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REAL ESTATE II
Applications

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HEWLETT PACKARD

HP-37E & HP-38E/38C

Real Estate II Income Property Analysis

Our Special Thanks To
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Introduction

This Real Estate II application book has been designed to supplement the HP-37E and HP-38E/38C Owner's Handbooks by providing a collection of applications specifically for income property analysis. Step by step keystroke procedures and/or programs with corresponding examples for 17 problem types are explained. Hopefully, this book will provide a reference guide to many of your problems, and show you how to redesign our examples to fit your specific needs.

It is sometimes necessary in these keystroke solutions to include operations which involve prefix keys, namely [f] on the HP-37E and [f] and [g] on the HP-38E/38C. For example, the operation [12×] is performed on the HP-37E as [f] [12×] and on the HP-38E/38C as [g] [12×]. In such cases, the keystroke solution omits the prefix key and indicates only the operation (as here, [12×]). As you work through the example problems, take care to press the appropriate prefix key (if any) for your calculator.

In addition, it should be noted that certain clearing functions on the two calculators have different key mnemonics. Clear finance on the HP-37E is represented as [CL FIN], and is represented as [FIN] on the HP-38E/38C. Clear all is represented as [CL ALL] and [ALL] on the HP-37E and HP-38E/38C respectively. Unless otherwise specified, this book will use the key mnemonics of the HP-37E, although the keystrokes are applicable to both machines.

All results are carried internally to ten significant digits. If intermediate answers are rounded by the user, slightly different final values may be obtained.

Contents

Annual Property Cash Flow Analysis

In most real estate investment and valuation problems (among others), it is necessary to calculate the future income and expense flows that are utilized in appraisal, financing and investment analysis.

The HP-37E and HP-38E/38C have an advantage in calculating Before-Tax Cash Flows because sequential or chain calculation capabilities can be utilized to work from Potential Gross Income to Cash Throw-Off to Equity in one continuous operation. With After-Tax Cash Flows, the ability of the HP-37E and HP-38E/38C to store values and to calculate schedules of depreciation and annual interest payments considerably shortens calculating time, as well as reduces the possibilities of error.

Before-Tax Cash Flows

The before-tax cash flows applicable to real estate analysis and problems are:

PGI: Potential Gross Income

EGI: Effective Gross Income

NOI: Net Operating Income (also called Net Income Before Recapture)

CTO: Cash Throw-Off to Equity (also called Gross Spendable Cash)

All are annual flows in real estate analysis.

These terms and symbols are further explained in the Appendix.

The derivation of these cash flows follows a set sequence:

1. Calculate Potential Gross Income by multiplying the rental per unit times the number of units, times the number of rental payment periods per year. This gives the rental income the property would generate if it were fully occupied.
2. Deduct Allowance for Vacancy and Rental Loss. The result is Rent Collections (which is also Effective Gross Income if there is no "Other Income").
3. Add "Other Income", such as receipts from concessions (laundry

equipment, etc.), produced from sources other than the rental of space. This is the Effective Gross Income.

4. Deduct Operating Expenses. These are expenditures the landlord-investor must make, by contract or custom, to preserve the property and keep it capable of producing the forecast gross income. The result is Net Operating Income.
5. Deduct Annual Debt Service on the mortgage. This produces Cash Throw-Off to Equity.

Thus:

$$\text{PGI} - \text{Vac} + \text{Other} = \text{EGI}$$

$$\text{EGI} - \text{OE} = \text{NOI}$$

$$\text{NOI} - \text{ADS} = \text{CTO}$$

Example:

A 60-unit apartment building has rentals of $250 per unit per month. Three units are currently vacant, which is a typical vacancy ratio for competitive properties. Concession income from coin-operated laundry equipment averages $6 per occupied unit per month.

Management fees are 3.5% of rent collections. Other operating expenses are: Property Taxes $27,350; Insurance $3,255; Repairs and Maintenance $14,285 plus a free apartment for the building superintendent; Utilities (sewer and water) $7,850; Heat and Air Conditioning $11,450; Replacements $3,975; Other (Miscellaneous) $3,125.

The property has just been financed with a $700,000 mortgage, fully amortized in level monthly payments at 9.5% interest over 20 years.

a. What is Effective Gross Income?

b. What is Net Operating Income?

c. What is Cash Throw-Off to Equity?

(a) Effective Gross Income:

Keystrokes:	Display:	
60 [ENTER↑]		# of units
250 [×]		rent/month
12 [×]	***180,000.00***	PGI
[ENTER↑]		

Keystrokes:	Display:	
ENTER↑		
3		average vacancy
ENTER↑		
60 ÷	0.05	Vacancy ratio
×		
−		
STO 2		
57 ENTER↑		rented units
6 ×		income/unit/month
12 ×	4,104.00	Laundry concessions
+	175,104.00	EGI
STO 3		

(b) Net Operating Income:

Keystrokes:	Display:	
RCL 2		
3.5 %	5,985.00	management fees
27350 +		property taxes/year
3255 +		insurance/year
14285 +		repairs & maintenance/year
250 ENTER↑		superintendent's rental
12 × +		
7850 +		utilities
11450 +		heating & air
3975 +		replacement & other
3125 +	80,275.00	OE
RCL 3	175,104.00	EGI
x≷y −	94,829.00	NOI
STO 5		

(c) Cash Throw-Off to Equity (CTO):

BEGIN ▮▮ END		
CL FIN		
20 12×	240.00	mortgage term
9.5 12÷	0.79	mortgage rate
700000 PV		property value

Keystrokes:	Display:	
PMT		
12 ×	-78,299.02	ADS, annual debt service (mortgage payments)
RCL 5		NOI
+	16,529.98	CTO

Before-Tax Reversions (Resale Proceeds)

The reversion receivable at the end of the income projection period is usually based on forecast or anticipated resale of the property at that time. The before-tax reversion amounts applicable to real estate analysis and problems are:

SP: Resale Price

CPR: Cash Proceeds of Resale

b: Outstanding Mortgage Balance

NCPR: Net Cash Proceeds of Resale to Equity

These terms and symbols are further explained in the Appendix.

The derivation of these reversions is as follows:

1. Forecast or estimate Resale Price. Deduct sales and disposition expenses (brokerage commission, legal fees, etc.). The result is Proceeds of Resale.
2. Calculate Outstanding Balance of the Mortgage at the end of the Income Projection Period and subtract it from Proceeds of Resale. The result is Net Cash Proceeds of Resale.

Thus: SP − Disp. Exp. = CPR

PR − b = NCPR

Example:

The apartment property in the preceding example is expected to be resold in 10 years. The forecast resale price is $800,000. The broker's commission is expected to be 6% and other selling or disposition expenses are 2.5%. The mortgage is the same as that indicated in the preceding example.

a. What will the Mortgage Balance be in 10 years?

b. What are the Forecast Cash Proceeds of Resale, Net Cash Proceeds of Resale?

Keystrokes:	**Display:**	
BEGIN ▮ END		
[CL FIN]		
20 [12×]	240.00	mortgage term
9.5 [12÷]	0.79	mortgage rate
700,000 [PV]		property value
[PMT]	-6,524.92	PMT
10 [12×]		projection period
[FV] [STO] 0	-504,253.59	−b
800,000 [ENTER↑]		estimated resale
6 [ENTER↑]		brokers fees
2.5 [+]		other selling expenses
[%]	68,000.00	
[−]	732,000.00	CPR
[RCL] 0 [+]	227,746.41	NCPR

After-Tax Cash Flows

After-tax cash flow is found for each year by deducting Income Tax Liability for that year from CTO,

where Taxable Income = NOI − Int. − Dep.

