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HEWLETT-PACKARD

HP-33E STATISTICS Applications

For Continuous Memory Models

Although this book refers specifically to the HP-33E or HP-38E, the programs and calculations contained herein apply equally well to the HP-33C or HP-38C. The user should note, however, that the display format and data register contents are retained by the calculator even though it has been turned off. It may be desirable to reset or clear these conditions before running programs or making calculations.

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HP-33E

Statistics Applications

February 1978

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Introduction

This Statistics Applications book was written to help you get the most from your HP-33E calculator. The programs were chosen to provide useful calculations for many of the common problems encountered in statistics.

They will provide you with immediate capabilities in your everyday calculations and you will find them useful as guides to programming techniques for writing your own customized software.

You will find general information on how to key in and run programs under "A Word about Program Usage" in the Applications book you received with your calculator.

We hope that this Statistics Applications book will be a valuable tool in your work and would appreciate your comments about it.

Contents

General Statistics

Covariance and Correlation Coefficient

For a set of given data points $\{(x_i, y_i), i = 1, 2, \ldots, n\}$, the covariance and the correlation coefficient are defined as:

$$\text{covariance } s_{xy} = \frac{1}{n(n-1)}(n\,\Sigma x_i\, y_i - \Sigma x_i\, \Sigma y_i)$$

$$\text{or } s_{xy}' = \frac{1}{n^2}(n\,\Sigma x_i\, y_i - \Sigma x_i\, \Sigma y_i)$$

$$\text{correlation coefficient } r = \frac{s_{xy}}{s_x\, s_y}$$

where s_x and s_y are standard deviations

$$s_x = \sqrt{\frac{n\,\Sigma x_i^2 - (\Sigma x_i)^2}{n(n-1)}}$$

$$s_y = \sqrt{\frac{n\,\Sigma y_i^2 - (\Sigma y_i)^2}{n(n-1)}}$$

Note: $-1 \leq r \leq 1$

KEY ENTRY	DISPLAY
f CLEAR PRGM	00
Σ+	01– 25
GTO 00	02– 13 00
f r	03– 14 23
R/S	04– 74
g s	05– 15 25
×	06– 61
×	07– 61

KEY ENTRY	DISPLAY
R/S	08– 74
RCL 2	09– 24 2
1	10– 1
−	11– 41
×	12– 61
RCL 2	13– 24 2
÷	14– 71
GTO 00	15– 13 00

REGISTERS			
R_0	R_1	R_2 n	R_3 Σx
R_4 Σx²	R_5 Σy	R_6 Σy²	R_7 Σxy

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Initialize		f PRGM f REG	
3	Perform this step for			
	i = 1, 2, … n	x_i		
		y_i	R/S	i
4	Correlation coefficient		GSB 03	r
5	Calculate covariance s_{xy}		R/S	s_{xy}
6	s_{xy}'		R/S	s_{xy}'
7	For a new case go to step 2.			

Example:

x_i	26	30	44	50	62	68	74
y_i	92	85	78	81	54	51	40

Solution:

$r = -0.9572$

$s_{xy} = -354.1429$

$s_{xy}' = -303.5510$

Keystrokes	**Display**
f PRGM f REG	
26 ENTER↑ 92 R/S	*1.0000*
30 ENTER↑ 85 R/S	*2.0000*
:	
:	
74 ENTER↑ 40 R/S	*7.0000*
GSB 03	*-0.9572*
R/S	*-354.1429*
R/S	*-303.5510*

Moments and Skewness

This program calculates the following statistics for a set of given data $\{x_1, x_2, \ldots, x_n\}$:

1st moment $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

2nd moment $m_2 = \frac{1}{n}\Sigma x_i^2 - \bar{x}^2$

3rd moment $m_3 = \frac{1}{n}\Sigma x_i^3 - \frac{3}{n}\bar{x}\,\Sigma x_i^2 + 2\bar{x}^3$

moment coefficient of skewness

$$\gamma_1 = \frac{m_3}{m_2^{3/2}}$$

KEY ENTRY	DISPLAY
f CLEAR PRGM	00
ENTER↑	01– 31
g x²	02– 15 0
Σ+	03– 25
GTO 00	04– 13 00
RCL 5	05– 24 5
RCL 2	06– 24 2
÷	07– 71
STO 6	08– 23 6
R/S	09– 74
RCL 3	10– 24 3
RCL 2	11– 24 2
÷	12– 71
RCL 6	13– 24 6
g x²	14– 15 0
−	15– 41
STO 1	16– 23 1
R/S	17– 74
RCL 7	18– 24 7
RCL 2	19– 24 2
÷	20– 71
RCL 3	21– 24 3
RCL 6	22– 24 6

KEY ENTRY	DISPLAY
×	23– 61
RCL 2	24– 24 2
÷	25– 71
3	26– 3
×	27– 61
−	28– 41
RCL 6	29– 24 6
ENTER↑	30– 31
g x²	31– 15 0
×	32– 61
2	33– 2
×	34– 61
+	35– 51
STO 0	36– 23 0
R/S	37– 74
RCL 0	38– 24 0
RCL 1	39– 24 1
1	40– 1
.	41– 73
5	42– 5
f yˣ	43– 14 3
÷	44– 71
GTO 00	45– 13 00

REGISTERS			
R_0 m_3	R_1 m_2	R_2 n	R_3 Σx
R_4 Σx^4	R_5 Σx^2	R_6 $\bar{x}$	R_7 Σx^3

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Initialize		[f] [PRGM] [f] [REG]	
3	Perform for i = 1, 2,..., n:			
	Input x-value	x_i	[R/S]	i
4	Delete erroneous data	x_k	[ENTER↑] [g] [x^2] [f]	
			[Σ-]	
5	Calculate the mean		[GSB] 05	$\bar{x}$
6	Calculate the second and			
	third moments		[R/S]	m_2
			[R/S]	m_3
7	Calculate the moment			
	coefficient of skewness		[R/S]	γ_1
8	For new case, go to step 2.			

