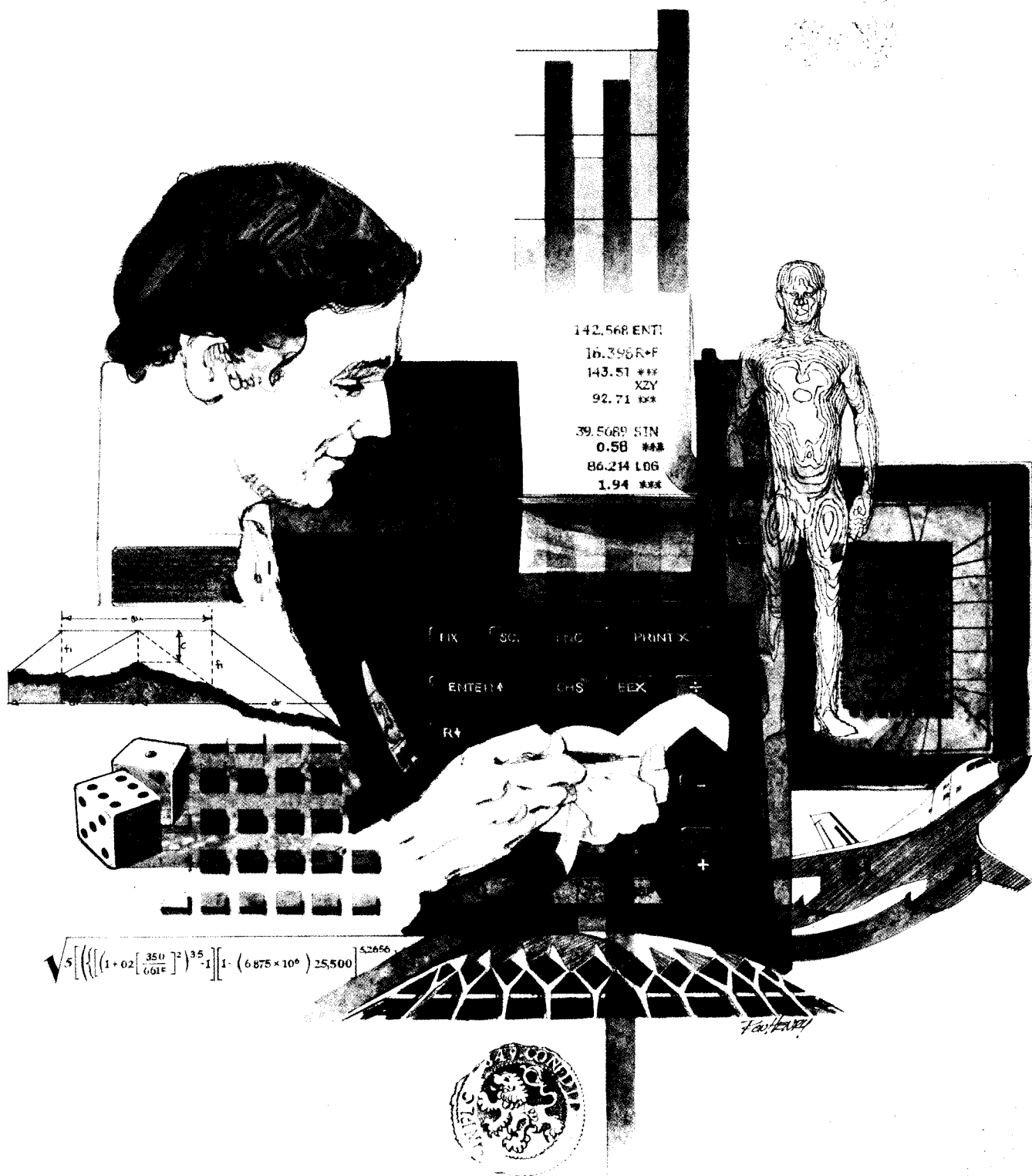


HEWLETT-PACKARD

# HP-67/HP-97

Users' Library Solutions  
Control Systems



## INTRODUCTION

In an effort to provide continued value to its customers, Hewlett-Packard is introducing a unique service for the HP fully programmable calculator user. This service is designed to save you time and programming effort. As users are aware, Programmable Calculators are capable of delivering tremendous problem solving potential in terms of power and flexibility, but the real genie in the bottle is program solutions. HP's introduction of the first handheld programmable calculator in 1974 immediately led to a request for program **solutions** — hence the beginning of the HP-65 Users' Library. In order to save HP calculator customers time, users wrote their own programs and sent them to the Library for the benefit of other program users. In a short period of time over 5,000 programs were accepted and made available. This overwhelming response indicated the value of the program library and a Users' Library was then established for the HP-67/97 users.

To extend the value of the Users' Library, Hewlett-Packard is introducing a unique service—a service designed to save you time and money. The Users' Library has collected the best programs in the most popular categories from the HP-67/97 and HP-65 Libraries. These programs have been packaged into a series of low-cost books, resulting in substantial savings for our valued HP-67/97 users.

We feel this new software service will extend the capabilities of our programmable calculators and provide a great benefit to our HP-67/97 users.

## A WORD ABOUT PROGRAM USAGE

Each program contained herein is reproduced on the standard forms used by the Users' Library. Magnetic cards are not included. The Program Description I page gives a basic description of the program. The Program Description II page provides a sample problem and the keystrokes used to solve it. The User Instructions page contains a description of the keystrokes used to solve problems in general and the options which are available to the user. The Program Listing I and Program Listing II pages list the program steps necessary to operate the calculator. The comments, listed next to the steps, describe the reason for a step or group of steps. Other pertinent information about data register contents, uses of labels and flags and the initial calculator status mode is also found on these pages. Following the directions in your HP-67 or HP-97 **Owners' Handbook and Programming Guide**, "Loading a Program" (page 134, HP-67; page 119, HP-97), key in the program from the Program Listing I and Program Listing II pages. A number at the top of the Program Listing indicates on which calculator the program was written (HP-67 or HP-97). If the calculator indicated differs from the calculator you will be using, consult Appendix E of your **Owner's Handbook** for the corresponding keycodes and keystrokes converting HP-67 to HP-97 keycodes and vice versa. No program conversion is necessary. The HP-67 and HP-97 are totally compatible, but some differences do occur in the keycodes used to represent some of the functions.

A program loaded into the HP-67 or HP-97 is not permanent—once the calculator is turned off, the program will not be retained. You can, however, permanently save any program by recording it on a blank magnetic card, several of which were provided in the Standard Pac that was shipped with your calculator. Consult your **Owner's Handbook** for full instructions. A few points to remember:

The Set Status section indicates the status of flags, angular mode, and display setting. After keying in your program, review the status section and set the conditions as indicated before using or permanently recording the program.

**REMEMBER!** To save the program permanently, **clip** the corners of the magnetic card once you have recorded the program. This simple step will protect the magnetic card and keep the program from being inadvertently erased.

As a part of HP's continuing effort to provide value to our customers, we hope you will enjoy our newest concept.

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# Program Description I

1

**Program Title** Frequency Response of a Transfer Function

**Contributor's Name** Hewlett-Packard

**Address** 1000 N.E. Circle Blvd.

**City** Corvallis

**State** Oregon

**Zip Code** 97330

**Program Description, Equations, Variables** For transfer function of the form:

$$G(S) = \frac{K_1 (\tau_2 S + 1)}{S^{N_3} (\tau_4 S + 1) (\tau_5 S + 1) \left( \frac{S^2}{\omega_7^2} + \frac{2\zeta_6 S}{\omega_7} + 1 \right)}$$

The program computes  $\angle G(j\omega)$ ,  $|G(j\omega)|$  and  $\log |G(j\omega)|$  for any input frequency  $\omega$ .

$\angle G(j\omega)$  is displayed upon completion of the calculation.

$|G(j\omega)|$  is stored in the stack (z and T) and in Register 8

$\log |G(j\omega)|$  is stored in the stack (y) and in register 9

Parameters  $K_1, \tau_2, N_3, \tau_4, \tau_5, \zeta_6$  and  $\omega_7$  are stored in registers 1, 2, 3, 4, 5, 6 and 7 respectively.

## Operating Limits and Warnings

For type 0 systems enter  $N_3=0$

$\tau_2, \tau_4$  and/or  $\tau_5$  can be entered as 0. If there is no quadratic term enter  $\zeta_6$  as 0 and  $\omega_7$  very large compared to  $\frac{1}{\tau_5}$ . Where  $\tau_5$  is the smallest (other than zero) first order term used.

This program has been verified only with respect to the numerical example given in *Program Description II*. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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# Program Description II

Sketch(es)

Sample Problem(s) Find  $\angle G(j\omega)$ ,  $|G(j\omega)|$  and  $\log |G(j\omega)|$  for

$$G(s) = \frac{12(s+.6)}{s(s+1)(s^2+6s+36)} = \frac{.2(1.67s + 1)}{s(s+1)\left(\frac{s^2}{36} + \frac{2 \times .5}{6}s + 1\right)}$$

For input frequencies of  $\omega = .01, .1, 1.0$  and  $10$  rad/sec.

$$K_1 = .2$$

$$\tau_2 = 1.67$$

$$N_3 = 1$$

$$\tau_4 = 1$$

$$\tau_5 = 0$$

$$\zeta_6 = .5$$

$$\omega_7 = 6$$

Solution(s)

$\omega$ (rad/sec)	$\angle G(j\omega)$ degrees	$ G(j\omega) $	$\log  G(j\omega) $
.01	-89.71	20.00	1.30 (26.02dB)
.1	-87.18	2.02	.30 (6.10dB)
1.0	-85.64	.28	-.55 (-11.09dB)
10.0	-224.56	.01	-1.86 (-37.29dB)

Reference(s) Automatic Control Engineering, F.H. Raven, McGraw-Hill, N.Y., 1968

This program is a translation of the HP-65 Users' Library program #00834A submitted by Eugene Bahniuk.

1 FREQUENCY RESPONSE 2

Param.  $\omega$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1.	Enter program		<input type="text"/>	<input type="text"/>	
2.	Initialize		<input type="text"/>	<input type="text"/>	
			<input type="text"/>	<input type="text"/>	
3.	Enter the gain	$k_1$	<input type="text"/>	<input type="text"/>	2.00 7.00
	Enter $\Gamma_2$ when 2.00 is displayed	$\Gamma_2, N_3$	<input type="text"/>	<input type="text"/>	2.00→7.00
	Repeat parameter entries in order $N_3, \Gamma_4, \Gamma_5$	$\Gamma_4, \Gamma_5, \zeta_6$	<input type="text"/>	<input type="text"/>	
	$\zeta_6$ and $\omega_7$	$\omega_7$	<input type="text"/>	<input type="text"/>	
			<input type="text"/>	<input type="text"/>	
4.	When 8.00 is displayed enter the frequency (rad/sec)	$\omega$	<input type="text"/>	<input type="text"/>	8.00
			<input type="text"/>	<input type="text"/>	
			<input type="text"/>	<input type="text"/>	
5.	Upon completion of the calculations / $G(j\omega)$ will be display		<input type="text"/>	<input type="text"/>	/ $G(j\omega)$
			<input type="text"/>	<input type="text"/>	
			<input type="text"/>	<input type="text"/>	
6.	Amplitude ratio $ G(j\omega) $ is stored in the stack (z and T) and in register 8		<input type="text"/>	<input type="text"/>	
	To find $ G(j\omega) $ REC 8		<input type="text"/>	<input type="text"/>	
			<input type="text"/>	<input type="text"/>	
7.	$\log G(j\omega) $ is in stack y and in register 9		<input type="text"/>	<input type="text"/>	
	To calculate decibels - REC 9, enter 20 and multiply		<input type="text"/>	<input type="text"/>	
			<input type="text"/>	<input type="text"/>	
8.	For another frequency go to Step 4		<input type="text"/>	<input type="text"/>	
			<input type="text"/>	<input type="text"/>	
9.	To change transfer function parameters enter all parameters or if only one parameter is to be changed enter the changed parameter(s) in the proper register(s) For $\Gamma_4$ in register 4 and so on.	$K_1, \Gamma_2, N_3$ $\Gamma_4, \Gamma_5, \zeta_6, \omega_7$	<input type="text"/>	<input type="text"/>	1.00 thru 7.00
			<input type="text"/>	<input type="text"/>	
			<input type="text"/>	<input type="text"/>	
			<input type="text"/>	<input type="text"/>	
10.	For frequency response of the new transfer function enter $\omega$ when 8.00 is displayed (if only one parameter was changed first enter B)	$\omega$	<input type="text"/>	<input type="text"/>	8.00
			<input type="text"/>	<input type="text"/>	
			<input type="text"/>	<input type="text"/>	
			<input type="text"/>	<input type="text"/>	
			<input type="text"/>	<input type="text"/>	
			<input type="text"/>	<input type="text"/>	