Tax Liability = Taxable Income × r

and ATCF = CTO − Tax Liability

The after-tax cash flow for each year may be calculated by means of the following HP-38E/38C program:

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
[g] [P/R] [g] [CL P]	00-	[+]	19- 51
[÷]	01- 71	[STO] 3	20- 21 3
[×]	02- 61	1	21- 1
1	03- 1	2	22- 2
[STO] 0	04- 21 0	[f] [AMORT]	23- 24 11
[g] [LAST X]	05- 25 31	[RCL] 0	24- 22 0
[STO] [−] 0	06- 21 41 0	[STO] [×] 1	25- 21 61 1
[−]	07- 41	[x⇄y]	26- 33
[÷]	08- 71	[RCL] 1	27- 22 1
[STO] 1	09- 21 1	[−]	28- 41
[g] [R↓]	10- 25 33	[RCL] 2	29- 22 2
1	11- 1	[×]	30- 61
[RCL] 2	12- 22 2	[CHS]	31- 32
[−]	13- 41	[RCL] 3	32- 22 3
[×]	14- 61	[+]	33- 51
[RCL] [PMT]	15- 22 14	[R/S]	34- 74
1	16- 1	[g] [GTO] 21	35- 25 7 21
2	17- 2	[g] [P/R]	
[×]	18- 61		

REGISTERS			
R_0 1-dep. rate	R_1 dep.	R_2 r	R_3 Used

1. Set the payment switch to END and press [f] [ALL].
2. Key in the annual interest rate and press [12÷].
3. Key in the principal and press [PV].
4. Key in the monthly payment and press [CHS] [PMT].
5. Key in the marginal tax rate, r, and press [STO] 2.
6. Key in the NOI and press [ENTER↑].
7. Key in the depreciation base and press [ENTER↑].
8. Key in the declining balance factor and press [ENTER↑].
9. Key in the useful life and press [R/S] to compute $ATCF_1$.
10. Continue pressing [R/S] to compute successive after tax cash flows.

Example 1:

An investor has recently purchased a new commercial property for $1,400,000. The value of the depreciable improvements is $1,200,000 with a 35 year economic life. The depreciation method used will be 150% declining balance.

The property is financed with a fully amortized loan of $1,050,000. The terms of the loan are 9.5% interest, monthly amortization, and a 25 year term. The monthly loan payment is $9,173.81.

The property generates net operating income of $135,000. The investor has a marginal tax rate of 50%.

What are the after-tax cash flows for this property in years 1-10?

Keystrokes:	Display:	
BEGIN/END switch to END		
[f] [ALL]		
9.5 [12÷]	**0.79**	
1050000 [PV]		
9173.81 [CHS] [PMT]		
.50 [STO] 2		
135000 [ENTER↑]		
1200000 [ENTER↑]		
1.50 [ENTER↑]		
35 [R/S]	**32,772.50**	$ATCF_1$
[R/S]	**31,134.63**	$ATCF_2$
[R/S]	**29,490.82**	$ATCF_3$
[R/S]	**27,833.74**	$ATCF_4$
[R/S]	**26,155.67**	$ATCF_5$
[R/S]	**24,448.40**	$ATCF_6$
[R/S]	**22,703.10**	$ATCF_7$
[R/S]	**20,910.39**	$ATCF_8$
[R/S]	**19,060.17**	$ATCF_9$
[R/S]	**17,141.58**	$ATCF_{10}$

Example 2:

The property was purchased for $900,000 of which $150,000 was allocated to land. Therefore the "depreciable amount" of investment in the buildings is $750,000. The buildings have an estimated remaining useful life of 25 years, and are to be depreciated on a 125% declining-balance basis.

The mortgage loan terms are: Principal of $700,000; Interest rate of 9.5%; Full amortization in level monthly payments over 20 years. The applicable income tax rate is 48%. NOI = $94,829.00.

What is the scheduled ATCF for 10 years?

Keystrokes:	Display:	
BEGIN/END switch to END		
[f] [ALL]		
9.5 [12÷]	**0.79**	
700000 [PV]		
20 [12×]		
[PMT]	**-6524.92**	monthly payments

Keystrokes:	Display:	
.48 [STO] 2		
94829 [ENTER↑]		
750000 [ENTER↑]		
1.25 [ENTER↑]		
25 [R/S]	**20,678.84**	$ATCF_1$
[R/S]	**19,191.61**	$ATCF_2$
[R/S]	**17,691.11**	$ATCF_3$
[R/S]	**16,169.29**	$ATCF_4$
[R/S]	**14,617.65**	$ATCF_5$
[R/S]	**13,027.20**	$ATCF_6$
[R/S]	**11,388.29**	$ATCF_7$
[R/S]	**9,690.66**	$ATCF_8$
[R/S]	**7,923.30**	$ATCF_9$
[R/S]	**6,074.32**	$ATCF_{10}$

After-Tax Cash Flows With Multiple Mortgages

The following program performs the same functions as the one preceding, but permits up to three mortgages to exist on the property.

KEY ENTRY	DISPLAY
[g] [P/R] [g] [CL P]	00–
0	01– 0
[STO] 4	02– 21 4
3	03– 3
[STO] 3	04– 21 3
[RCL] 3	05– 22 3
[g] [x=0]	06– 25 6
[g] [GTO] 39	07– 25 7 39
[RCL] 5	08– 22 5
[PMT]	09– 14
[RCL] 6	10– 22 6
[i]	11– 12
[RCL] 7	12– 22 7

KEY ENTRY	DISPLAY
[PV]	13– 13
[RCL] 8	14– 22 8
[STO] 5	15– 21 5
[RCL] 9	16– 22 9
[STO] 6	17– 21 6
[RCL] .0	18– 22 73 0
[STO] 7	19– 21 7
[RCL] .1	20– 22 73 1
[STO] 8	21– 21 8
[RCL] .2	22– 22 73 2
[STO] 9	23– 21 9
[RCL] .3	24– 22 73 3
[STO] .0	25– 21 73 0

KEY ENTRY	DISPLAY
1	26– 1
2	27– 2
[f] [AMORT]	28– 24 11
[STO] [+] 4	29– 21 51 4
[RCL] [PMT]	30– 22 14
[STO] .1	31– 21 73 1
[RCL] [i]	32– 22 12
[STO] .2	33– 21 73 2
[RCL] [PV]	34– 22 13
[STO] .3	35– 21 73 3
1	36– 1
[STO] [−] 3	37– 21 41 3

KEY ENTRY	DISPLAY
[g] [GTO] 05	38– 25 7 05
[RCL] 0	39– 22 0
[STO] [×] 1	40– 21 61 1
[RCL] 1	41– 22 1
[RCL] 4	42– 22 4
[−]	43– 41
[RCL] 2	44– 22 2
[g] [FRAC]	45– 25 61
[×]	46– 61
[RCL] 2	47– 22 2
[−]	48– 41
[g] [P/R]	

REGISTERS			
R_0 1-dep. rate	R_1 dep	R_2 Constant .r	R_3 Flag
R_4 Σint	R_5 PMT_1	R_6 i_1	R_7 PV_1
R_8 PMT_2	R_9 i_2	$R_{.0}$ PV_2	$R_{.1}$ PMT_3
$R_{.2}$ i_3	$R_{.3}$ PV_3	$R_{.4}$	$R_{.5}$

Instructions:

1. Press [f] [ALL].
2. Key in NOI and press [ENTER↑] 1 [ENTER↑].
3. Key in r and press [STO] 2 [−] [×].
4. Key in the total monthly payment and press [CHS] [ENTER↑].
5. Press 12 [×] [+] [f] [INTGR] [STO] [−] 2.
6. Key in the depreciation base and press [ENTER↑].
7. Key in the depreciation rate (converted to decimal) and press [ENTER↑].

