t_1, df_1
023	RCL1	
024	÷	
025	GSB1	
026	GSB0	
027	R/S	
028	RCL3	
029	RCL2	
030	X²	
031	RCL1	
032	÷	
033	-	
034	RCL1	
035	1	
036	-	
037	÷	
038	√X	
039	GSB0	
040	R/S	
041	RCL1	
042	√X	
043	÷	
044	÷	
045	GSB0	
046	R/S	
047	RCL1	
048	1	
049	-	
050	GSB0	
051	GSB1	
052	RTN	
053	*LBLD	
054	F0?	For correction of x_k, y_k.
055	GSB9	
056	-	
057	ST-2	
058	X²	
059	ST-3	
060	RCL1	
061	1	
062	-	
063	STO1	
064	GSB0	
065	RTN	
066	*LBLB	Set flag 0 for print.
067	SF0	
068	1	
069	RTN	
070	*LBL9	
071	X⇄Y	Print x_i, y_i.
072	SPC	
073	PRTX	
074	X⇄Y	
075	PRTX	
076	RTN	
077	*LBLa	
078	GSB0	Input x_i or y_i for t statistic for two means.
079	ST+2	
080	X²	
081	ST+3	
082	RCL1	
083	1	
084	+	
085	STO1	
086	GSB0	
087	GSB1	
088	RTN	
089	*LBLc	
090	STO7	
091	RCL1	
092	STO4	
093	RCL2	
094	STO5	
095	RCL3	
096	STO6	d
097	0	
098	STO1	
099	STO2	
100	STO3	
101	RCL7	
102	GSB0	
103	GSB1	
104	RTN	
105	*LBLd	
106	RCL6	
107	RCL5	
108	X²	
109	RCL4	
110	÷	
111	-	
112	RCL3	

REGISTERS

0	1	2	3	4	5	6	7	8	9
Used	n, n_1, n_2	Used	Used	n_1	Σx_i	Σx_i^2	d	n_1+n_2-2	
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I

Step	Key	Comment	Step	Key	Comment
113	+		169	*LBL1	
114	RCL2		170	F0?	
115	X²		171	SPC	Subroutine for space.
116	RCL1		172	RTN	
117	÷				
118	-				
119	RCL1				
120	RCL4				
121	+				
122	2	t_2, df_2			
123	-				
124	ST08				
125	÷				
126	√X				
127	1				
128	RCL1				
129	÷				
130	1				
131	RCL4				
132	÷				
133	+				
134	√X				
135	x				
136	RCL5				
137	RCL4				
138	÷				
139	RCL2				
140	RCL1				
141	÷				
142	-				
143	RCL7				
144	-				
145	X⇄Y				
146	÷				
147	GSB0				
148	R/S				
149	RCL8				
150	GSB0				
151	GSB1				
152	RTN				
153	*LBLb				
154	GSB0				
155	ST-2				
156	X²	For correction of x_k or y_k.			
157	ST-3				
158	RCL1				
159	1				
160	-				
161	ST01				
162	GSB0				
163	GSB1				
164	RTN				
165	*LBL0	Subroutine for print.			
166	F0?				
167	PRTX				
168	RTN				

LABELS

A Start	B Print	C $x_i \uparrow y_i$	D $x_k \uparrow y_k \rightarrow$	E $\rightarrow \bar{D}, S_D$ ...
a x_i or y_i	b x_k or y_k	c d→	d t_2 ; df_2	e
0 Print	1 Space	2	3	4
5	6	7	8	9 Print x_i, y_i

