HEWLETT PACKARD

HP-35

MATH PAC

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Your Math Pac

The Math Pac is a representative collection of key sequence routines for solving problems with your HP-35 Pocket Calculator.

We suggest that you first read the introductory material explaining the Standard Key Sequence Format. Then find the routine you want, and use it. Numerical examples are provided to enable you to try out routines before using them. In most cases, the given answers are rounded. An understanding of the HP-35 Operating Manual is also required if, in addition, you wish to track the changes in the memory register and stack registers on a step-by-step basis.

To make for easy reference, the body of the book is arranged in alphabetic sequence of key topics—those you are likely to think of when you want a routine. Thus, Complex Number Operations are presented in the "C" section, Progressions are presented in the "P" section, etc. Statistics (in the "S" section) includes the Mean and Standard Deviation. Functions that are thus grouped and presented under higher level headings are also given cross reference entries in the alphabetic sequence. Thus, if you happen to search for Mean in the "M" section, you will find there a reference to the page in the "S" section where mean is presented under "Statistics". The Contents section at the front is arranged logically (instead of by page order) to show all the routines available under each subject. The back cover contains an index.

Mr. Lee Skinner of Albuquerque, New Mexico (a prolific HP-35 user) contributed all but a few of the routines in this book. Two key sequence forms are included inside the back cover of this manual. You may wish to duplicate these forms and record your own key step programs.

Standard Key Sequence Format

Shown below is the key sequence routine for computing the roots of the Quadratic Equation:

$$AX^2 + BX + C = 0$$

Using A, B, and C, which you supply as data, the routine produces an intermediate result, D. If $D < 0$, one root is the complex conjugate of the other; the real and imaginary parts of one complex root develop on lines 7 and 8. Except for the opposite sign of the imaginary part, the other complex root is identical. If $D \geqslant 0$, the roots are real and develop on lines 5 and 6.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	A	↑	+	STO				
2	B	RCL	÷	CHS	↑	↑		
3		×						
4	C	↑	+	RCL	÷	−	D	If D < 0, go to 7
5		√x	STO	+			Root 1	
6		x⇄y	RCL	−			Root 2	Stop
7		CHS	√x	x⇄y			Real part	
8		x⇄y					Imag. part	

To execute the sequence, start with line 1 and read from left to right, making the appropriate keystrokes as you proceed. Interpret the respective columns as follows:

Data: Information to be supplied by you, the user. In the sample case, lines 1, 2, and 4 prompt the reader to enter coefficients A, B, and C.

Operations: The keys to be pressed after you enter any requested data item for the line. [↑] is the symbol used to denote the [ENTER ↑] key of the HP-35. All other key designations are identical to the HP-35 keys. Ignore any blank positions in the operations column.

Display: *Intermediate or final results which you should,* depending upon the case, *jot down.* In the sample case, D is developed so that the reader can decide which line (5 or 7) to execute next.

Remarks: Conditional and unconditional jumps to specified lines or other information for the reader. In the sample case, the reader is prompted to continue with line 7 (ignoring lines 5 and 6) if D is negative. If the condition fails, execution continues on the next line. In the sample case, the reader proceeds to line 5, if D is zero or positive.

Thus, lines are read in sequential order except where the remarks column directs otherwise (as in line 4 of the sample case). To assist the reader in distinguishing lines to be repeated, a sequence of lines making up an iterative process is outlined with a bold border. The following sequence for computing Chi-Square (lines 2, 3, and 4) illustrate this convention.

Formula:

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

where: O_i = observed frequency

E_i = expected frequency

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		CLX ↑		
2	O_i	↑		Perform lines 2–4 for i = 1, ..., n
3	E_i	STO − ↑ x RCL		
4		÷ +		

Contents

GEOMETRY AND TRIGONOMETRY

Amortization

See page 135

Angle Conversions

Degrees, minutes, and seconds to decimal degrees

Example:

$46°17'32.6'' = 46.29°$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	Seconds	↑	6	0	STO	÷		
2	Minutes	+	RCL	÷				
3	Degrees	+						

Decimal degrees to degrees, minutes and seconds

Note:

x = decimal degrees.

Example:

$23.32916667° = 23°19'45''$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	X	↑						Let D = integer part of X
2	D						Degrees	
3		−	6	0	STO	x	A	Let M = integer part of A
4	M						Minutes	
5		−	RCL	x			B	Let S = nearest integer to B
6	S						Seconds	

Bearing to azimuth

Note:

X = bearing in decimal degrees

Example:

S 42.6° E = 137.4°

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	X	↑		If S X° E, go to 2;
				If S X° W, go to 3;
				If N X° W, go to 4;
			Azimuth	If N X° E, stop
2		1 8 0 x⇄y –	Azimuth	Stop
3		1 8 0 +	Azimuth	Stop
4		CHS ↑ 3 6 0		
5		+	Azimuth	

Azimuth to bearing

Method:

If 0 < X < 90°, convert to N-° -′ -″ E;
if 90° < X < 180°, convert to S-° -′ -″ E;
if 180° < X < 270°, convert to S-° -′ -″ W;
if 270° < X < 360°, convert to N-° -′ -″ W.

Note:

X = azimuth in decimal degrees

Example:

226°23′ = 226.3833333° = S 46°23′ W

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	X	↑	D	If 0 < X < 90, go to 5;
				If 90 < X < 180, go to 2;
				If 180 < X < 270, go to 3;
				If 270 < X < 360, go to 4
2		1 8 0 x⇄y −	D	Go to 5
3		1 8 0 −	D	Go to 5
4		3 6 0 − CHS	D	
5	D[1]		Degrees	Note: D[1] is the integer part of D.
6		− 6 0 STO x	A	Let M = integer part of A
7	M		Minutes	
8		− RCL x	B	Let S = nearest integer to B
9	S		Seconds	

Radians to degrees

Examples:

1. 1 radian = 57.29577951°
2. $\frac{3}{4}\pi$ radians = 135°

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		1 8 0 π ÷		
2		↑ ↑ ↑		
3		CLX		
4	Radians	x	Degrees	Stop. For new case, go to 3.

Degrees to radians

Examples

1. $1^\circ = .0174532925$ radians
2. $266^\circ = 4.64$ radians

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		π	1	8	0	÷		
2		↑	↑	↑				
3		CLX						
4	Degrees	x					Radians	Stop. For new case, go to 3

Mils to degrees

Example:

1600 mils = 90°

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	Mils	↑	9	x	1	6		
2		0	÷				Degrees	

Degrees to mils

Example:

90° = 1600 mils

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	Degrees	↑	1	6	0	x		
2		9	÷				Mils	

Grads to degrees

Example:

300 grads = 270°

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	Grads	[↑] [.] [9] [x] [ ]	Degrees	

Degrees to grads

Example:

360° = 400 grads

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	Degrees	[↑] [.] [9] [÷] [ ]	Grads	

Angles of Triangle

See pages 147–151

Annuities

Present Value

The present value (PV) of an ordinary annuity is the total discounted value of a series of payments

$$PV = PMT \left[\frac{1 - (1 + i)^{-n}}{i} \right]$$

where PMT is the payment
i is the periodic interest rate
and n is the number of periods.

