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STUDENT ENGINEERING
Applications



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HP-33E

Student Engineering Applications

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Introduction

This Student Engineering Applications book was written to help you get the most from your HP-33E calculator. The programs were chosen to provide useful calculations for many common problems encountered in engineering.

They will provide you with immediate capabilities in your everyday calculations and you will find them useful as guides to programming techniques for writing your own customized software.

You will find general information on how to key in and run programs under "A Word about Program Usage" in the Applications book you received with your calculator.

We hope that this Student Engineering book will be a valuable tool in your work and would appreciate your comments about it.

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Electrical Engineering

Ohm's Law and Reactance Chart

This program provides interchangeable solutions for many of the simple relationships involved in electrical engineering problems. Specifically one may solve for:

Frequency: given inductance and capacitance.

Capacitance: given frequency and inductance

Inductance: given frequency and capacitance

Capacitive reactance: given frequency and capacitance

Inductive reactance: given frequency and inductance

Current or voltage: given resistance and E or I

Resistance: given voltage and current

Power dissipation: given I and R or E and R

Current or voltage: given power and resistance

Formulas Used:

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$X_c = \frac{1}{2\pi fC}$$

$$X_L = 2\pi fL$$

$$E = IR$$

$$P = I^2R = E^2/R$$

$$I = \sqrt{P/R}$$

$$E = \sqrt{RP}$$

where: f = resonant frequency in hertz

L = inductance in henrys

C = capacitance in farads

X_c = capacitive reactance in Ω

X_L = inductive reactance in Ω

E = voltage

I = current in amps

R = resistance in Ω

P = power in watts

KEY ENTRY	DISPLAY
f CLEAR (PRGM)	00
x	01- 61
f \sqrt{x}	02- 14 0
GSB 18	03- 12 18
GTO 00	04- 13 00
x^2y	05- 21
GSB 18	06- 12 18
g x^2	07- 15 0
x^2y	08- 21
÷	09- 71
GTO 00	10- 13 00
x	11- 61
GSB 18	12- 12 18
GTO 00	13- 13 00
x	14- 61
GSB 18	15- 12 18
g $1/x$	16- 15 3
GTO 00	17- 13 00
g π	18- 15 73

KEY ENTRY	DISPLAY
x	19- 61
2	20- 2
x	21- 61
g $1/x$	22- 15 3
g RTN	23- 15 12
g $1/x$	24- 15 3
x	25- 61
GTO 00	26- 13 00
g $1/x$	27- 15 3
x^2y	28- 21
g x^2	29- 15 0
x^2y	30- 21
x	31- 61
GTO 00	32- 13 00
g $1/x$	33- 15 3
x	34- 61
f \sqrt{x}	35- 14 0
GTO 00	36- 13 00

REGISTERS			
R ₀	R ₁	R ₂	R ₃
R ₄	R ₅	R ₆	R ₇

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program Reactance Chart			
2	To calculate f: Input L and C	L, henrys C, farads	ENTER ⬆ GSB 01	f, hertz
3	To calculate L or C: Input f Input C or L	f, hertz C or L, farads, henrys	ENTER ⬆ GSB 05	L or C, henrys, farads
4	To calculate X_C : Input f and C	f, hertz C, farads	ENTER ⬆ GSB 11	X_C
5	To calculate X_L : Input f and L	f, hertz L, henrys	ENTER ⬆ GSB 14	X_L
	Ohm's Law			
6	To calculate E: Input I and R	I, amps R, Ω	ENTER ⬆ GSB 25	E, volts
7	To calculate I or R: Input E and R or I	E, volts R or I, Ω , amps	ENTER ⬆ GSB 24	I or R, Ω , amps
	Power			
8	To calculate P: a) Input E and R or	E, volts R, Ω	ENTER ⬆ GSB 27	P, watts

	b) Input I and R	I, amps R, Ω	ENTER ⬆ GSB 28	P, watts
9	To calculate I: Input P and R	P, watts R, Ω	ENTER ⬆ GSB 33	I, amps
10	To calculate E: Input P and R	P, watts R, Ω	ENTER ⬆ GSB 34	E, volts

Example 1:

$C = 0.01 \mu\text{F}$, $L = 160 \mu\text{H}$, calculate f:

Keystrokes

f **ENG** 3
160 **EEX** **CHS** 6
ENTER⬆
.01 **EEX** **CHS** 6
GSB 01

Display

125.8 03 or $(125.8 \times 10^3 \text{ Hz})$

Example 2:

$L = 2.5 \text{ H}$, $f = 60 \text{ Hz}$, calculate C and X_L :

Keystrokes

60 **ENTER**⬆ 2.5
GSB 05
60 **ENTER**⬆ 2.5
GSB 14

Display

2.814 -06 or $(2.814 \times 10^{-6} \mu\text{F})$

942.5 00 or (942.5Ω)

Example 3:

$E = 345 \text{ V}$, $R = 1.25 \text{ M}\Omega$. Calculate I and P:

Keystrokes

345 **ENTER**⬆
1.25 **EEX** 6
GSB 24
345 **ENTER**⬆
1.25 **EEX** 6
GSB 27

Display

276.0 -06 or $(276 \times 10^{-6} \text{ amps})$

95.22 -03 or $(95.22 \times 10^{-3} \text{ watts})$

Resistors in Series or Parallel

This program calculates the total resistance of a group of resistors arranged in parallel or in series.

Formulas Used:

Resistors in Series: $R_T = R_1 + R_2 + R_3 \dots + R_n$

Resistors in Parallel: $R_T = \frac{1}{1/R_1 + 1/R_2 + 1/R_3 \dots + 1/R_n}$

- Note that this program can be used for summing capacitors in parallel or series instead of resistors. For series capacitors use instructions for parallel resistors. For parallel capacitors use instructions for series resistors.
- Several more advanced programs for circuit analysis may be found in: Anderson, L.H., "Calculator-Aided Circuit Analysis," *Ham Radio Magazine*, pp. 38-46, October 1977. Although written for the HP-25 they will work equally well on the HP-33E.

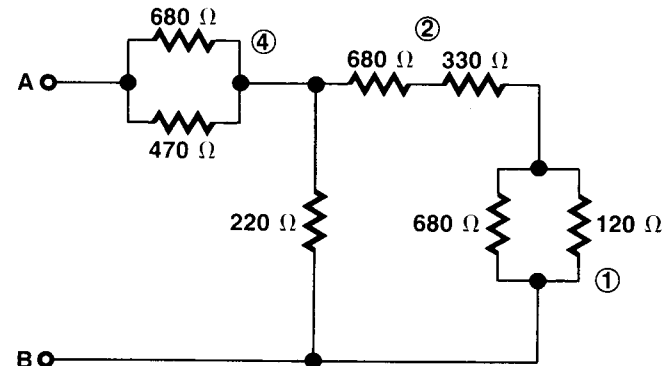
KEY ENTRY	DISPLAY
[F] CLEAR [PRGM]	00
0	01- 0
[GTO] 04	02- 13 04
1	03- 1
[STO] 0	04- 23 0
[R/S]	05- 74
[RCL] 0	06- 24 0
[9] [x=0]	07- 15 71
[GSB] 20	08- 12 20
[R+]	09- 22
[STO] [+] 1	10- 23 51 1
[GTO] 05	11- 13 05

KEY ENTRY	DISPLAY
[RCL] 1	12- 24 1
[RCL] 0	13- 24 0
[9] [x=0]	14- 15 71
[GSB] 20	15- 12 20
[CLX]	16- 34
[STO] 1	17- 23 1
[R+]	18- 22
[GTO] 00	19- 13 00
[R+]	20- 22
[9] [1/x]	21- 15 3
[ENTER+]	22- 31
[9] [RTN]	23- 15 12

REGISTERS			
R ₀ Code: 0 or 1	R ₁ Σ R, Σ1/R	R ₂	R ₃
R ₄	R ₅	R ₆	R ₇

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Initialize		[F] [REG] [F] [PRGM]	
3	a) Set for parallel resistors		[R/S]	0.0000
	or			
	b) Set for series resistors		[GSB] 03	1.0000
4	Optional: Retrieve last R _T			
	for use in next calculation		[R+]	Previous R _T
5	Input individual resistor values	R _i	[R/S]	
	(Repeat until all resistors in group have been input.)			
6	Calculate total resistance of the group		[GSB] 12	R _T , Ω
7	Optional: Store R _T for use in next calculation	R _T	[STO] 2	
8	Go to step 3a or 3b for next group.			