8. Key in the life and press [÷] [×] 1 [ENTER↑] [g] [LAST X] [−] [STO] 0 [÷] [STO] 1.
9. Key in the payment amount of the first mortgage, PMT_1, and press [CHS] [STO] 5.
10. Key in the interest rate on the first mortgage, i_1, and press [12÷] [STO] 6.
11. Key in the principal of the first mortgage, PV_1, and press [STO] 7.
12. Key in the payment amount of the second mortgage, PMT_2, and press [STO] 8. If there is no second mortgage, proceed to step 19.
13. Key in i_2 and press [12÷] [STO] 9.
14. Key in PV_2 and press [STO] .0.
15. Key in PMT_3 and press [STO] .1 (proceed to step 18 if there is no third mortgage).
16. Key in i_3 and press [12÷] [STO] .2.
17. Key in PV_3 and press [STO] .3.
18. Press [R/S] to compute ATCF for the first year.
19. Repeat step 18 to compute the ATCF for successive years.

Example:

Solve the previous problem using this program. Since there are no second or third mortgages, registers 8 through .3 will contain zeros and the ATCF's will be the same as those previously obtained (to the nearest dollar).

Keystrokes:	Display:	
BEGIN ▮▮ END		
[f] [ALL]		
94829 [ENTER↑]		
1 [ENTER↑]		
.48 [STO] 2 [−] [×]	***49,311.08***	
6524.92 [CHS] [ENTER↑]		from exp. 2, P. (11)
12 [×] [+] [f] [INTGR]		
[STO] [−] 2	***-28,987.00***	
750000 [ENTER↑]		
1.25 [ENTER↑]		from exp. 2, P. (11)

Keystrokes:	Display:	
25 [÷] [×] 1 [ENTER↑]		
[g] [LAST X] [−] [STO]		
0 [÷]		
[STO] 1	***39,473.68***	
6524.92 [CHS] [STO] 5		from exp. 2, P. (11)
9.5 [12÷] [STO] 6		
700000 [STO] 7		
[R/S]	***20,679.30***	$ATCF_1$
[R/S]	***19,192.07***	$ATCF_2$
[R/S]	***17,691.56***	$ATCF_3$
[R/S]	***16,169.75***	$ATCF_4$
:		

Note that these results are slightly higher than those previously obtained. The worst case difference is approximately 50 cents and the average difference (over different cases) is zero.

After-Tax Net Cash Proceeds of Resale

ATNCPR = NCPR − Tax Liability

To calculate Tax Liability, it is necessary to find the Gain on Resale. This is divided between Excess Depreciation, which is taxed (fully or partially, depending on whether any Excess Depreciation is "forgiven") as ordinary income; and the remainder, which is Capital Gain taxed at the capital gains tax rate.

The steps are:

1. Calculate total depreciation charged using the following keystroke sequence (see Appendix for the formula used):
   a) Key in 1, press [ENTER↑] [ENTER↑].
   b) Key in the percent declining balance depreciation and press [ENTER↑]. Key in 100, press [÷] to convert to decimal percent.
   c) Key in the useful life of the asset, press [÷] [−].
   d) Key in the depreciation accumulation period, press [y^x] [−].
   e) Key in the asset depreciation base (starting book value), press [×]. Subtract this from the original purchase price (Capital Outlay) to obtain Tax Basis.

2. Subtract Tax Basis from Proceeds of Resale. The result is Gain on Resale.
3. Subtract total straight-line depreciation over the income projection period from total depreciation charged. This produces Excess Depreciation.
4. Subtract Excess Depreciation from Gain on Resale to obtain Capital Gain.
5. Multiply Excess Depreciation by ordinary income tax rate. This produces ordinary income tax liability on resale.
6. Multiply Capital Gain by capital gains tax rate. This produces capital gains tax liability on resale.
7. Add the figures derived in steps 5 and 6 to obtain total Tax Liability on resale.
8. Subtract total Tax Liability from Net Cash Proceeds of Resale to obtain After-Tax Net Cash Proceeds of Resale.

Thus:

CO − Total Dep. = Tax Basis

CPR − Tax Basis = Gain on Resale

Total Dep. − S-L Dep. = Excess Dep.

Gain on Resale − Excess Dep. = Capital Gain

(Excess Dep. × Ord. Tax Rate) +
Cap. Gain × CG Tax Rate) = Tax Liability

NCPR − Tax Liability = ATNCPR

Example:

The apartment complex which has been used as an example throughout this chapter is forecast in year 10 to sell for $1,750,000. Disposition expenses will amount to 8%. The applicable ordinary income tax rate is 48% and the capital gain tax rate is 30%.

Recalling from previous examples:

P = $700,000 @ 9.5% for 20 years,

CO = $200,000

Estimated remaining useful life is 25 years, depreciated on a 125% declining balance basis.

What is the indicated ATNCPR? After Tax IRR? (with HP-38E/HP-38C)

Calculate and store NCPR:

Keystrokes:	Display:	
[CL FIN]		
20 [12×]	***240.00***	
9.5 [12÷]	***0.79***	
700000 [PV]		
[PMT]	***-6,524.92***	
10 [12×]	***120.00***	
[FV]	***-504,253.59***	b
1750000 [ENTER↑]		
8 [%] [−] [STO] 0	***1,610,000.00***	CPR
[+] [STO] 1	***1,105,746.41***	NCPR
Calculate and store accumulated depreciation over projection period:		
1 [ENTER↑] [ENTER↑]		
125 [ENTER↑] 100 [÷]	***1.25***	Declining balance factor
25 [÷] [−]		Assets useful life = 25
10 [y^x] [−]		Accumulation period = 10
750000 [×] [STO] 2	***300,947.30***	Accumulated depreciation
900000 [x⇄y] [−]	***599,052.70***	Tax basis
Subtract tax basis from proceeds of resale to obtain gain on resale:		
[RCL] 0	***1,610,000.00***	CPR
[x⇄y] [−]	***1,010,947.30***	Gain on Resale
Calculate excess depreciation and capital gains:		
750000 [ENTER↑]		
25 [÷] 10 [×]	***300,000.00***	Total straight-line depreciation
[RCL] 2	***300,947.30***	Accumulated depreciation
[x⇄y] [−] [STO] 3	***947.30***	Excess depreciation
[−]	***1,010,000.00***	Capital gain
Calculate ordinary and capital gain taxes to obtain total tax liability:		
[RCL] 3	***947.30***	
.48 [×]	***454.70***	Tax on excess depreciation
[x⇄y] .3 [×]	***303,000.00***	Capital gains tax
[+]	***303,454.70***	Total tax liability
Calculate after tax net cash proceeds of resale:		
[RCL] 1	***1,105,746.41***	NCPR
[x⇄y] [−]	***802,291.71***	ATNCPR

You may now calculate the internal rate of return on this investment with your HP-38E/HP-38C by going to the procedure on page (40).

Mortgage-Equity (Ellwood) Analysis

Analyzing and appraising real estate investment properties in terms of their mortgage and equity investment components constitutes Mortgage-Equity Analysis. It was formalized and popularized by the late L. W. Ellwood. This is why it is frequently referred to as "Ellwood Analysis."

This framework of analysis is used to estimate the Present Worth (Market Value or Investment Value) of the total property investment and of the equity investment position. Property value is estimated by capitalizing Net Operating Income at the Overall Rate:

$$V = \frac{NOI}{R}$$

Present Worth of the equity investment position is estimated by capitalizing Cash Throw-off to equity at the Equity Dividend Rate:

$$V_e = \frac{CTO}{R_e}$$

The Mortgage-Equity framework is also used to estimate the dollar amount of resale proceeds (CPR), or the percentage of increase (app.) or decrease (dep.) in resale proceeds over initial investment (Capital Outlay), required to achieve a given Basic Rate (r) or Equity Yield Rate (y).