FLAGS

0	Print
1	
2	
3	

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☐
3	☐	☒			n	2

Chi-Square Evaluation

Key	Comments
*LBLA	
CF0	Initialize
CF1	
0	
STO1	
STO2	
STO3	
RTN	
*LBLC	
F0?	
GSB9	
STO3	Σ+
-	
X²	
RCL3	
÷	
ST+2	
RCL1	
1	
+	
STO1	
GSB8	
RTN	
*LBLE	
F1?	
GTO1	χ_1^2
RCL2	
GSB7	
GSB8	
GSB7	
RTN	
*LBL1	
1	
RCL1	χ_2^2
RCL3	
×	
RCL2	
÷	
RCL2	
-	
GSB7	
GSB8	
RTN	
*LBLd	
RCL2	
RCL1	
÷	
GSB8	
GSB7	
RTN	
*LBLD	For correction O_j, E_j.
F0?	
GSB9	
STO3	
-	
X²	
RCL3	
÷	
ST-2	
RCL1	
1	
-	
STO1	
GSB8	
RTN	
*LBLa	For equal expected frequency.
GSB8	
ST+2	
X²	
ST+3	
RCL1	
1	
+	
STO1	
GSB8	
GSB7	
SF1	
RTN	
*LBLb	For correction of O_j.
GSB8	
ST-2	
X²	
ST-3	
RCL1	
1	
-	
STO1	
GSB8	
GSB7	
RTN	
*LBLB	Set flag 0 for print.
SF0	
1	
RTN	
*LBL9	Print O_j, E_j.
SPC	
X⇄Y	
PRTX	
X⇄Y	
PRTX	
RTN	
*LBLc	Set flag 1 for equal expected frequency.
SF1	
GTOE	
RTN	
*LBL8	Subroutine for print x.
F0?	
PRTX	
RTN	
*LBL7	Subroutine for print.
F0?	
SPC	

REGISTERS

0	1 n	2 Used	3 Used	4	5	6	7	8	9
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I

113 RTN

LABELS					FLAGS
A Start	B Print	C $O_i \uparrow E_i \rightarrow$	D $O_k \uparrow E_k$	E χ_1^2	0 Print
a O_j (Σ+)	b O_h (Σ−)	c χ_2^2	d E	e	1 Equal E_i
0	1 χ_2^2	2	3	4	2
5	6	7 Space	8 Print	9 Print	3

SET STATUS		
FLAGS	TRIG	DISP
ON OFF 0 ☐ ☒ 1 ☐ ☒ 2 ☐ ☒ 3 ☐ ☒	DEG ☒ GRAD ☐ RAD ☐	FIX ☒ SCI ☐ ENG ☐ n 2

Contingency Table

Step	Key	Comment	Step	Key	Comment
001	*LBLA		057	*LBLa	
002	CLRG	Initialize	058	SF0	Set flag for print.
003	CF0		059	1	
004	CF1		060	RTN	Print x_{1j}, x_{2j}.
005	0		061	*LBL9	
006	RTN		062	X⇄Y	
007	*LBLB	Input x_{1j}, x_{2j} for 2 x k.	063	PRTX	
008	F0?		064	X⇄Y	
009	GSB9		065	PRTX	
010	0		066	RTN	
011	STO7		067	*LBLb	3 x k
012	R↓		068	F0?	
013	GSB0		069	GSB8	Input x_{1j}, x_{2j}, x_{3j}.
014	RTN		070	STOC	
015	*LBLD		071	F1?	
016	RCL0		072	CHS	
017	RCL4	2 x k	073	ST+3	
018	×	χ^2	074	STO0	
019	RCL1		075	STO7	
020	÷		076	ENT↑	
021	RCL0		077	×	
022	RCL5		078	STO8	
023	RCL2		079	R↓	
024	÷		080	*LBL0	Input x_{1j}, x_{2j}, for 3 x k and 2 x k.
025	×		081	STOB	
026	+		082	F1?	
027	RCL0		083	CHS	
028	-		084	ST+2	
029	STOD		085	ST+0	
030	GSB7		086	ST+7	
031	R/S		087	ENT↑	
032	RCLD	2 x k	088	×	
033	RCL0	C_c	089	STO9	
034	RCLD		090	R↓	
035	+		091	STOA	
036	÷		092	F1?	
037	√X		093	CHS	
038	GSB7		094	ST+1	
039	GSB6		095	ST+0	
040	RTN		096	ST+7	
041	*LBLE	2 x k	097	ENT↑	
042	RCL1	R_1, R_2, T	098	×	
043	GSB7		099	RCL7	
044	R/S		100	÷	
045	RCL2		101	ST+4	
046	GSB7		102	RCL9	
047	R/S		103	RCL7	
048	+		104	÷	
049	GSB7		105	ST+5	
050	GSB6		106	RCL8	
051	RTN		107	RCL7	
052	*LBLC	2 x k correction.	108	÷	
053	SF1		109	ST+6	
054	GSBB		110	1	
055	CF1		111	F1?	
056	RTN		112	CHS	