Example:



Determine the total circuit resistance from A to B.

Keystrokes	Display	
f REG f PRGM		
Group 1: R/S	0.0000	Parallel mode
680 R/S 120 R/S		
GSB 12	102.0000	R_1, Ω
Group 2: GSB 03	1.0000	Series mode
R+	102.0000	Retrieve R_1
R/S 330 R/S		
680 R/S		
GSB 12	1,112.0000	R_2, Ω
Group 3: R/S	0.0000	Parallel mode
R+	1,112.0000	Retrieve R_2
R/S 220 R/S		
GSB 12	183.6637	R_3, Ω
STO 2		Save R_3
Group 4: R/S	0.0000	Parallel mode
680 R/S 470 R/S		
GSB 12	277.9130	R_4, Ω
Total R		
GSB 03	1.0000	Series mode
R+	277.9130	Retrieve R_4
R/S RCL 2 R/S		
GSB 12	461.5767	R_T, Ω

Exponential Growth or Decay

Many growth or decay phenomena encountered in science and engineering obey an exponential law of the general form:

$$X_t = X_{ss} - (X_{ss} - X_0) e^{-t/\tau}$$

where: X_t = Value at any time, t , (i.e., the instantaneous value)

X_{ss} = Steady state value (i.e., at $t = \infty$)

X_0 = Initial value (i.e., at $t = 0$)

t = Elapsed time (time after $t = 0$)

τ = Exponential time-constant for specific phenomena

This program provides interchangeable solutions for any one of the four variables X_t, X_{ss}, X_0 or t provided three variables and τ are known.

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
f CLEAR PRGM	00	RCL 2	23- 24 2
RCL 0	01- 24 0	=	24- 41
RCL 2	02- 24 2	X	25- 61
=	03- 41	RCL 2	26- 24 2
GSB 40	04- 12 40	+	27- 51
=	05- 71	GTO 00	28- 13 00
RCL 2	06- 24 2	RCL 0	29- 24 0
+	07- 51	RCL 2	30- 24 2
GTO 00	08- 13 00	=	31- 41
GSB 40	09- 12 40	RCL 1	32- 24 1
ENTER*	10- 31	RCL 2	33- 24 2
ENTER*	11- 31	=	34- 41
RCL 1	12- 24 1	=	35- 71
X	13- 61	f LN	36- 14 1
RCL 0	14- 24 0	RCL 4	37- 24 4
=	15- 41	X	38- 61
X\divY	16- 21	GTO 00	39- 13 00
1	17- 1	RCL 3	40- 24 3
=	18- 41	RCL 4	41- 24 4
=	19- 71	=	42- 71
GTO 00	20- 13 00	g e²	43- 15 1
GSB 40	21- 12 40	g RTN	44- 15 12
RCL 1	22- 24 1		

REGISTERS			
$R_0 X_0$	$R_1 X_t$	$R_2 X_{ss}$	$R_3 t$
$R_4 \tau$	R_5	R_6	R_7

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Store time constant	τ	[STO] 4	
3	Store variables:			
	Initial value	X_0	[STO] 0	
	Instantaneous value	X_t	[STO] 1	
	Steady state value	X_{ss}	[STO] 2	
	Elapsed time	t	[STO] 3	
	(Store any 3 of the 4 variables)			
4	To calculate:			
	X_0 , initial value		[GSB] 21	X_0
	X_t , instantaneous value		[GSB] 01	X_t
	X_{ss} , steady state value		[GSB] 09	X_{ss}
	t, elapsed time		[GSB] 29	t
5	For a new case go to step 2.			

Example 1:

Given a $5\mu\text{F}$ capacitor in series with a 1 megohm resistor. 1.5 seconds after the circuit is completed 125 volts are measured across R. To what voltage was the capacitor originally charged?

Note:

π = the RC time-constant, and the voltage at $t = \infty$ is zero.

Keystrokes

Display

5 [EEX] [CHS] 6
 [ENTER+] [EEX] 6
 [x] [STO] 4
 125 [STO] 1
 0 [STO] 2
 1.5 [STO] 3
 [GSB] 21

168.7324

volts

Example 2:

A cobalt 60 source (half life = 5.26 years) had an activity of 3.54 curies when purchased 8.5 years ago. What is its present activity?

Note:

Activity at $t = \infty$ will be zero, π = half life/ $\ln 2$.

Keystrokes

Display

5.26 [ENTER+]
 2 [f] [LN] [÷]
 [STO] 4
 3.54 [STO] 0
 0 [STO] 2
 8.5 [STO] 3
 [GSB] 01

1.1549

curies

Heat and Thermal Engineering

Black Body Thermal Radiation

Bodies with finite temperatures emit thermal radiation. The higher the absolute temperature, the more thermal radiation emitted. Bodies which emit the maximum possible amount of energy at every wavelength for a specified temperature are said to be black bodies. While black bodies do not actually exist in nature, many surfaces may be assumed to be black for engineering considerations.

Black body monochromatic emissive power

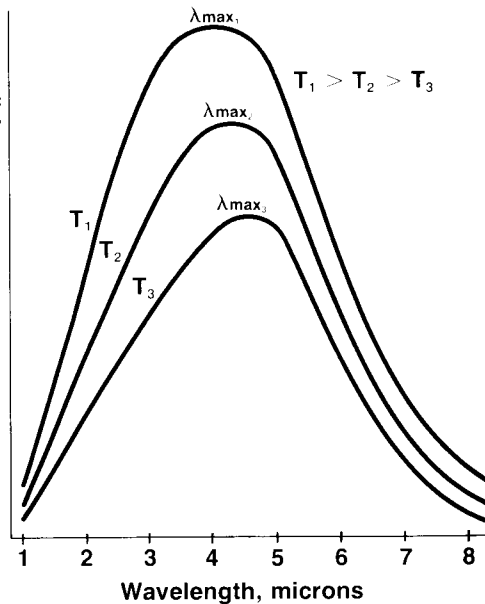


Figure 1 is a representation of black body thermal emission as a function of wavelength. Note that as temperature increases the area under the curves (total emissive power $E_{b(0-\lambda)}$) increases. Also note that the wavelength of maximum emissive power λ_{max} shifts to the left as temperature increases.

This program can be used to calculate the wavelength of maximum emissive power for a given temperature, the temperature corresponding to a particular wavelength of maximum emissive power, the total emissive power for all wavelengths and the emissive power at a particular wavelength.

Equations:

$$\lambda_{max} T = c_3$$

$$E_{b(0-\lambda)} = \sigma T^4$$

$$E_{b\lambda} = \frac{2\pi c_1}{\lambda^5 (e^{c_2/\lambda T} - 1)}$$

where: λ_{max} is the wavelength of maximum emissivity in microns;

T is the absolute temperature in °R or K;

$E_{b(0-\lambda)}$ is the total emissive power in Btu/hr-ft² or watts/cm²;

$E_{b\lambda}$ is the emissive power at λ in Btu/hr-ft²- μ m or watts/cm²- μ m;

$$\begin{aligned} c_1 &= 1.8887982 \times 10^7 \text{ Btu}\cdot\mu\text{m}^4/\text{hr}\cdot\text{ft}^2 \\ &= 5.9544 \times 10^3 \text{ W}\mu\text{m}^4/\text{cm}^2 \end{aligned}$$

$$c_2 = 2.58984 \times 10^4 \mu\text{m}\cdot^\circ\text{R} = 1.4388 \times 10^4 \mu\text{m}\cdot\text{K}$$

$$c_3 = 5.216 \times 10^3 \mu\text{m}\cdot^\circ\text{R} = 2.8978 \times 10^3 \mu\text{m}\cdot\text{K}$$

$$\sigma = 1.713 \times 10^{-9} \text{ Btu/hr}\cdot\text{ft}^2\cdot^\circ\text{R}^4 = 5.6693 \times 10^{-12} \text{ W/cm}^2\cdot\text{K}^4$$

$$\sigma_{exp} = 1.731 \times 10^{-9} \text{ Btu/hr}\cdot\text{ft}^2\cdot^\circ\text{R}^4 = 5.729 \times 10^{-12} \text{ W/cm}^2\cdot\text{K}^4$$

- Sources differ on values for constants. This could yield small discrepancies between published tables and HP-33E outputs.