## 97 Program Listing I

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11		057	RCL3	36 03	
002	1	01		058	9	09	
003	STOI	35 46		059	0	00	
004	*LBL9	21 09	Start of parameter	060	x	-35	
005	RCL1	36 46	entry display 1.00	061	+	-55	
006	R/S	51	and stop for entry	062	CHS	-22	
007	STOI	35 45	of $K_1$ Store $K_1$	063	RCL2	36 02	
008	ISZI	16 26 46		064	RCL9	36 08	
009	RCL1	36 46	Disply 2.00 and	065	x	-35	
010	9	09	stop for entry of $\tau_2$	066	RCL1	36 01	
011	X>Y?	16-34	etc.	067	x	-35	
012	GT09	22 09		068	RCL1	36 01	
013	RCL8	36 08	Compute $\angle G(j\omega)$ ,	069	+P	34	
014	*LBL6	21 12	$\angle G(j\omega)$ and $\log G(j\omega) $	070	R↑	16-31	Replaces $\omega$ with
015	STO3	35 08		071	=	-24	$ G(j\omega) $
016	RCL6	36 06		072	STO8	35 08	
017	RCL7	36 07		073	ENT↑	-21	Stores $\log G(j\omega) $
018	=	-24		074	LOG	16 32	
019	2	02	This is $2\zeta_6\omega/\omega_7$	075	STO9	35 09	
020	x	-35		076	R↑	16-31	Computes $\angle G(j\omega)$
021	RCL8	36 08		077	R↑	16-31	
022	x	-35		078	+	-55	
023	1	01		079	RTN	24	
024	RCL8	36 08		080	R/S	51	
025	RCL7	36 07					
026	=	-24					
027	ENT↑	-21	Results in $1-\omega^2/\omega_7^2$				
028	x	-35					
029	-	-45					
030	+P	34					
031	X<Y	-41					
032	RCL5	36 05					
033	RCL8	36 08					
034	x	-35		090			
035	1	01					
036	+P	34					
037	R↓	-31					
038	+	-55					
039	R↓	-31					
040	x	-35					
041	R↑	16-31					
042	RCL4	36 04					
043	RCL8	36 08					
044	x	-35		100			
045	1	01					
046	+P	34					
047	R↑	16-31					
048	x	-35					
049	R↓	-31					
050	+	-55					
051	R↑	16-31					
052	RCL8	36 08					
053	RCL3	36 03		110			
054	Y*	31					
055	x	-35					
056	X<Y	-41					

**REGISTERS**

0	1	2	3	4	5	6	7	8	9
	$K_1$	$\tau_2$	$N_3$	$\tau_4$	$\tau_5$	$\zeta_6$	$\omega_7$	$ G(j\omega) $	$\log G(j\omega) $
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

**SET STATUS**

FLAGS	TRIG	DISP
ON OFF		
0 <input type="checkbox"/> <input checked="" type="checkbox"/>	DEG <input checked="" type="checkbox"/>	FIX <input checked="" type="checkbox"/>
1 <input type="checkbox"/> <input checked="" type="checkbox"/>	GRAD <input type="checkbox"/>	SCI <input type="checkbox"/>
2 <input type="checkbox"/> <input checked="" type="checkbox"/>	RAD <input type="checkbox"/>	ENG <input type="checkbox"/>
3 <input type="checkbox"/> <input checked="" type="checkbox"/>		n <u>2</u>

# Program Description I

**Program Title** BODE OF TRANSFER FUNCTION THAT HAS EACH POLE AND ZERO GIVEN

**Contributor's Name** Hewlett-Packard Company

**Address** 1000 N.E. Circle Boulevard

**City** Corvallis

**State** Oregon

**Zip Code** 97330

## Program Description, Equations, Variables

Given a laplace transfer function:

$$F(s) = K \frac{(s+z_1+jz_1')(s+z_2+jz_2') \cdots (s+z_N+jz_N')}{(s+p_1+jp_1')(s+p_2+jp_2') \cdots (s+p_M+jp_M')}$$

$$F(j\omega) = \text{Magnitude} \angle \theta$$

$$\text{where magnitude} = |K| \frac{\sqrt{(z_1'+\omega)^2+z_1'^2} \sqrt{(z_2'+\omega)^2+z_2'^2} \cdots \sqrt{(z_N'+\omega)^2+z_N'^2}}{\sqrt{(p_1'+\omega)^2+p_1'^2} \sqrt{(p_2'+\omega)^2+p_2'^2} \cdots \sqrt{(p_M'+\omega)^2+p_M'^2}}$$

$$\text{and } \theta = \tan^{-1} \frac{z_1'+\omega}{z_1} + \tan^{-1} \frac{z_2'+\omega}{z_2} + \cdots + \tan^{-1} \frac{z_N'+\omega}{z_N}$$

$$-\tan^{-1} \frac{p_1'+\omega}{p_1} + \tan^{-1} \frac{p_2'+\omega}{p_2} + \cdots + \tan^{-1} \frac{p_M'+\omega}{p_M} + 2 \frac{|K|-K}{K} \tan^{-1} 1$$

also decibels = 20 log<sub>10</sub> magnitude

N.B. zero<sub>i</sub> = -(z<sub>i</sub>+jz<sub>i</sub>')  
 pole<sub>i</sub> = -(p<sub>i</sub>+jp<sub>i</sub>')  
 -----  
 -----

## Operating Limits and Warnings

K must be real and non-zero but may be negative.

About 4 seconds of computing time is required for each pole and each zero,  
 whereas 00361A takes half as long.

This program has been verified only with respect to the numerical example given in *Program Description II*. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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# Program Description II

## Sketch(es)

## Sample Problem(s)

Find gain ( in decibels and volts/volt) and phase shift in degrees

For  $F(s) = 0.09091 \frac{s + 62890}{s + 5717}$  at 3333 Hz.

## Solution(s)

3333 [A]	→ 0.
62890 [↑] 0 [B]	→ 1.
5717 [↑] 0 [C]	→ -1.
.09091 [R/S]	→ -1.
[D]	→ -11.1 Decibels
[R/S]	→ 2.77590 ( $10^{-1}$ ) Volts/Volt
[E]	→ -56.3 Degrees

## Reference(s)

HP-65 Program 00361A

This program is a translation of the HP-65 Users' Library Program  
#02582A submitted by E. P. Sansing.



# 97 Program Listing I

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11	Frequency input	057	X*Y	-41	
002	CLRG	16-53	Subroutine	058	+P	34	
003	ST06	35 06		059	GT00	22 00	
004	2	02	Convert Hertz to	060	*LBLD	21 14	Subroutine to
005	X	-35	routine/second	061	RCL1	36 01	compute decibels 12
006	Pi	16-24		062	LOG	16 32	
007	X	-35		063	2	02	
008	ST05	35 05	Store rads/sec k	064	0	00	
009	1	01	will default to 1	065	X	-35	
010	ST01	35 01	unless step 9 used	066	DSP1	-63 01	
011	RCL2	36 02		067	R/S	51	
012	FIX	-11		068	RCL1	36 01	Display magnitude
013	DSP0	-63 00		069	DSP5	-63 05	
014	R/S	51	Display 0.	070	SCI	-12	
015	RCL6	36 06	If input was	071	RTN	24	
016	ST05	35 05	rads/sec recall	072	*LBLE	21 15	Subroutine to call
017	RCL1	36 01	input store rads/	073	RCL2	36 02	phase angle
018	RTN	24	sec display 1.	074	DSP1	-63 01	
019	*LBLB	21 12	Subroutine for	075	FIX	-11	
020	RCL3	36 03	zero inputs	076	RTN	24	
021	1	01	Increment N counter	077	R/S	51	
022	+	-55					
023	ST03	35 03					
024	ST07	35 07					
025	R↓	-31					
026	RCL5	36 05					
027	+	-55					
028	X*Y	-41					
029	+P	34					
030	GT00	22 00					
031	*LBLC	21 13	Subroutine for pole				
032	RCL4	36 04	inputs				
033	1	01	Decrement -M pole				
034	-	-45	inputs	090			
035	ST04	35 04					
036	ST07	35 07					
037	R↓	-31					
038	RCL5	36 05					
039	+	-55					
040	CHS	-22					
041	X*Y	-41					
042	+P	34					
043	1/X	52					
044	*LBL0	21 00	Subroutine to	100			
045	STx1	35-35 01	update magnitude				
046	R↓	-31	and angle				
047	RCL2	36 02					
048	+	-55					
049	1	01					
050	+R	44					
051	+P	34					
052	R↓	-31					
053	ST02	35 02					
054	RCL7	36 07	Display N or -M	110			
055	R/S	51					
056	0	00					

SET STATUS			
FLAGS		TRIG	DISP
0	<input type="checkbox"/> ON <input checked="" type="checkbox"/> OFF	DEG <input checked="" type="checkbox"/>	FIX <input checked="" type="checkbox"/>
1	<input type="checkbox"/> <input checked="" type="checkbox"/>	GRAD <input type="checkbox"/>	SCI <input type="checkbox"/>
2	<input type="checkbox"/> <input checked="" type="checkbox"/>	RAD <input type="checkbox"/>	ENG <input type="checkbox"/>
3	<input type="checkbox"/> <input checked="" type="checkbox"/>		n 2

REGISTERS									
0	magnitude M	2 phase ang. θ	3 zero counts i < N	4 pole counts -i > -M	5 radians per second	6 first input, frequency	7 last updated (N or -M)	8	9
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A	B	C	D	E	I				

# Program Description I

9

**Program Title** BODE OF SECOND-ORDER OVER THIRD-ORDER TRANSFER FUNCTION

**Contributor's Name** Hewlett-Packard Company

**Address** 1000 N.E. Circle Boulevard

**City** Corvallis

**State** Oregon

**Zip Code** 97330

## Program Description, Equations, Variables

$$F(s) = \frac{Qs^3 + Gs^2 + Hs + K}{Rs^4 + Ls^3 + Ms^2 + Ns + P}$$

where Q and R are options, limited to [-1, 0, +1]  
G, H, L, M, N, and P may assume any real value

$$|\text{magnitude}| = \frac{\sqrt{(K-G\omega^2)^2 + (H\omega-Q\omega^2)^2}}{\sqrt{(P-M\omega^2+R\omega^4)^2 + (N\omega-L\omega^3)^2}}$$

where  $\omega = 2\pi F$

$$\text{Decibels} = 20 \log_{10} |\text{magnitude}|$$

$$\text{Phase} = \tan^{-1} \frac{H\omega-Q\omega^3}{K-G\omega^2} - \tan^{-1} \frac{N\omega-L\omega^3}{P-M\omega^2+R\omega^4}$$

in degrees (unless angular mode is set to radians or grads).

**Operating Limits and Warnings** Frequency, F, can not equal zero. Coefficient of  $s^3$  in numerator is -1, 0, or +1. Coefficient of  $s^4$  in denominator is -1, 0, or +1. Display of phase will be between  $-180^\circ$  and  $+180^\circ$  ( $-\pi$  and  $+\pi$  radians or -200 and +200 grads).

This program has been verified only with respect to the numerical example given in *Program Description II*. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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# Program Description II

## Sketch(es)

## Sample Problem(s)

Find gain (in decibels and volts/volt) and phase shift in degrees

For  $F(s) = \frac{8s^3-1}{3s^3+8}$  at DC and  $f=0.1$  Hz

Option 3 (see page 4) is required but the coefficient of  $s^3$  in the numerator can be only -1, 0, or +1.

Rewritten,  $F(s) = \frac{s^3-0.125}{0.375s^3+1}$

$G = 0$

$H = 0$

$k = -0.125$

$L = 0.375$

$M = 0$

$N = 0$

$p = 1$

$f = 10^{-50}$  (zero is not allowed) and 0.1

**Solution(s)** Switch to Run: [GTO] 1 Switch to w/prgm: [g][DEL][RCL] 8[-]

Switch to Run: 0[↑] 0[↑] .125[CHS][A] → 0.00

.375 [↑] 0 [↑] 0 [↑] 1 [B] → 0.38

[EEX] 50 [CHS][D] →

[R/S] →

[E] →

.1 [D] →

[R/S] →

[E] →

$$F = 0 \text{ Hz} \begin{cases} \text{Gain} = -18.06 \text{ db} \\ \text{Gain} = 0.13 \text{ volts/volt} \\ \text{Phase} = 180.00 \text{ degrees} \end{cases}$$

$$F = 0.1 \text{ Hz} \begin{cases} \text{Gain} = -11.16 \text{ db} \\ \text{Gain} = 0.28 \text{ volts/volt} \\ \text{Phase} = -111.43 \text{ degrees} \end{cases}$$

**Reference(s)** This program is a translation of the HP-65 Users' Library Program #02272A submitted by Edward P. Sansing.