REGISTERS

0	1	2	3	4	5	6	7	8	9
T	R_1	R_2	R_3	$\Sigma^2\, x_{1j}/C_j$	$\Sigma^2\, x_{2j}/C_j$	$\Sigma^2\, x_{3j}/C_j$	C_j	x_{3j}^2	x_{2j}^2
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
x_{1j}	x_{2j}	x_{3j}	χ^2	k	

Step	Key	Comment
113	RCLE	
114	+	
115	STOE	
116	GSB7	
117	GSB6	
118	CF1	
119	R/S	
120	RCL7	
121	GSB7	
122	GSB6	
123	RTN	
124	*LBLd	
125	RCL1	
126	RCL2	
127	RCL3	
128	+	
129	+	
130	STO0	3 x k
131	RCL4	χ^2
132	RCL1	
133	÷	
134	STO9	
135	RCL5	
136	RCL2	
137	÷	
138	ST+9	
139	RCL6	
140	RCL3	
141	÷	
142	ST+9	
143	RCL9	
144	1	
145	-	
146	RCL0	
147	x	
148	GSB7	
149	R/S	
150	ENT↑	
151	ENT↑	3 x k
152	RCL0	C_c
153	+	
154	÷	
155	√X	
156	GSB7	
157	GSB6	
158	R/S	
159	*LBLe	3 x k
160	RCL1	R_1, R_2, R_3, T
161	GSB7	
162	R/S	
163	RCL2	
164	GSB7	
165	R/S	
166	RCL3	
167	GSB7	
168	R/S	
169	RCL0	
170	GSB7	
171	RTN	
172	*LBLc	3 x k correction.
173	SF1	
174	GSBb	
175	CF1	
176	RTN	
177	*LBLE	Print x_{1j}, x_{2j}, x_{3j}.
178	R↓	
179	R↓	
180	PRTX	
181	R↑	
182	PRTX	
183	R↑	
184	PRTX	
185	RTN	
186	*LBL7	Subroutine for print.
187	F0?	
188	PRTX	
189	RTN	
190	*LBL6	Subroutine for space.
191	F0?	
192	SPC	
193	RTN	

LABELS

A Start	B x_{1j}, x_{2j}	C Correction	D χ^2; C_c	E R_1; R_2; T
a Print	b Input 3 x k	c Correction	d χ^2; C_c	e R_1; R_2
0 Used	1	2	3	4
5	6 Space	7 Print	8 Print	9 Print

FLAGS

0	Print
1	Correction
2	
3	

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☐
3	☐	☒			n	2

Spearman's Rank Correlation Coefficient

Step	Instruction	Comment
001	*LBLA	
002	CLRG	
003	CF0	Initialize
004	CF1	
005	0	
006	RTN	
007	*LBLC	
008	F0?	
009	GSB9	Input R_i, S_i.
010	-	
011	X²	
012	F1?	
013	CHS	
014	ST+2	
015	RCL1	
016	1	
017	F1?	
018	CHS	
019	+	
020	STO1	
021	GSB0	
022	GSB1	
023	RTN	
024	*LBLE	
025	1	
026	RCL2	
027	6	
028	×	
029	RCL1	
030	X²	
031	1	
032	-	
033	RCL1	
034	×	
035	÷	
036	-	
037	GSB0	r_s, z
038	R/S	
039	RCL1	
040	1	
041	-	
042	√X	
043	×	
044	GSB0	
045	GSB1	
046	RTN	
047	*LBLD	
048	SF1	For correction.
049	GSBC	
050	CF1	
051	RTN	
052	*LBLB	
053	SF0	Set flag 0 for print.
054	1	
055	RTN	
056	*LBL9	
057	X⇄Y	
058	PRTX	
059	X⇄Y	Print R_i, S_i.
060	PRTX	
061	RTN	
062	*LBL0	Subroutine for print.
063	F0?	
064	PRTX	
065	RTN	Subroutine for space.
066	*LBL1	
067	F0?	
068	SPC	
069	RTN	