Reference:

Robert Siegel and John R. Howell, *Thermal Radiation Heat Transfer*, Volume I, National Aeronautics and Space Administration, 1968.

KEY ENTRY	DISPLAY
\boxed{f} CLEAR \boxed{PRGM}	00
\boxed{STO} 5	01- 23 5
\boxed{GSB} 34	02- 12 34
\boxed{STO} 6	03- 23 6
$\boxed{R/S}$	04- 74
\boxed{STO} 6	05- 23 6
\boxed{GSB} 34	06- 12 34
\boxed{STO} 5	07- 23 5
$\boxed{R/S}$	08- 74
\boxed{RCL} 5	09- 24 5
4	10- 4
\boxed{f} $\boxed{y^x}$	11- 14 3
\boxed{RCL} 4	12- 24 4
\boxed{x}	13- 61
\boxed{f} PAUSE	14- 14 74
\boxed{RCL} 1	15- 24 1
2	16- 2
\boxed{x}	17- 61
\boxed{g} $\boxed{\pi}$	18- 15 73

KEY ENTRY	DISPLAY
\boxed{x}	19- 61
\boxed{RCL} 6	20- 24 6
5	21- 5
\boxed{f} $\boxed{y^x}$	22- 14 3
$\boxed{=}$	23- 71
\boxed{RCL} 2	24- 24 2
\boxed{RCL} 6	25- 24 6
$\boxed{=}$	26- 71
\boxed{RCL} 5	27- 24 5
$\boxed{=}$	28- 71
\boxed{g} $\boxed{e^x}$	29- 15 1
1	30- 1
$\boxed{=}$	31- 41
$\boxed{=}$	32- 71
\boxed{GTO} 00	33- 13 00
\boxed{RCL} 3	34- 24 3
\boxed{x} \boxed{y}	35- 21
$\boxed{=}$	36- 71
\boxed{g} \boxed{RTN}	37- 15 12

REGISTERS

R_0	$R_1 C_1$	$R_2 C_2$	$R_3 C_3$
$R_4 \sigma$	$R_5 T$	$R_6 \lambda$	R_7

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Set display			
3	Store constants		\boxed{f} \boxed{SCI} 4	
	a) For SI (W, μm , cm, K)			

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	$c_1 = 5.9544 \times 10^3$	c_1	\boxed{STO} 1	
	$c_2 = 1.4388 \times 10^4$	c_2	\boxed{STO} 2	
	$c_3 = 2.8978 \times 10^3$	c_3	\boxed{STO} 3	
	$\sigma = 5.6693 \times 10^{-12}$	σ	\boxed{STO} 4	
	or			
	$\sigma_{exp} = 5.729 \times 10^{-12}$	σ_{exp}	\boxed{STO} 4	
	b) For English (Btu, μm , hr, ft, $^{\circ}R$)			
	$c_1 = 1.8887982 \times 10^7$		\boxed{STO} 1	
	$c_2 = 2.58984 \times 10^4$		\boxed{STO} 2	
	$c_3 = 5.216 \times 10^3$		\boxed{STO} 3	
	$\sigma = 1.713 \times 10^{-9}$		\boxed{STO} 4	
	or			
	$\sigma_{exp} = 1.731 \times 10^{-9}$		\boxed{STO} 4	
4	To calculate λ_{max}	T	\boxed{GSB} 01	$\lambda_{max}, \mu m$
5	To calculate temp. at which λ is maximum	λ	\boxed{GSB} 05	T, $^{\circ}$
6	To calculate black body total emissive power and total emissive power at any λ .			
	a) For λ_{max} , do step 3 or step 4 then		\boxed{GSB} 09	$(E_{b(0-\infty)})^*$ $E_b \lambda_{max}$
	b) For other λ	λ	\boxed{STO} 6 \boxed{GSB} 09	$(E_{b(0-\infty)})^*$ $E_b \lambda$
	* $(E_{b(0-\infty)})$ displayed by pause only.			

Example:

If the human eye was designed to work most efficiently in sunlight and the visible spectrum peaks at about $.550 \mu\text{m}$, what is the sun's temperature in K? Assume that the sun is a black body. What is the total emissive power and the emissive power at λ_{max} ? What is the emissive power at $\lambda = 0.400 \mu\text{m}$ (ultraviolet limit) and $0.700 \mu\text{m}$ (infrared limit).

Keystrokes**Display**

f SCI 4		
5.9544 EEX 3		
STO 1		
1.4388 EEX 4		
STO 2		
2.8978 EEX 3		
STO 3		
5.6693 EEX CHS 12		
STO 4		
.55 GSB 05	5.2687 03	$(5.2698 \times 10^3 \text{ K})$
GSB 09	4.3687 03	$(4.3687 \times 10^3 \text{ watts/cm}^2,$ $E_{b(0-x)})$ (Pause only)
	5.2229 03	$(5.2229 \times 10^3 \text{ watts/cm}^2$ $-\mu\text{m}, E_{b\lambda\mu\alpha\chi})$
.4 STO 6 GSB 09	4.3687 03	(Ignore)
	3.9649 03	$(3.9649 \times 10^3 \text{ watts/cm}^2$ $-\mu\text{m}, E_{b\lambda})$
.7 STO 6 GSB 09	4.3687 03	(Ignore)
	4.5934 03	$(4.5934 \times 10^3 \text{ watts/cm}^2$ $-\mu\text{m}, E_{b\lambda})$

Ideal Gas Equation of State

Many gases obey the ideal gas laws quite closely at reasonable temperatures and pressures. This program calculates any one of the four variables when data for the other three and the universal gas constant are entered. Likewise, the value of the universal gas constant can be determined by entering data for the four variables.

Equation:

$$PV = nRT$$

where: P is the absolute pressure
V is the volume
n is the number of moles present
R is the universal gas constant
T is the absolute temperature

TABLE 1

Values of the Universal Gas Constant

Value of R	Units of R	Units of P	Units of V	Units of T
8.314	N-m/g mole- K	N/m ²	m ³ /g mole	K
83.14	cm ³ -bar/g mole- K	bar	cm ³ /g mole	K
82.05	cm ³ -atm/g mole- K	atm	cm ³ /g mole	K
0.08205	ℓ -atm/g mole- K	atm	ℓ /g mole	K
0.7302	atm-ft ³ /lb mole-°R	atm	ft ³ /lb mole	°R
10.73	psi-ft ³ /lb mole-°R	psi	ft ³ /lb mole	°R
1545	psf-ft ³ /lb mole-°R	psf	ft ³ /lb mole	°R

Remarks:

- At low temperatures or high pressures the ideal gas law does not represent the behavior of real gases.
- The value of R used must be compatible with the units of P, V, T.
- In running the program be sure to enter zero for the variable to be calculated.