Finally, the analysis can be used to calculate the Basic Rate (r) on the total property investment, or the Equity Yield Rate (y) on the equity investment.

If all figures were available in dollar amounts, it would be unnecessary to have a separate Mortgage-Equity framework. However, often the dollar value of Present Worth, Resale Proceeds (reversion) and Mortgage Principal are unknown. Only NOI is given as a dollar figure, with mortgage loan terms and capital gain (app.) or loss (dep.) on resale given as percentages. Thus it is necessary to calculate the Basic Rate (r) and the Overall Rate (R) to apply to NOI to estimate value.

Calculation of Basic Rate and Overall Rate

The formulas used for calculation of Basic Rate (r), and Overall Rate (R) are to be found in the Appendix.

The given values required are:

- i = mortgage interest rate
- m = loan-to-value ratio of mortgage
- n_t = total number of mortgage payments to full amortization
- y = equity yield rate
- n = income projection period (investment holding period)
- dep./app. = capital loss or gain on resale as a percentage of present worth or value of property

With these values, it is then possible to calculate:

$$f = \text{mortgage constant}\left(f = \frac{ADS}{V}\right)$$

p = percentage of mortgage principal paid off over the income projection period ($p = 1 - b$)

After these values are calculated, r and R can be calculated.

Example:

To illustrate all the required calculations to derive the Basic Rate (r) and the Overall Rate (R), the following conditions are assumed: An investor plans to purchase an income property, hold it for 10 years, and then resell it. It is estimated that the proceeds of resale will result in a 15% capital loss. A 25-year mortgage loan with level monthly payments at 8.75% interest can be obtained, with a loan-to-value ratio of 70%. The investor is seeking a 14% yield on his equity investment.

$i = 8.75\%$

$m = .70$

$n_t = 25$ years

$y = 14\%$

$n = 10$ years

dep. $= .15$

Keystrokes:	Display:	
BEGIN [switch] END		
[CL FIN]		
[f] 6		set display mode
25 [12×]	*300.000000*	
8.75 [12÷]	*0.729167*	
.70 [PV]		
[PMT]	*-0.005755*	
12 [×] [CHS] [STO] 0	*0.069060*	f
10 [12×] [FV]		
[CHS]	*0.575818*	b
1 [x⇄y] [−] [FV]	*0.424182*	p
10 [n]		
14 [i]		
.3 [CHS] [PV]		
[PMT]		
[RCL] 0 [+]	*0.104638*	r, Basic Rate
[RCL] [FV]		
.15 [−]	*0.274182*	NCPR
[FV]		
[PMT]		
[RCL] 0 [+]	*0.112395*	R, Overall Rate

Note:

NOI must be a level annuity. The total income stream is thus a level annuity plus a reversion receivable at the end of the payment period. Also, all cash flows (NOI, ADS and CTO) are before-tax cash flows, and all rates of return (4, i and y), as well as all capitalization rates (R, R and R_e) are before-tax annual rates.

To calculate the mortgage constant, f, and mortgage balance, b, at the end of the income projection period:

1. Set the payment switch to END and press [CL FIN].
2. Key in the amortization period, n_t (years), and press [12×].
3. Key in the annual mortgage interest, i, and press [12÷].
4. Key in the loan to value ratio, m, and press [PV].

5. Press [PMT].
6. Press 12 [×] [CHS] [STO] 0 to obtain (and store) f, the mortgage constant.
7. Key in the income projection period, n, and press [12×] [FV] [CHS] to obtain b, the % mortgage balance.
8. Key in 1 [x⇄y] [−] [FV] to obtain and store p, the % mortgage principal paid off over the projection period.

To calculate the basic rate, 4, and overall rate R:

9. Key in n, the income projection period, and press [n].
10. Key in y, the equity yield rate, and press [i].
11. Key in (1-m) and press [CHS] [PV].
12. Press [PMT].
13. Add f, from above, by pressing [RCL] 0 [+], to obtain r.
14. Press [RCL] [FV].
15. Add the appreciation, expressed as a decimal (or subtract the depreciation), to obtain NCPR, the net cash proceeds of resale.
16. Press [FV].
17. Press [PMT].
18. Add f, from above, by pressing [RCL] 0 [+] to obtain R.

Calculation of Value (Present Worth) With *R*, Given Only NOI

When *R* is calculated, as above, Value is estimated by the formula:

$$V = \frac{NOI}{R}$$

Example 1:

In the preceding examples R = .112395. If NOI is forecast at $33,500, what is the estimated value or present worth of the property?

Keystrokes:	Display:	
[f] 2		
33500 [ENTER↑]		
.112395 [÷]	***298,055.96***	V or PW

Example 2:

A property is forecast to produce NOI of $24,550 annually. The most probable mortgage loan terms are an 82% loan with level monthly payments at 9.25% interest over a maturity of 22 years 8 months. The investor expects to hold the property for 12 years and then sell it at 20% above its present value. If the investor is looking for a 15.35% rate of return on equity investment, what is the value (present worth) of the property? What is the indicated present worth of the equity investment position?

The given values are:

n_t = 272 months	n = 12 years
m = .82	app. = .20
y = 15.35%	NOI = $24,550
i = 9.25%	

Keystrokes:	Display:	
BEGIN / END (switch)		
[CL FIN]		
[f] 6		
272 [n]	***272.000000***	
9.25 [12÷]	***0.770833***	
.82 [PV]		
[PMT]		
12 [×] [CHS] [STO] 0	***0.086573***	f
12 [12×] [FV]		
[CHS] [STO] 1	***0.585670***	b Store for future use.

(Because in this example the selling price is specified as 120% of the present worth, r is not calculated and the instructions are modified as follows: Skip steps 12 and 13, and enter 1.00 − b in place of step 14).

12 [n]		
15.35 [i]		
.18 [CHS] [PV]		(100-82)% = 18% down payment
1 [ENTER↑]		
[RCL] 1 [−]		Recall b and subtract
.2 [+]	***0.614330***	NCPR
[FV]		
[PMT]		
[RCL] 0 [+]	***0.099547***	R

To calculate V,

[f] 2		
24550 [x⇄y] [÷]	***246,618.15***	V

To calculate V_e and P,

[ENTER↑]		
[ENTER↑]	***246,618.15***	
[RCL] [PV] [CHS]	***0.18***	
[×]	***44,391.27***	$V_e = (1 - m) \times V$
[−]	***202,226.88***	$P = V - V_e$

Calculation of Equity Dividend Rate, R_e

The Equity Dividend Rate, R_e is applied directly to Cash Throw-Off to Equity to find the present worth of the equity investment position:

$$V_e = \frac{CTO}{R_e}$$

The Equity Dividend Rate is calculated when CTO and the amount of the equity investment are known in dollar amounts by the formula:

$$R_e = \frac{CTO}{V_e}$$

Example:

The equity investment in an income property is $44,391. NOI is forecast at $24,550, while Annual Debt Service is $21,350. What is the indicated Equity Dividend Rate, R_e?

Keystrokes:	Display:	
[f] 6		
24550 [ENTER↑]		
21350 [−]	***3,200.000000***	CTO = NOI − ADS
44391 [÷]	***0.072087***	R_e

When dollar amounts are not available, the Equity Dividend Rate can be calculated with all the data used to calculate R.

Calculation of Cash Throw-Off to Equity, CTO:

1. Calculate NCPR, (see page 7) and press [FV].
2. Key in n and press [n].
3. Key in y and press [i].
4. Key in (1 − m) and press [CHS] [PV].
5. Press [PMT] to obtain CTO.