REGISTERS

0	1 n	2 ΣD_i^2	3	4	5	6	7	8	9
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A	B	C	D	E	I				

LABELS					FLAGS	SET STATUS		
A Start	B Print	C (Σ+)	D (Σ−)	E →r_s, z	0 Print	FLAGS	TRIG	DISP
a	b	c	d	e	1 (Σ−)	ON OFF 0 ☐ ☒ 1 ☐ ☒ 2 ☐ ☒ 3 ☐ ☒	DEG ☒ GRAD ☐ RAD ☐	FIX ☒ SCI ☐ ENG ☐ n 2
0	1	2	3	4	2			
5	6	7	8	9	3			

$\bar{x}$ and R Control Chart

Step	Instruction	Comment
001	*LBLA	
002	CLRG	
003	CF0	Initialize
004	CF1	
005	0	
006	RTN	Store 1 in R_E for print.
007	*LBLB	
008	1	
009	STOE	
010	RTN	
011	*LBLC	
012	STO0	
013	RCL4	
014	STOA	
015	RCL5	
016	STOB	
017	RCL0	
018	GSB9	
019	F1?	
020	GTO1	
021	0	Input x_{ij}.
022	STO1	
023	STO2	
024	STO3	
025	X⇄Y	
026	STO4	
027	STO5	
028	SF1	
029	*LBL1	
030	RCL4	
031	X⇄Y	
032	X>Y?	
033	STO4	
034	RCL5	
035	X⇄Y	
036	X≤Y?	
037	STO5	
038	F0?	
039	CHS	
040	ST+2	
041	X²	
042	F0?	
043	CHS	
044	ST+3	
045	RCL1	
046	1	
047	F0?	
048	CHS	
049	+	
050	STO1	
051	GSB9	
052	RTN	
053	*LBLE	
054	GSB7	x_{max}, x_{min}
055	RCL4	
056	GSB9	
057	R/S	
058	*LBLE	
059	RCL5	
060	GSB9	
061	RTN	
062	*LBLa	
063	CF1	
064	RCL6	
065	1	
066	+	
067	STO6	
068	RCL2	$\bar{x}_i$
069	RCL1	
070	÷	
071	GSB9	
072	ST+7	
073	R/S	
074	*LBLa	
075	RCL4	
076	RCL5	
077	-	R_i
078	ST+8	
079	GSB9	
080	GSB7	
081	R/S	
082	*LBLb	
083	RCL7	
084	RCL6	$\bar{\bar{x}}$
085	÷	
086	GSB9	
087	RTN	
088	*LBLb	
089	RCL8	
090	RCL6	
091	÷	
092	STO3	$\bar{R}$
093	GSB9	
094	GSB7	
095	R/S	
096	*LBLc	
097	RCL3	
098	x	
099	RCL7	$L_{\bar{x}}$
100	RCL6	
101	÷	
102	X⇄Y	
103	-	
104	GSB9	
105	R/S	
106	*LBLc	
107	LSTX	$U_{\bar{x}}$
108	2	
109	x	
110	+	
111	GSB9	
112	GSB7	

REGISTERS

0	1	2	3	4	5	6	7	8	9
x_{ij}	n	Σx_{ij}	Σx_{ij}^2, $\bar{R}$	x_{max}	x_{min}	m	$\Sigma \bar{x}_i$	ΣR_i	Used
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
Last x_{max}	Last x_{min}			1 for print	

Program	Comments
113 R/S	
114 *LBLd	
115 RCL3	L_R
116 x	
117 GSB9	
118 GSB7	
119 R/S	
120 *LBLe	
121 RCL3	U_R
122 x	
123 GSB9	
124 GSB7	
125 RTN	
126 *LBLD	Correction x_{ik}.
127 STO0	
128 RCLA	
129 STO4	
130 RCLB	
131 STO5	
132 RCL0	
133 SF0	
134 GSBC	
135 RCLA	
136 STO4	
137 RCLB	
138 STO5	
139 CF0	
140 RCL1	
141 RTN	
142 *LBL9	Subroutine for print.
143 RCLE	
144 X>0?	
145 GTO8	
146 R↓	
147 RTN	
148 *LBL8	
149 R↓	
150 PRTX	
151 RTN	
152 *LBL7	Subroutine for space.
153 RCLE	
154 X>0?	
155 SPC	
156 R↓	
157 RTN	