Example:

What is the present value of \$200 received at the end of each year for 10 years if the money earns 3% annually? (PMT = 200, i = .03, n = 10)

Answer: \$1706.04

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	i	[↑] [STO] [1] [+] [ ]		
2	n	[x⇄y] [x^y] [1/x] [1] [x⇄y]		
3		[−] [RCL] [÷] [ ] [ ]		
4	PMT	[×] [ ] [ ] [ ] [ ]	PV	

Anti-Log$_{10}$ of X

Example:

Antilog$_{10}$ 2 = 100.

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	X	[↑] [1] [0] [x^y] [ ]		

Arc Cosecant

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Arc Cosine, Secondary

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Arc Cotangent

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Arc Secant

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Arc Sine, Secondary

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Arc Tangent, Secondary

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Area of Triangle

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Arithmetic Mean

See page 138

Arithmetic Progressions

See pages 113, 115-116

Average

See pages 138, 140–142

Azimuth to Bearing

See page 12

Base Conversion

Base conversion algorithms are given for positive values only. To convert a negative number, (1) change the sign, (2) convert the number, and (3) change the sign of result.

$(A)_b$ means, "A is a number in base b."

Decimal integer to integer in any base

$$(I)_{10} \rightarrow (J)_b$$

In the following key sequence, f + 1 is the number of digits in $(J)_b$. d_i (i = 1, f + 1) represents the i^{th} digit in $(J)_b$, counting from left to right, i.e.

$$(J)_b = (d_1 d_2 \cdots d_{f+1})_b$$

Notes: 1. If $d_i = 0$, line 7 can be replaced by the [×] key.

2. For large numbers, f is the exponent for base exponentiation.

$$(J)_b = (d_1 \cdot d_2 \ldots d_{f+1})_b \times b^f$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	b	[↑] [↑] [ ] [ ] [ ]		
2	I	[STO] [ln] [x⇄y] [ln] [÷]	D	Let f be the largest integer ≤ D
3	f	[x⇄y] [CLX] [+] [ ] [ ]	f	
4		[x⇄y] [↑] [↑] [RCL] [R↓]		
5		[R↓] [x^y] [÷] [ ] [ ]	E_1	Let d_i = integer part of E_i
		[ ] [ ] [ ] [ ] [ ]		(i = 1, ..., f)
6	d_1	[−] [×] [ ] [ ] [ ]	E_2	
7	d_i	[−] [×] [ ] [ ] [ ]	E_{i+1}	Perform line 7 for i = 2, ..., f
8		[ ] [ ] [ ] [ ] [ ]		d_{f+1} = nearest integer to E_{f+1}

Example 1:

Convert $(1206)_{10}$ to hexadecimal (base 16).
(The Hexadecimal digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F).

Answer:

$(1206)_{10} = (4B6)_{16}$ (f = 2)

Example 2:

Convert 513 to octal (base 8).

Answer:

$(513)_{10} = (1001)_8$ (f = 3)

Example 3:

Convert 6.023×10^{23} to octal.

Answer:

$6.023 \times 10^{23} = (1.7743)_8 \times 8^{26}$

Note:

If we consider 6.023×10^{23} to be a scientific measurement good only to 4 significant digits, it is meaningless for the octal representation to contain more than 5 significant digits. Therefore, we stop before the loop is completed.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	8	↑	↑					
2	6.023	EEX						
3	23	STO	ln	x⇄y	ln	÷	26.33160947	
4	26	x⇄y	CLX	+			26	f = 26
5		x⇄y	↑	↑	RCL	R↓		
6		R↓	x^y	÷			1.992843519	$d_1 = 1$
7	1	−	x				7.942748152	$d_2 = 7$
8	7	−	x				7.541985216	$d_3 = 7$
9	7	−	x				4.335881728	$d_4 = 4$
10	4	−	x				2.687053824	$d_5 = 3$ (rounded), Stop

Integer without exponent in base b to decimal integer

$$(d_1 d_2 \dots d_{n-1}\ d_n)_b \rightarrow (I)_{10}$$

Examples:

1. $(730020461)_8 = (123740465)_{10}$
2. $(7D0F)_{16} = (32015)_{10}$
   (A = 10, B = 11, C = 12, D = 13, E = 14, F = 15 in the hexadecimal system)

Note: If $d_i = 0$, line 3 can be replaced by the [×] key.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	[↑]	[↑]	[↑]	[ ]	[ ]		
2	d_1	[x]	[ ]	[ ]	[ ]	[ ]		
3	d_i	[+]	[x]	[ ]	[ ]	[ ]		Perform line 3 for i = 2, ..., n−1
4	d_n	[+]	[ ]	[ ]	[ ]	[ ]	$(I)_{10}$	

Integer with exponent in base b to decimal integer

$$d_1 \cdot d_2 \cdots d_n \times b^{Exp} \rightarrow (I)_{10}$$

Examples:

1. $(3.0002)_8 \times 8^{11} = 2.577399803 \times 10^{10}$ (b = 8, Exp = 11, n = 5)
2. $(D2EE4)_{16} \times 16^{28} = (D.2EE4)_{16} \times 16^{32}$
   $= 4.485999088 \times 10^{39}$

Note: If $d_i = 0$, line 7 can be replaced by the [×] key.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	[↑]	[↑]	[ ]	[ ]	[ ]		
2	Exp	[↑]	[ ]	[ ]	[ ]	[ ]		
3	n	[−]	[1]	[+]	[x⇄y]	[x^y]		
4		[STO]	[ ]	[ ]	[ ]	[ ]		
5	b	[↑]	[↑]	[↑]	[ ]	[ ]		
6	d_1	[X]	[ ]	[ ]	[ ]	[ ]		
7	d_i	[+]	[x]	[ ]	[ ]	[ ]		Perform line 7 for i = 2, ..., n−1
8	d_n	[+]	[RCL]	[x]	[ ]	[ ]	$(I)_{10}$	

Fractional decimal number to base b

$$(x)_{10} \rightarrow (y)_b$$

Method:

In the following algorithm, c is the number of significant digits in x.

$d_i(i = 1, ..., c)$ represents the i^{th} digit in $(y)_b$, counting from left to right.

$$(y)_b = (d_1 \cdot d_2 \ldots d_c)_b \times b^{-H}$$

Examples:

1. $(0.2937)_{10} = (2.263)_8 \times 8^{-1} = (.2263)_8$ (c = 4)
2. $(3.688)_{10} \times 10^{-54} = (5.A6E)_{16} \times 16^{-45}$
   (c = 4, in hexadecimal system A = 10, E = 14)

Note:

If $d_i = 0$, line 8 can be replaced by the [×] key.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	↑	↑					
2	x	STO	1/x	ln	x⇄y	ln		
3		÷					D	Let H be the smallest integer ≥ D
4	H	x⇄y	CLX	+				
5		x⇄y	↑	↑	RCL	R↓		
6		R↓	x^y	x			E_1	d_i = integer part of E_i
								(i = 1, ..., c−1)
7	d_1	−	x				E_2	
8	d_i	−	x				E_{i+1}	Perform line 8 for i = 2, ..., c−1
9								d_c = nearest integer to E_c

Fractional number in base b to decimal number

$$(.\, d_1\, d_2 \cdots d_n)_b \rightarrow (A)_{10}$$

or

$$d_1 \,.\, d_2 \cdots d_n \times b^{-Exp} \rightarrow (A)_{10}$$

Examples:

1. $(.0E728)_{16} = (.0564)_{10}$ (In hexadecimal system, E = 14)
2. $(7.200067)_8 \times 8^{-29} = 4.6853 \times 10^{-26}$ (Exp = 29)

Note: Exponents are in base 10.