Example:

An income property has an 82% mortgage with level monthly payments at 9.25% interest fully amortized in 22 years 8 months. The equity investor is seeking a 15.35% Equity Yield Rate over the income projection of 12 years. What is the indicated Equity Dividend Rate, if the proceeds of resale are forecast to be 20% above present value of the property?

n_t = 272 months	i = 9.25%
m = .82	n = 12 years
y = 15.35%	app. = .20

Keystrokes:	Display:	
[CL FIN]		
272 [n]		
9.25 [12÷]		
.82 [PV]		
[PMT]		

Skip step 6, since f is not needed

12 [12×] [FV]		
[CHS]	***0.585670***	b

Compute NCPR = present worth + appreciation (or − depreciation) − b

Keystrokes:	Display:	
1		PW
[x⇄y] [−]		
.20 [+]	***0.614330***	NCPR
[FV]		
12 [n]		
15.35 [i]		
.18 [CHS] [PV]		
[PMT]	***0.012974***	CTO
[RCL] [PV]		
[CHS]	***0.180000***	V_e
[÷]	***0.072078***	R_e

Calculation of Value (Present Worth) With Dollar Amounts Given

This procedure involves calculating the present worth of the future income stream and reversion, to derive the present worth of the investment. It can be used to estimate property value using NOI, the discount or basic rate (r), and the Proceeds of Resale. It can also be used to estimate the present worth of the equity investment position using CTO, y, and the Net Cash Proceeds of Resale.

1. Compute the annual debt service, ADS.
2. Compute the mortgage balance, b.
3. Compute the net cash proceeds of resale, NCPR, and press [FV].

4. Key in n and press [n].
5. Key in y and press [i].
6. Key in CTO and press [PMT].
7. Press [FV] [CHS] to obtain V_e.
8. Add the mortgage principal, P, to obtain the present worth, V.

Example:

An income property is forecast to produce NOI of $20,575 per year. It has just been financed with a $160,000 mortgage, to be fully amortized in level monthly payments at 8.75% interest over 25 years. The anticipated proceeds of resale of the property in 10 years is $191,250. The equity investor expects an Equity Yield Rate of 14.75%.

What is the present worth of the equity investment position? What is the present worth (value) of the property?

Keystrokes:	**Display:**	
[CL FIN]		
[f] 2		
25 [12×]		
8.75 [12÷]		
160000 [CHS] [PV]		
[PMT]		
12 [×]	***15,785.16***	ADS
10 [12×]		
[FV]	***131,615.55***	b
191250 [−] [CHS] [FV]	***59,634.45***	NCPR
10 [n]		
14.75 [i]		
20575 [ENTER↑]		
15785.16 [−] [PMT]	***4,789.84***	CTO
[PV] [CHS]	***39,334.98***	V_e
160000 [+]	***199,334.98***	V

So far in this chapter, we have shown how to calculate and use quantities like f (and its dollar value equivalent, ADS), b, NCPR, CPR, n, y, 1 − m (and its dollar value equivalent, CO), and NOI. In the remainder

of the chapter, the same procedure is invariably used: four of the five quantities, n, y, 1 − m (or CO), CTO, and NCPR are stored in the financial registers n, i, PV, PMT, and FV respectively and the fifth quantity is calculated.

Calculation of Capital Appreciation or Depreciation on Resale, Plus Resale Price Required to Achieve a Given Equity Yield Rate:

The percent of capital appreciation or depreciation on resale required to achieve a given equity yield rate can be calculated using either rates or dollar amounts. In addition, when dollar amounts are available, it is possible to calculate the dollar amount of resale proceeds required. The calculations can be applied to either NOI or CTO cash flows.

a. Calculation of dep. or app. Using Rates

The formula for dep. or app. is:

$$\%\ \text{app. or dep.} = \left(\frac{\text{CPR} - \text{V}}{\text{V}}\right) \times 100$$

Where positive results represent app., negative results dep.

Example:

An investment is producing NOI at an Overall Rate (R) of 10.25%. It has just been financed with a 70% mortgage at 9% interest, fully amortized in level monthly payments over 20 years. What must resale proceeds be at the end of 10 years for the investor to earn a 13% rate of return on the equity investment?

Keystrokes:	**Display:**
[CL FIN]	
[f] 6	
20 [12×]	
9 [12÷]	
.7 [PV]	

Keystrokes:	Display:	
[PMT]		
12 [×] [ENTER↑]	**-0.075577**	– f
.1025 [+] [STO] 0	**0.026923**	CTO

(note that CTO is calculated using the usual relationship CTO = NOI – ADS, where NOI = V × R and ADS – f × y, with V = 1.00 (100%)).

10 [12×]		
[FV] [CHS] [STO] 1	**0.497181**	b
10 [n]		
[RCL] 0 [PMT]		CTO
13 [i]		y
.3 [CHS] [PV]		1 – m
[FV]	**0.522455**	NCPR
[RCL] 1 [+]	**1.019636**	CPR

Next, calculate appreciation, again using V = 1.00.

1 [x⇄y]		
[f] [Δ%]	**1.963618**	% appreciation

What if the desired yield rate is 16%?

16 [i]		
[FV]	**0.749392**	NCPR
[RCL] 1 [+]	**1.246573**	CPR
1 [x⇄y]		
[f] [Δ%]	**24.657342**	% appreciation

To earn an Equity Yield Rate of 13%, Proceeds of Resale must be 1.96% higher than the original purchase price or value.

To earn an Equity Yield Rate of 16%, Proceeds of Resale must be 24.66% higher than the original purchase price or value.

b. Calculations of dep. or app. Using Dollar Figures.

These procedures calculate the dollar amount of the reversion (CPR or NCPR) needed to achieve a given or desired rate of return (r or y).

In one procedure, the net amount of Future Worth is derived as the amount of the reversion. In another procedure, the net amount of Present

Worth of the investment position not covered by periodic income is derived and the amount of revision required to cover that net amount of Present Worth is then calculated.

Example:

Find the Future Sales Price, Amount of Equity Reversion, and app./dep. required to achieve a given Equity Yield Rate.

An investment property is for sale for $100,000. It is expected to produce NOI of $11,000 per year. It can be financed with a $70,000 mortgage at 9% interest, fully amortized in level monthly payments over 20 years. What must the property sell for in 10 years for the investor to earn a 13% rate of return (y) on the equity investment? What must the equity reversion be? What percentage of dep. or app. is involved?

Keystrokes:	Display:	
[CL FIN]		
[f] 2		
20 [12×]		
9 [12÷]		
70000 [PV]		
[PMT]		
12 [×] [CHS]	**7,557.70**	ADS
10 [12×]		
[FV] [CHS]	**49,718.12**	b
10 [n]		
13 [i]		
11000 [ENTER↑]		NOI
7557.70 [–] [PMT]	**3,442.30**	CTO
30000 [CHS] [PV]		CO
[FV]	**38,430.72**	NCPR
49718.12 [+]	**88,148.84**	CPR
100000 [x⇄y]		
[f] [Δ%]	**-11.85**	% depreciation

Continuing with this example, if the investor had instead desired a 15% rate of return on his equity investment, what must the cash proceeds from the sale of the property be in 10 years? Appreciation or depreciation?

Keystrokes:	Display:	
15 [i]		
[FV]	***51,475.24***	NCPR
49718.12 [+]	***101,193.36***	CPR
100000 [x⇄y]		
[f] [Δ%]	***1.19***	% appreciation

Example 2:

Find the Future Sales Price (Resale Proceeds) and Percentage app. or dep. Required to Achieve a Given Discount Rate (r).

An investment property was recently acquired for $65,800. NOI is forecast to be $6,350 per year. What must it resell for (net) in 12 years to produce a rate of return (r) of 10.45% on the total property investment? What percentage app. or dep. over the original purchase price does this represent?