Example:

i	1	2	3	4	5	6	7	8	9
x_i	2.1	3.5	4.2	6.5	4.1	3.6	5.3	3.7	4.9

Solution:

$\bar{x} = 4.21$

$m_2 = 1.39$

$m_3 = 0.39$

$\gamma_1 = 0.24$

Keystrokes	Display
[f] [PRGM] [f] [REG]	
2.1 [R/S] 3.5 [R/S]	
4.2 [R/S] ...	
4.9 [R/S]	*9.0000*
[GSB] 05	*4.2111*
[R/S]	*1.3899*
[R/S]	*0.3864*
[R/S]	*0.2358*

Partial Correlation Coefficients

The partial correlation coefficient measures the relationship between any two of the variables when all other are kept constant.

For the case of 3 variables, the partial correlation coefficient between X_1 and X_2 keeping X_3 constant is

$$r_{12\cdot 3} = \frac{r_{12} - r_{13}\, r_{23}}{\sqrt{(1 - r_{13}^2)(1 - r_{23}^2)}}$$

where r_{ij} denotes the correlation coefficient of X_i and X_j.

Similarly, for the case of 4 variables, the partial correlation coefficient between X_1 and X_2 keeping X_3 and X_4 constant is

$$r_{12\cdot 34} = \frac{r_{12\cdot 4} - r_{13\cdot 4}\, r_{23\cdot 4}}{\sqrt{(1 - r_{13\cdot 4}^2)(1 - r_{23\cdot 4}^2)}} = \frac{r_{12\cdot 3} - r_{14\cdot 3}\, r_{24\cdot 3}}{\sqrt{(1 - r_{14\cdot 3}^2)(1 - r_{24\cdot 3}^2)}}$$

Any partial correlation coefficient can be calculated by means of these formulas (using this program) if correlation coefficients r_{12}, r_{13}, r_{23}, ... are given.

Note:

This program finds $r_{13\cdot 2}$, $r_{23\cdot 1}$ by similar formulas.

Reference:

S. Wilks. *Mathematical Statistics,* John Wiley and Sons, 1962.

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
f CLEAR PRGM	00	STO 0	13– 23 0
STO 2	01– 23 2	RCL 1	14– 24 1
g x^2	02– 15 0	RCL 2	15– 24 2
1	03– 1	×	16– 61
−	04– 41	−	17– 41
x⇄y	05– 21	x⇄y	18– 21
STO 1	06– 23 1	÷	19– 71
g x^2	07– 15 0	R/S	20– 74
1	08– 1	RCL 1	21– 24 1
−	09– 41	RCL 2	22– 24 2
×	10– 61	RCL 0	23– 24 0
f $\sqrt{x}$	11– 14 0	GTO 01	24– 13 01
x⇄y	12– 21		

REGISTERS			
R_0 r_{12}, r_{13}, r_{23}	R_1 r_{13}, r_{23}, r_{12}	R_2 r_{23}, r_{12}, r_{13}	R_3
R_4	R_5	R_6	R_7

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Input data and calculate			
	correlation coefficients	r_{12}	ENTER↑	
		r_{13}	ENTER↑	
		r_{23}	GSB 01	$r_{12 \cdot 3}$
			R/S	$r_{13 \cdot 2}$
			R/S	$r_{23 \cdot 1}$
3	For a new case, go to step 2.			

Example:

Suppose $r_{12} = -0.96$, $r_{13} = -0.1$, $r_{23} = 0.12$, then the partial correlation coefficients are:

$r_{12 \cdot 3} = -.96$

$r_{13 \cdot 2} = .05$

$r_{23 \cdot 1} = .09$

Keystrokes	Display
.96 CHS ENTER↑ .1	
CHS ENTER↑ .12	
GSB 01	-0.9597
R/S	0.0547
R/S	0.0861

Probability

Factorial

This program calculates factorials for positive integers between 2 and 69.

$$n! = n\,(n-1)\,(n-2)\ldots(2)\,(1)$$

Notes:

1. For large values of n, the program will take some time to arrive at a result, up to a maximum of about 20 seconds for n = 69.
2. The program checks for the input values automatically. ***"Error 0"*** will be displayed for $n < 2$ or $n > 69$, and non-integer numbers.

KEY ENTRY	DISPLAY
[f] CLEAR [PRGM]	00
[GSB] 13	01– 12 13
[ENTER↑]	02– 31
1	03– 1
[STO] 0	04– 23 0
[x≷y]	05– 21
[STO] [×] 0	06– 23 61 0
1	07– 1
[−]	08– 41
[f] [x≠y]	09– 14 61
[GTO] 06	10– 13 06
[RCL] 0	11– 24 0
[GTO] 00	12– 13 00
6	13– 6
9	14– 9

KEY ENTRY	DISPLAY
[x≷y]	15– 21
[f] [x>y]	16– 14 51
[GTO] 28	17– 13 28
2	18– 2
[f] [x>y]	19– 14 51
[GTO] 28	20– 13 28
[x≷y]	21– 21
[ENTER↑]	22– 31
[g] [FRAC]	23– 15 33
[g] [x≠0]	24– 15 61
[GTO] 28	25– 13 28
[x≷y]	26– 21
[g] [RTN]	27– 15 12
0	28– 0
[÷]	29– 71

REGISTERS			
R_0 Used	R_1	R_2	R_3
R_4	R_5	R_6	R_7

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Key in n ($2 \le n \le 69$)	n	[GSB] 01	n!
3	For a new n, go to step 2.			