Note: If $d_i = 0$, line 3 can be replaced by the [×] key.

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	b	1/x ↑ ↑ ↑		
2	d_n	x		
3	d_i	+ x		Perform line 3 for i = n−1, ..., 1
4			$(A)_{10}$	If there is no exponent, stop;
				Otherwise go to 5
5	Exp	↑ 1 −		
6	b	x^y 1/x x	$(A)_{10}$	

Bearing to Azimuth

See page 12

Bernoulli Numbers

The Bernoulli numbers B_1, B_2, B_3 --- are defined by

$$B_n = \frac{(2n)!}{\pi^{2n} 2^{2n-1}} \left[1 + \frac{1}{2^{2n}} + \frac{1}{3^{2n}} + \frac{1}{4^{2n}} + \ldots \right].$$

Specifically,

$$\frac{1}{6}, \frac{1}{30}, \frac{1}{42}, \frac{1}{30}, \frac{5}{66}, \frac{691}{2730}, \frac{7}{6}, \ldots$$

Example:

The 8^{th} Bernoulli number = 7.092

(i takes the values 1, 2, and
j takes the values 15, 14, ..., 2 in the example.)

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	n	↑ 2 x ↑ ↑		
2		↑ 1		
3		STO		Perform lines 3–5 for i = 1, 2, ..
4	i	↑ 2 x 1 +		until A_i does not change
5		x^y 1/x RCL +	A_i	
6		R↓	D	If D ⩽ 2, go to 8
7	j	x		Perform line 7 for j = D-1,
				D-2, ..., 2
8		2 x RCL x R↓		
9		R↓ 2 x^y 1 −		
10		x⇄y π x^y x ÷		

Centigrade to Fahrenheit

See page 55

Centimeters to Feet, Inches

See page 56

Chi-Square Evaluation

Formula:

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

where O_i = observed frequency

E_i = expected frequency

Example:

O_i	8	50	47	56	5	14
E_i	9.6	46.75	51.85	54.4	8.25	9.15

Answer:

$\chi^2 = 4.844$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		CLX ↑		
2	O_i	↑		Perform lines 2–4 for i = 1, ..., n
3	E_i	STO − ↑ × RCL		
4		÷ +		

Combinations

y objects taken x at a time (binominal coefficient)

Formula:

$$\binom{y}{x} = {}_yC_x = C^y_x = C(y,x) = \frac{y!}{x!(y-x)!} = \frac{y \cdot (y-1) \cdot \ldots \cdot (y-x+1)}{1 \cdot 2 \cdot \ldots \cdot x}$$

Notes:

1. This algorithm is for $y \geqslant x \geqslant 2$.
2. ${}_yC_0 = {}_yC_y = 1;\ {}_yC_1 = {}_yC_{y-1} = y$
3. ${}_yC_x = {}_yC_{y-x}$; so if $y - x < x$, compute ${}_yC_{y-x}$ instead of ${}_yC_x$.

Example:

${}_7C_5 = 21$. (Compute ${}_7C_2$ to get the answer)

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	y	↑		
2	i	×		Perform line 2 for i = y − x + 1,
				..., y − 1
3	x			If x = 2, go to 6.
4		↑		
5	j	×		Perform line 5 for j = x − 1,
				x − 2, ..., 2
6		÷		

Complex Number Operations

Complex add

Formula:

$$(a_1 + ib_1) + (a_2 + ib_2) = (a_1 + a_2) + i(b_1 + b_2)$$
$$= u + iv$$

Example:

$$(3 + 4i) + (7.4 - 5.6i) = 10.4 - 1.6i$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	a_1	[↑] [ ] [ ] [ ] [ ]		
2	a_2	[+] [ ] [ ] [ ] [ ]	u	
3	b_1	[↑] [ ] [ ] [ ] [ ]		
4	b_2	[+] [ ] [ ] [ ] [ ]	v	

Complex subtract

Formula:

$$(a_1 + ib_1) - (a_2 + ib_2) = (a_1 - a_2) + i(b_1 - b_2)$$
$$= u + iv$$

Example:

$$(3 + 4i) - (7.4 - 5.6i) = -4.4 + 9.6i$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	a_1	[↑] [ ] [ ] [ ] [ ]		
2	a_2	[−] [ ] [ ] [ ] [ ]	u	
3	b_1	[↑] [ ] [ ] [ ] [ ]		
4	b_2	[−] [ ] [ ] [ ] [ ]	v	

Complex multiply

(Use for short data input)

Formula:

$$(a_1 + ib_1)(a_2 + ib_2) = (a_1 a_2 - b_1 b_2) + i(a_1 b_2 + a_2 b_1)$$
$$= u + iv$$

Example:

$$(3 + 4i)(7 - 2i) = 29 + 22i$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	b_1	↑		
2	a_1	↑ ↑		
3	a_2	x		
4	b_2	R↓ R↓ R↓ STO x		
5		−	u	
6	a_2	RCL x R↓ R↓ R↓		
7	b_2	x +	v	

Complex multiply, alternate method

(Data are entered only once, use for long data input)

Method:

If a_1, a_2, b_1, b_2 are all non-zero, and if $\theta = \theta_1 + \theta_2 \neq 90°$ or $-90°$, then

$$(a_1 + ib_1)(a_2 + ib_2) = \frac{b_1 \cdot b_2}{\sin\theta_1 \cdot \sin\theta_2}(\cos\theta + i\sin\theta) = u + iv.$$

Note: $a_1 + ib_1 = r_1 e^{i\theta_1}$,

$a_2 + ib_2 = r_2 e^{i\theta_2}$

Example:

$$(3 + 4i)(5 - 12i) = 63 - 16i$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	a_1	↑		
2	b_1	STO x⇄y ÷ arc tan		
3	a_2	↑		
4	b_2	↑ R↓ x⇄y ÷ arc		
5		tan RCL R↓ R↓ R↓		
6		x STO R↓ ↑ R↓		
7		+	D	If D = 90 or −90, stop.
				Use other method.
8		RCL R↓ STO R↓ sin		
9		x⇄y sin x ÷ RCL		
10		cos x ↑ ↑ RCL		
11		tan x x⇄y	u	
12		x⇄y	v	

Complex divide

(Use for short data input)