KEY ENTRY	DISPLAY
\boxed{f} CLEAR \boxed{PRGM}	00
\boxed{GSB} 28	01- 12 28
\boxed{STO} 2	02- 23 2
$\boxed{R+}$	03- 22
\boxed{GSB} 28	04- 12 28
\boxed{STO} 1	05- 23 1
$\boxed{R+}$	06- 22
$\boxed{R/S}$	07- 74
\boxed{GSB} 28	08- 12 28
\boxed{STO} 5	09- 23 5
$\boxed{R+}$	10- 22
\boxed{GSB} 28	11- 12 28
\boxed{STO} 4	12- 23 4
$\boxed{R+}$	13- 22
\boxed{GSB} 28	14- 12 28
\boxed{STO} 3	15- 23 3
\boxed{RCL} 3	16- 24 3

KEY ENTRY	DISPLAY
\boxed{RCL} 4	17- 24 4
\boxed{X}	18- 61
\boxed{RCL} 5	19- 24 5
\boxed{X}	20- 61
\boxed{RCL} 1	21- 24 1
\boxed{RCL} 2	22- 24 2
\boxed{X}	23- 61
$\boxed{+}$	24- 71
$\boxed{R/S}$	25- 74
\boxed{g} $\boxed{\sqrt{x}}$	26- 15 3
\boxed{GTO} 00	27- 13 00
\boxed{g} $\boxed{x=0}$	28- 15 61
\boxed{g} \boxed{RTN}	29- 15 12
$\boxed{R+}$	30- 22
1	31- 1
\boxed{g} \boxed{RTN}	32- 15 12

REGISTERS

R_0	R_1 P	R_2 V	R_3 n
R_4 R	R_5 T	R_6	R_7

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Set display and initialize		\boxed{f} \boxed{FIX} 2 \boxed{f} \boxed{PRGM}	
3	Input variables*			
	Pressure	P	$\boxed{ENTER+}$	
	Volume	V	$\boxed{R/S}$	
	Number of moles	n	$\boxed{ENTER+}$	
	Universal gas constant**	R	$\boxed{ENTER+}$	
	Absolute temp.	T		
4	a) To calculate P or V		$\boxed{R/S}$	P or V
	or			
	b) To calculate n, R or T		$\boxed{R/S}$ $\boxed{R/S}$	n, R or T
5	To change conditions:			
	a) Go to step 3, or,			
	b) Store new variable in			
	in proper register (see			
	register contents) and			
	store 1 in register of			
	variable to be calculated,			
	then,			
	for P or V		\boxed{GSB} 16	P or V
	or,			
	for n, R or T		\boxed{GSB} 16 $\boxed{R/S}$	n, R or T
	* Note: variables must be in-			
	put in order shown. Input			
	zero for variable to be calculated			
	** Be sure R is in units com-			
	patible with units of			
	variables			

Example:

0.63 moles of air are enclosed in 25000 cm³ at 1200K. What is the pressure in bars? In atmospheres? Assume an ideal gas.

(R = 83.14 cm³-bar/g mole-K or 82.05 cm³- atm/g mole-K)

Keystrokes	Display	
f FIX 2 f PRGM		
0 ENTER ↵		
25000 R/S		
.63 ENTER ↵		
83.14 ENTER ↵	2.51	bars
1200 R/S		
82.05 STO 4		
GSB 16	2.48	atm.

Mechanical Engineering

Equations of Motion

This program provides solutions for many problems involving motion of an object given a constant acceleration and initial velocity. Velocity, distance traveled and acceleration may be found if time is known. Given the distance traveled, velocity or time may be calculated or, given velocity, time or acceleration may be calculated.

Equations:

$$V = V_0 + at$$

$$V = (V_0^2 + 2 aS)^{1/2}$$

$$S = V_0 t + \frac{1}{2} at^2$$

$$t = \frac{V - V_0}{a} \quad \text{or} \quad a = \frac{V - V_0}{t}$$

where: V = velocity

V₀ = initial velocity

a = acceleration

S = distance

t = time

These same equations also hold for circular motion where:

ω [angular velocity (radians/sec)] replaces V

ω_0 replaces V₀

α replaces a

θ [angular displacement (radians)] replaces S

- Generally accepted values for the frequently used gravitational constant are 980.665 cm/sec² or 32.17398 ft/sec².

KEY ENTRY	DISPLAY
\boxed{f} CLEAR \boxed{PRGM}	00
\boxed{STO} 2	01- 23 2
$\boxed{R+}$	02- 22
\boxed{STO} 0	03- 23 0
$\boxed{R/S}$	04- 74
\boxed{RCL} 2	05- 24 2
\boxed{x}	06- 61
\boxed{RCL} 0	07- 24 0
$\boxed{+}$	08- 51
\boxed{g} \boxed{RTN}	09- 15 12
2	10- 2
\boxed{x}	11- 61
\boxed{RCL} 2	12- 24 2
\boxed{x}	13- 61
\boxed{RCL} 0	14- 24 0
\boxed{g} $\boxed{x^2}$	15- 15 0
$\boxed{+}$	16- 51
\boxed{f} $\boxed{\sqrt{x}}$	17- 14 0
\boxed{g} \boxed{RTN}	18- 15 12
\boxed{STO} 3	19- 23 3
2	20- 2
$\boxed{\div}$	21- 71
\boxed{GSB} 05	22- 12 05

KEY ENTRY	DISPLAY
\boxed{RCL} 3	23- 24 3
\boxed{x}	24- 61
$\boxed{R/S}$	25- 74
$\boxed{x\&y}$	26- 21
\boxed{RCL} 0	27- 24 0
$\boxed{-}$	28- 41
$\boxed{x\&y}$	29- 21
$\boxed{\div}$	30- 71
$\boxed{R/S}$	31- 74
\boxed{GSB} 10	32- 12 10
\boxed{STO} 7	33- 23 7
\boxed{RCL} 0	34- 24 0
\boxed{CHS}	35- 32
$\boxed{+}$	36- 51
\boxed{RCL} 2	37- 24 2
$\boxed{\div}$	38- 71
\boxed{g} $\boxed{x<0}$	39- 15 41
\boxed{GTO} 42	40- 13 42
$\boxed{R/S}$	41- 74
\boxed{RCL} 7	42- 24 7
\boxed{CHS}	43- 32
\boxed{GTO} 34	44- 13 34

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Input initial velocity and acceleration	V_0 a	$\boxed{ENTER+}$ \boxed{GSB} 01	
	To solve for velocity:			
3	Input time or Input distance	t S	\boxed{GSB} 05 \boxed{GSB} 10	V V
	To solve for distance:			
4	Input time	t	\boxed{GSB} 19	S
	To solve for time:			
5a	Input velocity and acceleration or or	V a	$\boxed{ENTER+}$ \boxed{GSB} 26 \boxed{RCL} 2 \boxed{GSB} 26	t t
5b	Input distance	S	\boxed{GSB} 32	t
	To solve for acceleration:			
6	Input velocity and time	V t	$\boxed{ENTER+}$ \boxed{GSB} 26	a

Example:

A stone is thrown from a 100 meter high bridge with an initial velocity of 15 meters per second. What will be the velocity when it strikes the river below? How long will it take to hit the water? (The acceleration of gravity is 9.80665 m/sec²)

Keystrokes

Display

15 $\boxed{ENTER+}$
9.80665 \boxed{GSB} 01
100 \boxed{GSB} 10
100 \boxed{GSB} 32

46.7582
3.2384

V, meters/sec
t, seconds

REGISTERS

R_0 V_0	R_1	R_2 a	R_3 t
R_4	R_5	R_6	R_7 Used

Natural Frequency of Oscillators

This program solves for the natural frequency, the spring constant or the mass (alternatively, weight) of a simple oscillator. Examples of this are a spring or torsional pendulum obeying Hooke's law or a pendulum undergoing small oscillations.

Equations:

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \text{ for spring}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k_T}{J}} \text{ for torsional pendulum}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{1}{\ell/g}} \text{ for pendulum in small oscillation (i.e., } \theta \approx \sin\theta)$$

$$m = \frac{W}{g}$$

where: m = mass, W = weight

g = acceleration due to gravity

f = natural frequency of the oscillator (hertz)

k = spring constant (wt/ℓ)

k_T = torsional constant (wt ℓ/radian)

J = mass moment of inertia (wt ℓ sec²)

ℓ = length

- Note that for a simple pendulum, length is equivalent to weight.
- In running the program be sure to enter zero for the variable to be calculated.