Examples:

1. 5! = 120
2. 10! = 3628800

Keystrokes	Display
5 [GSB] 01	***120.0000***
10 [GSB] 01	***3,628,800.000***

Permutation

A permutation is an ordered subset of a set of distinct objects. The number of possible permutations, each containing n objects, that can be formed from a collection of m distinct objects is given by

$${}_mP_n = \frac{m!}{(m-n)!} = m(m-1)\ldots(m-n+1)$$

where m, n are integers and $0 \le n \le m$.

Notes:

1. ${}_mP_n$ can also be denoted by P_n^m, $P(m,n)$ or $(m)_m$.
2. ${}_mP_0 = 1$, ${}_mP_1 = m$, ${}_mP_m = m!$

KEY ENTRY	DISPLAY
[f] CLEAR [PRGM]	00
[STO] 1	01– 23 1
[x⇄y]	02– 21
[STO] 0	03– 23 0
[RCL] 0	04– 24 0
[RCL] 1	05– 24 1
[g] [x=0]	06– 15 71
[GTO] 31	07– 13 31
[f] [x=y]	08– 14 71
[GTO] 33	09– 13 33
[f] [x>y]	10– 14 51
[GTO] 41	11– 13 41
1	12– 1
[f] [x=y]	13– 14 71
[GTO] 43	14– 13 43
[R↓]	15– 22
[−]	16– 41
1	17– 1
[+]	18– 51
[×]	19– 61
[f] [LST x]	20– 14 73
[RCL] 0	21– 24 0
1	22– 1

KEY ENTRY	DISPLAY
[−]	23– 41
[f] [x=y]	24– 14 71
[GTO] 28	25– 13 28
[R↓]	26– 22
[GTO] 17	27– 13 17
[R↓]	28– 22
[R↓]	29– 22
[GTO] 00	30– 13 00
1	31– 1
[GTO] 00	32– 13 00
1	33– 1
[−]	34– 41
[g] [x=0]	35– 15 71
[GTO] 39	36– 13 39
[STO] [×] 0	37– 23 61 0
[GTO] 33	38– 13 33
[RCL] 0	39– 24 0
[GTO] 00	40– 13 00
0	41– 0
[÷]	42– 71
[R↓]	43– 22
[R↓]	44– 22
[GTO] 00	45– 13 00

REGISTERS			
R_0 m	R_1 n	R_2	R_3
R_4	R_5	R_6	R_7

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Input m, n and	m	[ENTER↑]	
	calculate permutations	n	[GSB] 01	$_mP_n$
3	For a new case, go to step 2.			

Examples:

1. $_{43}P_3 = 74046$
2. $_{73}P_4 = 26122320$

Keystrokes	Display
[f] [FIX] 0	
43 [ENTER↑] 3 [GSB] 01	***74,046.***
73 [ENTER↑] 4	
[GSB] 01	***26,122,320.***

Combination

A combination is a selection of one or more of a set of distinct objects without regard to order. The number of possible combinations, each containing n objects, that can be formed from a collection of m distinct objects is given by

$$_mC_n = \frac{m!}{(m-n)!\,n!} = \frac{m(m-1)\ldots(m-n+1)}{1\cdot 2\cdot \ldots \cdot n}$$

where m, n are integers and $0 \leqslant n \leqslant m$.

This program calculates $_mC_n$ using the following algorithm:

1. If $n \leqslant m - n$

$$_mC_n = \frac{m-n+1}{1}\cdot\frac{m-n+2}{2}\cdot \ldots \cdot\frac{m}{n}.$$

2. If $n > m - n$, program calculates $_mC_{m-n}$.

Notes:

1. ${}_mC_n$, which is also called the binomial coefficient, can be denoted by C_n^m, C(m,n), or $\binom{m}{n}$.
2. ${}_mC_n = {}_mC_{m-n}$
3. ${}_mC_0 = {}_mC_m = 1$
4. ${}_mC_1 = {}_mC_{m-1} = m$

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
f CLEAR PRGM	00	x⇄y	17– 21
−	01– 41	f x>y	18– 14 51
f LST x	02– 14 73	GTO 24	19– 13 24
f x≤y	03– 14 41	f x=y	20– 14 71
x⇄y	04– 21	GTO 24	21– 13 24
STO 0	05– 23 0	RCL 2	22– 24 2
1	06– 1	GTO 00	23– 13 00
STO 1	07– 23 1	x⇄y	24– 21
+	08– 51	RCL 0	25– 24 0
STO 2	09– 23 2	+	26– 51
R↓	10– 22	RCL 1	27– 24 1
g x=0	11– 15 71	÷	28– 71
GTO 32	12– 13 32	STO × 2	29– 23 61 2
1	13– 1	R↓	30– 22
RCL 1	14– 24 1	GTO 13	31– 13 13
+	15– 51	1	32– 1
STO 1	16– 23 1	GTO 00	33– 13 00

REGISTERS			
R_0 max	R_1 Used	R_2 Used	R_3
R_4	R_5	R_6	R_7

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Key in m, n and	m	ENTER↑	
	calculate combination	n	GSB 01	${}_mC_n$
3	For a new case, go to step 2.			