Formula:

$$(a_1 + ib_1) \div (a_2 + ib_2) = \frac{(a_1 a_2 + b_1 b_2) + i(a_2 b_1 - a_1 b_2)}{a_2^2 + b_2^2}$$

$$= u + iv$$

Example:

$$\frac{3 + 4i}{7 - 2i} = .245 + .64i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b_1	↑						
2	a_1	↑	↑					
3	a_2	x	·					
4	b_2	R↓	R↓	R↓	STO	x		
5		+						
6	a_2	RCL	x	R↓	R↓	R↓		
7	b_2	x	–	STO				
8	a_2	↑	x					
9	b_2	↑	x	+	RCL	x⇄y		
10		STO	÷	x⇄y	RCL	÷	u	
11		x⇄y					v	

Complex divide, alternate method

(Data are entered only once; use for long data input)

Formula:

If a_1, a_2, b_1, b_2 are all non-zero, and if $\theta = \theta_1 - \theta_2 \neq 90^\circ$ or -90°, then

$$(a_1 + ib_1) \div (a_2 + ib_2) = \left(\frac{b_1}{b_2} \Big/ \frac{\sin\theta_1}{\sin\theta_2}\right)(\cos\theta + i\sin\theta) = u + iv$$

Note: $a_1 + ib_1 = r_1 e^{i\theta_1}$,

$a_2 + ib_2 = r_2 e^{i\theta_2}$.

Example:

$$(63 - 16i) \div (5 - 12i) = 3 + 4i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a_1	↑						
2	b_1	STO	x⇄y	÷	arc	tan		
3	a_2	↑						
4	b_2	↑	R↓	x⇄y	÷	arc		
5		tan	RCL	R↓	R↓	R↓		
6		÷	STO	R↓	↑	R↓		
7		–					D	If D = 90 or –90, stop.
								Use other method.
8		RCL	R↓	STO	R↓	sin		
9		x⇄y	sin	÷	÷	RCL		
10		cos	x	↑	↑	RCL		
11		tan	x	x⇄y			u	
12		x⇄y					v	

Complex reciprocal

Formula:

$$\frac{1}{a + ib} = \frac{a - ib}{a^2 + b^2} = u + iv$$

Example:

$$\frac{1}{2 + 3i} = .15 - .23i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	↑						
2	a	↑	↑	x	STO	CLX		
3		+	R↓	x	RCL	+		
4		STO	÷	CHS	x⇄y	RCL		
5		÷					u	
6		x⇄y					v	

Complex absolute value

Formula:

$$|a + ib| = \sqrt{a^2 + b^2}$$

Example:

$$|3 + 4i| = 5$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	a	[↑] [×] [ ] [ ] [ ]		
2	b	[↑] [×] [+] [√x] [ ]		

Complex square

Formula:

$$(a + ib)^2 = (a^2 - b^2) + i(2ab) = u + iv$$

Example:

$$(7 - 2i)^2 = 45 - 28i$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	a	[↑] [↑] [×] [ ] [ ]		
2	b	[STO] [↑] [×] [−] [ ]	u	
3		[x⇄y] [RCL] [×] [2] [×]	v	

Complex square root

Formula:

$$\sqrt{a + ib} = \pm\left[\sqrt{\frac{a + \sqrt{a^2 + b^2}}{2}} + i\,\frac{b}{2\sqrt{\frac{a + \sqrt{a^2 + b^2}}{2}}}\right] = \pm(u + iv)$$

Example:

$$\sqrt{7 + 6i} = \pm(2.85 + 1.05i)$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	b	↑		
2	a	STO x⇄y		If b = 0 and a < 0, go to 7;
				if b = 0 and a ⩾ 0, go to 9.
3		↑ ↑ x RCL ↑		
4		x + $\sqrt{x}$ RCL +		
5		2 ÷ $\sqrt{x}$ STO	u	
6		2 RCL x ÷	v	Stop
7		RCL CHS $\sqrt{x}$ x⇄y	u	
8		x⇄y	v	Stop
9		RCL $\sqrt{x}$	u	
10		x⇄y	v	

Complex natural logarithm (base e)

Formula:

$$\ln(a + ib) = \ln(\sqrt{a^2 + b^2}) + i\left(\arc\tan \frac{b}{a}\right)\frac{\pi}{180}$$

$$= u + iv$$

Example:

$$\ln i = 1.57i$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	b	↑		
2	a			If a ≠ 0, go to 4
3		CLX EEX CHS 9 9		
4		STO x⇄y ↑ ↑ x		
5		RCL ↑ x + $\sqrt{x}$		
6		ln	u	
7		x⇄y RCL ÷ arc tan		If a ⩾ 0, go to 9
8		1 8 0 +		
9		π x 1 8 0		
10		÷	v	

Complex exponential

Formula:

$$e^{(a+ib)} = e^{a} (\cos\theta + i \sin\theta) = u + iv$$

where

$$\theta = \frac{180b}{\pi}$$

Example:

$$e^{1.570796327i} = i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	↑						
2	a	e^x	STO	x⇄y	1	8		
3		0	x	π	÷	↑		
4		cos	RCL	x			u	
5		x⇄y	sin	RCL	x		v	

Complex exponential (t^{a+ib})

Formula:

$$t^{a+ib} = e^{(a+ib)\ln t} = u + iv$$

Restriction:

$$(t > 0)$$

Example:

$$2^{3+4i} = -7.46 + 2.89i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	↑						
2	a	x⇄y						
3	t	ln	STO	x	x⇄y	RCL		
4		x	e^x	STO	x⇄y	1		
5		8	0	x	π	÷		
6		↑	cos	RCL	x		u	
7		x⇄y	sin	RCL	x		v	

Complex number to integral power

Formula:

$$(a + ib)^n = r^n (\cos n\theta + i \sin n\theta) = u + iv$$

where: $r = \sqrt{a^2 + b^2}$,

$\theta = \arctan \frac{b}{a}$, and

n is a positive integer.

Example:

$$(3 + 4.5i)^5 = 926.44 - 4533.47i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	↑						
2	a							If a ≠ 0, go to 4
3		CLX	EEX	CHS	9	9		
4		STO	x⇄y	↑	↑	x		
5		RCL	↑	x	+	√x		
6		x⇄y	RCL	÷	arc	tan		If a ≥ 0, go to 8
7		1	8	0	+			
8		STO						
9	n	↑	↑	R↓	R↓	R↓		
10		x^y	RCL	x⇄y	STO	CLX		
11		+	x	↑	cos	RCL		
12		x					u	
13		x⇄y	sin	RCL	x		v	

Integral roots of complex number

Formula:

$$(a + ib)^{\frac{1}{n}} = r^{\frac{1}{n}} \left(\cos \frac{\theta + 360k}{n} + i \sin \frac{\theta + 360k}{n} \right)$$

$$= u_k + iv_k$$

where: n is a positive integer, and

k = 0, 1, ..., n − 1.