## 11

[illegible]



# 97 Program Listing I

13

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS	
001	*LBLA	21 11	Numerator coefficients	057	XZY	-41	Compute Phase	
002	ST03	35 03		058	=	-24		
003	R↓	-31		059	ENT↑	-21		
004	ST02	35 02		060	LOG	16 32		
005	R↓	-31		061	2	02		
006	ST01	35 01		062	0	00		
007	RTN	24		063	x	-35		
008	*LBLB	21 12		Denominator coefficients	064	R/S		51
009	ST07	35 07			065	XZY		-41
010	R↓	-31			066	RTN		24
011	ST06	35 06	067		*LBLE	21 15		
012	R↓	-31	068		R↑	16-31		
013	ST05	35 05	069		R↑	16-31		
014	R↓	-31	070		RCL9	36 09		
015	ST04	35 04	071		-	-45		
016	RTN	24	072		CHS	-22		
017	*LBLD	21 14	Convert degrees to radians		073	LSTX		16-63
018	2	02		074	XZY	-41		
019	x	-35		075	1	01		
020	Pi	16-24		076	→R	44		
021	x	-35		077	→F	34		
022	ST08	35 08		Compute  F(s)	078	R↓		-31
023	RCL2	36 02			079	R/S		51
024	RCL8	36 08			080	R↓		-31
025	=	-24			081	RTN		24
026	*LBL1	21 01			082	R/S		51
027	RCL3	36 03						
028	RCL8	36 08						
029	X²	53						
030	=	-24						
031	RCL1	36 01						
032	-	-45						
033	→P	34						
034	ST09	35 09	090					
035	R↓	-31						
036	RCL6	36 06						
037	RCL8	36 08						
038	=	-24						
039	RCL4	36 04						
040	RCL8	36 08						
041	x	-35						
042	-	-45						
043	RCL7	36 07						
044	RCL8	36 08	100					
045	X²	53						
046	=	-24						
047	*LBL2	21 02						
048	RCL5	36 05						
049	-	-45						
050	RCL9	36 09						
051	R↓	-31						
052	→P	34						
053	R↓	-31						
054	R↓	-31	110					
055	ST09	35 09						
056	R↓	-31						

SET STATUS					
FLAGS		TRIG	DISP		
0	<input type="checkbox"/> ON <input checked="" type="checkbox"/> OFF	DEG <input checked="" type="checkbox"/>	FIX <input checked="" type="checkbox"/>		
1	<input type="checkbox"/> ON <input checked="" type="checkbox"/> OFF	GRAD <input type="checkbox"/>	SCI <input type="checkbox"/>		
2	<input type="checkbox"/> ON <input checked="" type="checkbox"/> OFF	RAD <input type="checkbox"/>	ENG <input type="checkbox"/>		
3	<input type="checkbox"/> ON <input checked="" type="checkbox"/> OFF		n <u>2</u>		

REGISTERS									
0	1	2	3	4	5	6	7	8	9
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A	B	C	D	E	I				



# Program Description I

**Program Title** BODE OF 2ND - OVER 2ND - ORDER TIMES S\*\*N TRANSFER FUNCTION

**Contributor's Name** Hewlett-Packard Company

**Address** 1000 N. E. Circle Boulevard

**City** Corvallis **State** Oregon **Zip Code** 97330

## Program Description, Equations, Variables

$$F(s) = \frac{Qs^3 + Gs^2 + Hs + K}{s^N(Rs^3 + Ls^2 + Ms + P)}$$

where: Q and R are options, limited to [-1, 0, +1]  
 G, H, K, L, M, and P may assume any real value  
 N is a non-negative integer [0, 1, 2, ...]

$$|\text{Magnitude}| = \frac{\sqrt{(K - G\omega^2)^2 + (H\omega - Q\omega^3)^2}}{\omega^N \sqrt{(P - L\omega^2)^2 + (M\omega - R\omega^3)^2}}$$

where:  $\omega = 2\pi F$

$$\text{Decibels} = 20 \log_{10} |\text{Magnitude}|$$

$$\text{Phase} = \tan^{-1} \frac{H\omega - Q\omega^3}{K - G\omega^2} - N \sin^{-1} 1.0 - \tan^{-1} \frac{M\omega - R\omega^3}{P - L\omega^2}$$

in degrees (unless angular mode is set to radians or grads).

**Operating Limits and Warnings** Frequency, F, can not equal zero. Power, N, is a non-negative integer. Coefficient of  $s^3$  in numerator or denominator is -1, 0, or +1 display of phase will be between  $-180^\circ$  and  $+180^\circ$  ( $-\pi$  and  $+\pi$  radians or  $-200$  and  $+200$  grads).

This program has been verified only with respect to the numerical example given in *Program Description II*. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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# Program Description II

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## Sketch(es)

## Sample Problem(s)

Find gain (in decibels and volts/volt) and phase shift in degrees

$$\text{For } F(s) = \frac{8s^3 - 1}{s^2(s^2 - 2)} \quad \text{at } F = 0.1 \text{ Hz and } 1.0 \text{ Hz}$$

Option 3 (see page 4) is required but the coefficient of  $s^3$  can be only -1, 0, or +1

$$\text{Rewritten, } F(s) = \frac{s^3 - 0.125}{s^2(0.125s^2 - 0.25)}$$

So	$G = 0$	$H = 0$
	$K = -0.125$	$N = 2$
	$L = 0.125$	$M = 0$
	$P = -0.25$	$F = 0.1 \text{ and } 1.0$

**Solution(s)** Switch to Run: [GTO] 1 Switch to w/prgm: [g][DEL][g][DEL][RCL]8[F<sup>-9</sup>][ $\sqrt{\phantom{x}}$ ][-]

Switch to Run: 0 [↑] 0 [↑] .125 [CHS][A] → 0.00

2 [↑] .125 [↑] 0 [↑] .25 [CHS][B] → 2.00

.1 [D] → Gain = 7.42 db

[R/S] → F = 0.1 Hz Gain = 2.35 volts/volt

[E] → Phase = -116.74 degrees

1 [D] → Gain = 1.67 db

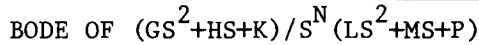
[R/S] → F = 1.0 Hz Gain = 1.21 volts/volt

[E] → Phase = -90.03 degrees

**Reference(s)** This program is a translation of the HP-65 Users' Library Program  
#02273A submitted by Edward P. Sansing.

[illegible]

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STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
	OPTIONS				
	Any combination of the four may be used; however, order of selection must be as follows:				
1.	To use radians/sec. inputs instead of Hz				
	Switch to run		GTO	D	
	Switch to W/PRGM				21 14
	Perform 4 times		{ SST	f	35
			{ DEL		14
	Switch to run				
2.	For dB-10 x log <sub>10</sub> Magnitude				
	Instead of 20 x log <sub>10</sub> Magnitude				
	Switch to run		GTO	1	
	Switch to W/PRGM				23 01
	Single step 16 times		SST		02
			f	DEL	16 32
		1			01
	Switch to run				
3.	For S <sup>3</sup> + GS <sup>2</sup> + HS + k as numerator				
	Switch to run		GTO	1	
	Switch to W/PRGM		RCL	8	34 08
				√	
	(for -S <sup>3</sup> , use +)			x <sup>2</sup>	53
				-	-45
	Switch to RUN				
4.	For S <sup>N</sup> (S <sup>3</sup> +LS <sup>2</sup> +MS+P)				
	As denominator switch to run		GTO	2	
	Switch to W/PRGM				23 02
			RCL	8	34 08
			x <sup>2</sup>		53
	(for -S <sup>3</sup> , use +)			-	-45
	Switch to RUN				

# 97 Program Listing I

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11		057	RCL9	36 09	
002	ST07	35 03		058	R↓	-31	
003	R↓	-31		059	+P	34	
004	ST02	35 02	Store numerator	060	R↑	16-31	
005	R↓	-31	coefficients	061	=	-24	
006	ST01	35 01		062	R↓	-31	
007	RTN	24		063	-	-45	
008	*LBLB	21 12		064	CHS	-22	
009	ST07	35 07		065	R↑	16-31	
010	R↓	-31		066	ENT↑	-21	
011	ST06	35 06	Store denominator	067	LOG	16 32	Total phase
012	R↓	-31	coefficients &	068	2	02	magnitude
013	ST05	35 05	power	069	0	00	
014	R↓	-31		070	X	-35	
015	ST04	35 04		071	R/S	51	
016	RTN	24		072	X↔Y	-41	
017	*LBLD	21 14	Compute gain	073	RTN	24	
018	2	02		074	*LBL E	21 15	
019	X	-35		075	R↑	16-31	
020	P↑	16-24	These four	076	1	01	Compute phase
021	X	-35	program steps	077	+R	44	
022	ST08	35 08	are deleted	078	+P	34	
023	RCL6	36 06	for inputting	079	R↓	-31	
024	*LBL2	21 02	radians/second	080	RTN	24	
025	RCL7	36 07		081	R/S	51	
026	RCL8	36 08					
027	=	-24					
028	RCL5	36 05					
029	RCL8	36 08					
030	X	-35					
031	-	-45					
032	+P	34					
033	RCL8	36 08					
034	RCL4	36 04		090			
035	Y↔X	31					
036	X	-35	Denominator				
037	1	01	magnitude				
038	SIN <sup>-1</sup>	16 41					
039	RCL4	36 04					
040	X	-35					
041	X↔Y	-41					
042	R↓	-31					
043	+	-55					
044	R↑	16-31		100			
045	ST09	35 09					
046	R↓	-31					
047	RCL3	36 03					
048	RCL8	36 08					
049	=	-24					
050	RCL1	36 01					
051	RCL8	36 08					
052	X	-35					
053	-	-45					
054	RCL2	36 02		110			
055	*LBL1	21 01					
056	X↔Y	-41					

REGISTERS									
0 G,coeff of S <sup>2</sup>	1 G,coeff of S <sup>2</sup>	2 H,coeff of S <sup>1</sup>	3 k,coeff of S <sup>0</sup>	4 N,power of S	5 L,coeff of S <sup>2</sup>	6 M,coeff. of S <sup>1</sup>	7 p,coeff of S <sup>0</sup>	8 ω RAD/SEC	9 DENOM MAGNIT
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A	B	C	D	E	I				

SET STATUS		
FLAGS	TRIG	DISP
ON OFF		
0 <input type="checkbox"/> <input checked="" type="checkbox"/>	DEG <input checked="" type="checkbox"/>	FIX <input checked="" type="checkbox"/>
1 <input type="checkbox"/> <input checked="" type="checkbox"/>	GRAD <input type="checkbox"/>	SCI <input type="checkbox"/>
2 <input type="checkbox"/> <input checked="" type="checkbox"/>	RAD <input type="checkbox"/>	ENG <input type="checkbox"/>
3 <input type="checkbox"/> <input checked="" type="checkbox"/>		n <u>2</u>

# Program Description I

**Program Title** Pole-Zero to Group Delay

**Contributor's Name** Hewlett-Packard

**Address** 1000 N.E. Circle Blvd.

**City** Corvallis

**State** Oregon

**Zip Code** 97330

**Program Description, Equations, Variables** Given a transmission function in the S-Plane:

$$\frac{V_0}{E_i} = \frac{k[s-(\sigma_{z1} + j\Omega_{z1})][s-(\sigma_{z2} + j\Omega_{z2})] \cdots [s-(\sigma_{zn} + j\Omega_{zn})]}{[s-(\sigma_{p1} + j\Omega_{p1})][s-(\sigma_{p2} + j\Omega_{p2})] \cdots [s-(\sigma_{pm} + j\Omega_{pm})]}$$

in which poles & zeros are of the form:  $\sigma + j\Omega$

This program evaluates the expression for time response:

$$T = \sum_{k=1}^m d_{pk} - \sum_{k=1}^n d_{zk} \quad \text{where } d_k = \frac{\sigma_k}{\sigma_k^2 + (\omega - \Omega_k)^2} \quad \text{for}$$

either poles or zeros. A negative quantity for T indicates delay so that for positive values of time delay

$$TD = -T = \sum_{k=1}^n d_{zk} - \sum_{k=1}^m d_{pk}$$

Group delay:  $GD = TD - TD_0$  where is the time delay at reference frequency  $\omega_0$ .  
As  $\omega_0 \rightarrow 0$ ,  $TD_0 \rightarrow$  insertion delay.

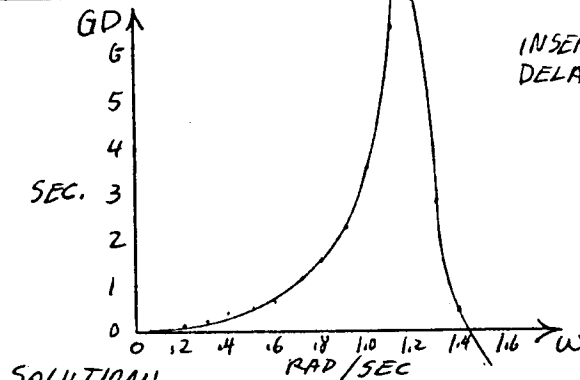
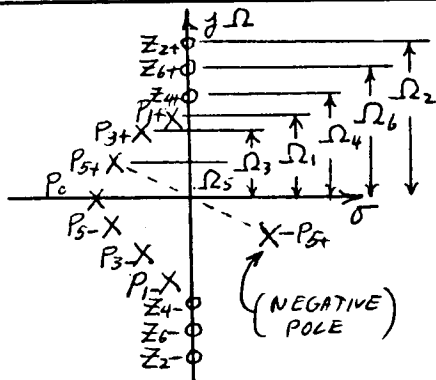
## Operating Limits and Warnings

This program has been verified only with respect to the numerical example given in *Program Description II*. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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# Program Description II

Sketch(es)



Sample Problem(s)

A 7 pole Caver - Chebyshev filter normalized to  $R = 1\Omega$  and $\omega_0 = 1$  rad/sec has pole & zero coordinates as follows:

$\sigma_0$	$\sigma_1$	$\sigma_3$	$\sigma_5$	$\Omega_1$	$\Omega_2$	$\Omega_3$	$\Omega_4$	$\Omega_5$	$\Omega_6$
-.78683	-.12406	-.3904	-.65874	1.16504	4.3544	.99847	2.0445	.60818	2.4903

Tabulate the group delay referred to  $\omega=.001$  from  $\omega=.1$  to  $\omega=1.5$  and generate a normalized group delay curve.

**SOLUTION:** Note that all zeros lie on the  $j\Omega$  axis resulting in no contribution to delay. Register loading is shown below. Pole zero data is called up with the following sequence written in step 2 of the instructions. Results are tabulated below and graphed at top of page.