LABELS					FLAGS
A Start	B Print	C x_{ij} (Σ+)	D x_{ik} (Σ−)	E x_{max}, x_{min}	0 Correction
a $\bar{x}_{ij}$, R_i	b $\bar{\bar{x}}$, $\bar{R}$	c $L_{\bar{x}}$, $U_{\bar{x}}$	d L_R	e U_R	1 1st data
0	1 j > 1	2	3	4	2
5	6	7 Space	8 Print	9 Print?	3

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☐
3	☐	☒			n	2

Operating Characteristic Curves

001	*LBLA	
002	CF0	
003	CLRG	Initialize
004	0	
005	RTN	
006	*LBLB	Set flag 0 for print.
007	SF0	
008	1	
009	RTN	
010	*LBLC	Store N.
011	GSB3	
012	STO1	
013	RTN	
014	*LBLD	
015	STO3	Store n, c for finite lot size.
016	X⇄Y	
017	STO2	
018	GSB3	
019	X⇄Y	
020	GSB3	
021	RTN	
022	*LBLE	
023	GSB4	
024	GSB3	
025	RCL1	
026	×	
027	INT	
028	STO4	
029	RCL1	
030	RCL2	Finite lot size.
031	GSBa	
032	RCL1	
033	RCL4	
034	-	
035	RCL2	
036	GSBa	
037	R↑	
038	÷	
039	STO5	
040	STO6	
041	STO7	
042	RCL3	
043	0	
044	STO8	
045	X=Y?	
046	GSB5	
047	*LBL0	
048	RCL4	
049	-	
050	RCL8	
051	RCL2	
052	-	
053	×	
054	RCL8	
055	1	
056	+	

057	÷	
058	LSTX	
059	RCL1	
060	RCL4	
061	-	
062	RCL2	
063	-	
064	+	
065	÷	
066	RCL6	
067	×	
068	STO6	
069	ST+7	
070	RCL3	
071	1	
072	RCL8	
073	+	
074	STO8	
075	X≠Y?	
076	GTO0	
077	1	
078	RCL7	
079	X>Y?	
080	X⇄Y	
081	GSB3	
082	R/S	
083	*LBLa	
084	-	
085	LSTX	
086	X≤Y?	
087	GSB9	
088	STO5	
089	1	
090	STO7	
091	+	
092	STO6	
093	CLX	
094	X=Y?	
095	GTO8	
096	*LBL1	
097	R↓	
098	1	
099	RCL7	
100	+	
101	STO7	
102	X>Y?	
103	GTO7	
104	RCL5	
105	X⇄Y	
106	+	
107	LSTX	
108	÷	
109	RCL6	
110	×	
111	STO6	
112	GTO1	

REGISTERS

0	1 N, n	2 n, p	3 c, f(0)	4 M	5 f(0)	6 , c	7 , counter	8 Used	9
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A	B	C	D	E	I				

Step	Key	Comments
113	RTN	
114	*LBL7	
115	RCL6	
116	RTN	
117	*LBL8	
118	1	
119	GSB3	
120	RTN	
121	*LBL9	
122	STO6	
123	X⇄Y	
124	RTN	
125	*LBLd	Input n, c for infinite lot size.
126	STO6	
127	X⇄Y	
128	STO1	
129	GSB3	
130	X⇄Y	
131	GSB3	
132	RTN	
133	*LBLe	
134	STO2	
135	GSB4	
136	GSB3	
137	STO2	
138	RCL2	
139	1	
140	-	
141	CHS	
142	÷	
143	STO8	
144	LSTX	
145	RCL1	
146	Y^X	
147	STO3	
148	RCL6	
149	0	
150	STO7	
151	X=Y?	
152	GSB6	
153	CLX	
154	RCL1	
155	X⇄Y	
156	X>Y?	
157	GTOb	
158	RCL3	
159	STO4	
160	STO5	
161	*LBL2	
162	RCL1	
163	RCL7	
164	-	
165	RCL7	
166	1	
167	+	
168	÷	
169	RCL8	
170	×	
171	RCL4	
172	×	
173	STO4	
174	ST+5	
175	RCL7	
176	1	
177	+	
178	STO7	
179	RCL6	
180	X≠Y?	
181	GTO2	
182	1	
183	RCL5	
184	X>Y?	
185	X⇄Y	
186	GSB3	
187	R/S	
188	*LBL6	
189	RCL3	
190	GSB3	
191	R/S	
192	RTN	
193	*LBL5	
194	RCL5	
195	GSB3	
196	R/S	
197	RTN	
198	*LBL3	Subroutine for print.
199	F0?	
200	PRTX	
201	RTN	
202	*LBL4	Subroutine for space.
203	F0?	
204	SPC	
205	RTN	