Keystrokes:	Display:	
[CL FIN]		
12 [n]		
10.45 [i]		
6350 [PMT]		
65800 [CHS] [PV]		
[FV]	***77,359.15***	selling price
65800 [x⇄y]		
[f] [Δ%]	***17.57***	% appreciation

Rather than purchasing the building outright, if the investor had instead financed the property with 20% down and the balance at 8.75% for 25 years, what must the property app./dep. be to earn the desired 10.45%?

Keystrokes:	Display:	
[CL FIN]		
25 [12×]		
8.75 [12÷]		
65800 [ENTER↑]		
20 [%]	***13,160.00***	CO
[−]	***52,640.00***	P
[PV]		
[PMT]	***−432.78***	

Keystrokes:	Display:	
12 [×] [CHS] [STO] 0	***5,193.32***	ADS
12 [12×]		
[FV] [CHS] [STO] 1	***40,244.02***	b
12 [n]		
10.45 [i]		
6350		NOI
[RCL] 0 [−]	***1,156.68***	CTO
[PMT]		
65800 [ENTER↑]		
20 [%] [CHS]	***−13,160.00***	CO
[PV]		
[FV]	***17,961.56***	NCPR
[RCL] 1 [+]	***58,205.58***	CPR
65800 [x⇄y]		
[f] [Δ%]	***−11.54***	% depreciation

Calculation of Equity Yield Rate (y) From Dollar Figures

The income stream conventionally forecast in Mortgage-Equity or Ellwood Analysis is a level annuity plus a reversion. It is either a level NOI flow plus Proceeds of Resale, or a level CTO flow plus NCPR.

The equity yield rate (y) can be calculated on the equity investment using CTO and NCPR; the discount rate or basic rate (r) can be calculated on the property investment using NOI and CPR.

Example:

An investor has just purchased an income property for $123,750. A mortgage of $95,000 was obtained financed at 9¼% for 25 years. NOI is forecast at $13,200. The investor plans to hold the property for 12 years and then resell it. Anticipated resale proceeds are $135,000. What is the indicated equity yield rate?

Keystrokes:	Display:
[CL FIN]	
25 [12×]	
9.25 [12÷]	
95000 [PV]	

Keystrokes:	Display:	
PMT	-813.56	
12× CHS STO 0	9,762.75	ADS
12 12×		
FV	-73,687.08	−b
135000 +	61,312.92	NCPR
FV		
12 n		
13200 ENTER↑		NOI
RCL 0 −	3,437.25	CTO
PMT		
123750 ENTER↑		
95000 −		
CHS	-28,750.00	CO
PV		
i	15.70	y

Investment and Feasibility Analysis

Decision making involves making a choice from among two or more alternative courses of action. The features found on the HP-37E and HP-38E/38C make it possible for the analyst to consider almost any combination of outcomes, and compare them with one another to select the "best" alternative, or to compare them with some standard of acceptability to make an accept-reject decision.

Feasibility Analysis is a process of measuring and testing whether a proposed investment is expected to meet an investor's minimum standard(s) of acceptability.

Investment Analysis consists essentially of comparing alternative investment or project proposals, and ranking them according to the results of their feasibility tests. The highest-ranking alternative is the "best" in terms of the investor's standard(s) of acceptability.

Feasibility Tests

A feasibility test measures whether a project or investment is likely to meet an investor's standard of acceptability. These standards of acceptability include:

1. The investment should be worth to the investor at least as much as it will cost the investor to acquire it. This criterion is tested by calculating the Present Worth of the Forecast Future Cash Flows from the investment at a rate of discount reflecting the rate of return minimally acceptable to the investor, and comparing that Present Worth to the Capital Outlay required. This procedure uses Present Worth, Net Present Value and the Profitability Index.
2. The investment should produce a rate of return to the investor at least as high as the rate of return desired or required. This criterion is tested by calculating the Internal Rate of Return or Modified Internal Rate of Return on the investment, and comparing it with the investor's desired or required rate of return.
3. The investment should provide for full recovery of the investor's Capital Outlay within the time period desired or required by the

investor. This criterion is tested by calculating the Payback Period and comparing it with the investor's desired or required payback period.

Present Worth

The PW of any investment is calculated by discounting the Forecast Future Cash Flows at a specified rate. For feasibility analysis, that specified rate is the minimally acceptable rate of return to the investor. It is y for estimating equity investment value, r for estimating total property value, and i for estimating the present worth of a mortgage.

1. Level Annuity, No Reversion

- Key in the number of payments and press [n].
- Key in the rate of return per period and press [i].
- Key in the cash flow per period and press [PMT].
- Press [PV] to obtain the present value, PW.

2. Level Annuity with Reversion or Balloon Payment

The Present Worth of the level cash flows is added to the Present Worth of the reversion, both at the investor's minimally acceptable rate of return. The sum is the Present Worth of the investment.

- Key in the income projection period, n, and press [n].
- Key in the basic rate, r, and press [i].
- Key in the selling price, PR, and press [FV].
- Key in the net operating income, NOI, and press [PMT].
- Press [PV] [CHS] to obtain the present worth, PW.

Example:

An income property purchased for $62,500 is forecast to produce NOI of $7,357 per year. The investor expects to hold it for 10 years, and then sell it. The Forecast Proceeds of Resale are $60,000.

The property has just been financed with a $50,000 mortgage at 9% interest, with level monthly payments over a 25-year term.

What is the Present Worth of the property assuming no mortgage and a Basic Rate (r) of 10.5%?

Keystrokes:	Display:	
[CL FIN]		
10 [n]		
10.5 [i]		
60000 [FV]		
7537 [PMT]		
[PV] [CHS]	***67,440.27***	PW

What is the Present Worth of the equity investment position with an Equity Yield Rate of 14%?

The procedure for solving this type of problem is detailed in the previous chapter.

Keystrokes:	Display:	
[CL FIN]		
25 [12×]		
9 [12÷]		
50000 [PV]		
[PMT]		
12 [×] [STO] 0	***−5,035.18***	ADS
10 [12×]		
[FV]	***−41,369.62***	−b
60000 [+]	***18,630.38***	NCPR
[FV]		
7537		NOI
[RCL] 0 [+]	***2,501.82***	CTO
[PMT]		
10 [n]		
14 [i]		
[PV] [CHS]	***18,075.22***	PW

3. Variable (Increasing/Decreasing) Annuity

The majority of real estate financing arrangements deal with equal periodic payments. It is possible however to consider a payment stream where the payment amounts change, such as income, repair costs or price changes reflecting inflation or deflation. The present worth of such a payment stream may be easily calculated by summing each individual payment. A quicker and easier way to sum the payments is with the following keystrokes:

a) Set the Payment switch to the END position and press [CL FIN].

b) Key in the total number of payment periods, press [n].

c) Key in the payment percentage increase per period expressed as one plus the decimal interest rate, press [ENTER↑]. If there is a percentage decrease, express it as one minus the decimal interest rate.

d) Key in the discount (interest) rate per period expressed as one plus the decimal interest rate, press [f] [Δ%] [i].

e) Press [CLx], key in the starting payment amount, press [x≷y] [÷] [PMT].*

f) Press [PV] to obtain the present worth of the payment stream.

Note:

This procedure assumes that payments occur at the end of the period.

Example 1:

Repair costs on the new duplex that you just purchased are expected to start at $200 one year from now and increase at the rate of 12% per year. What sum must be put in the bank today to cover the next three years of maintenance if the bank pays 5.5% interest?

Keystrokes:	Display:	
BEGIN ▮ END		
[CL FIN]		
3 [n]		Years
1.12 [ENTER↑]		Payment percentage increase
1.055 [f] [Δ%]	**-5.80**	Interest rate per period
[i]		Adjusted i
[CLx]		
200 [x≷y]		Starting payment amount
[÷] [PMT]	**178.57**	
[PV]	**-604.48**	Present worth of cash outflows

* Positive for cash received, negative for cash paid out.