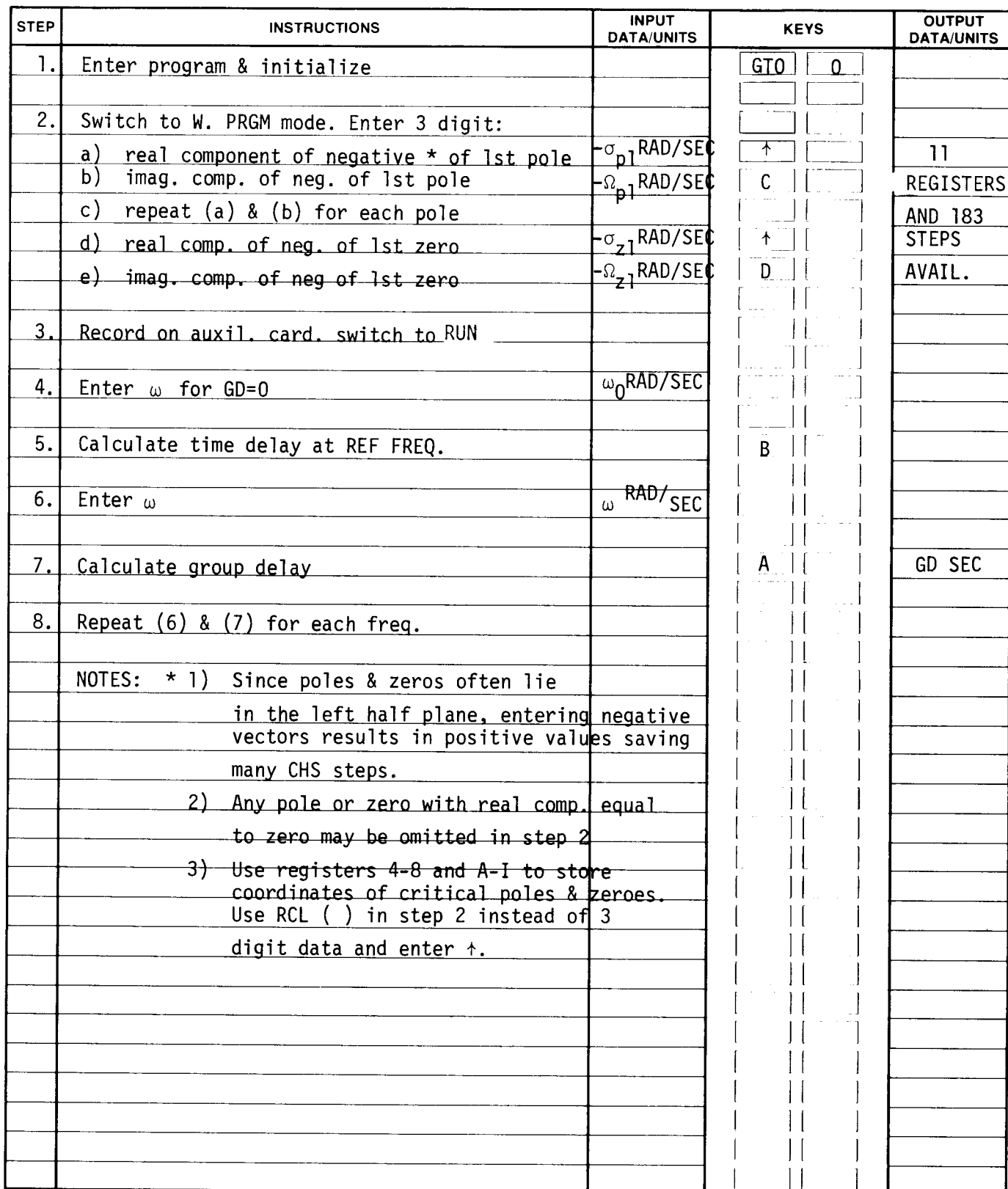
Solution(s)	In W. PRGM MODE:	$\omega$ RAD/SEC	TD/GD SEC	$\omega$ RAD/SEC	GD SEC
REGISTER:	-P <sub>0</sub> : .787↑OC	.001	TD=3.770	.8	1.544
R4=- $\sigma_1$ =0.12406	-P <sub>1+</sub> : RCL 4 RCL 5 CHS C	.1	GD=0.015	.9	2.282
R5= $\Omega_1$ =1.16504	-P <sub>1-</sub> : RCL 4 RCL 5 C	.2	0.064	1.0	3.647
R6=- $\sigma_3$ =0.3904	-P <sub>3+</sub> : RCL 6 RCL 7 CHS C	.3	0.150	1.1	6.663
R7= $\Omega_3$ =0.99847	-P <sub>3-</sub> : RCL 6 RCL 7 C	.4	0.281	1.2	7.220
R8=- $\sigma_5$ =0.65874	-P <sub>5+</sub> : RCL 8 .608 CHS C	.5	0.462	1.3	2.842
	-P <sub>5-</sub> : RCL 8 .608 C	.6	0.708	1.4	0.390
	TOTAL: 34 steps	.7	1.052	1.5	-0.808

Reference(s)

Handbook of Filter Synthesis - A. Zverev 1967 p. 149.

This program is a translation of the HP-65 program #04775A submitted by Charles R. Olson.

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[illegible]

# Program Description I

Program Title ROOT TEST FOR CONTINUOUS AND DISCRETE TIME SYSTEM STABILITY

Contributor's Name JAMES V. WISEMAN JR.

Address 3926 CANYON RD

City LAFAYETTE

State CALIF

Zip Code 94549

Program Description, Equations, Variables DESIGNED TO TEST THE LOCATION OF TIME ROOTS OF A CHARACTERISTIC EQUATION IN TIME FORM:

$$G_0 p^n + G_1 p^{n-1} + \dots + G_{n-1} p + G_n = 0 \quad p \text{ is 's' or 'z' } \begin{matrix} \text{DEPENDENT ON SYSTEM} \\ \text{AT HAND} \end{matrix}$$

ROOT TEST FOR RHP ROOTS: REPEATED  $n+1$  TIMES:  $\# \text{RHP ROOTS} = \sum (\# \text{TIMES } R_i < 0)$

For  $i = 1, 3, 5, \dots$  Loop:  $R_i = G_0 / G_i$

$\begin{cases} a_{i-1} = a_i \\ a_i = G_{i+1} - R_i G_i \end{cases}$

AXIS SHIFT FOR GIVEN OFFSET  $d$ : ( $p \leftarrow p' + d$ )

$$\begin{bmatrix} a_1 = G_1 + G_0 d \\ G_2 = G_2 + G_1 d \\ \vdots \\ G_{n-1} = G_{n-1} + G_{n-2} d \end{bmatrix} \quad \begin{bmatrix} a_1 = G_1 + G_0 d \\ G_2 = G_2 + G_1 d \\ \vdots \\ G_{n-1} = G_{n-1} + G_{n-2} d \end{bmatrix} \quad \dots \quad \begin{bmatrix} G_1 = G_1 + G_0 d \\ G_2 = G_2 + G_1 d \end{bmatrix} \quad \begin{bmatrix} G_1 = G_1 + G_0 d \end{bmatrix}$$

$z = (s+1)/(s-1)$  SUBSTITUTION:

WHERE:  $R_j = \frac{(-1)^j i!}{(i-j)! j!}$

$$\begin{bmatrix} a_0' = G_0 + R_0' a_1 \\ G_1' = G_0 + R_1' a_1 \\ G_2' = G_1 + R_2' a_2 \end{bmatrix} \quad \begin{bmatrix} a_0' = G_0 + R_0^2 a_2 \\ G_1' = G_0 + G_1 + R_1^2 a_2 \\ G_2' = G_1 + R_2^2 a_2 \end{bmatrix} \quad \dots \quad \begin{bmatrix} G_0 = G_0 + R_0^n a_n \\ G_1' = G_0 + G_1 + R_1^n a_n \\ G_2' = G_1 + G_2 + R_2^n a_n \\ \vdots \\ G_n = G_{n-1} + R_n^n a_n \end{bmatrix}$$

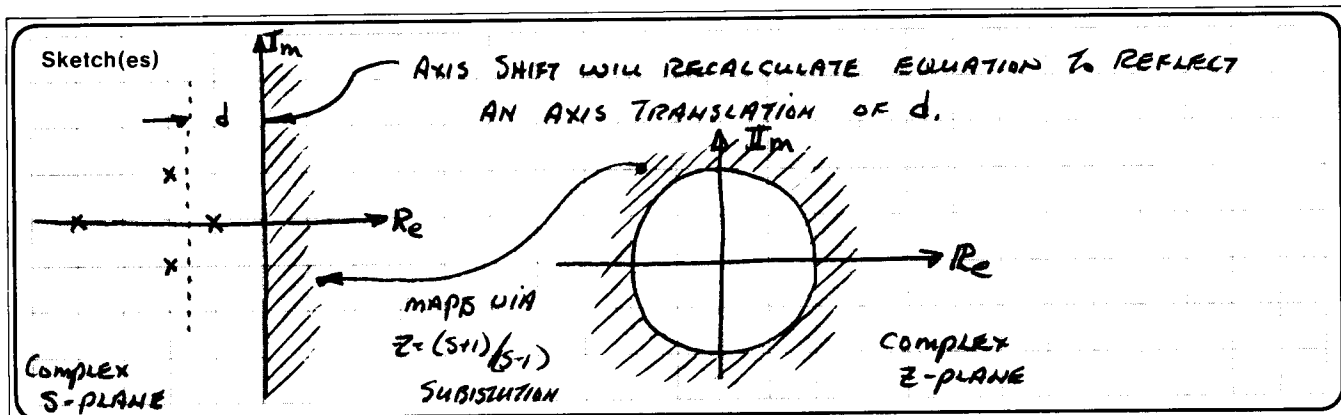
Operating Limits and Warnings

- 1) LIMITED BY CALCULATOR REGISTERS <sup># IS</sup> TO 20<sup>th</sup> ORDER
- 2) ROOT TEST WITH VARIOUS ROOTS ALONG IMAGINARY AXIS MAY INDICATE INSTABILITY. SHIFT AXIS SLIGHTLY TO CHECK ACTUAL LOCATION OF ROOTS.

This program has been verified only with respect to the numerical example given in Program Description II. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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# Program Description II



## Sample Problem(s)

- 1) Does  $s^4 + 2s^3 + s^2 + 4s + 4$  REPRESENT A STABLE CONTINUOUS TIME SYSTEM?
  - a) ENTER POLY. BY PRESSING 'P'. IN RESPONSE TO THE FLASHING '0' KEY IN Q. (1), IN RESPONSE TO THE FLASHING '1' KEY IN Q. (2) ETC ETC UNTIL Q. (4) IS ENTERED IN RESPONSE TO THE FLASHING '4'. STORE POLY. BY PRESSING 'S' (IGNORE FLASHING '5').
  - b) EXECUTE ROUTH TEST BY PRESSING 'A'. CALCULATOR WILL PAUSE DISPLAYING 1<sup>st</sup> COLUMN OF ROUTH ARRAY: 1, 2, -1, 2, 4 AND FLASH (OR PRINT) THE NUMBER OF RHP ROOTS: 2. UNSTABLE
- 2) RECALCULATE  $p^4 + 9p^3 + 29p^2 + 41p + 20$  TO REFLECT AN AXIS SHIFT OF  $d = -1.5$  (SEE ABOVE SKETCH)
  - a) ENTER POLY. AS IN PROB. #1 PART (a)
  - b) KEY-IN -1.5 PRESS 'C'. AT END OF CALCULATION, PRINT OUT NEW POLYNOMIAL BY 'E':  $p^4 + 3p^3 + 2p^2 + 1.25p - 1.5625$
- 3) Does  $z^4 + 3z^3 + .1z^2 + .05z + .001$  REPRESENT A STABLE DISCRETE TIME SYSTEM?
  - a) ENTER POLY. AS IN PROB #1 PART (a)
  - b) MAKE  $z = \frac{s+1}{s-1}$  SUBSTITUTION BY 'C'. RESULTS CAN BE FOUND BY 'E' TO BE:  $1.451s^4 + 4.496s^3 + 5.806s^2 + 3.496s + .751$
  - c) EXECUTE ROUTH TEST BY PRESSING 'A'. AS IN PROB #1 PART (b) CALCULATOR WILL PAUSE AT 1.451, 4.496, 4.678, 2.774, 0.751 STOPPING WITH "0." IN THE DISPLAY, THE #RHP  $\therefore$  STABLE

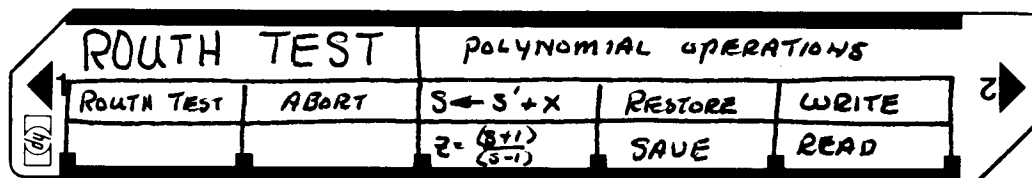
Reference(s) 1) Y. TAKAHASHI, M. RABINS, AND D. AUSLANDER, CONTROL, ADDISON-WESLEY, READING, MASS., 1970

2) JURY, E.I., THEORY AND APPLICATION OF THE Z TRANSFORM METHOD, NEW YORK: WILEY 1964

3) J. WISEMAN, RECURSIVE ALGORITHMS FOR ROUTH TEST IN CONT. & DISCRETE TIME TRANSACTIONS OF ASME, JDSMIL TO BE PUBLISHED.