Example:

5 + 3i has three cube roots:

$u_0 + iv_0 = 1.77 + .32i$

$u_1 + iv_1 = -1.16 + 1.37i$

$u_2 + iv_2 = -.61 - 1.69i$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	b	↑		
2	a			If a ≠ 0, go to 4
3		CLX EEX CHS 9 9		
4		STO x⇄y ↑ ↑ x		
5		RCL ↑ x + √x		
6		x⇄y RCL ÷ arc tan		If a ≥ 0, go to 8
7		1 8 0 +		
8		x⇄y		
9	n	↑ 1/x R↓ R↓ x⇄y		
10		R↓ x^y STO CLX +		
11		÷ ↑ ↑ cos RCL		
12		x	u_0	
13		x⇄y sin RCL x	v_0	
14		R↓ R↓		Perform lines 14–18 for k = 1,
				2, ..., n−1
15		3 6 0 ↑		
16	n	÷ + ↑ ↑ cos		
17		RCL x	u_k	
18		x⇄y sin RCL x	v_k	

Complex number to a complex power

Formula:

$$\text{If } a_1 + ib_1 \neq 0,$$
$$(a_1 + ib_1)^{(a_2+ib_2)} = e^{(a_2+ib_2)\ln(a_1+ib_1)} = u + iv$$

Example:

$$(1 + i)^{(2-i)} = 1.49 + 4.13i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b_1	↑						
2	a_1							If $a_1 \neq 0$, go to 4
3		CLX	EEX	CHS	9	9		
4		STO	x⇄y	↑	↑	x		
5		RCL	↑	x	+	√x		
6		ln	x⇄y	RCL	÷	arc		
7		tan						If $a_1 \geq 0$, go to 9
8		1	8	0	+			
9		π	x	1	8	0		
10		÷	x⇄y	↑	↑			
11	a_2	x						
12	b_2	R↓	R↓	R↓	STO	x		
13		−						
14	a_2	RCL	x	R↓	R↓	R↓		
15	b_2	x	+	x⇄y	e^x	STO		
16		x⇄y	1	8	0	x		
17		π	÷	↑	cos	RCL		
18		x					u	
19		x⇄y	sin	RCL	x		v	

Complex root of a complex number

Formula:

If $a_1 + ib_1 \neq 0$

$$(a_1 + ib_1)^{\frac{1}{a_2+ib_2}} = e^{[\ln(a_1+ib_1)]/(a_2+ib_2)}$$

$$= u + iv$$

Example:

Find the $(2 - i)^{th}$ root of $1.49 + 4.126i$.

Answer: $1 + i$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	b_1	[↑] [ ] [ ] [ ] [ ]		
2	a_1	[ ] [ ] [ ] [ ] [ ]		If $a_1 \neq 0$, go to 4
		[ ] [ ] [ ] [ ] [ ]		
3		[CLX] [EEX] [CHS] [9] [9]		
4		[STO] [x⇄y] [↑] [↑] [×]		
5		[RCL] [↑] [×] [+] [√x]		
6		[ln] [x⇄y] [RCL] [÷] [arc]		
7		[tan] [ ] [ ] [ ] [ ]		If $a_1 \geq 0$, go to 9
		[ ] [ ] [ ] [ ] [ ]		
8		[1] [8] [0] [+] [ ]		
9		[π] [×] [1] [8] [0]		
10		[÷] [x⇄y] [↑] [↑] [ ]		
11	a_2	[×] [ ] [ ] [ ] [ ]		
12	b_2	[R↓] [R↓] [R↓] [STO] [×]		
13		[+] [ ] [ ] [ ] [ ]		
14	a_2	[RCL] [×] [R↓] [R↓] [R↓]		
15	b_2	[×] [−] [STO] [ ] [ ]		
16	a_2	[↑] [×] [ ] [ ] [ ]		
17	b_2	[↑] [×] [+] [RCL] [x⇄y]		
18		[STO] [÷] [x⇄y] [RCL] [÷]		
19		[e^x] [STO] [x⇄y] [1] [8]		
20		[0] [×] [π] [÷] [↑]		
21		[cos] [RCL] [×] [ ] [ ]	u	
22		[x⇄y] [sin] [RCL] [×] [ ]	v	

Logarithm of a complex number to a complex base

Formula:

$$\log_{(a_1+ib_1)}(a_2 + ib_2) = \frac{\ln(a_2 + ib_2)}{\ln(a_1 + ib_1)} = u + iv$$

$a_1 + ib_1 \neq 0$

Example: $\log_{(1+i)} (1.49 + 4.126i) = 2 - i$

$(a_3 = .34657359,\ b_3 = .7853981633)$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b_1	↑						
2	a_1							If $a_1 \neq 0$, go to 4
3		CLX	EEX	CHS	9	9		
4		STO	x⇄y	↑	↑	x		
5		RCL	↑	x	+	√x		
6		ln					a_3	
7		x⇄y	RCL	÷	arc	tan		If $a_1 \geq 0$, go to 9
8		1	8	0	+			
9		π	x					
10		1	8	0	÷		b_3	
11		CLR						
12	b_2	↑						
13	a_2							If $a_2 \neq 0$, go to 15
14		CLX	EEX	CHS	9	9		
15		STO	x⇄y	↑	↑	x		
16		RCL	↑	x	+	√x		
17		ln	x⇄y	RCL	÷	arc		
18		tan						If $a_2 \geq 0$, go to 20
19		1	8	0	+			
20		π	x	1	8	0		
21		÷	x⇄y	↑	↑			
22	a_3	x						
23	b_3	R↓	R↓	R↓	STO	x		
24		+						
25	a_3	RCL	x	R↓	R↓	R↓		
26	b_3	x	−	STO				
27	a_3	↑	x					
28	b_3	↑	x	+	RCL	x⇄y		
29		STO	÷	x⇄y	RCL	÷	u	
30		x⇄y					v	

Complex Trigonometric and Hyperbolic Functions

In this section all angles in the equations are in radians.

Complex sine

Formula:

$$\sin(a + ib) = \sin a \cosh b + i \cos a \sinh b$$
$$= u + iv$$

Example:

$$\sin(2 + 3i) = 9.154 - 4.1689i$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	b	[↑] [ ] [ ] [ ] [ ]		
2	a	[↑] [1] [8] [0] [x]		
3		[π] [÷] [STO] [sin] [x⇄y]		
4		[↑] [↑] [R↓] [R↓] [e^x]		
5		[↑] [1/x] [+] [2] [÷]		
6		[x] [RCL] [x⇄y] [STO] [ ]	u	
7		[CLX] [+] [cos] [x⇄y] [e^x]		
8		[↑] [1/x] [–] [2] [÷]		
9		[x] [ ] [ ] [ ] [ ]	v	

Complex cosine

Formula:

$$\cos(a + ib) = \cos a \cosh b - i \sin a \sinh b$$
$$= u + iv$$

Example:

$$\cos(2 + ei) = -4.189 - 9.109i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	↑						
2	a	↑	1	8	0	x		
3		π	÷	STO	cos	x⇄y		
4		↑	↑	R↓	R↓	e^x		
5		↑	1/x	+	2	÷		
6		x	RCL	x⇄y	STO		u	
7		CLX	+	sin	x⇄y	e^x		
8		↑	1/x	–	2	÷		
9		x	CHS				v	