LABELS					FLAGS	SET STATUS		
A Start	B Print	C N	D n↑c→	E p→P_a	0 Print	FLAGS (ON OFF)	TRIG	DISP
a f(x)	b	c	d n↑c→	e p→P_a	1	0 ☐ ☒	DEG ☒	FIX ☒
0 P_a	1 f(x + 1)	2 Used	3 Print	4 Space	2	1 ☐ ☒	GRAD ☐	SCI ☐
5 c, f(0)	6 f(0)	7 c	8 1	9 STO 6	3	2 ☐ ☒	RAD ☐	ENG ☐
						3 ☐ ☒		n 2

Single- and Multi-Server Queues

Step	Key	Comment
001	*LBLA	
002	GSB9	
003	STO1	
004	STOI	
005	R↓	ρ
006	STO2	
007	X⇄Y	
008	STO5	
009	÷	
010	STO3	
011	PRTX	
012	R/S	
013	*LBLB	
014	1	
015	STO4	
016	0	
017	*LBL1	
018	RCL4	
019	+	
020	LSTX	
021	RCL3	
022	×	
023	RCL1	
024	RCLI	
025	-	
026	1	
027	+	
028	÷	
029	STO4	P_0, P_b
030	R↓	
031	DSZI	
032	GTO1	
033	1	
034	RCL3	
035	RCL1	
036	÷	
037	-	
038	RCL4	
039	X⇄Y	
040	÷	
041	STOI	
042	+	
043	1/X	
044	SPC	
045	PRTX	
046	R/S	
047	*LBLB	
048	RCLI	
049	×	
050	STOI	
051	PRTX	
052	R/S	
053	*LBLC	
054	RCLI	
055	RCL3	
056	×	
057	RCL1	
058	RCL3	
059	-	
060	÷	
061	STO4	Lq, L
062	SPC	
063	PRTX	
064	R/S	
065	*LBLC	
066	RCL3	
067	+	
068	STO6	
069	PRTX	
070	R/S	
071	*LBLD	
072	RCL4	
073	RCL2	Tq, T
074	÷	
075	SPC	
076	PRTX	
077	R/S	
078	*LBLD	
079	RCL6	
080	RCL2	
081	÷	
082	PRTX	
083	R/S	
084	*LBLE	
085	SPC	P(t)
086	PRTX	
087	RCL1	
088	RCL5	
089	×	
090	RCL2	
091	-	
092	×	
093	CHS	
094	e^x	
095	RCLI	
096	×	
097	PRTX	
098	SPC	
099	R/S	
100	*LBL9	Print u, λ, n.
101	R↓	
102	R↓	
103	SPC	
104	PRTX	
105	R↑	
106	PRTX	
107	R↑	
108	PRTX	
109	RTN	
110	*LBLa	m, n
111	GSB8	
112	STO2	

REGISTERS

0	1 n, m	2 λ, n	3 ρ	4 Lq, k	5 μ, Q_k, L	6 L, ΣQ_k	7 ΣkQ_k, −F	8 a	9 Used
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
					Used, P_b