# User Instructions

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STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1)	READ-IN BOTH SIDES OF CARD		<input type="text"/> <input type="text"/>	0.
2)	READ IN POLYNOMIAL BY 'fe' (READ)		<input type="text"/> f <input type="text"/> e	
	'm' WILL BE FLASHED FOR EACH $G_m$ , REQUESTING USER TO KEY-IN $G_m$ DURING PAUSE, ONCE ENTRY HAS BEEN MADE NEXT 'm' IS FLASHED. STOP DATA ENTRY REQUEST BY 'fd' (SAVE) - THIS WILL SAVE $G_m$ 'S IN SECONDARY REG'S IF $n \leq 10$ , OTHERWISE IT WILL STORE $G_m$ 'S ON SCRATCH CARD VIA 'CD'	$G_m$	<input type="text"/> <input type="text"/>	m
			<input type="text"/> f <input type="text"/> d	
			<input type="text"/> <input type="text"/>	0.
3)	IF DISCRETE TIME SYSTEM, MAKE $Z = (S+1)/(S-1)$ SUBSTITUTION BY 'fc'. THIS WILL GENERATE A NEW POLYNOMIAL IN PLACE OF ORIGINAL, REFLECTING SUBSTITUTION		<input type="text"/> f <input type="text"/> c	
			<input type="text"/> <input type="text"/>	0.
4)	IF AXIS SHIFT IS DESIRED, ENTER LOCATION OF NEW ORIGIN IN X. EXECUTE SHIFT BY 'C' ( $S \leftarrow S' + X$ ). THIS WILL GENERATE A NEW POLYNOMIAL IN PLACE OF ORIGINAL, REFLECTING AXIS SHIFT	OFFSET	<input type="text"/> C <input type="text"/>	
			<input type="text"/> <input type="text"/>	0.
5)	ROUTH TEST IS EXECUTED BY 'A' (ROUTHTEST) PROGRAM PAUSES DURING EXECUTION TO DISPLAY ELEMENTS OF 1 <sup>ST</sup> COLUMN IN ROUTH ARRAY. NO. OF SIGN CHANGES INDICATES NO. OF RHP ROOTS. PROGRAM HALTS, PRINTING # OF RHP ROOTS.		<input type="text"/> A <input type="text"/>	
			<input type="text"/> <input type="text"/>	
			<input type="text"/> <input type="text"/>	
			<input type="text"/> <input type="text"/>	# RHP Roots
	<u>NOTE:</u>		<input type="text"/> <input type="text"/>	
	a) RECURSIVE ALGORITHMS ARE DESTRUCTIVE IN THAT THEY DESTROY THE ORIGINAL POLYNOMIAL. FOR ADDITIONAL RUNS, DATA MUST BE RESTORED VIA 'D' (RESTORE) IF $n \leq 10$ OTHERWISE BY SCRATCH CARD FROM STEP 2.		<input type="text"/> D <input type="text"/>	
	b) POLYNOMIAL MAY BE REVIEWED OR PRINTED FOR RECORDS BY 'E' (WRITE).		<input type="text"/> <input type="text"/>	
	c) IF AN ERROR IS MADE DURING $G_m$ ENTRY AND DATA IS 'TAKEN', PRESS 'B' (ABORT) AND START OVER WITH 'fe'		<input type="text"/> B <input type="text"/>	
			<input type="text"/> f <input type="text"/> e	

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS												
001	* LBL A	31 25 11			STO (I)	33 24													
	SCI	32 23			DSZ	31 33													
	DSP 3	23 03			R↓	35 53													
	Ø	00		060	STO (I)	33 24													
	STO D	33 14			RCL E	34 15													
	* LBL Ø	31 25 00			X>Ø?	31 81													
	RCL Ø	34 00			GTO Ø	22 00													
	PAUSE	35 72			RCL Ø	34 00													
	RCL I	34 01			PAUSE	35 72													
010	X≠Ø?	31 61			RCL D	34 14													
	GTO a	22 31 11			FIX	31 23													
	CLX	44			DSP Ø	23 00													
	FEX	43			PRINTX	31 84													
	CHS	42		070	SPACE	35 84													
	S	05			R/S	84													
	a	00			GTO B	22 12													
	STO I	33 01			* LBL C	31 25 13													
	* LBL a	32 25 11			X=Ø?	31 51													
	÷	81		GTO B	22 12														
020	STO C	33 13	ROUTIN TEST		SCI	32 23	AXIS SHIFT												
	X>Ø?	31 81			DSP 4	23 04													
	GTO a	22 31 11			PRINTX	31 84													
	RCL D	34 14			STO D	33 14													
	I	01		080	RCL E	34 15													
	+	61			STO C	33 13													
	STO D	33 14			* LBL 2	31 25 02													
	* LBL a	32 25 11			Ø	00													
	I	01			ST I	35 33													
	ST I	35 33			* LBL 3	31 25 03													
030	* LBL I	31 25 01			RCL (I)	34 24													
	RCL (I)	34 24			RCL D	34 14													
	DSZ	31 33			X	71													
	STO (I)	33 24			ISZ	31 34													
	STO (D)	33 24		090	STO+(G)	33 61 24													
	ISZ	31 34			RC I	35 34													
	ISZ	31 34			RCL C	34 13													
	RCL (I)	34 24			X≠Y?	32 61													
	ISZ	31 34		GTO 3	22 03														
	RCL (I)	34 24		I	1														
040	RCL C	34 13		-	51														
	X	71		STO C	33 13														
	-	51		X≠Ø?	31 61														
	DSZ	31 33		GTO Z	22 02														
	DSZ	31 33		GTO B	22 12														
	STO (I)	33 24		* LBL c	32 25 13														
	ISZ	31 34		CF2	35 61 02														
	ISZ	31 34		I	01														
	RC I	35 34		STO D	33 14														
	RCL E	34 15		* LBL b	32 25 12														
050	X>Y?	32 81		Ø	00														
	GTO I	22 01		ST I	35 33														
	I	01		* LBL 4	31 25 04														
	-	51		GSB 5	31 22 05														
	STO E	33 15		GSB 6	31 22 06	110													
	RCL (I)	34 24		RCL (I)	34 24														
	Ø	00		GSB 6	31 22 06														
REGISTERS																			
0	U <sub>0</sub>	1	Q <sub>1</sub>	2	Q <sub>2</sub>	3	Q <sub>3</sub>	4	...	5	...	6	...	7	...	8	...	9	....
S <sub>0</sub>	Q <sub>10</sub> OR Q <sub>0</sub>	S <sub>1</sub>	Q <sub>11</sub> OR Q <sub>1</sub>	S <sub>2</sub>	Q <sub>12</sub> OR Q <sub>2</sub>	S <sub>3</sub>	Q <sub>13</sub> OR Q <sub>3</sub>	S <sub>4</sub>	...	S <sub>5</sub>	....	S <sub>6</sub>	....	S <sub>7</sub>	....	S <sub>8</sub>	....	S <sub>9</sub>	....
A	Q <sub>20</sub>	B	SCRATCH	C	SCRATCH	D	SCRATCH	E	N	I RESERVED									

# 67 Program Listing II

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STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
	X	71			9	09	
	+	61			RCL E	34 15	
	RCL (I)	34 24		170	X = Y?	32 71	
	X $\leftrightarrow$ Y	35 52			GTO d	22 31 14	
	STO + (E)	33 61 24			W/DATA	31 41	
	ISZ	31 34			GTO B	22 12	
	RCL D	34 14			* LBL d	32 25 14	
120	RC I	35 34			STO B	33 12	
	X $\neq$ Y?	32 61			ST I	35 33	
	SFZ	35 51 02			* LBL 9	31 25 09	
	R A	35 54			RCL (I)	34 24	
	FZ ?	35 71 02		180	P $\neq$ S	31 42	
	GTO 4	22 04			STO (I)	33 24	
	GSBS	31 22 05			P $\neq$ S	31 42	
	RCL (E)	34 24			DSZ	31 33	
	X	71			GTO 9	22 09	
	+	61			RCL 0	24 00	
130	STO (I)	33 24			P $\neq$ S	31 42	
	RCL E	34 15			STO 0	33 00	
	RCL D	34 14			GTO B	22 12	
	X = Y?	32 51			* LBL e	32 25 15	
	GTO B	22 12		190	CF3	35 61 03	
	I	01			0	00	
	+	61			ST I	35 33	
	STO D	33 14			* LBL 7	31 25 07	
	GTO b	22 31 12			PAUSE	35 72	
	* LBL 5	31 25 05			F3 ?	35 71 03	
140	RC I	35 34			GTO e	22 31 15	
	X $\neq$ 0?	31 61			GTO 7	22 07	
	GTO 5	22 05			* LBL e	32 25 15	
	I	01			STO (I)	33 24	
	STO C	33 13		200	RC I	35 34	
	RTN	35 22			STO E	33 15	
	* LBL 5	31 25 05			ISZ	31 34	
	RCL D	34 14			RC I	35 34	
	-	51			GTO 7	22 07	
	I	01			* LBL E	31 25 15	
150	-	51			SPACE	35 84	
	RCL C	34 13			SCI	32 23	
	X	71			DSP 5	23 05	
	STO C	33 13			0	00	
	RC I	35 34		210	ST I	35 33	
	N!	35 81			* LBL B	31 25 08	
	$\div$	81			RCL (I)	34 24	
	RTN	35 22			PRINT X	31 84	
	* LBL 6	31 25 06			ISZ	31 34	
	RCL D	34 14			RCL E	34 15	
160	X $\neq$ I	35 24			RC I	35 34	
	STO D	33 14			X = Y?	32 71	
	R $\downarrow$	35 33			GTO B	22 08	
	RTN	35 22			SPACE	35 84	
	* LBL D	31 25 14		220	* LBL B	31 25 12	
	RCL B	34 12			CLX	44	
	STO E	33 15			FIX	31 23	
	P $\neq$ S	31 42			DSP 0	23 00	
	* LBL d	32 25 14			R/S	84	

LABELS					FLAGS	SET STATUS		
A ROUTH	B ABORT	C $S \leftarrow S + X$	D RESTORE	E WRITE	0	FLAGS	TRIG	DISP
a X	b X	c $Z = (S+1)/(S-1)$	d SAVE	e READ	1	ON OFF		
0 X	1 X	2 X	3 X	4 X	2 SUB.	0 <input type="checkbox"/> <input checked="" type="checkbox"/>	DEG <input checked="" type="checkbox"/>	FIX <input checked="" type="checkbox"/>
5 X	6 X	7 X	8 X	9 X	3 DATA ENTRY	1 <input type="checkbox"/> <input checked="" type="checkbox"/>	GRAD <input type="checkbox"/>	SCI <input type="checkbox"/>
						2 <input type="checkbox"/> <input checked="" type="checkbox"/>	RAD <input type="checkbox"/>	ENG <input type="checkbox"/>
						3 <input type="checkbox"/> <input checked="" type="checkbox"/>		n 0

# Program Description I

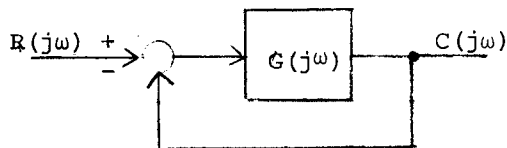
**Program Title** CONVERT FREQUENCY RESPONSE, OPEN LOOP, CLOSED LOOP

**Contributor's Name** Hewlett-Packard Company

**Address** 1000 N.E. Circle Boulevard

**City** Corvallis **State** Oregon **Zip Code** 97330

**Program Description, Equations, Variables** For a linear, unity feedback control system this program converts open loop frequency response data  $\angle G(j\omega)$  (or  $\log|G(j\omega)|$  or  $20\log|G(j\omega)|$ ) to closed loop data  $\angle \frac{C}{R}(j\omega)$  and  $|\frac{C}{R}(j\omega)|$  (or  $\log|\frac{C}{R}(j\omega)|$  or  $20\log|\frac{C}{R}(j\omega)|$ )



where  $\frac{C}{R}(j\omega) = \frac{G(j\omega)}{1+G(j\omega)}$

This program also converts from closed loop data

$$\angle \frac{C}{R}(j\omega) \text{ and } |\frac{C}{R}(j\omega)|$$

$$\angle G(j\omega) \text{ and } |G(j\omega)|.$$

The relationship used is:

$$g(j\omega) = \frac{\frac{C}{R}(j\omega)}{1 - \frac{C}{R}(j\omega)}$$

## Operating Limits and Warnings

- When the input phase angle cosine ( $\angle \frac{C}{R}(j\omega)$  or  $\angle G(j\omega)$ ) is zero (e.g.  $\angle \frac{C}{R}(j\omega) = -90$ ) the conversion is inaccurate beyond the 6th decimal place.
- When input is to be  $|G(j\omega)|$  or  $|\frac{C}{R}(j\omega)|$  label C is used and  $|\frac{C}{R}(j\omega)|$  or  $|G(j\omega)|$  is calculated. Similarly when the input is  $\log|G(j\omega)|$  or  $\log|\frac{C}{R}(j\omega)|$  then  $\log|\frac{C}{R}(j\omega)|$  or  $\log|G(j\omega)|$ . Similarly when input is in decibels the result is decibels.

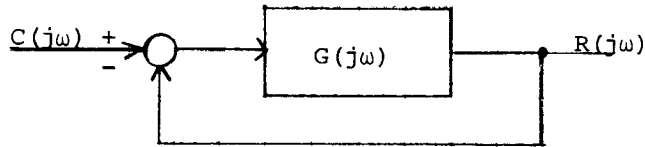
This program has been verified only with respect to the numerical example given in *Program Description II*. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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# Program Description II

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## Sketch(es)



Systems having feedback elements can be converted to an equivalent unity feedback system. See the reference for methods of determining equivalent unity feedback systems.

**Sample Problem(s)** For the system  $G(j\omega) = \frac{4}{j\omega(1 + .25j\omega)(1 + .0625j\omega)}$

For  $\omega = .1, 1$  and  $10$  rad/sec  $\angle G(j\omega)$  and  $|G(j\omega)|$  are:

$\omega$ rad/sec	$\angle G(j\omega)$ degrees	$ G(j\omega) $
.1	- 91.79	39.99
1	-107.61	3.87
10	-190.20	.13

Determine closed loop frequency response for  $\omega = .1, 1, 10$  rad/sec

**Solution(s)** For each frequency

$\omega$	$\angle \frac{C}{R}(j\omega)$ degrees	$ \frac{C}{R}(j\omega) $
.1	- 1.43	1.00
1	- 14.96	1.05
10	-191.71	.15

**Reference(s)** Raven, F.H., Automatic Control Engineering, McGraw-Hill, New York.1968.

This program is a translation of the HP-65 Users' Library Program #00892A submitted by Eugene Bahniuk.