This program is based on an HP-65 Users' Library program by Lane R. Pendleton.

KEY ENTRY	DISPLAY
[F] CLEAR [PRGM]	00
[STO] 3	01- 23 3
[R*]	02- 22
[STO] 2	03- 23 2
[R*]	04- 22
[STO] 1	05- 23 1
[G] [X=0]	06- 15 71
[GTO] 18	07- 13 18
[RCL] 2	08- 24 2
[G] [X=0]	09- 15 71
[GTO] 14	10- 13 14
[GSB] 27	11- 12 27
[÷]	12- 71
[GTO] 00	13- 13 00
[GSB] 27	14- 12 27
[RCL] 3	15- 24 3
[X]	16- 61
[GTO] 00	17- 13 00
[RCL] 2	18- 24 2
[RCL] 3	19- 24 3

KEY ENTRY	DISPLAY
[±]	20- 71
[F] [X]	21- 14 0
[G] [π]	22- 15 73
2	23- 2
[X]	24- 61
[±]	25- 71
[GTO] 00	26- 13 00
[RCL] 1	27- 24 1
[G] [π]	28- 15 73
[X]	29- 61
2	30- 2
[X]	31- 61
[G] [X ²]	32- 15 0
[G] [RTN]	33- 15 12
[RCL] 0	34- 24 0
[÷]	35- 71
[GTO] 00	36- 13 00
[RCL] 0	37- 24 0
[X]	38- 61
[GTO] 00	39- 13 00

REGISTERS			
R ₀ g	R ₁ f	R ₂ k	R ₃ m
R ₄	R ₅	R ₆	R ₇

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Store the gravitational acceleration constant:			
	a) If SI	980.665, (cm/sec ²)	[STO] 0	g, cm/sec ²
	b) If English	386.088, (in/sec ²)	[STO] 0	g, in/sec ²
3	Input data in order shown (use zero for unknown variable):			
	Frequency	f, (sec ⁻¹)	[ENTER+]	
	Spring constant	k (wt/l)	[ENTER+]	
	Mass	m	[f] [PRGM]	
	or Weight	W	[GSB] 34	
4	Calculate unknown variable:		[R/S]	
	Frequency			f, Hz.
	or spring const.			k, (wt/l)
	or mass			m
5	To convert mass to weight	m	[GSB] 37	W
6	To convert weight to mass	W	[GSB] 34	m

Example 1:

A weight of 10 lbs. is attached to a spring whose constant is 100 lbs/in. Find the frequency of the system.

Keystrokes	Display
386.088 [STO] 0	
0 [ENTER+]	
100 [ENTER+]	10
[GSB] 34	
[R/S]	9.8892 Hz

Example 2:

A torsional pendulum has a natural frequency of 200 Hz. Its mass moment of inertia is 400 kg in. sec². Find the torsional constant.

Keystrokes	Display
980.665 [STO] 0	
200 [ENTER+]	
0 [ENTER+]	
400 [f] [PRGM]	
[R/S] [f] [SCI] 4	6.3165 08 (6.3165 × 10 ⁸) kg in/radian

Example 3:

Find the length of a pendulum which has a 1Hz natural frequency.

Note:

The length of the pendulum is equivalent to its weight.)

Keystrokes	Display
386.088 [STO] 0	
1 [ENTER+]	
1 [ENTER+]	0
[R/S]	
[GSB] 37	9.7797 inches

Kinetic Energy

This program calculates an interchangeable solution among the variables weight (or mass), velocity, and kinetic energy, for an object moving at constant velocity. The program operates in either English or metric units. For metric units, any consistent set of units may be used; the quantity mass must be used. For English units, the energy must be in foot-pounds, the velocity in feet per second, and the quantity weight in pounds.

- where: K.E. = kinetic energy
 W = weight (lb)
 m = mass (kg, g)
 v = velocity
 g = acceleration due to gravity = 980.665 cm/sec²
 or 32.17398 ft/sec²

Equations:

English	Metric
$K.E. = \frac{1}{2} \frac{W}{g} v^2$	$K.E. = \frac{1}{2} mv^2$
$1 \text{ ft-lb} = 1.98 \times 10^6 \text{ hp}$	

KEY ENTRY	DISPLAY
[F] CLEAR [PRGM]	00
2	01- 2
[STO] 0	02- 23 0
[GTO] 13	03- 13 13
6	04- 6
4	05- 4
.	06- 73
3	07- 3
4	08- 4
7	09- 7
9	10- 9
6	11- 6
[STO] 0	12- 23 0
[R/S]	13- 74
[9] [x ²]	14- 15 0
[x]	15- 61
[RCL] 0	16- 24 0
[÷]	17- 71
[GTO] 13	18- 13 13

KEY ENTRY	DISPLAY
1	19- 1
9	20- 9
8	21- 8
[EEEX]	22- 33
4	23- 4
[±]	24- 71
[GTO] 13	25- 13 13
[9] [x ²]	26- 15 0
[x ² y]	27- 21
[RCL] 0	28- 24 0
[x]	29- 61
[x ² y]	30- 21
[±]	31- 71
[GTO] 13	32- 13 13
[±]	33- 71
[RCL] 0	34- 24 0
[x]	35- 61
[f] [x ²]	36- 14 0
[GTO] 13	37- 13 13

REGISTERS			
R ₀ Used	R ₁	R ₂	R ₃
R ₄	R ₅	R ₆	R ₇

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Initialize:			
	for metric (SI)		GSB 01	2.0000
	for English		GSB 04	64.3480
	To calculate Kinetic Energy:			
3	Input weight (mass)	W (m)	ENTER+	
	and velocity	V	R/S	K.E.
	To calculate Weight (mass):			
4	Input kinetic energy	K.E.	ENTER+	
	and velocity	V	GSB 26	W (m)
	To calculate Velocity:			
5	Input kinetic energy	K.E.	ENTER+	
	and weight (mass)	W (m)	GSB 33	V
	Optional:			
	Convert K.E. (ft-lb) to K.E.			
	(hp)	K.E. (ft-lb)	GSB 19	K.E. (hp)
7	For a new case go to step 2.			

Example 1:

The slider of a slider-crank mechanism is used to punch holes in a slab of metal. It is found that the work required to punch a hole is 775 ft-lb. If the slider weighs 5.25 lbs., how fast must it be moving when it strikes the metal? What is the required work in horsepower?

Keystrokes

Display

GSB 04
 775 **ENTER+**
 5.25 **GSB** 33 **97.4627** ft/sec
 775 **ENTER+**
GSB 19
f **SCI** 4 **3.9141-04** 3.9141×10^{-4} hp

Example 2:

An object weighing 4.8 kg is moving with constant velocity of 3.5 m/sec. Find its kinetic energy.

Keystrokes

Display

f **FIX** 4 **GSB** 01
 4.8 **ENTER+** 3.5 **R/S** **29.4000** joules

RPM/Torque/Power

This program provides an interchangeable solution for RPM, torque, and power in both Systeme International (metric) and English units.