Examples:

1. ${}_{73}C_4 = 1088430$
2. ${}_{43}C_3 = 12341$

Keystrokes	Display
f FIX 0	
73 ENTER↑ 4 GSB 01	***1,088,430.***
43 ENTER↑ 3 GSB 01	***12,341.***

Random Number Generator

This program calculates uniformly distributed pseudo random numbers u_i in the range

$$0 \leq u_i \leq 1$$

using the following formula:

$$u_i = \text{Fractional part of } \left[(\pi + u_{i-1})^3\right].$$

The user has to specify the starting value u_0 (the "seed" of the sequence) such that

$$0 \leq u_0 \leq 1.$$

KEY ENTRY	DISPLAY
[f] CLEAR [PRGM]	00
[g] [π]	01– 15 73
[RCL] 0	02– 24 0
[+]	03– 51
3	04– 3
[f] [y^x]	05– 14 3
[g] [FRAC]	06– 15 33
[STO] 0	07– 23 0
[GTO] 00	08– 13 00

REGISTERS			
R_0 u_i	R_2	R_3	R_4
R_5	R_6	R_7	

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Store seed	u_0	[STO] 0 [f] [PRGM]	
3	Generate random number		[R/S]	u_i
4	Repeat step 3 as many times			
	as desired			
5	For new sequence, go to			
	step 2.			

Example:

Find the sequence of two digit random numbers generated from a seed of 0.192743568.

Solution:

0.07, 0.14, 0.34, 0.37, 0.17, 0.46, 0.82, ...

Keystrokes	Display
[f] [PRGM] [f] [FIX] 2	
.192743568 [STO] 0	
[R/S]	*0.07*
[R/S]	*0.14*
[R/S]	*0.34*
[R/S]	*0.37*

etc.

Distributions

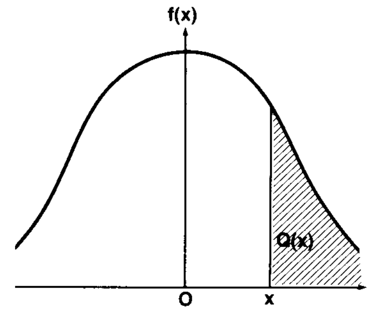
Normal Distribution

The density function for a standard normal variable is

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

The upper tail area is

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt.$$



For $x \geq 0$, polynomial approximation is used to calculate Q(x):

$$Q(x) = f(x)\,(b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5) + \epsilon(x)$$

where: $|\epsilon(x)| < 7.5 \times 10^{-8}$

$$t = \frac{1}{1 + rx}, \quad r = 0.2316419$$

$b_1 = .31938153$, $b_2 = -.356563782$

$b_3 = 1.781477937$, $b_4 = -1.821255978$

$b_5 = 1.330274429$

Note:

The program only works for $x \geq 0$. Equations $f(-x) = f(x)$, $Q(-x) = 1 - Q(x)$, where $x \geq 0$, can be used to find f and Q for negative numbers.

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, 1968.

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
f CLEAR PRGM	00	ENTER↑	21– 31
ENTER↑	01– 31	ENTER↑	22– 31
STO 6	02– 23 6	ENTER↑	23– 31
×	03– 61	RCL 5	24– 24 5
2	04– 2	×	25– 61
÷	05– 71	RCL 4	26– 24 4
CHS	06– 32	+	27– 51
g e^x	07– 15 1	×	28– 61
g π	08– 15 73	RCL 3	29– 24 3
2	09– 2	+	30– 51
×	10– 61	×	31– 61
f $\sqrt{x}$	11– 14 0	RCL 2	32– 24 2
÷	12– 71	+	33– 51
STO 7	13– 23 7	×	34– 61
R/S	14– 74	RCL 1	35– 24 1
RCL 0	15– 24 0	+	36– 51
RCL 6	16– 24 6	×	37– 61
×	17– 61	RCL 7	38– 24 7
1	18– 1	×	39– 61
+	19– 51	GTO 00	40– 13 00
g 1/x	20– 15 3		

REGISTERS			
R_0 r	R_1 b_1	R_2 b_2	R_3 b_3
R_4 b_4	R_5 b_5	R_6 x	R_7 f(x)

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Store constants	r	STO 0	
		b_1	STO 1	
		b_2	STO 2	
		b_3	STO 3	
		b_4	STO 4	
		b_5	STO 5	
3	Input x and calculate f(x)		GSB 01	f(x)
4	Calculate Q(x)		R/S	Q(x)
5	For a new case, go to step 3.			

Examples:

1. x = 1.18
2. x = 2.28

Solutions:

1. f(x) = 0.1989

   Q(x) = 0.1190
2. f(x) = 0.0297

   Q(x) = 0.0113

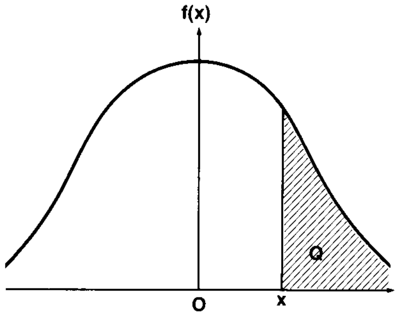
Keystrokes	Display
0.2316419 STO 0	
0.31938153 STO 1	
0.356563782 CHS	
STO 2	
1.781477937 STO 3	
1.821255978 CHS	
STO 4	
1.330274429 STO 5	
1.18 GSB 01	*0.1989*
R/S	*0.1190*
2.28 GSB 01	*0.0297*
R/S	*0.0113*

Inverse Normal Integral

This program determines the value of x such that

$$Q = \int_x^{\infty} \frac{e^{-\frac{t^2}{2}}}{\sqrt{2\pi}}\, dt$$

where Q is given and $0 < Q \leq 0.5$.



The following rational approximation is used:

$$x = t - \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3} + \epsilon(Q)$$

where: $|\epsilon(Q)| < 4.5 \times 10^{-4}$

$$t = \sqrt{\ln \frac{1}{Q^2}}$$

$c_0 = 2.515517 \quad d_1 = 1.432788$

$c_1 = 0.802853 \quad d_2 = 0.189269$

$c_2 = 0.010328 \quad d_3 = 0.001308$

Reference:

Abramowitz and Stegun, *Handbook of Mathematical Functions,* National Bureau of Standards, 1968.