## User Instructions

1 ← FREQUENCY RESPONSE CONVERSION → 2

(a) Open to Close    Closed to Open    Magn.    Log Magn.    DB

[illegible]

# 97 Program Listing I

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STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11	Ind. that conv.	057	*LBL2	21 02	
002	0	00	from open to close	058	CF0	16 22 00	
003	ST06	35 06	loop is to be	059	LOG	16 32	
004	*LBL5	21 05	performed	060	2	02	
005	0	00	Enter phase	061	0	00	
006	R/S	51		062	A	-35	
007	ST04	35 04		063	ST01	35 01	
008	RCL6	36 06		064	R/S	51	
009	-	-45	These steps determ.				
010	1	01	if vector add. or				
011	R/S	51	vecotr sub. is				
012	*LBLB	21 12	performed				
013	1	01	For closed to open				
014	8	08	loop conv. Note:				
015	0	00	Vector sub. occurs	070			
016	ST06	35 06	by reversing vector				
017	GT05	22 05	direction then				
018	*LBLC	21 13	add resultin vector				
019	X*Y	-41					
020	R+	-31					
021	ST05	35 05					
022	+R	44	Steps 22 through 34				
023	1	01	perform the				
024	+	-55	complex algebra	080			
025	+P	34					
026	RCL5	36 05					
027	X*Y	-41					
028	÷	-24					
029	ST01	35 01					
030	X*Y	-41					
031	CHS	-22					
032	RCL4	36 04					
033	+	-55					
034	ST02	35 02		090			
035	X*Y	-41					
036	F12	16 23 01					
037	GT01	22 01					
038	F02	16 23 00					
039	GT02	22 02					
040	R/S	51					
041	*LBLD	21 14	Magnitudes are				
042	10*	16 33	logarithmic				
043	SF1	16 21 01					
044	GT0C	22 13		100			
045	*LBL1	21 01					
046	CF1	16 22 01					
047	LOG	16 32					
048	ST01	35 01					
049	R/S	51					
050	*LBLE	21 15	Indicates magnitudes				
051	2	02	are decibels				
052	0	00					
053	÷	-24					
054	10*	16 33		110			
055	SF0	16 21 00					
056	GT0C	22 13					

SET STATUS		
FLAGS	TRIG	DISP
ON OFF		
0 <input type="checkbox"/> <input checked="" type="checkbox"/>	DEG <input checked="" type="checkbox"/>	FIX <input checked="" type="checkbox"/>
1 <input type="checkbox"/> <input checked="" type="checkbox"/>	GRAD <input type="checkbox"/>	SCI <input type="checkbox"/>
2 <input type="checkbox"/> <input checked="" type="checkbox"/>	RAD <input type="checkbox"/>	ENG <input type="checkbox"/>
3 <input type="checkbox"/> <input checked="" type="checkbox"/>		n2

## REGISTERS

0	1 computed magnitude	2 computed phase	3	4 entered phase	5 Ent.magn	6 0	7	8	9
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A	B	C	D	E	I				

# Program Description I

**Program Title** AID TO ROOT LOCUS PLOTS I - REAL POLES  
**Contributor's Name** Hewlett-Packard Company  
**Address** 1000 N.E. Circle Boulevard  
**City** Corvallis **State** Oregon **Zip Code** 97330

**Program Description, Equations, Variables** Given the forward transfer function of unity gain feedback system  $KG(s) = \frac{K(s+z_1)(s+z_2)}{s(s+p_1)(s+p_2)(s+p_3)(s+p_4)}$

where  $s = \sigma + i\omega$  is the complex frequency variable and  $z_1, z_2, p_1, p_2, p_3$  and  $p_4$  are real numbers, the program helps in finding the roots of  $1 + KG(s) = 0$ , which determine the poles of the closed-loop system. It follows that at any point in the  $s$ -plane, which is a root of the above equation for some value of  $K$ ,  $G(\sigma + i\omega) = \frac{1}{K} 180^\circ$ .

Since the rules for approximate construction of root locus plots are well-known [1,2], this program can be used to obtain the exact location of the roots in certain regions of the  $s$ -plane. The user would select a value of  $\sigma$ , say  $\sigma_1$  and assume a trial value for  $\omega$ , say  $\omega_1$ . The program then determines  $G^{-1}(\sigma_1 + i\omega_1) = Ke^{i\phi}$ , and  $\phi$  is displayed. If  $\phi$  is not equal to  $180^\circ$ , a new trial value of  $\omega = \omega_2$  is obtained. The process is repeated until  $\phi$  is as closed to  $180^\circ$  as desired. The equation for searching the correct value of  $\omega$  is

$$\omega_2 = \omega_1 \left(4 - \frac{\phi}{60}\right), \text{ where } \phi \text{ is in degrees.}$$

The convergence may be slow when  $\omega$  approaches zero; in this case the user may often extrapolate mentally to accelerate the convergence. Normally 4 to 8 iterations are sufficient.

The value of the gain constant  $K$  required for this location of the root is obtained from  $y$ -register.

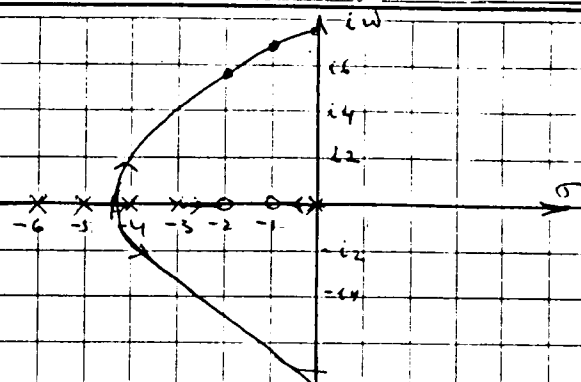
**Operating Limits and Warnings** The search equation is based on the assumption that  $\omega_1$  is greater than zero. This is no limitation since the root locus is always symmetrical about the real axis of the  $s$ -plane.

This program has been verified only with respect to the numerical example given in *Program Description II*. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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# Program Description II

Sketch(es)



Sample Problem(s)

$$KG(s) = \frac{K(s+1)(s+2)}{s(s+3)(s+4)(s+5)(s+6)}$$

It is desired to obtain the complex roots for  $\sigma = 0$ ,  $\sigma = -1$ , and  $\sigma = -2.5$

- Solution(s)**
- i) For  $\sigma = 0$ , starting with  $\omega = 5$ , after 4 iterations  $\phi = 180.01$ , for  $\omega = 8.545$ , and  $K = 999.51$
  - ii) For  $\sigma = -1$ , starting  $\omega=6$ , after 4 iterations,  $\phi = 179.99$  for  $\omega = 6.823$  and  $K = 519.61$
  - iii) For  $\sigma = -2.5$ , starting  $\omega = 4$ , after 5 iterations  $\phi = 180.01$ , for  $\omega = 4.35$  and  $K = 140.52$ .

**Reference(s)** <sup>(1)</sup> D'Azzo, J.J., Houpis, C.H. "Linear Control System Analysis and Design", McGraw-Hill Book Co. 3rd Edition. 1975. pp.202-242.

<sup>(2)</sup> Evans, W.R. "Control-System Dynamics", McGraw-Hill Book Co. 1954.

This program is a translation of the HP-65 Users' Library Program

#04561A submitted by Naresh K. Sinha.

[illegible]

## 35

REGISTERS									
0	1	2	3	4	5	6	7	8	9
	$z_1$	$z_2$	$p_1$	$p_2$	$p_3$	$p_4$	$\sigma_1$	$\omega_1$	Used
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A		B		C		D		I	

# Program Description I

**Program Title** AID TO ROOT LOCUS PLOTS II - COMPLEX POLES

**Contributor's Name** Hewlett-Packard Company

**Address** 1000 N.E. Circle Boulevard

**City** Corvallis

**State** Oregon

**Zip Code** 97330

**Program Description, Equations, Variables** Given the forward transfer function of a unity feedback system

$$KG(s) = \frac{K(s+z_1)(s+z_2)}{s(s+p_1)(s+p_2)(s+\alpha+i\beta)(s+\alpha-i\beta)}$$

where  $s = \sigma + i\omega$  is the complex frequency variable, and  $z_1, z_2, p_1, p_2$ , and  $\alpha, \beta$  are real numbers, the program helps in finding the roots of  $1 + KG(s) = 0$ , which determine the poles of the closed-loop system. It follows that at any point in the  $s$ -plane, which is a root of the above equation for some value of  $K$ , the argument of  $G(s)$  is equal to 180 degrees.

Since the rules for the construction of the approximate root locus plot are well known, this program can be used to obtain the exact location of the roots in critical regions of the  $s$ -plane. The user would select a value of  $\sigma$ , say  $\sigma_1$  and assume a trial value for  $\omega$ , say  $\omega_1$ . The program then determines the modules  $M$ , and the argument,  $\phi$  of  $G^{-1}(s)$  at this point, and the argument  $\phi$  is displayed. If  $\phi \neq 180$ , a new trial value of  $\omega = 2$  is obtained using the equation

$$\omega_2 = \omega_1 (4 - \phi/180)$$

The process is repeated until  $\phi$  is as close to 180 as desired. The convergence may be slow when  $\omega$  approaches zero. In this case, the user may often extrapolate mentally to accelerate convergence. Normally 4 to 8 iterations are sufficient.

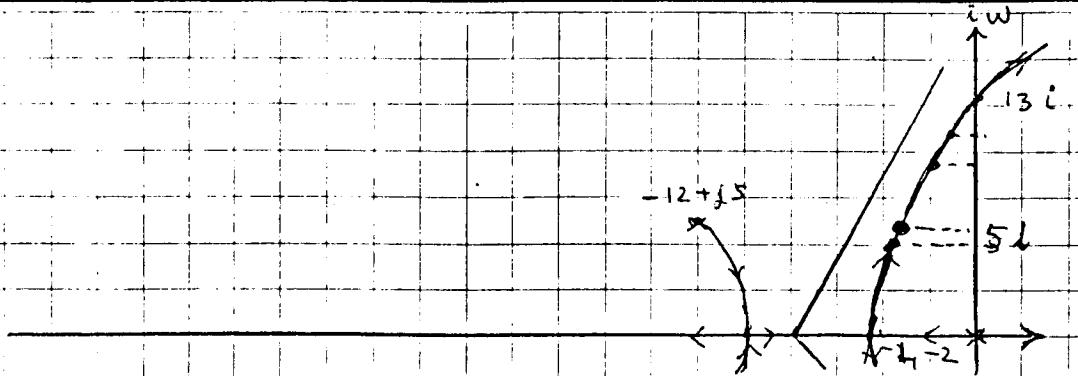
**Operating Limits and Warnings** The search equation is based on the assumption that  $\omega$  is positive. This is no limitation since the root-locus plot is always symmetrical about the real-axis of the  $s$ -plane.

This program has been verified only with respect to the numerical example given in *Program Description II*. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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# Program Description II

## Sketch(es)



## Sample Problem(s) Consider

$$KG(s) = \frac{K}{s(s+12+5j)(s+12-5j)} = \frac{K(s+1)(s+2)}{s(s+1)(s+2)(s+12+5j)(s+12-5j)}$$

The approximate sketch of the root locus is easily obtained following standard rules (see references 1 or 2). It is shown above.

It is desired to obtain the exact locations of the roots for  $\sigma = -1, -2$  and  $-4$ .

**Solution(s)** (1) For  $\sigma = -1$ , starting with  $\omega = 12$ , after 3 iterations the root is located at  $\omega = 11.14$  with  $\phi = 180.00$  and  $K = 2750$

(2) For  $\sigma = -2$ , starting with  $\omega = 10$ , after 3 iterations the root is located at  $\omega = 9.22$  with  $\phi = 180.01$  and  $K = 1780.7$

(3) For  $\sigma = -4$ , starting with  $\omega = 5.5$ , after 8 iterations the root is located at  $\omega = 5.00$  with  $\phi = 180.01$  and  $K = 656.3$

- Reference(s)**
1. Evans, W.R. "Control Systems Dynamics". McGraw-Hill Book Co., 1954.
  2. D'Azzo, J.J. and Houpis, C.H., "Linear Control System Analysis and Design". McGraw-Hill Book Co. 3rd Edition. 1975.

This program is a translation of the HP-65 Users' Library Program #04562A submitted by Naresh K. Sinha.