Step	Key	Comment
113	R↓	
114	STO1	
115	R/S	
116	*LBLb	ρ
117	GSB8	
118	STO8	
119	÷	
120	STO3	
121	PRTX	
122	R/S	
123	*LBLc	
124	CLX	
125	STO7	
126	1	
127	STO4	
128	STO5	
129	STO6	
130	*LBL3	
131	RCL2	
132	RCL4	
133	X>Y?	
134	X⇄Y	
135	RCL3	
136	X⇄Y	
137	÷	
138	RCL1	
139	RCL4	
140	-	
141	1	L
142	+	
143	x	
144	RCL5	
145	x	
146	STO5	
147	EEX	
148	CHS	
149	9	
150	0	
151	X>Y?	
152	GTO2	
153	R↓	
154	ST+6	
155	RCL4	
156	x	
157	ST+7	
158	RCL1	
159	RCL4	
160	1	
161	+	
162	STO4	
163	X≤Y?	
164	GTO3	
165	*LBL2	
166	RCL7	
167	RCL6	
168	÷	
169	STO5	
170	SPC	
171	PRTX	
172	R/S	
173	*LBLc	T
174	RCL8	
175	x	
176	PRTX	
177	R/S	
178	*LBLd	L_q
179	RCL5	
180	RCL1	
181	÷	
182	1	
183	-	
184	RCL3	
185	1	
186	+	
187	x	
188	STO7	
189	1	
190	+	
191	RCL1	
192	x	
193	SPC	
194	PRTX	
195	R/S	
196	*LBLd	T_q
197	RCL8	
198	x	
199	PRTX	
200	R/S	
201	*LBLe	
202	RCL7	
203	CHS	F
204	SPC	
205	PRTX	
206	SPC	
207	R/S	
208	*LBL8	Print m, n, s, a.
209	R↓	
210	SPC	
211	PRTX	
212	R↑	
213	PRTX	
214	RTN	

LABELS

A … →ρ	B →P_o, →P_b	C →L_q, →L	D →T_q, →T	E t→P(t)
a m↑n→	b S↑a→ρ	c →L, →T	d →L_q, →T_q	e →F
0	1 P_o, P_b	2 L	3 K	4
5	6	7	8 Print	9 Print

FLAGS

0	
1	
2	
3	

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☐
3	☐	☒			n	2

Appendix A
MAGNETIC CARD SYMBOLS AND CONVENTIONS

SYMBOL OR CONVENTION	INDICATED MEANING
White mnemonic: x A	White mnemonics are associated with the user-definable key they are above when the card is inserted in the calculator's window slot. In this case the value of x could be input by keying it in and pressing A.
Gold mnemonic: y x f E	Gold mnemonics are similar to white mnemonics except that the gold f key must be pressed before the user-definable key. In this case y could be input by pressing f E.
x ↑ y A	↑ is the symbol for ENTER↑. In this case ENTER↑ is used to separate the input variables x and y. To input both x and y you would key in x, press ENTER↑, key in y and press A.
[x] A	The box around the variable x indicates input by pressing STO A.
(x) A	Parentheses indicate an option. In this case, x is not a required input but could be input in special cases.
➔x A	➔ is the symbol for calculate. This indicates that you may calculate x by pressing key A.
➔x, y, z A	This indicates that x, y, and z are calculated by pressing A once. The values would be printed in x, y, z order.
➔x; y; z A	The semi-colons indicate that after x has been calculated using A, y and z may be calculated by pressing R/S.
➔"x ", y A	The quote marks indicate that the x value will be "paused" or held in the display for one second. The pause will be followed by the display of y.
◁▷ x A	The two-way arrow ◁▷ indicates that x may be either output or input when the associated user-definable key is pressed. If numeric keys have been pressed between user-definable keys, x is stored. If numeric keys have not been pressed, the program will calculate x.

SYMBOLS AND CONVENTIONS (Continued)

SYMBOL OR CONVENTION	INDICATED MEANING
P? [A]	The question mark indicates that this is a mode setting, while the mnemonic indicates the type of mode being set. In this case a print mode is controlled. Mode settings typically have a 1.00 or 0.00 indicator displayed after they are executed. If 1.00 is displayed, the mode is on. If 0.00 is displayed, it is off.
START [A]	The word START is an example of a command. The start function should be performed to begin or start a program. It is included when initialization is necessary.
DEL [A]	This special command indicates that the last value or set of values input may be deleted by pressing [A].
→x; ... [A]	Three dots (...) indicate that additional output follows. See User Instructions for complete description of variables output.

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A ● C D E