	SI	English
RPM	RPM	RPM
Torque	nt-m	ft-lb
Power	watts	hp

Equations:

$$\text{RPM} \times \text{Torque} = \text{Power}$$

$$1 \text{ hp} = 745.7 \text{ watts}$$

$$1 \text{ ft-lb} = 1.356 \text{ joules}$$

$$1 \text{ hp} = 550 \frac{\text{ft-lb}}{\text{sec}}$$

$$1 \text{ RPM} = \pi/30 \text{ radians/sec}$$

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
\boxed{f} CLEAR \boxed{PRGM}	00	$\boxed{[x]y}$	21-- 21
3	01-- 3	\boxed{g} $\boxed{1/x}$	22-- 15 3
0	02-- 0	\boxed{STO} 5	23-- 23 5
\boxed{g} $\boxed{\pi}$	03-- 15 73	$\boxed{+}$	24-- 71
$\boxed{+}$	04-- 71	$\boxed{[x]}$	25-- 61
\boxed{STO} 7	05-- 23 7	\boxed{STO} 7	26-- 23 7
7	06-- 7	$\boxed{R/S}$	27-- 74
4	07-- 4	$\boxed{+}$	28-- 71
5	08-- 5	\boxed{RCL} 7	29-- 24 7
.	09-- 73	$\boxed{[x]}$	30-- 61
7	10-- 7	$\boxed{R/S}$	31-- 74
\boxed{STO} 5	11-- 23 5	\boxed{RCL} 6	32-- 24 6
1	12-- 1	$\boxed{+}$	33-- 71
.	13-- 73	\boxed{GTO} 27	34-- 13 27
3	14-- 3	$\boxed{[x]}$	35-- 61
5	15-- 5	\boxed{RCL} 7	36-- 24 7
6	16-- 6	$\boxed{-}$	37-- 71
\boxed{STO} 6	17-- 23 6	$\boxed{R/S}$	38-- 74
$\boxed{R/S}$	18-- 74	\boxed{RCL} 5	39-- 24 5
\boxed{g} $\boxed{1/x}$	19-- 15 3	$\boxed{+}$	40-- 71
\boxed{STO} 6	20-- 23 6	\boxed{GTO} 27	41-- 13 27

REGISTERS

R ₀	R ₁	R ₂	R ₃
R ₄	R ₅ Used	R ₆ Used	R ₇ Used

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Initialize:			
	For metric (SI)		\boxed{GSB} 01	1.3560
	For English		\boxed{GSB} 01 $\boxed{R/S}$	5,251.4089
	To calculate RPM			
3	Input power	power	\boxed{ENTER} *	
	Input torque	torque	\boxed{GSB} 28	RPM
	To calculate Torque:			
4	Input power	power	\boxed{ENTER} *	
	Input RPM	RPM	\boxed{GSB} 28	torque
	Optional:			
4b	Convert torque to other system		$\boxed{R/S}$	torque
	To calculate Power:			
5a	Input torque	torque	\boxed{ENTER} *	
	Input RPM	RPM	\boxed{GSB} 35	power
	Optional:			
5b	Convert power to other system		$\boxed{R/S}$	power

Example 1:

Compute the torque from an engine developing 11 hp at 6500 RPM. Find the SI equivalent.

Keystrokes

\boxed{GSB} 01 $\boxed{R/S}$
 11 \boxed{ENTER} *
 6500 \boxed{GSB} 28
 $\boxed{R/S}$

Display

8.8870
12.0508

ft-lb, English
 nt-m, SI

Example 2:

A generator is turning at 1600 RPM with a torque of 20 nt-m. If it is 90% efficient what is the power input in watts? In horsepower?

Keystrokes

G/S 01

20 ENTER .9 ±

1600 G/S 35

R/S

Display**3,723.3691**

watts

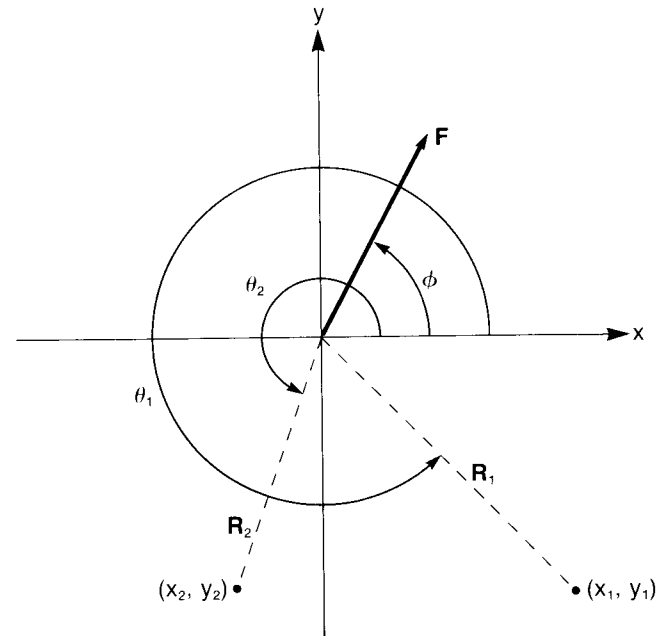
4.9931

hp

Stress Analysis

Static Equilibrium at a Point

This program calculates the two reaction forces necessary to balance a given two-dimensional force vector. The direction of the reaction forces may be specified as a vector of arbitrary length or by Cartesian coordinates using the point of force application as the origin.



Equations:

$$R_1 \cos \theta_1 + R_2 \cos \theta_2 = F \cos \phi$$

$$R_1 \sin \theta_1 + R_2 \sin \theta_2 = F \sin \phi$$

where: F is the known force;

ϕ is the direction of the known force;

R_1 is one reaction force;

θ_1 is the direction of R_1 ;

R_2 is the second reaction force;

θ_2 is the direction of R_2 ;

The coordinates x_1 and y_1 are referenced from the point where F is applied to the end of the member along which R_1 acts; x_2 and y_2 are the coordinates referenced from the point where F is applied to the end of the member along which R_2 acts.

Remarks:

This program assumes the calculator is set in DEG mode.

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
f CLEAR PRGM	00	RCL 2	25- 24 2
g +P	01- 15 4	x	26- 61
x2y	02- 21	-	27- 41
1	03- 1	RCL 1	28- 24 1
f +R	04- 14 4	RCL 2	29- 24 2
STO 0	05- 23 0	x	30- 61
x2y	06- 21	RCL 0	31- 24 0
STO 1	07- 23 1	RCL 3	32- 24 3
g RTN	08- 15 12	x	33- 61
g +P	09- 15 4	-	34- 41
x2y	10- 21	÷	35- 71
1	11- 1	R/S	36- 74
f +R	12- 14 4	f LST x	37- 14 73
STO 2	13- 23 2	STO 6	38- 23 6
x2y	14- 21	RCL 5	39- 24 5
STO 3	15- 23 3	RCL 0	40- 24 0
g RTN	16- 15 12	x	41- 61
f +R	17- 14 4	RCL 4	42- 24 4
STO 4	18- 23 4	RCL 1	43- 24 1
x2y	19- 21	x	44- 61
STO 5	20- 23 5	-	45- 41
RCL 4	21- 24 4	RCL 6	46- 24 6
RCL 3	22- 24 3	÷	47- 71
x	23- 61	g RTN	48- 15 12
RCL 5	24- 24 5		

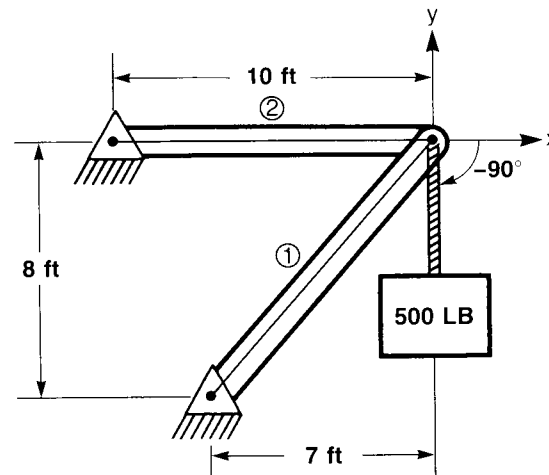
REGISTERS

$R_0 \cos \theta_1$	$R_1 \sin \theta_1$	$R_2 \cos \theta_2$	$R_3 \sin \theta_2$
$R_4 F \cos \phi$	$R_5 F \sin \phi$	R_6 Used	R_7

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Define reaction directions as Cartesian coordinates or as vectors of arbitrary magnitude. (Use the point of force application as the origin):			
	Define direction one in rectangular form	y_1	ENTER+	y_1
		x_1	GSB 01	$\sin \theta_1$
	or			
	in polar form	θ_1	GSB 03	$\sin \theta_1$
	and			
	Define direction two in rectangular form	y_2	ENTER+	
		x_2	GSB 09	$\sin \theta_2$
	or			
	polar form	θ_2	GSB 11	$\sin \theta_2$
3	Key in known force: direction, then magnitude and compute reactions.	ϕ	ENTER+	
		F	GSB 17	R_1
			R/S	R_2
4	To change force, go to step 3.			
	To change either or both reaction directions, go to step 2.			