# 97 Program Listing I

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STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11		057	RCL8	36 08	
002	ST02	35 02	$z_2$	058	X	-35	
003	X $\div$ Y	-41		059	ST08	35 08	
004	ST01	35 01	$z_1$	060	RCL7	36 07	
005	R/S	51		061	RCL8	36 08	
006	ST06	35 06	$\beta$	062	RTN	24	Display $\omega_2$
007	R↓	-31		063	*LBLD	21 14	
008	ST05	35 05	$\alpha$	064	RCL7	36 07	
009	R↓	-31		065	+	-55	
010	ST04	35 04	$P_2$	066	RCL8	36 08	
011	R↓	-31		067	X $\div$ Y	-41	
012	ST03	35 03	$P_1$	068	+P	34	
013	RTN	24		069	X $\div$ Y	-41	
014	*LBLB	21 12		070	R↓	-31	
015	ST08	35 08	$\omega_1$	071	$\div$	-24	
016	X $\div$ Y	-41		072	R↓	-31	
017	ST07	35 07	$\alpha_1$	073	-	-45	
018	RCL5	36 05		074	CHS	-22	
019	+	-55		075	R↑	16-31	
020	X $\div$ Y	-41		076	RTN	24	
021	RCL6	36 06		077	*LBL E	21 15	
022	+	-55		078	X $\div$ Y	-41	
023	RCL8	36 08		079	+P	34	
024	RCL6	36 06		080	X $\div$ Y	-41	
025	-	-45		081	R↓	-31	
026	R↑	16-31		082	X	-35	
027	+P	34		083	R↓	-31	
028	R↓	-31		084	+	-55	
029	R↓	-31		085	R↑	16-31	
030	GSBE	23 15		086	RTN	24	
031	RCL4	36 04		087	R/S	51	
032	RCL7	36 07					
033	+	-55		090			
034	RCL8	36 08					
035	GSBE	23 15					
036	RCL3	36 03					
037	RCL7	36 07					
038	+	-55					
039	RCL8	36 08					
040	GSBE	23 15					
041	RCL7	36 07					
042	RCL8	36 08					
043	GSBE	23 15					
044	RCL2	36 02		100			
045	GSBD	23 14					
046	RCL1	36 01					
047	GSBD	23 14					
048	X $\div$ Y	-41					
049	RTN	24	Display $\phi$				
050	*LBLC	21 13					
051	6	06					
052	0	00					
053	$\div$	-24					
054	4	04		110			
055	-	-45					
056	CHS	-22					

REGISTERS									
0	1	2	3	4	5	6	7	8	9
	$z_1$	$z_2$	$P_1$	$P_2$	$\alpha$	$\beta$	$\sigma_1$	$\omega_1$	Used
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A	B	C	D	E	I				

SET STATUS		
FLAGS	TRIG	DISP
ON OFF		
0 <input type="checkbox"/> <input checked="" type="checkbox"/>	DEG <input checked="" type="checkbox"/>	FIX <input checked="" type="checkbox"/>
1 <input type="checkbox"/> <input checked="" type="checkbox"/>	GRAD <input type="checkbox"/>	SCI <input type="checkbox"/>
2 <input type="checkbox"/> <input checked="" type="checkbox"/>	RAD <input type="checkbox"/>	ENG <input type="checkbox"/>
3 <input type="checkbox"/> <input checked="" type="checkbox"/>		n <u>2</u>

# Program Description I

**Program Title** Classical Control Gains

**Contributor's Name** Hewlett-Packard

**Address** 1000 N.E. Circle Blvd.

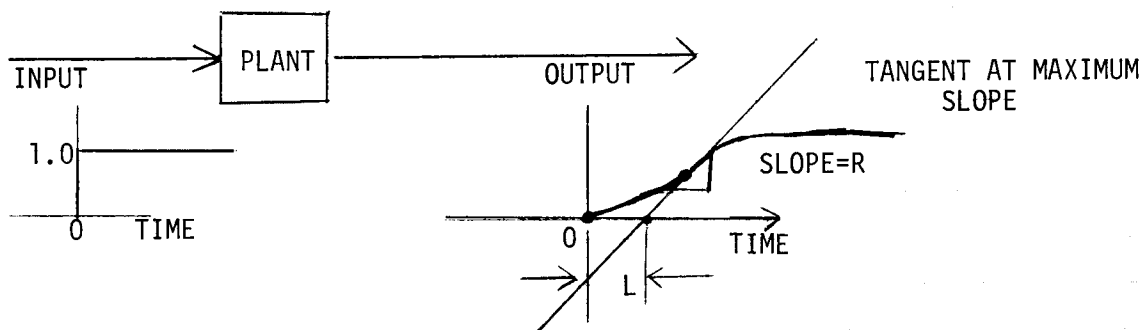
**City** Corvallis

**State** Oregon

**Zip Code** 97330

**Program Description, Equations, Variables** This program computes the Ziegler-Nichols recommended settings for P, PI, and PID control. Data is required from one of two tests: the general control form is  $G_c = k_c [1 + \frac{1}{T_i s} + T_D s]$

A) OPEN LOOP TEST - input step function, measure response, draw tangent at point of maximum response slope, scale L & R.



R = maximum slope

L = time intercept of maximum slopeline

Then for P control,  $k_c = 1/RL$ ; PI control,  $k_c = 0.9/RL$ ,  $T_i = 3.3L$ ;

for PID control,  $k_c = 1.2/RL$ ,  $T_i = 2L$ ,  $T_D = L/2$

## Operating Limits and Warnings

Note that plant must be greater than first order for open loop test  
(s is Laplace operator)

This program has been verified only with respect to the numerical example given in *Program Description II*. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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# Program Description I

**Program Title** Classical Control Gains

**Contributor's Name** Hewlett-Packard

**Address** 1000 N.E. Circle Blvd.

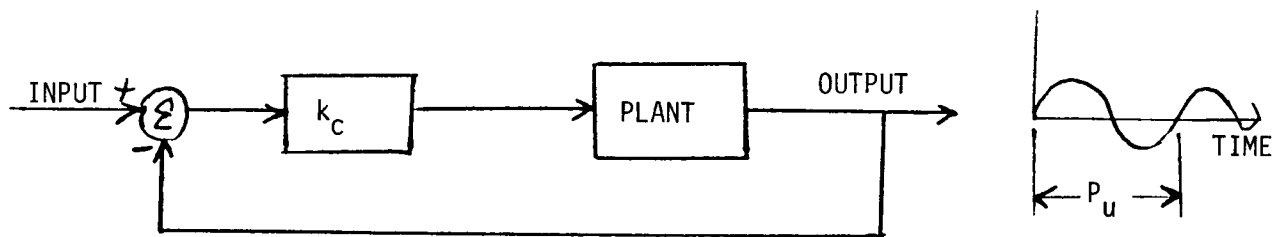
**City** Corvallis

**State** Oregon

**Zip Code** 97330

## Program Description, Equations, Variables

B CLOSED LOOP TEST - Increase  $k_c$  until plant is near instability (oscillating output). Let the magnitude of  $k_c$  at that point be called  $k_u$ , and the period of oscillation be called  $P_u$ .



Then for P control, let  $k_c = .5k_u$

PI control, let  $k_c = .45k_u$ ,  $T_i = 0.83P_u$

PID control, let  $k_c = 0.6k_u$ ,  $T_i = .5P_u$ ,  $T_0 = P_u/8$

General control form is  $G_c = k_c \left[ 1 + \frac{1}{T_i s} + T_0 s \right]$

## Operating Limits and Warnings

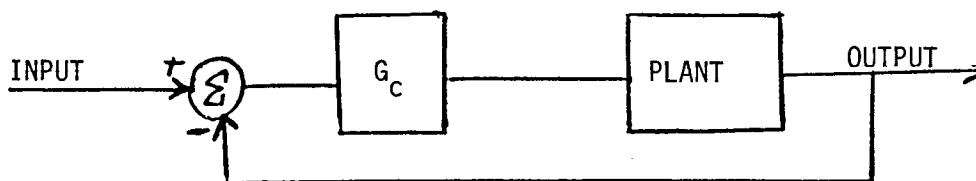
Plant must be greater than second order for closed loop test.

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# Program Description II

## Sketch(es)



Final form of controlled plant

P control  $\rightarrow G_c = k_c$ ; PI control  $\rightarrow G_c = k_c [1 + \frac{1}{T_i s}]$

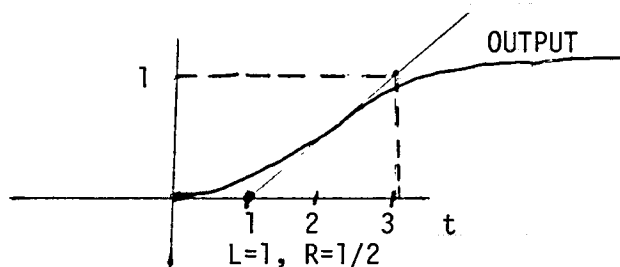
PID control  $\rightarrow G_c = k_c [1 + \frac{1}{T_i s} + T_0 s]$

## Sample Problem(s)

1. For the following open loop test data, compute the control coefficients for:

- P control
- PI control
- PID control

(unit step input)



2. During a closed loop P control test, the plant output became oscillatory with  $k_c = k_u = 10$ , the period of oscillations was 50 sec. Compute coefficients for:

- p control
- PI control
- PID control

## Solution(s)

1.  $L=1, R=1/2$

- $k_c = 2$
- $k_c = 1.8, T_i = 3.3$
- $k_c = 2.4, T_i = 2.0, T_0 = 0.5$

2.  $k_u = 10, P_u = 50$

- $k_c = 5$
- $k_c = 4.5, T_i = 41.5$
- $k_c = 6, T_i = 25, T_0 = 6.25$

## Reference(s)

OGATA; Modern Control Engineering, Prentice-Hall.

This program is a translation of the HP-65 Users' Library program #04463A submitted by Randy A. Coverstone.

[illegible]

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11	Open loop test data	057	.	-62	P control
002	ST02	35 02		058	8	08	
003	X	-35		059	3	03	
004	1/X	52		060	X	-35	
005	ST01	35 01		061	ST06	35 06	
006	ST03	35 03		062	CLX	-51	
007	.	-62		063	RTN	24	
008	9	09		064	*LBLC	21 13	
009	X	-35		065	RCL3	36 03	
010	ST04	35 04		066	RTN	24	
011	RCL1	36 01		067	*LBLD	21 14	
012	1	01		068	RCL4	36 04	
013	.	-62		069	R/S	51	
014	2	02		070	RCL6	36 06	
015	X	-35		071	RTN	24	
016	ST05	35 05		072	*LBLE	21 15	
017	RCL2	36 02		073	RCL5	36 05	
018	2	02		074	R/S	51	
019	X	-35		075	RCL7	36 07	
020	ST07	35 07		076	R/S	51	
021	1	01		077	RCL8	36 08	
022	.	-62		078	RTN	24	
023	6	06		079	R/S	51	
024	5	05					
025	X	-35					
026	ST06	35 06					
027	RCL2	36 02					
028	2	02					
029	=	-24					
030	ST08	35 08					
031	CLX	-51					
032	RTN	24					
033	*LBLB	21 12					
034	ST02	35 02	090				
035	R4	-31					
036	ST01	35 01					
037	2	02					
038	=	-24					
039	ST03	35 03					
040	.	-62					
041	9	09					
042	X	-35					
043	ST04	35 04					
044	RCL1	36 01	100				
045	.	-62					
046	6	06					
047	X	-35					
048	ST05	35 05					
049	RCL2	36 02					
050	2	02					
051	=	-24					
052	ST07	35 07					
053	4	04					
054	=	-24	110				
055	ST08	35 08					
056	RCL2	36 02					

Closed loop test data

SET STATUS		
FLAGS	TRIG	DISP
ON OFF		
0 <input type="checkbox"/> <input checked="" type="checkbox"/>	DEG <input checked="" type="checkbox"/>	FIX <input checked="" type="checkbox"/>
1 <input type="checkbox"/> <input checked="" type="checkbox"/>	GRAD <input type="checkbox"/>	SCI <input type="checkbox"/>
2 <input type="checkbox"/> <input checked="" type="checkbox"/>	RAD <input type="checkbox"/>	ENG <input type="checkbox"/>
3 <input type="checkbox"/> <input checked="" type="checkbox"/>		n <u>2</u>

REGISTERS

0	1 A-T/RL	2 A-L	3 P-k <sub>C</sub>	4 PI-k <sub>C</sub>	5 PID-k <sub>C</sub>	6 PI-T <sub>i</sub>	7 PID-T <sub>i</sub>	8 PID-T <sub>0</sub>	9
S0	S1 B-k <sub>C</sub>	S2 B-P <sub>u</sub>	S3	S4	S5	S6	S7	S8	S9

A

B

C

D

E

I

# Program Description I

**Program Title** FIRST ORDER REGULATOR

**Contributor's Name** Hewlett-Packard Company

**Address** 1000 N.E. Circle Boulevard

**City** Corvallis

**State** Oregon

**Zip Code** 97330

**Program Description, Equations, Variables** Given a system;

$$\dot{x} = \frac{dx}{dt} = ax + bu$$

where  $x$  = system state

$u$  = system control

$a, b$  constant.

This program solves the regulator problem i.e. determines the optimal feedback gain to minimize the following performance index:

$$\text{performance index} = J = 1/2 \int_0^{\infty} (qx^2 + ru^2) dt$$

the solution is:

$$u = -cx, \text{ where } c = \frac{b}{r} S \text{ and } S \text{ is the positive solution to the Riccati equation:}$$

$$0 = -2as + \frac{s^2 b^2}{r} - q$$

Then:

$$\dot{x} = \bar{a}x \text{ where } \bar{a} = a - bc$$

$$\text{and } x = x_0 e^{-\frac{t}{\tau}} \text{ where } \tau = -\frac{1}{\bar{a}}$$

**Operating Limits and Warnings**

$$q \geq 0$$

$$r > 0$$

$$b \neq 0$$

This program has been verified only with respect to the numerical example given in *Program Description II*. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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# Program Description II

## Sketch(es)

## Sample Problem(s)

$$\dot{x} = ax + bu, \quad J = 1/2 \int_0^{\infty} \left[ \left( \frac{x}{x_{\max}} \right)^2 + \left( \frac{u}{u_{\max}} \right)^2 \right] dt$$

$$1. \quad a = -1, \quad b = 1, \quad q = \left( \frac{1}{x_{\max}} \right)^2 = 0, \quad r = \left( \frac{1}{u_{\max}} \right)^2 = 1$$

$$2. \quad a = 1, \quad b = 1, \quad q = 0, \quad r = 1$$

$$3. \quad a = 1, \quad b = 1, \quad q = 1, \quad r = 1$$

$$4. \quad a = 1, \quad b = 2, \quad q = 3, \quad r = 4$$

## Solution(s)

$$1. \quad c = 0, \quad s = 0, \quad \bar{a} = -1, \quad \tau = 1$$

$$2. \quad c = 2, \quad s = 2, \quad \bar{a} = -1, \quad \tau = 1$$

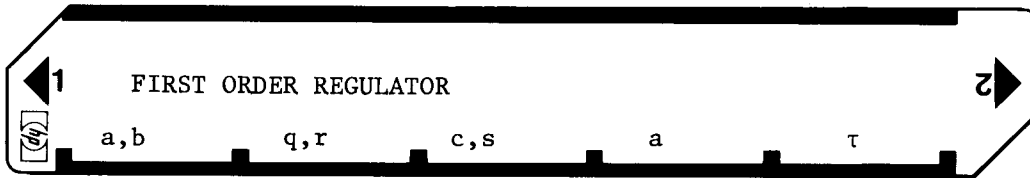
$$3. \quad c = 2.41, \quad s = 2.41, \quad \bar{a} = -1.41, \quad \tau = 0.707$$

$$4. \quad c = 1.5, \quad s = 3.0, \quad \bar{a} = -2, \quad \tau = 1/2$$

**Reference(s)** Shultz and Melsa, State Functions and Linear Control Systems, McGraw-Hill, 1967.

This program is a translation of the HP-65 Users' Library Program #04464A submitted by Randy A. Coverstone.