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
f CLEAR PRGM	00	×	17– 61
ENTER↑	01– 31	1	18– 1
×	02– 61	+	19– 51
g 1/x	03– 15 3	STO 7	20– 23 7
f LN	04– 14 1	CLX	21– 34
f √x	05– 14 0	RCL 2	22– 24 2
STO 6	06– 23 6	×	23– 61
ENTER↑	07– 31	RCL 1	24– 24 1
ENTER↑	08– 31	+	25– 51
ENTER↑	09– 31	×	26– 61
RCL 5	10– 24 5	RCL 0	27– 24 0
×	11– 61	+	28– 51
RCL 4	12– 24 4	RCL 7	29– 24 7
+	13– 51	÷	30– 71
×	14– 61	−	31– 41
RCL 3	15– 24 3	GTO 00	32– 13 00
+	16– 51		

REGISTERS			
R_0 c_0	R_1 c_1	R_2 c_2	R_3 d_1
R_4 d_2	R_5 d_3	R_6 t	R_7 $1 + d_1 t + \ldots$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Store constants	c_0	STO 0	
		c_1	STO 1	
		c_2	STO 2	
		d_1	STO 3	
		d_2	STO 4	
		d_3	STO 5	
3	Input Q to calculate x	Q	GSB 01	x
4	For a new case, go to step 3.			

Examples:

1. Q = 0.12
2. Q = 0.05

Solutions:

1. x = 1.1751
2. x = 1.6452

Keystrokes	Display
2.515517 STO 0	
0.802853 STO 1	
0.010328 STO 2	
1.432788 STO 3	
0.189269 STO 4	
0.001308 STO 5	
0.12 GSB 01	*1.1751*
0.05 GSB 01	*1.6452*

Curve Fitting

Exponential Curve Fit

This program calculates the least squares fit of n pairs of data points $\{(x_i, y_i), i = 1, 2, \ldots, n\}$, where $y_i > 0$, for an exponential function of the form

$$y = ae^{bx} \quad (a > 0).$$

The equation is transformed into

$$\ln y = \ln a + bx.$$

The following statistics are calculated:

1. Coefficients a, b

$$b = \frac{n \Sigma x_i \ln y_i - (\Sigma x_i)(\Sigma \ln y_i)}{n \Sigma x_i^2 - (\Sigma x_i)^2}$$

$$a = \exp\left[\frac{\Sigma \ln y_i}{n} - b\frac{\Sigma x_i}{n}\right]$$

2. Coefficient of determination

$$r^2 = \frac{\left[n \Sigma x_i \ln y_i - \Sigma x_i \Sigma \ln y_i\right]^2}{\left[n \Sigma x_i^2 - (\Sigma x_i)^2\right]\left[n \Sigma (\ln y_i)^2 - (\Sigma \ln y_i)^2\right]}$$

3. Estimated value $\hat{y}$ for a given x

$$\hat{y} = a e^{bx}$$

Note:

n is a positive integer and $n \neq 1$.

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
f CLEAR PRGM	00	R/S	07– 74
f LN	01– 14 1	f r	08– 14 23
x⇄y	02– 21	g x^2	09– 15 0
Σ+	03– 25	R/S	10– 74
GTO 00	04– 13 00	f ŷ	11– 14 22
f L.R.	05– 14 24	g e^x	12– 15 1
g e^x	06– 15 1	R/S	13– 74

REGISTERS			
R_0	R_1	R_2 n	R_3 Σx
R_4 Σx^2	R_5 Σ ln y	R_6 $\Sigma (\ln y)^2$	R_7 Σ x ln y

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Initialize		f REG f PRGM	
3	Perform for i = 1 ,..., n:			
	Input x-value and y-value	x_i	ENTER↑	
		y_i	R/S	i
4	Calculate constants		GSB 05	a
			x⇄y	b
5	Calculate coefficient of			
	determination		R/S	r^2
6	To calculate ŷ, input x	x	GSB 11	ŷ
7	Perform step 6 as many			
	times as desired			
8	For new case, go to step 2.			

Example:

x_i	.72	1.31	1.95	2.58	3.14
y_i	2.16	1.61	1.16	.85	0.5

Solution:

a = 3.4451, b = −0.5820

$y = 3.45\ e^{-0.58x}$

$r^2 = 0.9803$

For x = 1.5, ŷ = 1.4389

Keystrokes

f REG f PRGM	
.72 ENTER↑ 2.16	
R/S	***1.0000***
1.31 ENTER↑ 1.61	
R/S	***2.0000***
1.95 ENTER↑ 1.16	
R/S	***3.0000***
2.58 ENTER↑ 0.85	
R/S	***4.0000***
3.14 ENTER↑ 0.5	
R/S	***5.0000***
GSB 05	***3.4451***
x⇄y	***−0.5820***
R/S	***0.9803***
1.5 GSB 11	***1.4389***

Logarithmic Curve Fit

This program fits a logarithmic curve

$$y = a + b \ln x$$

to a set of data points

$$\{(x_i, y_i), i = 1, 2, \ldots, n\}$$

where: $x_i > 0$.

Program calculates:

1. Regression coefficients

$$b = \frac{n \Sigma y_i \ln x_i - \Sigma \ln x_i \Sigma y_i}{n \Sigma (\ln x_i)^2 - (\Sigma \ln x_i)^2}$$

$$a = \frac{1}{n}(\Sigma y_i - b \Sigma \ln x_i)$$

2. Coefficient of determination

$$r^2 = \frac{\left[n \Sigma y_i \ln x_i - \Sigma \ln x_i \Sigma y_i\right]^2}{\left[n \Sigma (\ln x_i)^2 - (\Sigma \ln x_i)^2\right]\left[n \Sigma y_i^2 - (\Sigma y_i)^2\right]}$$

3. Estimated value $\hat{y}$ for given x

$$\hat{y} = a + b \ln x$$

Note:

n is a positive integer and $n \neq 1$.