When the payment amounts change by a constant amount instead of by a constant ratio, as in the previous problem, the following keystrokes will give the present value:

1. Set the Payment switch to END and press [CL FIN].
2. Key in the periodic discount (interest) rate as a percent; press [i].
3. Key in the starting payment; press [ENTER↑].
4. Key in the amount that the payment increases each period; press [ENTER↑].
5. Key in the periodic discount (interest) rate as a decimal; press [÷] [STO] 0 [+] [PMT].
6. Key in the total number of payment periods; press [ENTER↑] [n] [RCL] 0 [×] [CHS] [FV].
7. Press [PV] to obtain the present worth of the payments.

Example 2:

If the repair costs in the previous example increased each year by $35, what would be the sum required?

Keystrokes:	Display:	
BEGIN ▮ END		
[CL FIN]		Periodic interest rate
5.5 [i]		Starting payment
200 [ENTER↑]		Periodic payment increase
35 [ENTER↑]		
.055 [÷]	**636.36**	
[STO] 0		
[+]	**836.36**	Adjusted payment
[PMT]		Years
3 [ENTER↑]		
[n]		
[RCL] 0		
[×]		
[CHS]	**-1,909.09**	Adjusted future value
[FV]		
[PV]	**-630.65**	Present worth of cash out flows

Net Present Value

Net Present Value is the difference between Present Worth and Capital Outlay required: NPV = PW − CO.

The test of feasibility is NPV ≥ 0.

If Present Worth at the investor's required or desired rate of return is equal to or greater than the Capital Outlay required to acquire the investment position, then the investment is "feasible".

With the HP-38E/38C you can solve directly for the net present value of up to twenty groups of cash flows. The following keystrokes are used to find the net present value of an investment: (The position of the payment switch is irrelevant.)

1. Press [f] [ALL].
2. Key in the given interest rate and press [i].
3. Key in the initial investment amount and press [g] [CF_0]. If there is no initial investment, key in a zero for the amount.

Note:

Use the cash flow sign convention: positive values for cash received; negative values for cash paid out.

4. Key in the first cash flow amount and press [g] [CF_j]. Then key in the number of times that cash flow occurs, if other than 1, and press [g] [N_j].
5. Key in the remaining cash flows in the same manner.
6. Press [f] [NPV] to compute the net present value.

Example:

A rental property has 7 years remaining on the lease to the single tenant. The property is for sale for $200,000. A mortgage in the amount of $137,500 can be obtained.

A potential investor seeking an after-tax rate of return on his equity investment of 12% has forecast the after-tax cash flows and reversion, based on lease terms, as follows:

Year	ATCF
1	$11,846
2	$ 9,000
3	$ 9,000
4	$ 9,000
5	$ 5,000
6	$ 5,000
7 (Reversion included)	$51,883

Keystrokes:	Display:	
[f] [ALL]		
12 [i]		
200000 [ENTER↑]		
137500 [−]	*62,500.00*	
[CHS] [g] [CF_0]		
11846 [g] [CF_j]		
9000 [g] [CF_j]		
3 [g] [N_j]		
5000 [g] [CF_j]		
2 [g] [N_j]		
51833 [g] [CF_j]		
[f] [NPV]	*−3,805.88*	Net present value

Because the NPV is negative, the property does not reach the investor's desired return. What is the actual internal rate of return? Press [f] [IRR] and you'll find that the yield or return on the investment is 10.44%. Solve for the NPV again and the answer will be very close to zero.

Profitability Index

The Profitability Index is the Ratio of Present Worth to Capital Outlay:

$$PI = \frac{PW}{CO}$$

The test of feasibility is: PI ≥ 1.

Example:

An investment property has just been purchased for $62,500, including a $50,000 mortgage. NOI is forecast at $7537 annually, while CTO is $2502. The property is expected to be resold in 10 years for $60,000, at which time NCPR would be $18,630. What is the Profitability Index for the property investment at a Basic Rate of 10.5%?

Keystrokes:	Display:	
[CL FIN]		
10 [n]		
10.5 [i]		
60000 [FV]		
7537 [PMT]		
[PV]	*-67,440.27*	PW
[CHS]		
62500		CO
[÷]	*1.08*	PI

Internal Rate of Return

As noted previously, an Internal Rate of Return is that rate of discount at which the Present Worth of Forecast Future Cash Flows from an investment exactly equals the required Capital Outlay.

The Test of feasibility is: IRR ≥ Desired Rate of Return

The calculated IRR from the property investment and forecast cash flow data must be equal to or greater than the rate of return required or desired by the investor.

Using your HP-38E/38C, the keystroke procedure for solving IRR is as follows: (refer to HP-38E/38C Owner's Manual for a more detailed treatment.)

1. Press [f] [ALL].
2. Key in the amount of the initial investment and press [g] [CF_0]. If there is no initial investment, key in zero for the amount.

Note:

Use the cash flow sign convention: positive values for cash received; negative values for cash paid out.

3. Key in the amount of the first cash flow and press [g] [CF_j]. Then key in the number of times that cash flow occurs, if other than 1, and press [g] [N_j].
4. Enter the remaining cash flows in the same manner.
5. Press [f] [IRR] to compute the internal rate of return.

Example 1:

An investment property is forecast to produce the following after-tax cash flows over a 10-year income projection period.

Year	ATCF
1	$ 2,861
2	$ 2,000
3	$ 2,000
4	$ 2,000
5	$ 2,000
6	$ 2,000
7	$ 1,000
8	$ 1,000
9	$ 1,000
10 (Reversion included)	$14,765

The investor can acquire the equity investment position for $12,500. What is the after-tax rate of return, calculated as an IRR?

Keystrokes:	Display:	
[f] [ALL]		
12500 [CHS] [g] [CF_0]		
2861 [g] [CF_j]		
2000 [g] [CF_j]		
5 [g] [N_j]		
1000 [g] [CF_j]		
3 [g] [N_j]		
14765 [g] [CF_j]		
[f] [IRR]	*15.79*	% IRR

Check with NPV

[f] [NPV]	*0.000001*	

Example 2:

The property used in the examples on pages (10) and (16) was sold after the tenth year.

The initial cash outlay was $200,000. The after tax cash flows, from page (11), and ATNCPR from page (17), were:

Year	ATCF
1	$ 20,678.84
2	$ 19,191.61
3	$ 17,691.11
4	$ 16,169.29
5	$ 14,617.25
6	$ 13,017.20
7	$ 11,388.29
8	$ 9,690.66
9	$ 7,923.30
10	$ 6,074.32
ATNCPR	$802,291.71

Calculate the IRR on the investment.

Note:

The ATNCPR must be added to the 10th year ATCF to obtain the total 10^{th} year ATCF after resale.

Keystrokes:	Display:	
f ALL		
200000 CHS g CF_0	*-200,000.00*	Initial cash outlay
20678.84 g CF_j		
19191.61 g CF_j		
17691.11 g CF_j		
16169.29 g CF_j		
14617.25 g CF_j		
13017.20 g CF_j		
11388.29 g CF_j		
9690.66 g CF_j		
7923.30 g CF_j		
6074.32 ENTER↑		
802291.71 +	*808,366.03*	10^{th} year total ATCF
g CF_j		
f IRR	*19.72*	%, after tax IRR

Payback Period

The Payback Period is the number of years required to return or "pay-back" the amount of Capital Outlay, disregarding any rate of discount. It is n when:

$$\sum_{t=1}^{n} (CF_t) \geq CO$$

The test of feasibility is: $n \leq$ Target

An investment is feasible when the Capital Outlay is forecast to be repaid within the period required or desired by the investor.