## 47

[illegible]

[illegible]

# Program Description I

**Program Title** SECOND ORDER REGULATOR

**Contributor's Name** Hewlett-Packard Company

**Address** 1000 N.E. Circle Boulevard

**City** Corvallis

**State** Oregon

**Zip Code** 97330

**Program Description, Equations, Variables** Given a system and a quadratic performance index as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} u$$

$$J = 1/2 \int_0^{\infty} \{ [x_1 \ x_2] \begin{bmatrix} q_{11} & q_{12} \\ q_{12} & q_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + u^2 r \} dt$$

where:  $x_1, x_2$  are the system states  
 $u$  is the control  
 $J$  is the performance index to be minimized.

The optimal control is given by:

$$u = -c_1 x_1 - c_2 x_2 \quad \text{and} \quad [\dot{\bar{x}}] = [\bar{A}][x] \quad [A] = [A] - [b][c_1 \ c_2]$$

$$\text{where: } c_1 = \frac{1}{b} (c_1^* + a_{11} c_2^*) \quad c_2 = \frac{1}{b} (a_{12} c_2^*)$$

$$c_1^* = a_1 + \sqrt{a_1^2 + q_1 \frac{b^2}{r}} \quad c_2^* = a_2 + \sqrt{a_2^2 + 2c_1^* + q_3 \frac{b^2}{r}}$$

$$\begin{aligned} a_1 &= a_{12} a_{21} - a_{11} a_{22} & q_1 &= q_{11} - 2 \frac{q_{12} a_{11}}{a_{12}} + \frac{q_{22} a_{11}^2}{a_{12}^2} \\ a_2 &= a_{11} + a_{22} & q_3 &= q_{22} \end{aligned}$$

## Operating Limits and Warnings

$$q_{11}, q_{12}, q_{13} \geq 0 ; \quad q_{11} + q_{12} + q_{13} \neq 0$$

$$r > 0$$

This program has been verified only with respect to the numerical example given in *Program Description II*. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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# Program Description II

## Sketch(es)

**Sample Problem(s)**  $\ddot{y} + \theta \dot{y} + \phi = u$  · second order system

let  $x_1 = y$

$x_2 = \dot{y}$

$$\text{so } \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\phi & -\theta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

suppose  $\phi = 2$   
 $\theta = 3$

Find the optimal control that minimizes:

1.  $J = 1/2 \int_0^\infty (x_1^2 + x_2^2 + u^2) dt$  , i.e.  $q_{11} = 1, q_{12} = 0, q_{22} = 1, r = 1$

2.  $J = 1/2 \int_0^\infty (4x_1^2 + u^2) dt$  , i.e.  $q_{11} = 4, q_{12} = 0, q_{22} = 0, r = 1$

3.  $J = 1/2 \int_0^\infty u^2 dt$  - note that this case violates  $q_1 + q_2 + q_3 \neq 0$

## Solution(s)

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$b = 1$$

$$u = -c_1 x_1 - c_2 x_2$$

1.  $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, r = 1 \rightarrow u = -.236x_1 -.236x_2$

2.  $Q = \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix}, r = 1 \rightarrow u = -.828x_1 -.264x_2$

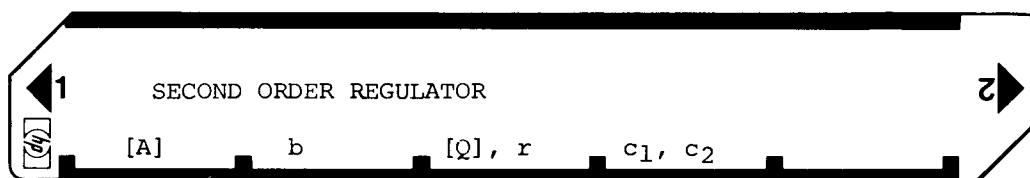
3.  $Q = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, r = 1 \rightarrow u = 0$

**Reference(s)** Shultz and Melsa, State Functions and Linear Control Systems, McGraw-Hill, 1967.

This program is a translation of the HP-65 Users' Library Program

#04465A submitted by Randy A. Coverstone.

## 51



STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program		[ ]	[ ]	
2	Input A matrix: input $a_{11}$	$a_{11}$	[↑]	[ ]	$a_{11}$
	then $a_{12}$	$a_{12}$	[↑]	[ ]	$a_{12}$
	then $a_{21}$	$a_{21}$	[↑]	[ ]	$a_{21}$
	then $a_{22}$	$a_{22}$	[A]	[ ]	$a_{11}$
3	Input b	b	[B]	[ ]	b
4	Input Q and r: input $q_{11}$	$q_{11}$	[↑]	[ ]	$q_{11}$
	then $q_{12}$	$q_{12}$	[↑]	[ ]	$q_{12}$
	then $q_{22}$	$q_{22}$	[↑]	[ ]	$q_{22}$
	then r	r	[C]	[ ]	$q_1/r$
5	Calculate feedback coefficients		[D]	[ ]	$c_1$
			[R/S]	[ ]	$c_2$
	(Optional)* Calculate $\bar{A}$ (closed loop dynamics matrix)		[R/S]	[ ]	
	Recall $\bar{A}$		[RCL]	1	$a_{11}$
			[RCL]	2	$a_{12}$
			[RCL]	3	$a_{21}$
			[RCL]	4	$a_{22}$
6	To change Q and r go to step 4				
7	For new case go to step 2				
**	DO NOT CALCULATE $\bar{A}$ IF STEP #6 IS TO BE EXECUTED, AS [A] REPLACES THE ORIGINAL [A] MATRIX AND STEP 5 MUST OPERATE ON [A], NOT $\bar{A}$				
	$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} u$				
	$J = 1/2 \int_0^\infty (q_{11}x_1^2 + 2q_{12}x_1x_2 + q_{22}x_2^2 + ru^2) dt$				
	minimize $J \rightarrow u = -c_1x_1 - c_2x_2$				
	$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$				

# 97 Program Listing I

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS		
001	*LBLA	21 11	Store [A]	057	X <sup>2</sup>	53	c2*		
002	ST04	35 04		058	RCL8	36 08			
003	R↓	-31		059	x	-35			
004	ST03	35 03		060	+	-55			
005	R↓	-31		061	JX	54			
006	ST02	35 02		062	RCL1	36 01			
007	R↓	-31		063	RCL4	36 04			
008	ST01	35 01		064	+	-55			
009	R/S	51		065	ST07	35 07			
010	*LBLB	21 12		066	+P	34			
011	ST05	35 05	Store b	067	RCL7	36 07	Display c <sub>1</sub>		
012	RTN	24		068	+	-55			
013	*LBLC	21 13		069	ST07	35 07			
014	ST07	35 07		070	RCL1	36 01			
015	=	-24		071	x	-35			
016	ST08	35 08		072	RCL6	36 06			
017	LSTX	16-63		073	+	-55			
018	x	-35		074	ST06	35 06			
019	RCL1	36 01		075	RCL5	36 05			
020	x	-35		076	÷	-24			
021	RCL2	36 02		077	R/S	51	Display c <sub>2</sub>		
022	÷	-24		078	RCL7	36 07			
023	X <sup>2</sup> Y	-41		079	RCL2	36 02			
024	2	02		080	x	-35			
025	x	-35		081	ST07	35 07			
026	-	-45		082	RCL5	36 05			
027	RCL1	36 01		083	÷	-24			
028	x	-35		084	R/S	51			
029	RCL2	36 02		085	RCL2	36 02			
030	÷	-24		086	RCL3	36 03			
031	+	-55		087	RCL6	36 06	Compute [A]		
032	RCL7	36 07		088	-	-45			
033	÷	-24		089	RCL4	36 04			
034	ST07	35 07		090	RCL7	36 07			
035	RTN	24		091	-	-45			
036	*LBLD	21 14		092	RCL1	36 01			
037	RCL2	36 02		093	R↓	-31			
038	RCL3	36 03		094	GTOA	22 11			
039	x	-35		095	R/S	51			
040	RCL1	36 01	c <sub>1</sub> *						
041	RCL4	36 04							
042	x	-35							
043	-	-45							
044	ST06	35 06		100					
045	RCL7	36 07							
046	JY	54							
047	RCL5	36 05							
048	x	-35							
049	RCL6	36 06							
050	+P	34					SET STATUS		
051	RCL6	36 06							
052	+	-55							
053	ST06	35 06							
054	2	02		110					
055	x	-35							
056	RCL5	36 05							
REGISTERS									
0	1 a <sub>11</sub>	2 a <sub>12</sub>	3 a <sub>21</sub>	4 a <sub>22</sub>	5 b	6 temp	7 temp	8 temp	9 R→P
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A	B	C	D	E	I				

## NOTES



## NOTES

## **Hewlett-Packard Software**

In terms of power and flexibility, the problem-solving potential of the Hewlett-Packard line of fully programmable calculators is nearly limitless. And in order to see the practical side of this potential, we have several different types of software to help save you time and programming effort. Every one of our software solutions has been carefully selected to effectively increase your problem-solving potential. Chances are, we already have the solutions you're looking for.

### **Application Pacs**

To increase the versatility of your fully programmable Hewlett-Packard calculator, HP has an extensive library of "Application Pacs". These programs transform your HP-67 and HP-97 into specialized calculators in seconds. Each program in a pac is fully documented with commented program listing, allowing the adoption of programming techniques useful to each application area. The pacs contain 20 or more programs in the form of prerecorded cards, a detailed manual, and a program card holder. Every Application Pac has been designed to extend the capabilities of our fully programmable models to increase your problem-solving potential.

You can choose from:

**Statistics**  
**Mathematics**  
**Electrical Engineering**  
**Business Decisions**  
**Clinical Lab and Nuclear Medicine**

**Mechanical Engineering**  
**Surveying**  
**Civil Engineering**  
**Navigation**  
**Games**

### **Users' Library**

The main objective of our Users' Library is dedicated to making selected program solutions contributed by our HP-67 and HP-97 users available to you. By subscribing to our Users' Library, you'll have at your fingertips, literally hundreds of different programs. No longer will you have to: research the application; program the solution; debug the program; or complete the documentation. Simply key your program to obtain your solution. In addition, programs from the library may be used as a source of programming techniques in your application area.

A one-year subscription to the Library costs \$9.00. You receive: a catalog of contributed programs; catalog updates; and coupons for three programs of your choice (a \$9.00 value).

### **Users' Library Solutions Books**

Hewlett-Packard recently added a unique problem-solving contribution to its existing software line. The new series of software solutions are a collection of programs provided by our programmable calculator users. Hewlett-Packard has currently accepted over 6,000 programs for our Users' Libraries. The best of these programs have been compiled into 40 Library Solutions Books covering 39 application areas (including two game books).

Each of the Books, containing up to 15 programs without cards, is priced at \$10.00, a savings of up to \$35.00 over single copy cost.

The Users' Library Solutions Books will compliment our other applications of software and provide you with a valuable new tool for program solutions.

**Options/Technical Stock Analysis**  
**Portfolio Management/Bonds & Notes**  
**Real Estate Investment**  
**Taxes**  
**Home Construction Estimating**  
**Marketing/Sales**  
**Home Management**  
**Small Business**  
**Antennas**  
**Butterworth and Chebyshev Filters**  
**Thermal and Transport Sciences**  
**EE (Lab)**  
**Industrial Engineering**  
**Aeronautical Engineering**  
**Control Systems**  
**Beams and Columns**  
**High-Level Math**  
**Test Statistics**  
**Geometry**  
**Reliability/QA**

**Medical Practitioner**  
**Anesthesia**  
**Cardiac**  
**Pulmonary**  
**Chemistry**  
**Optics**  
**Physics**  
**Earth Sciences**  
**Energy Conservation**  
**Space Science**  
**Biology**  
**Games**  
**Games of Chance**  
**Aircraft Operation**  
**Aviation**  
**Calendars**  
**Photo Dark Room**  
**COGO-Surveying**  
**Astrology**  
**Forestry**

## CONTROL SYSTEMS

These programs incorporate many of the important calculations from control theory. Bode plots, stability criteria, root-locus plots, and optimization are included.

FREQUENCY RESPONSE OF A TRANSFER FUNCTION  
BODE OF TRANSFER FUNCTION THAT HAS EACH POLE AND  
ZERO GIVEN  
BODE OF SECOND-ORDER OVER THIRD-ORDER TRANSFER  
FUNCTION  
BODE OF SECOND-ORDER OVER SECOND-ORDER TIMES  
S\*\*N TRANSFER FUNCTION  
POLE-ZERO TO GROUP DELAY  
ROUTH TEST FOR CONTINUOUS AND DISCRETE TIME SYSTEM  
ANALYSIS  
CONVERT FREQUENCY RESPONSE — OPEN LOOP, CLOSED  
LOOP  
AID TO ROOT LOCUS PLOTS I — REAL POLES  
AID TO ROOT LOCUS PLOTS II — COMPLEX POLES  
CLASSICAL CONTROL GAINS  
FIRST ORDER REGULATOR  
SECOND ORDER REGULATOR



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