Complex tangent

Formula:

$$\tan(a + ib) = \frac{\sin 2a + i \sinh 2b}{\cos 2a + \cosh 2b}$$

$$= u + iv$$

Example:

$$\tan(2 + 3i) = -.00376 + 1.003i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	↑						
2	a	↑	2	x	STO	↑		
3		1	8	0	x	π		
4		÷	cos	x⇄y	2	x		
5		↑	↑	R↓	R↓	e^x		
6		↑	1/x	+	2	÷		
7		+	RCL	x⇄y	STO	CLX		
8		+	x⇄y	e^x	↑	1/x		
9		–	2	÷	RCL	÷		
10		x⇄y	1	8	0	x		
11		π	÷	sin	RCL	÷	u	
12		x⇄y					v	

Complex cotangent

Formula:

$$\cot(a + ib) = \frac{\sin 2a - i \sinh 2b}{\cosh 2a - \cos 2b}$$

$$= u + iv$$

Example:

$$\cot(2 + 3i) = -.0037 - .9968i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	↑						
2	a	↑	2	x	STO	↑		
3		1	8	0	x	π		
4		÷	cos	x⇄y	2	x		
5		↑	↑	R↓	R↓	e^x		
6		↑	1/x	+	2	÷		
7		x⇄y	−	RCL	x⇄y	STO		
8		CLX	+	x⇄y	e^x	↑		
9		1/x	−	2	÷	RCL		
10		÷	CHS	x⇄y	1	8		
11		0	x	π	÷	sin		
12		RCL	÷				u	
13		x⇄y					v	

Complex cosecant

Formula:

$$\csc(a + ib) = \frac{1}{\sin(a + ib)}$$

$$= u + iv$$

Example:

$$\csc(2 + 3i) = .09 + .0412i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	↑						
2	a	↑	1	8	0	x		
3		π	÷	STO	sin	x⇄y		
4		↑	↑	R↓	R↓	e^x		
5		↑	$^1/_x$	+	2	÷		
6		x	RCL	x⇄y	STO	CLX		
7		+	cos	x⇄y	e^x	↑		
8		$^1/_x$	–	2	÷	x		
9		RCL	↑	x	x⇄y	↑		
10		↑	x	RCL	R↓	x⇄y		
11		R↓	+	STO	÷		u	
12		x⇄y	RCL	÷	CHS		v	

Complex secant

Formula:

$$\sec(a + ib) = \frac{1}{\cos(a + ib)}$$

$$= u + iv$$

Example:

$$\sec(2 + 3i) = -.04 + .09i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	↑						
2	a	↑	1	8	0	x		
3		π	÷	STO	cos	x⇄y		
4		↑	↑	R↓	R↓	e^x		
5		↑	$^1/_x$	+	2	÷		
6		x	RCL	x⇄y	STO	CLX		
7		+	sin	x⇄y	e^x	↑		
8		$^1/_x$	–	2	÷	x		
9		CHS	RCL	↑	x	x⇄y		
10		↑	↑	x	RCL	R↓		
11		x⇄y	R↓	+	STO	÷	u	
12		x⇄y	RCL	÷	CHS		v	

Complex arc sine

Formula:

$$\arc \sin (a + ib) = \arc \sin \beta + i\, \mathrm{sgn}(b) \ln (\alpha + \sqrt{\alpha^2 - 1})$$

$$= u + iv$$

where: $\alpha = \frac{1}{2}\sqrt{(a+1)^2 + b^2} + \frac{1}{2}\sqrt{(a-1)^2 + b^2}$

$$\beta = \frac{1}{2}\sqrt{(a+1)^2 + b^2} - \frac{1}{2}\sqrt{(a-1)^2 + b^2}$$

$$\mathrm{sgn}(b) = \begin{cases} 1 \text{ if } b \geqslant 0 \\ -1 \text{ if } b < 0 . \end{cases}$$

Example:

$$\arc \sin (5 + 8i) = .556 + 2.9387i$$

Note: Inverse trigonometric and inverse hyperbolic functions are multiple-valued functions, but only one answer will be given for each of them.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑						
2	b	↑	x	STO	x⇄y	↑		
3		↑	1	+	↑	x		
4		x⇄y	1	−	↑	x		
5		RCL	+	√x	2	÷		
6		STO	R↓	+	√x	2		
7		÷	↑	↑	RCL	+		
8		x⇄y	RCL	−	arc	sin		
9		π	x	1	8	0		
10		÷					u	
11		R↓	↑	↑	x	1		
12		−	√x	+	ln			If b ⩾ 0, go to 14
13		CHS						
14							v	

Complex arc cosine

Formula:

$$\text{arc cos }(a + ib) = \text{arc cos }\beta - i \text{ sgn }(b) \ln(\alpha + \sqrt{\alpha^2 - 1})$$

$$= u + iv$$

where: $\alpha = \frac{1}{2}\sqrt{(a+1)^2 + b^2} + \frac{1}{2}\sqrt{(a-1)^2 + b^2}$

$$\beta = \frac{1}{2}\sqrt{(a+1)^2 + b^2} - \frac{1}{2}\sqrt{(a-1)^2 + b^2}$$

$$\text{sgn }(b) = \begin{cases} 1 \text{ if } b \geqslant 0 \\ -1 \text{ if } b < 0 \end{cases}$$

Example:

$$\text{arc cos }(5 + 8i) = 1.0147 - 2.9387i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑						
2	b	↑	x	STO	x⇄y	↑		
3		↑	1	+	↑	x		
4		x⇄y	1	–	↑	x		
5		RCL	+	√x	2	÷		
6		STO	R↓	+	√x	2		
7		÷	↑	↑	RCL	+		
8		x⇄y	RCL	–	arc	cos		
9		π	x	1	8	0		
10		÷					u	
11		R↓	↑	↑	x	1		
12		–	√x	+	ln			If b < 0, go to 14
13		CHS						
14							v	

Complex arc tangent

Formula:

$$\text{arc tan}\,(a + ib) = \frac{1}{2}\left[\pi - \text{arc tan}\,\frac{1+b}{a} - \text{arc tan}\,\frac{1-b}{a}\right] + \frac{i}{4}\ln\left[\frac{(1+b)^2 + a^2}{(1-b)^2 + a^2}\right]$$

$$= u + iv$$

where: $(a + ib)^2 \neq -1$.