Example 1:

Find the reaction forces in the pin-jointed structure shown below.



Keystrokes

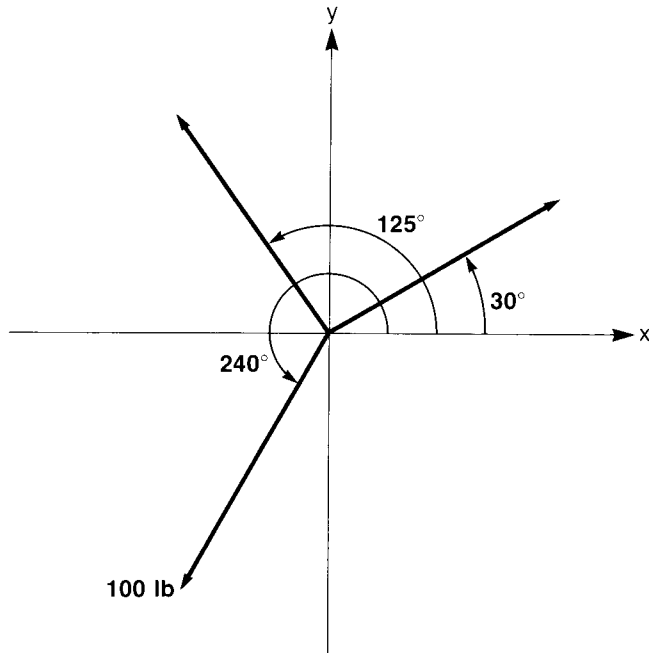
8 **CHS** **ENTER+**
 7 **CHS**
GSB 01
 0 **ENTER+** 10 **CHS**
GSB 09
 90 **CHS** **ENTER+**
 500 **GSB** 17
R/S

Display

-0.7526 $\sin \theta_1$
0.0000 $\sin \theta_2$
-664.3841 R_1
437.5000 R_2

Example 2:

Find the reaction forces for the diagram below:



Keystrokes

30 **[GSB]** 03
 125 **[GSB]** 11
 240 **[ENTER]**
 100 **[GSB]** 17
[R/S]

Display

0.5000
0.8192
90.9770
50.1910

R₁
 R₂

Vector Cross Product

If $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ are two dimensional vectors then the cross product of A and B is denoted by $A \times B$ and is calculated as follows:

$$A \times B = \left(\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}, - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}, \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \right) = (a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1)$$

Let the solution be represented by (c_1, c_2, c_3) .

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
[F] CLEAR [PRGM]	00	[RCL] 6	13- 24 6
[RCL] 2	01- 24 2	[X]	14- 61
[RCL] 6	02- 24 6	[-]	15- 41
[X]	03- 61	[R/S]	16- 74
[RCL] 3	04- 24 3	[RCL] 1	17- 24 1
[RCL] 5	05- 24 5	[RCL] 5	18- 24 5
[X]	06- 61	[X]	19- 61
[-]	07- 41	[RCL] 2	20- 24 2
[R/S]	08- 74	[RCL] 4	21- 24 4
[RCL] 3	09- 24 3	[X]	22- 61
[RCL] 4	10- 24 4	[-]	23- 41
[X]	11- 61	[GTO] 00	24- 13 00
[RCL] 1	12- 24 1		

REGISTERS			
R ₀	R ₁ a ₁	R ₂ a ₂	R ₃ a ₃
R ₄ b ₁	R ₅ b ₂	R ₆ b ₃	R ₇

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Store A	a ₁	[STO] 1	
		a ₂	[STO] 2	
		a ₃	[STO] 3	
3	Store B	b ₁	[STO] 4	
		b ₂	[STO] 5	
		b ₃	[STO] 6	
4	Calculate cross product		[GSB] 01	c ₁
			[R/S]	c ₂
			[R/S]	c ₃
5	For a new case, go to step 2.			

Example:

Let A = (2, 5, 2) and B = (3, 3, -4).

Solution:

A × B = (-26, 14, -9)

Keystrokes	Display	
2 [STO] 1		
5 [STO] 2		
2 [STO] 3		
3 [STO] 4		
3 [STO] 5		
4 [CHS] [STO] 6		
[GSB] 01	-26.0000	c ₁
[R/S]	14.0000	c ₂
[R/S]	-9.0000	c ₃

Angle Between, Norm and Dot Product of Vectors

Let $\vec{a} = (a_1, a_2, \dots, a_n)$ and $\vec{b} = (b_1, b_2, \dots, b_n)$ be two vectors.

The norm of a is denoted by $|\vec{a}|$ and is calculated by the following

formula:

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + \dots + a_n^2}$$

similarly,

$$|\vec{b}| = \sqrt{b_1^2 + b_2^2 + \dots + b_n^2}$$

The dot product of \vec{a} and \vec{b} is denoted by $\vec{a} \cdot \vec{b}$ and is calculated by the following formula:

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + \dots + a_n b_n$$

The angle between \vec{a} and \vec{b} is denoted by θ and is calculated by the following formula:

$$\theta = \cos^{-1} \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} \right)$$

The angle is calculated in any angular mode. When calculated in degrees, decimal degrees are assumed.

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
[f] CLEAR [PRGM]	00	[STO] [+] 2	11- 23 51 2
[ENTER*]	01- 31	[GTO] 00	12- 13 00
[g] [x ²]	02- 15 0	[RCL] 2	13- 24 2
[STO] [+] 1	03- 23 51 1	[RCL] 0	14- 24 0
[R*]	04- 22	[RCL] 1	15- 24 1
[x ₂ y]	05- 21	[x]	16- 61
[ENTER*]	06- 31	[f] [√x]	17- 14 0
[g] [x ²]	07- 15 0	[÷]	18- 71
[STO] [+] 0	08- 23 51 0	[g] [COS ⁻¹]	19- 15 8
[R*]	09- 22	[GTO] 00	20- 13 00
[x]	10- 61		

REGISTERS			
R ₀ ∑ a _i ²	R ₁ ∑ b _i ²	R ₂ ∑ a _i b _i	R ₃
R ₄	R ₅	R ₆	R ₇

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Initialize		f REG f PRGM	
3	Perform for $i = 1, \dots, n$:			
	Key in a_i and b_i	a_i	ENTER	
		b_i	R/S	
4	Find norm of \vec{a}		RCL 0 f \sqrt{x}	$ \vec{a} $
5	Find norm of \vec{b}		RCL 1 f \sqrt{x}	$ \vec{b} $
6	Find $ \vec{a} \cdot \vec{b} $		RCL 2	$ \vec{a} \cdot \vec{b} $
7	Calculate angle between \vec{a} and \vec{b}		GSB 13	θ

Example:

Let $\vec{a} = (2, 5, 2)$ and $\vec{b} = (3, 3, -4)$

Solution:

$$|\vec{a}| = 5.7446$$

$$|\vec{b}| = 5.8310$$

$$\vec{a} \cdot \vec{b} = 13.0000$$

$$\theta = 67.1635$$

Keystrokes

f **REG** **f** **PRGM**
 2 **ENTER** 3 **R/S**
 5 **ENTER** 3 **R/S**
 2 **ENTER** 4 **CHS**
R/S
RCL 0 **f** **\sqrt{x}**
RCL 1 **f** **\sqrt{x}**
RCL 2
GSB 13

Display

5.7446
5.8310
13.0000
67.1635

$|\vec{a}|$
 $|\vec{b}|$
 $|\vec{a} \cdot \vec{b}|$
 θ

Engineering Economics

Discounted Cash Flow: Net Present Value, Internal Rate of Return

The primary purpose of this program is to compute the net present value of a series of cash flows. In general, an initial investment V_0 is made in some enterprise which is expected to bring in periodic cash flows C_1, C_2, \dots, C_n . Given a discount rate i , which must be entered as a decimal, then for each cash flow C_k , the program will compute the net present value at period k , NPV_k . A negative value for NPV_k indicates that the enterprise has not yet been profitable. A positive NPV_k means that the enterprise has been profitable, to the extent that a rate of return i on the original investment has been exceeded.