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
[f] CLEAR [PRGM]	00	[f] [r]	07– 14 23
[x⇄y]	01– 21	[g] [x²]	08– 15 0
[f] [LN]	02– 14 1	[R/S]	09– 74
[Σ+]	03– 25	[f] [LN]	10– 14 1
[GTO] 00	04– 13 00	[f] [ŷ]	11– 14 22
[f] [L.R.]	05– 14 24	[R/S]	12– 74
[R/S]	06– 74		

REGISTERS			
R_0	R_1	R_2 n	R_3 $\Sigma \ln x$
R_4 $\Sigma (\ln x)^2$	R_5 Σy	R_6 Σy^2	R_7 $\Sigma (\ln x)(y)$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Initialize		[f] [REG] [f] [PRGM]	
3	Perform for i = 1 ,..., n:			
	Input x-value and y-value	x_i	[ENTER↑]	
		y_i	[R/S]	i
4	Calculate constants		[GSB] 05	a
			[x⇄y]	b
5	Calculate coefficient of			
	determination		[R/S]	r^2
6	To calculate $\hat{y}$, input x	x	[GSB] 10	$\hat{y}$
7	Perform step 6 as many			
	times as desired			
8	For new case, go to step 2.			

Example:

x_i	3	4	6	10	12
y_i	1.5	9.3	23.4	45.8	60.1

Solution:

$a = -47.0212$, $b = 41.3945$

$y = -47.02 + 41.39 \ln x$

$r^2 = 0.9798$

For $x = 8$, $\hat{y} = 39.0562$

For $x = 14.5$, $\hat{y} = 63.67$

Keystrokes	Display
[f] [REG] [f] [PRGM]	
3 [ENTER↑] 1.5 [R/S]	*1.0000*
4 [ENTER↑] 9.3 [R/S]	*2.0000*
6 [ENTER↑] 23.4 [R/S]	*3.0000*
10 [ENTER↑] 45.8 [R/S]	*4.0000*
12 [ENTER↑] 60.1 [R/S]	*5.0000*
[GSB] 05	*-47.0212*
[x⇄y]	*41.3945*
[R/S]	*0.9798*
8 [GSB] 10	*39.0562*
14.5 [GSB] 10	*63.6738*

Power Curve Fit

This program fits a power curve

$$y = ax^b \quad (a > 0)$$

to a set of data points

$$\{(x_i, y_i), i = 1, 2, \ldots, n\}$$

where: $x_i > 0$, $y_i > 0$.

By writing this equation as

$$\ln y = b \ln x + \ln a$$

the problem can be solved as a linear regression problem.

Output statistics are:

1. Regression coefficients

$$b = \frac{n \Sigma (\ln x_i)(\ln y_i) - (\Sigma \ln x_i)(\Sigma \ln y_i)}{n \Sigma (\ln x_i)^2 - (\Sigma \ln x_i)^2}$$

$$a = \exp\left[\frac{\Sigma \ln y_i}{n} - b\frac{\Sigma \ln x_i}{n}\right]$$

2. Coefficient of determination

$$r^2 = \frac{\left[n \Sigma (\ln x_i)(\ln y_i) - (\Sigma \ln x_i)(\Sigma \ln y_i)\right]^2}{\left[n \Sigma (\ln x_i)^2 - (\Sigma \ln x_i)^2\right]\left[n \Sigma (\ln y_i)^2 - (\Sigma \ln y_i)^2\right]}$$

3. Estimated value $\hat{y}$ for given x

$$\hat{y} = ax^b$$

Note:
n is a positive integer and $n \neq 1$.

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
[f] CLEAR [PRGM]	00	[R/S]	08- 74
[f] [LN]	01- 14 1	[f] [r]	09- 14 23
[x⇄y]	02- 21	[g] [x²]	10- 15 0
[f] [LN]	03- 14 1	[R/S]	11- 74
[Σ+]	04- 25	[f] [LN]	12- 14 1
[GTO] 00	05- 13 00	[f] [ŷ]	13- 14 22
[f] [L.R.]	06- 14 24	[g] [eˣ]	14- 15 1
[g] [eˣ]	07- 15 1	[R/S]	15- 74

REGISTERS			
R_0	R_1	R_2 n	R_3 $\Sigma \ln x_i$
R_4 $\Sigma (\ln x_i)^2$	R_5 $\Sigma \ln y_i$	R_6 $\Sigma (\ln y_i)^2$	R_7 Used

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Initialize		[f] [REG] [f] [PRGM]	
3	Perform for i = 1 ,…, n:			
	Input x-value and y-value	x_i	[ENTER↑]	
		y_i	[R/S]	i
4	Calculate constants		[GSB] 06	a
			[x⇄y]	b
5	Calculate coefficient of			
	determination		[R/S]	r^2
6	Input x-value and calculate $\hat{y}$	x	[GSB] 12	$\hat{y}$
7	Perform step 6 as many			
	times as desired			
8	For new case, go to step 2.			