If cash flows are level, $n \geq \dfrac{CO}{CTO}$

Example:

An investor has just purchased an income property for $62,500, of which $12,500 was equity. CTO is forecast at $2502 annually. What is the Payback Period?

Keystrokes:	Display:	
12500 ENTER↑		
2502 ÷	*5.00*	Payback Period

Appendix

Real Estate Symbols and Terminology

Income Symbols

PGI: Potential Gross Income (Number of rental units times rental per unit, at 100% occupancy, annually)

v: Allowance for vacancy and income loss (annual)

EGI: Effective Gross Income: Rent Collections plus "Other Income" (PGI − v + "Other" = EGI, annual)

OE: Operating Expenses (annual)

NOI: Net Operating Income (annual: EGI − OE = NOI)

ADS: Annual Debt Service (Monthly mortgage payment × 12)

CTO: Cash Throw-Off to Equity (annual: NOI − ADS = CTO); Gross Spendable Income

ATCF: After-Tax Cash Flow (annual: NOI − Income Tax Liability = ATCF); Net Spendable Income

Value (Present Worth, Reversion) Symbols

V: Value (Present Worth)

PW: Present Worth (Value, Present Value)

SP: Sales Price

V_m: Value, Principal, Present Worth of Mortgage

V_e: Value, Present Worth of Equity ($V_e = (1 - m) \times V$)

P: Principal of Mortgage ($P = V - V_e$)

CPR: Cash Proceeds of Resale; Reversion (forecast; before tax) CPR = SP − Selling of Disposition Expenses

b: Balance of Mortgage Outstanding

NCPR: Net Cash Proceeds of Resale (to equity; before tax); Equity Reversion NCPR = CPR − b

ATNCPR: After-Tax Net Cash Proceeds of Resale (to equity); After-Tax Equity Reversion

FW: Future Worth (Reversion; Resale Proceeds)

Rate, Capitalization Rate, Rate of Return Symbols

R: Overall Rate (on property investment): annual(NOI ÷ V; NOI ÷ SP)

r: Basic Rate; annual

f: Mortgage Constant: annual (ADS ÷ V)

i: Mortgage Interest Rate (contract): annual

R_e: Equity Dividend Rate: annual (CTO ÷ V_e)

y: Equity Yield Rate; Rate of Return on Equity Investment: annual

IRR: Internal Rate of Return: annual

n: Income Projection Period; Investment Holding Period

Mortgage-Equity (Ellwood) Analysis Symbols

dep.: "Depreciation", Capital Loss on Resale as a Percentage of Value

app.: "Appreciation", Capital Gain on Resale as a Percentage of Value

CPR: Cash Proceeds of Resale; Reversion at End of Income Projection Period (n)

NCPR: Net Cash Proceeds of Resale to Equity; Equity Reversion at End of Income Projection Period (n): NCPR = CPR − b

m: Loan-to-Value Ratio; Mortgage Principal as a Percentage of Value

CO: Capital Outlay; Investment in Time Period 0

(1 − m): Equity as a Percentage of Value

Financial Formulas

After-Tax Cash Flows

$ATCF_k$ = after-tax cash flow for k^{th} year

Int_k = interest for k^{th} year

Dep_k = depreciation for k^{th} year

r = appropriate tax rate

NOI = net operating income

$$ATCF_k = NOI(1 - r) - 12 \times PMT + r(Int_k + Dep_k)$$

if Dep_k and Dep_{k-1} are the depreciation taken for year k and the year before k, respectively:

$$Dep_k = (1 - Dep_{k-1})$$

The declining balance rate is computed as follows:

$$\text{declining balance rate} = \frac{\text{declining balance factor}}{\text{asset's useful life expectancy}}$$

The depreciation for the first year is:

$$Dep_1 = \text{declining balance rate} \times \text{depreciable base}$$

After-Tax Cash Flows with Multiple Mortgages

$$ATCF_k = NOI(1 - r) - 12\Sigma PMT + r(\Sigma Int_k + Dep_k)$$

where ΣPMT and ΣInt are total monthly payments and total annual interest.

After-Tax Net Cash Proceeds of Resale

where

Dep_k = total depreciation

SBV = starting book value

DBF = declining balance factor

LIFE = assets useful life

k = depreciation accumulation period

$$Dep_k = SBV\left[1\left(1 - \frac{DBF}{100\ LIFE}\right)^k\right]$$

Mortgage-Equity (Ellwood) Analysis

Basic Rate:

$$r = mf + (1 - m)y - mp\ 1/s_n$$

Overall Rate:

$$R = r + dep.\ 1/s_n$$

$$R = r - app.\ 1/s_n + dep.\ 1/s_n$$

$$R = mf + (1 - m)y - mp\ 1/s_n - app.\ 1/s_n$$

where

i = mortgage interest rate

m = loan-to-value ratio of mortgage

y = equity yield rate

dep./app. = capital loss or gain on resale as a percentage of present worth or value of property

$1/s_n$ = sinking fund factor at the equity yield rate over the income projection period

f = mortgage constant

p = percentage of mortgage principal paid off over the income projection period

R = Overall Rate; annual

r = Basic Rate; annual

OTHER APPLICATIONS BOOKS WHICH ARE AVAILABLE

LENDING, SAVINGS, AND LEASING APPLICATIONS (00038-90025)

APR with Fees; Discounted Mortgages; Constant Principal Loans; Add-On Rate Converted to APR; Add-On Loan With Credit Life; Rule of 78's; Nominal Rate to Effective Rate; Number of Periods to Deplete a Savings Account; Periodic Deposits and Withdrawals; Savings Account Compounded Daily; Compounding Periods Different from Payment Periods; Advance Payments With Residual; Skipped Payments

INVESTMENT ANALYSIS AND STATISTICS APPLICATIONS FOR BUSINESS PROFESSIONALS AND STUDENTS (00038-90026)

Modified IRR (FMRR); Lease vs Purchase; Break-Even Analysis; Bonds; Exponential, Logarithmic and Power Curve Fits; Exponential Smoothing; Standard Error of the Mean; Grouped Data; Chi-Square; Normal Distribution; Covariance; Permutation; Combination; Random Number Generator

REAL ESTATE APPLICATIONS (00038-90024)

APR With Fees; Discounted Mortgages; Present Value and Yield of a Mortgage With Balloon Payment One Period After Last Payment; Deferred Annuities; Present Value of Increasing/Decreasing Annuity; Equity Yield Rate; Equity Investment Value and Present Value; Future Sales Price and Overall Depreciation/Appreciation Rate; Mortgage Constant; Refinancing; Wrap-Around Mortgages; Modified IRR (FMRR); Canadian Mortgages; Depreciation; Exponential Curve Fit

MARKETING AND FORECASTING APPLICATIONS (00038-90049)

Moving Average; Seasonal Variation Factors; Exponential Curve Fit; Gompertz Curve Fit; Forecasting with Exponential Smoothing; Breakeven Analysis; Operating Leverage; Profit and Loss Analysis; Markup and Margin Calculations; List and Net Prices; Learning Curve; Queuing and Waiting Line Theory; Cash Flow Loader; Percentage Tabulator.

PERSONAL FINANCE APPLICATIONS (00038-90052) (HP-38E/38C ONLY)

IRA or Keogh Plan; Stock Portfolio Evaluation; U.S. Treasury Bill Valuation; True Annual Growth Rate of an Investment Portfolio; Bond Purchased Between Coupons; The True Cost of an Insurance Policy; Real Estate Equity Investment Analysis; Homeowner's Monthly Payment Estimator; True Annual Percentage Interest Rate on a Mortgage with Fees; Rent versus Buy.