Example:

$$\text{arc tan}\,(5 + 8i) = 1.5142 + .0898i$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	b	[↑] [↑] [1] [+] [ ]		
2	a	[STO] [÷] [arc] [tan] [x⇄y]		
3		[↑] [1] [x⇄y] [−] [↑]		
4		[↑] [RCL] [÷] [arc] [tan]		
5		[x⇄y] [R↓] [+] [x⇄y] [CLX]		
6		[1] [8] [0] [x⇄y] [−]		
7		[2] [÷] [π] [x] [1]		
8		[8] [0] [÷] [ ] [ ]	u	
9		[R↓] [2] [x⇄y] [−] [↑]		
10		[x] [RCL] [↑] [x] [STO]		
11		[+] [x⇄y] [↑] [x] [RCL]		
12		[+] [÷] [ln] [4] [÷]	v	

Complex arc cotangent

Formula:

$$\text{arc cot}\,(a + ib) = \frac{\pi}{2} - \text{arc tan}\,(a + ib) = u + iv$$

Example:

$$\text{arc cot}\,(5 + 8i) = .0566 - .0898i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	↑	↑	1	+			
2	a	STO	÷	arc	tan	x⇄y		
3		↑	1	x⇄y	–	↑		
4		↑	RCL	÷	arc	tan		
5		x⇄y	R↓	+	x⇄y	CLX		
6		+	2	÷	π	x		
7		1	8	0	÷		u	
8		R↓	2	x⇄y	–	↑		
9		x	RCL	↑	x	STO		
10		+	x⇄y	↑	x	RCL		
11		+	÷	ln	4	÷		
12		CHS					v	

Complex arc cosecant

Formula:

$$\text{arc csc}\,(a + ib) = \text{arc sin}\left(\frac{1}{a + ib}\right)$$

$$= u + iv$$

Example:

$$\text{arc csc}\,(5 + 8i) = .05598 - .0899i$$

$$(D < 0)$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	↑						
2	a	↑	↑	x	STO	CLX		
3		+	R↓	x	RCL	+		
4		STO	÷	CHS	x⇄y	RCL		
5		÷	x⇄y				D	
6		↑	x	STO	x⇄y	↑		
7		↑	1	+	↑	x		
8		x⇄y	1	−	↑	x		
9		RCL	+	√x	2	÷		
10		STO	R↓	+	√x	2		
11		÷	↑	↑	RCL	+		
12		x⇄y	RCL	−	arc	sin		
13		π	x	1	8	0		
14		÷					u	
15		R↓	↑	↑	x	1		
16		−	√x	+	ln			If D > 0, go to 18
17		CHS						
18							v	

Complex arc secant

Formula:

$$\text{arc sec }(a + ib) = \text{arc cos}\left(\frac{1}{a + ib}\right)$$

$$= u + iv$$

Example:

$$\text{arc sec }(5 + 8i) = 1.5148 + .0899i$$

$$(D < 0)$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	↑						
2	a	↑	↑	x	STO	CLX		
3		+	R↓	x	RCL	+		
4		STO	÷	CHS	x⇄y	RCL		
5		÷	x⇄y				D	
6		↑	x	STO	x⇄y	↑		
7		↑	1	+	↑	x		
8		x⇄y	1	−	↑	x		
9		RCL	+	√x	2	÷		
10		STO	R↓	+	√x	2		
11		÷	↑	↑	RCL	+		
12		x⇄y	RCL	−	arc	cos		
13		π	x	1	8	0		
14		÷					u	
15		R↓	↑	↑	x	1		
16		−	√x	+	ln			If D < 0, go to 18
17		CHS						
18							v	

Complex hyperbolic sine

Formula:

$$\sinh(a + ib) = -i \sin i(a + ib)$$
$$= u + iv$$

Example:

$$\sinh(3 - 2i) = -4.1689 - 9.154i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑						
2	b	CHS	↑	1	8	0		
3		x	π	÷	STO	sin		
4		x⇄y	↑	↑	R↓	R↓		
5		e^x	↑	1/x	+	2		
6		÷	x	RCL	x⇄y	STO		
7		CLX	+	cos	x⇄y	e^x		
8		↑	1/x	−	2	÷		
9		x	RCL	CHS	x⇄y		u	
10		x⇄y					v	

Complex hyperbolic cosine

Formula:

$$\cosh(a + ib) = \cos i(a + ib)$$
$$= u + iv$$

Example:

$$\cosh(1 + 2i) = -.6421 + 1.0686i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑						
2	b	CHS	↑	1	8	0		
3		x	π	÷	STO	cos		
4		x⇄y	↑	↑	R↓	R↓		
5		e^x	↑	1/x	+	2		
6		÷	x	RCL	x⇄y	STO	u	
7		CLX	+	sin	x⇄y	e^x		
8		↑	1/x	−	2	÷		
9		x	CHS				v	

Complex hyperbolic tangent

Formula:

$$\tanh(a + ib) = -i \tan i(a + ib)$$

$$= u + iv$$

Example:

$$\tanh(1 + 2i) = 1.1667 - .243i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑						
2	b	CHS	↑	2	x	STO		
3		↑	1	8	0	x		
4		π	÷	cos	x⇄y	2		
5		x	↑	↑	R↓	R↓		
6		e^x	↑	1/x	+	2		
7		÷	+	RCL	x⇄y	STO		
8		CLX	+	x⇄y	e^x	↑		
9		1/x	−	2	÷	RCL		
10		÷	x⇄y	1	8	0		
11		x	π	÷	sin	RCL		
12		÷	CHS	x⇄y			u	
13		x⇄y					v	

Complex hyperbolic cotangent

Formula:

$$\coth(a + ib) = i \cot i(a + ib)$$
$$= u + iv$$

Example:

$$\coth(1 + 2i) = .8213 + .17138i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑						
2	b	CHS	↑	2	x	STO		
3		↑	1	8	0	x		
4		π	÷	cos	x⇄y	2		
5		x	↑	↑	R↓	R↓		
6		e^x	↑	1/x	+	2		
7		÷	x⇄y	−	RCL	x⇄y		
8		STO	CLX	+	x⇄y	e^x		
9		↑	1/x	−	2	÷		
10		RCL	÷	CHS	x⇄y	1		
11		8	0	x	π	÷		
12		sin	RCL	÷	x⇄y	CHS	u	
13		x⇄y					v	

Complex hyperbolic cosecant

Formula:

$$\operatorname{csch}(a + ib) = i \csc i(a + ib)$$
$$= u + iv$$

Example:

$$\operatorname{csch}(1 + 2i) = -.2215 - .63549i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑						
2	b	CHS	↑	1	8	0		
3		x	π	÷	STO	sin		
4		x⇄y	↑	↑	R↓	R↓		
5		e^x	↑	1/x	+	2		
6		÷	x	RCL	x⇄y	STO		
7		CLX	+	cos	x⇄y	e^x		
8		↑	1/x	–	2	÷		
9		x	RCL	↑	x	x⇄y		
10		↑	↑	x	RCL	R↓		
11		x⇄y	R↓	+	STO	÷		
12		x⇄y	RCL	÷			u	
13		x⇄y					v	

Complex hyperbolic secant

Formula:

$$\operatorname{sech}(a + ib) = \sec i\,(a + ib)$$

$$= u + iv$$

Example:

$$\operatorname{sech}(1 + 2i) = -.4131 - .6875i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑						
2	b	CHS	↑	1	8	0		
3		x	π	÷	STO	cos		
4		x⇄y	↑	↑	R↓	R↓		
5		e^x	↑	1/x	+	2		
6		÷	x	RCL	x⇄y	STO		
7		CLX	+	sin	x⇄y	e^x		
8		↑	1/x	–	2	÷		
9		x	CHS	RCL	↑	x		
10		x⇄y	↑	↑	x	RCL		
11		R↓	x⇄y	R↓	+	STO		
12		÷					u	
13		x⇄y	RCL	÷	CHS		v	