Example:

x_i	10	12	15	17	20	22	25	27	30	32	35
y_i	0.95	1.05	1.25	1.41	1.73	2.00	2.53	2.98	3.85	4.59	6.02

Solution:

a = 0.0262, b = 1.4556

$y = .03x^{1.46}$

$r^2 = 0.9355$

For x = 18, $\hat{y}$ = 1.7609

For x = 23, $\hat{y}$ = 2.5159

Keystrokes	Display
[f] [REG] [f] [PRGM]	
10 [ENTER↑] 0.95 [R/S]	*1.0000*

Keystrokes	Display
12 [ENTER↑] 1.05 [R/S]	*2.0000*
:	
:	
35 [ENTER↑] 6.02 [R/S]	*11.0000*
[GSB] 06	*0.0262*
[x⇄y]	*1.4556*
[R/S]	*0.9355*
18 [GSB] 12	*1.7609*
23 [GSB] 12	*2.5159*

Test Statistics

Chi-Square Evaluation

This program calculates the value of the χ^2 statistic for the goodness of fit test by the equation

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

where: O_i = observed frequency

E_i = expected frequency.

The χ^2 statistic measures the closeness of the agreement between the observed frequencies and expected frequencies.

Notes:

1. In order to apply this test to a set of given data, it may be necessary to combine some classes to make sure that each expected frequency is not too small (say, not less than 5).
2. If the expected frequencies E_i are all equal to some value E, then E should be calculated beforehand as

$$E = \frac{\Sigma O_i}{n}$$

and then input at each step as the expected frequency E_i.

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
f CLEAR PRGM	00	STO 0	14– 23 0
0	01– 0	GTO 04	15– 13 04
STO 0	02– 23 0	STO 2	16– 23 2
STO 1	03– 23 1	−	17– 41
R/S	04– 74	g x²	18– 15 0
STO 2	05– 23 2	RCL 2	19– 24 2
−	06– 41	÷	20– 71
g x²	07– 15 0	STO − 1	21– 23 41 1
RCL 2	08– 24 2	RCL 0	22– 24 0
÷	09– 71	1	23– 1
STO + 1	10– 23 51 1	−	24– 41
RCL 0	11– 24 0	STO 0	25– 23 0
1	12– 1	GTO 04	26– 13 04
+	13– 51		

REGISTERS			
R_0 n	R_1 χ^2	R_2 E_i	R_3
R_4	R_5	R_6	R_7

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Initialize		[f] [PRGM] [R/S]	0.0000
3	Perform for i = 1 ,..., n:			
	Input observed and expected			
	frequencies	O_i	[ENTER↑]	
		E_i	[R/S]	i
4	Delete erroneous data	O_k	[ENTER↑]	
		E_k	[GSB] 16	
5	Display χ^2		[RCL] 1	χ^2
6	For new case go to step 2.			

Example:

O_i	8	50	47	56	5	14
E_i	9.6	46.75	51.85	54.4	8.25	9.15

Solution:

$\chi^2 = 4.8444$

Keystrokes	Display
[f] [PRGM] [R/S]	*0.0000*
8 [ENTER↑] 9.6 [R/S]	*1.0000*
50 [ENTER↑] 46.75	
[R/S]	*2.0000*
:	
:	
14 [ENTER↑] 9.15	
[R/S]	*6.0000*
[RCL] 1	*4.8444*

Paired t Statistic

Given a set of paired observations from two normal populations with means μ_1, μ_2 (unknown)

x_i	x_1	x_2	...	x_n
y_i	y_1	y_2	...	y_n

let

$$D_i = x_i - y_i$$

$$\bar{D} = \frac{1}{n}\sum_{i=1}^{n} D_i$$

$$s_D = \sqrt{\frac{n \Sigma D_i^2 - (\Sigma D_i)^2}{n(n-1)}}$$

$$s_{\bar{D}} = \frac{s_D}{\sqrt{n}}$$

The test statistic

$$t = \frac{\bar{D}}{s_{\bar{D}}},$$

which has n − 1 degrees of freedom (df), can be used to test the null hypothesis

$$H_0: \mu_1 = \mu_2.$$

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
f CLEAR PRGM	00	RCL 0	10– 24 0
−	01– 41	÷	11– 71
Σ+	02– 25	R/S	12– 74
GTO 00	03– 13 00	RCL 2	13– 24 2
g s	04– 15 25	1	14– 1
RCL 2	05– 24 2	−	15– 41
f √x	06– 14 0	GTO 00	16– 13 00
÷	07– 71	−	17– 41
STO 0	08– 23 0	f Σ-	18– 14 25
g x̄	09– 15 24	GTO 00	19– 13 00

REGISTERS			
R_0 Used	R_1	R_2 n	R_3 ΣD_i
R_4 ΣD_i^2	R_5	R_6	R_7

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Initialize		f REG f PRGM	
3	Perform for i = 1 ,..., n:			
	Input one pair of observation	x_i	ENTER↑	
		y_i	R/S	i
4	Delete erroneous data	x_k	ENTER↑	
		y_k	GSB 17	
5	Calculate t and df		GSB 04	t
			GSB 13	df
6	For new case, go to step 2.			

Example:

x_i	14	17.5	17	17.5	15.4
y_i	17	20.7	21.6	20.9	17.2

Solution:

t = −7.1554

df = 4

Keystrokes	Display
f REG f PRGM	
14 ENTER↑ 17 R/S	1.0000
17.5 ENTER↑ 20.7	
R/S	2.0000
:	
:	
15.4 ENTER↑ 17.2	
R/S	5.0000
GSB 04	−7.1554
GSB 13	4.0000

t Statistic For Two Means

Suppose $\{x_1, x_2, \ldots, x_{n_1}\}$ and $\{y_1, y_2, \ldots, y_{n_2}\}$ are independent random samples from two normal populations having means μ_1, μ_2 (unknown) and the same unknown variance σ^2.

We want to test the null hypothesis

$$H_o: \mu_1 - \mu_2 = D$$

where: D is a given number.