The program may also be used iteratively to calculate an internal rate of return. The objective here is to find the discount rate i which will make the final net present value, NPV_n , equal to zero. The procedure, then, is to store V_0 and a first guess at the rate of return i , input the cash flows C_1 through C_n , and thus find NPV_n . If NPV_n is negative, the estimated rate of return was too high; if NPV_n is positive, the estimate for i was too low. Adjust the estimate for i accordingly, store the new i , and input the cash flows again. Inspect the new value of NPV_n to obtain a new estimate for i and repeat the process. The entire procedure is repeated until NPV_n is zero, or very close to it. The last value of i input is then regarded as the internal rate of return.

Each figure for net present value is found by

$$NPV_k = V_0 + \sum_{j=1}^k \frac{C_j}{(1+i)^j}$$

This program employs the convenient sign convention used in the most recent HP calculators and programs. Cash received is represented by a positive value (+). Cash paid out is represented by a negative value (-).

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
\square CLEAR \square PRGM	00	\square STO 3	12- 23 3
\square RCL 1	01- 24 1	\square R/S	13- 74
1	02- 1	\square RCL 2	14- 24 2
\square STO 4	03- 23 4	\square RCL 4	15- 24 4
\square +	04- 51	1	16- 1
\square STO 2	05- 23 2	\square +	17- 51
\square -	06- 71	\square STO 4	18- 23 4
\square RCL 0	07- 24 0	\square \square \square \square	19- 14 3
\square +	08- 51	\square -	20- 71
\square RCL 4	09- 24 4	\square RCL 3	21- 24 3
\square \square PAUSE	10- 14 74	\square +	22- 51
\square \square \square	11- 21	\square GTO 09	23- 13 09

REGISTERS

$R_0 V_0$	$R_1 i$	$R_2 (1 + i)$	$R_3 NPV_k$
$R_4 k$	R_5	R_6	R_7

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	Store initial investment and discount rate			
	Initial investment	V_0^*	\square STO 0	
	Interest rate	i (decimal)	\square STO 1	
3	Perform for $k = 1, \dots, n$:		\square \square PRGM	
	Input C_k cash flow and compute NPV_k	C_k^*	\square R/S	(k)
				NPV_k^*
4	For a new case, go to step 2.			
	* Note: Cash received is represented by a positive value (+). Cash paid out is represented by a negative value (-).			

Example:

You are contemplating installing a processing machine for \$150,000 at a capital cost of 10% after taxes. Based on the following cash flows, will this investment be profitable?

Year	Cash Flow
1	\$30,000
2	26,300
3	50,000
4	55,600
5	45,200

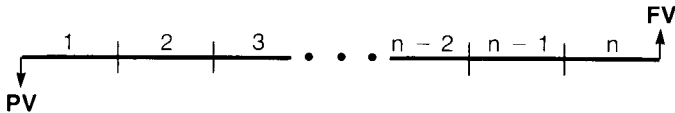
Keystrokes**Display**

f FIX 2
 15 CHS EEX 4
 STO 0

0.1 STO 1 f PRGM

3 EEX 4 R/S	-122,727.27	1 st year
26300 R/S	-100,991.74	2 nd year
5 EEX 4 R/S	-63,426.00	3 rd year
55600 R/S	-25,450.45	4 th year
45200 R/S	2,615.20	5 th year

Since C_3 is positive the investment is profitable to the extent that the cost of capital is 10%.

Compound Amount

This program applies to an amount of principal that has been placed into an account and compounded periodically, with no further deposits. The important variables in this case are the number of compounding periods n , the periodic interest rate i , the principal or present value PV , the future value of the account FV , and the amount of interest accrued I . Any of these may be calculated from the others by these formulas:

$$n = \frac{\ln |FV/PV|}{\ln (1 + i)}$$

$$i = \left| \frac{FV}{PV} \right|^{1/n} - 1$$

$$PV = -FV (1 + i)^{-n}$$

$$FV = -PV (1 + i)^n$$

$$I = -PV [(1 + i)^n - 1]$$

where: n = total number of periods

i = periodic interest rate (expressed as decimal)

i.e., an annual interest rate of 6% is expressed as 0.06,

which is a monthly rate of $\frac{0.06}{12} = 0.005$

PV = present value (value at beginning of first period)

FV = future value (value at end of last period)

This program employs the convenient sign convention used in the most recent HP calculators and programs. Cash received is represented by a positive value (+). Cash paid out is represented by a negative value (-).

KEY ENTRY	DISPLAY	KEY ENTRY	DISPLAY
f CLEAR PRGM	00	RCL 5	24- 24 5
RCL 5	01- 24 5	CHS	25- 32
RCL 4	02- 24 4	x	26- 61
÷	03- 71	GTO 00	27- 13 00
g ABS	04- 15 34	GSB 44	28- 12 44
f LN	05- 14 1	RCL 1	29- 24 1
GSB 44	06- 12 44	f y^x	30- 14 3
f LN	07- 14 1	RCL 4	31- 24 4
÷	08- 71	CHS	32- 32
GTO 00	09- 13 00	x	33- 61
RCL 5	10- 24 5	GTO 00	34- 13 00
RCL 4	11- 24 4	GSB 44	35- 12 44
÷	12- 71	RCL 1	36- 24 1
g ABS	13- 15 34	f y^x	37- 14 3
RCL 1	14- 24 1	1	38- 1
g √x	15- 15 3	±	39- 41
f y^x	16- 14 3	RCL 4	40- 24 4
1	17- 1	CHS	41- 32
±	18- 41	x	42- 61
GTO 00	19- 13 00	GTO 00	43- 13 00
GSB 44	20- 12 44	RCL 2	44- 24 2
RCL 1	21- 24 1	1	45- 1
CHS	22- 32	+	46- 51
f y^x	23- 14 3	g RTN	47- 15 12

REGISTERS

R ₀	R ₁ n	R ₂ i	R ₃
R ₄ PV	R ₅ FV	R ₆	R ₇

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Key in the program			
2	To compute number of periods	i (decimal)	STO 2	
		PV*	STO 4	
		FV*	STO 5	
			GSB 01	n
3	To compute periodic interest rate	n	STO 1	
		PV*	STO 4	
		FV*	STO 5	
			GSB 10	i (decimal)
4	To compute principal	n	STO 1	
		i (decimal)	STO 2	
		FV*	STO 5	
			GSB 20	PV
5	To compute future value	n	STO 1	
		i (decimal)	STO 2	
		PV*	STO 4	
			GSB 28	FV
6	To compute accrued interest	n	STO 1	
		(i decimal)	STO 2	
		PV*	STO 4	
			GSB 35	1
7	For a new case, go to step 2, 3, 4, 5, or 6.			
	* Note: Cash received is represented by a positive value (+). Cash paid out is represented by a negative value (-).			

Example 1:

Find the rate of return on \$1000 compounded quarterly if it amounts to \$1500 in 5 years.

Note:

$$n = 20$$

Keystrokes**Display**20 **[STO]** 11000 **[CHS]** **[STO]** 41500 **[STO]** 5 **[GSB]** 10 **0.0205** (quarterly)4 **[x]** **0.0819** (8.19% annually)**Example 2:**

How much will you need to invest today at 5¼% compounded quarterly to have \$3000 in 5 years?

Note:

$$n = 20$$

Keystrokes:**Display****[f]** **[FIX]** 220 **[STO]** 1.0575 **[ENTER]**4 **[=]** **[STO]** 23000 **[STO]** 5 **[GSB]** 20 **-2,255.02** (\$)