Define

$$\bar{x} = \frac{1}{n_1} \sum_{i=1}^{n_1} x_i$$

$$\bar{y} = \frac{1}{n_2} \sum_{i=1}^{n_2} y_i$$

$$t = \frac{\bar{x} - \bar{y} - D}{\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \sqrt{\frac{\Sigma x_i^2 - n_1 \bar{x}^2 + \Sigma y_i^2 - n_2 \bar{y}^2}{n_1 + n_2 - 2}}}$$

We can use this t statistic, which has the t distribution with $n_1 + n_2 - 2$ degrees of freedom, to test the null hypothesis H_0.

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
f CLEAR PRGM	00	+	25– 51
STO 1	01– 23 1	2	26– 2
g $\bar{x}$	02– 15 24	−	27– 41
STO 5	03– 23 5	÷	28– 71
RCL 4	04– 24 4	f $\sqrt{x}$	29– 14 0
STO 6	05– 23 6	RCL 1	30– 24 1
0	06– 0	g 1/x	31– 15 3
STO 2	07– 23 2	RCL 2	32– 24 2
STO 3	08– 23 3	g 1/x	33– 15 3
STO 4	09– 23 4	+	34– 51
R/S	10– 74	f $\sqrt{x}$	35– 14 0
STO 0	11– 23 0	×	36– 61
g $\bar{x}$	12– 15 24	RCL 5	37– 24 5
STO 3	13– 23 3	RCL 3	38– 24 3
RCL 6	14– 24 6	−	39– 41
RCL 1	15– 24 1	RCL 0	40– 24 0
RCL 5	16– 24 5	−	41– 41
GSB 45	17– 12 45	x⇄y	42– 21
RCL 4	18– 24 4	÷	43– 71
RCL 2	19– 24 2	R/S	44– 74
RCL 3	20– 24 3	g x^2	45– 15 0
GSB 45	21– 12 45	×	46– 61
+	22– 51	−	47– 41
RCL 1	23– 24 1	g RTN	48– 15 12
RCL 2	24– 24 2		

REGISTERS			
R_0 D	R_1 n_1	R_2 n_1, n_2	R_3 Σx, Σy, $\bar{y}$
R_4 Σx^2, Σy^2	R_5 $\bar{x}$	R_6 Σx^2	R_7

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Initialize		[f] [REG]	
3	Perform for i = 1 ,..., n_1:			
	Input x-value	x_i	[Σ+]	i
4	Initialize for y		[GSB] 01	0.0000
5	Perform for i = 1 ,..., n_2:			
	Input y-value	y_i	[Σ+]	i
6	Input D and calculate t	D	[R/S]	t
7	To find the means of x- and			
	y-values		[RCL] 5	$\bar{x}$
			[RCL] 3	$\bar{y}$
8	For a new case, go to step 2.			

Example:

x: 79, 84, 108, 114, 120, 103, 122, 120

y: 91, 103, 90, 113, 108, 87, 100, 80, 99, 54

$n_1 = 8$

$n_2 = 10$

$D = 0$ (i.e., H_0: $\mu_1 = \mu_2$)

Solution:

$t = 1.7316$

$\bar{x} = 106.2500$

$\bar{y} = 92.5000$

Keystrokes	Display
[f] [REG]	
79 [Σ+] 84 [Σ+] ···	
120 [Σ+]	*8.0000*
[GSB] 01	*0.0000*
91 [Σ+] 103 [Σ+] ···	
54 [Σ+]	*10.0000*
0 [R/S]	*1.7316*
[RCL] 5	*106.2500*
[RCL] 3	*92.5000*

One Sample Test Statistics For The Mean

For a normal population (x_1, x_2 ..., x_n) with a known variance σ^2, a test of the null hypothesis

$$H_0: \text{mean } \mu = \mu_0$$

is based on the z statistic (which has a standard normal distribution)

$$z = \frac{\sqrt{n}\,(\bar{x} - \mu_0)}{\sigma}$$

If the variance σ^2 is unknown, then

$$t = \frac{\sqrt{n}\,(\bar{x} - \mu_0)}{s}$$

is used instead. This t statistic has the t distribution with n − 1 degrees of freedom. x and s are sample mean and standard deviation.

KEY ENTRY	DISPLAY
f CLEAR PRGM	00
STO 1	01– 23 1
g x̄	02– 15 24
RCL 1	03– 24 1
−	04– 41
RCL 2	05– 24 2
f √x	06– 14 0
×	07– 61
STO 0	08– 23 0
CLX	09– 34

KEY ENTRY	DISPLAY
R/S	10– 74
g s	11– 15 25
RCL 0	12– 24 0
x⇄y	13– 21
÷	14– 71
R/S	15– 74
RCL 0	16– 24 0
x⇄y	17– 21
÷	18– 71
GTO 00	19– 13 00

REGISTERS			
R_0 t · s	R_1 μ_0	R_2 n	R_3 Σx
R_4 Σx^2	R_5 Σy	R_6 Σy^2	R_7 Σxy

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Initialize		f REG	
3	Perform for i = 1 ,..., n:			
	Input value	x_i	Σ+	i
4	Input μ_0	μ_0	f PRGM R/S	0.0000
5	Calculate t		GSB 11	t
	or			
5	Input σ and calculate z	σ	GSB 16	z
6	For new case, go to step 2.			

Example:

Suppose $\mu_0 = 2$, for the following set of data

{2.73, 0.45, 2.52, 1.19, 3.51, 2.75, 1.79, 1.83, 1, 0.87, 1.9, 1.62, 1.74, 1.92, 1.24, 2.68}

Solution:

test statistic t = −.6919 or z = −0.5650 if σ = 1.

Keystrokes	Display
f REG	
2.73 Σ+ 0.45 Σ+ …	
2.68 Σ+	16.0000
2 f PRGM R/S	0.0000
GSB 11	-0.6919
1 GSB 16	-0.5650