Complex inverse hyperbolic sine

Formula:

$$\sinh^{-1}(a + ib) = -i \arcsin i(a + ib)$$
$$= u + iv$$

Example:

$$\sinh^{-1}(8 - 5i) = 2.9387 - .556i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑						
2	b	CHS	x⇄y	↑	x	STO		
3		x⇄y	↑	↑	1	+		
4		↑	x	x⇄y	1	–		
5		↑	x	RCL	+	√x		
6		2	÷	STO	R↓	+		
7		√x	2	÷	↑	↑		
8		RCL	+	x⇄y	RCL	–		
9		arc	sin	π	x	1		
10		8	0	÷	CHS	STO		
11		R↓	↑	↑	x	1		
12		–	√x	+	ln			If a ⩾ 0, go to 14
13		CHS						
14							u	
15		RCL					v	

Complex inverse hyperbolic cosine

Formula:

$$\cosh^{-1}(a + ib) = i \arccos(a + ib)$$
$$= u + iv$$

Example:

$$\cosh^{-1}(5 + 8i) = 2.9387 + 1.0147i$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	a	↑		
2	b	↑ x STO x⇄y ↑		
3		↑ 1 + ↑ x		
4		x⇄y 1 − ↑ x		
5		RCL + √x 2 ÷		
6		STO R↓ + √x 2		
7		÷ ↑ ↑ RCL +		
8		x⇄y RCL − arc cos		
9		π x 1 8 0		
10		÷ STO R↓ ↑ ↑		
11		x 1 − √x +		
12		ln		If b ≥ 0, go to 14
13		CHS		
14			u	
15		RCL	v	

Complex inverse hyperbolic tangent

Formula:

$$\tanh^{-1}(a + ib) = -i \text{ arc tan } i(a + ib)$$

$$= u + iv$$

Example:

$$\tanh^{-1}(8 - 5i) = .0898 - 1.5142i$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	a	↑ ↑ 1 +		
2	b	CHS STO ÷ arc tan		
3		x⇄y ↑ 1 x⇄y −		
4		↑ ↑ RCL ÷ arc		
5		tan x⇄y R↓ + x⇄y		
6		CLX 1 8 0 x⇄y		
7		− 2 ÷ π x		
8		1 8 0 ÷ CHS	v	
9		R↓ 2 x⇄y − ↑		
10		x RCL ↑ x STO		
11		+ x⇄y ↑ x RCL		
12		+ ÷ ln 4 ÷	u	

Complex inverse hyperbolic cotangent

Formula:

$$\coth^{-1}(a + ib) = i \operatorname{arc} \cot i(a + ib)$$
$$= u + iv$$

Example:

$$\coth^{-1}(8 - 5i) = .0898 + .0566i$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑	↑	1	+			
2	b	CHS	STO	÷	arc	tan		
3		x⇄y	↑	1	x⇄y	–		
4		↑	↑	RCL	÷	arc		
5		tan	x⇄y	R↓	+	x⇄y		
6		CLX	+	2	÷	π		
7		x	1	8	0	÷	v	
8		R↓	2	x⇄y	–	↑		
9		x	RCL	↑	x	STO		
10		+	x⇄y	↑	x	RCL		
11		+	÷	ln	4	÷	u	

Complex inverse hyperbolic cosecant

Formula:

$$\operatorname{csch}^{-1}(a + ib) = i \text{ arc csc } i\,(a + ib)$$
$$= u + iv$$

Example:

$$\operatorname{csch}^{-1}(8 - 5i) = .0899 + .05598i$$
$$(D < 0)$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑						
2	b	CHS	↑	↑	x	STO		
3		CLX	+	R↓	x	RCL		
4		+	STO	÷	CHS	x⇄y		
5		RCL	÷	x⇄y			D	
6		↑	x	STO	x⇄y	↑		
7		↑	1	+	↑	x		
8		x⇄y	1	–	↑	x		
9		RCL	+	√x	2	÷		
10		STO	R↓	+	√x	2		
11		÷	↑	↑	RCL	+		
12		x⇄y	RCL	–	arc	sin		
13		π	x	1	8	0		
14		÷	STO	R↓	↑	↑		
15		x	1	–	√x	+		
16		ln						If D < 0, go to 18.
17		CHS						
18							u	
19		RCL					v	

Complex inverse hyperbolic secant

Formula:

$$\text{sech}^{-1}(a + ib) = i \text{ arc sec } (a + ib)$$
$$= u + iv$$

Example:

$$\text{sech}^{-1}(5 + 8i) = -.0899 + 1.5148i$$
$$(D < 0)$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	↑						
2	a	↑	↑	x	STO	CLX		
3		+	R↓	x	RCL	+		
4		STO	÷	CHS	x⇄y	RCL		
5		÷	x⇄y				D	
6		↑	x	STO	x⇄y	↑		
7		↑	1	+	↑	x		
8		x⇄y	1	−	↑	x		
9		RCL	+	√x	2	÷		
10		STO	R↓	+	√x	2		
11		÷	↑	↑	RCL	+		
12		x⇄y	RCL	−	arc	cos		
13		π	x	1	8	0		
14		÷	STO	R↓	↑	↑		
15		x	1	−	√x	+		
16		ln						If D ≥ 0, go to 18.
17		CHS						
18							u	
19		RCL					v	

Compound Interest

See page 96

Conversions

Repetitive use of formulas with two constants.

The technique used here is to store both constants—one in the S register, and the other in the T register. After initially storing the two constants, the formulas can be used repeatedly without limit.

Centigrade to Fahrenheit (a°C → b°F)

Formula:

$$b = \frac{9}{5} a + 32$$

Examples:

a	–30	0	28	100	539
b	–22.	32.	82.4	212.	1002.2

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		[3] [2] [STO] [9] [↑]		
2		[5] [÷] [↑] [↑] [↑]		
3		[CLX] [ ] [ ] [ ] [ ]		
4	a	[×] [RCL] [+] [ ] [ ]	b	Stop. For new case, go to 3

Fahrenheit to centigrade (b°F → a°C)

Formula:

$$a = \frac{5}{9} (b - 32)$$

Examples:

b	–460	–40	0	32	212
a	–273.33	–40.	–17.78	0.	100.

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		[3] [2] [STO] [5] [↑]		
2		[9] [÷] [↑] [↑] [↑]		
3		[CLX] [ ] [ ] [ ] [ ]		
4	b	[RCL] [–] [×] [ ] [ ]		Stop. For new case, go to 3

Feet and inches to centimeters

Formulas:

1 foot = 12 inches

1 inch = 2.54 cm

Examples:

4′ 8″ = 142.24 cm
5′ 5″ = 165.1 cm
6′ 3″ = 190.5 cm

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		2	.	5	4	STO		
2		1	2	↑	↑	↑		
3		CLX						
4	Feet	x						
5	Inches	+	RCL	x			Centimeters	Stop. For new case, go to 3

Centimeters to feet and inches

Examples:

164 cm = 5′ 4.6″
180 cm = 5′ 10.9″

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		1	2	STO	2	.		
2		5	4	1/x	↑	↑		
3		↑						
4		CLX						
5	Centimeters	x	RCL	÷			D	Let f = integer part of D
6	f						Feet	
7		–	RCL	x			Inches	Stop. For new case, go to 4

Coordinate Conversions

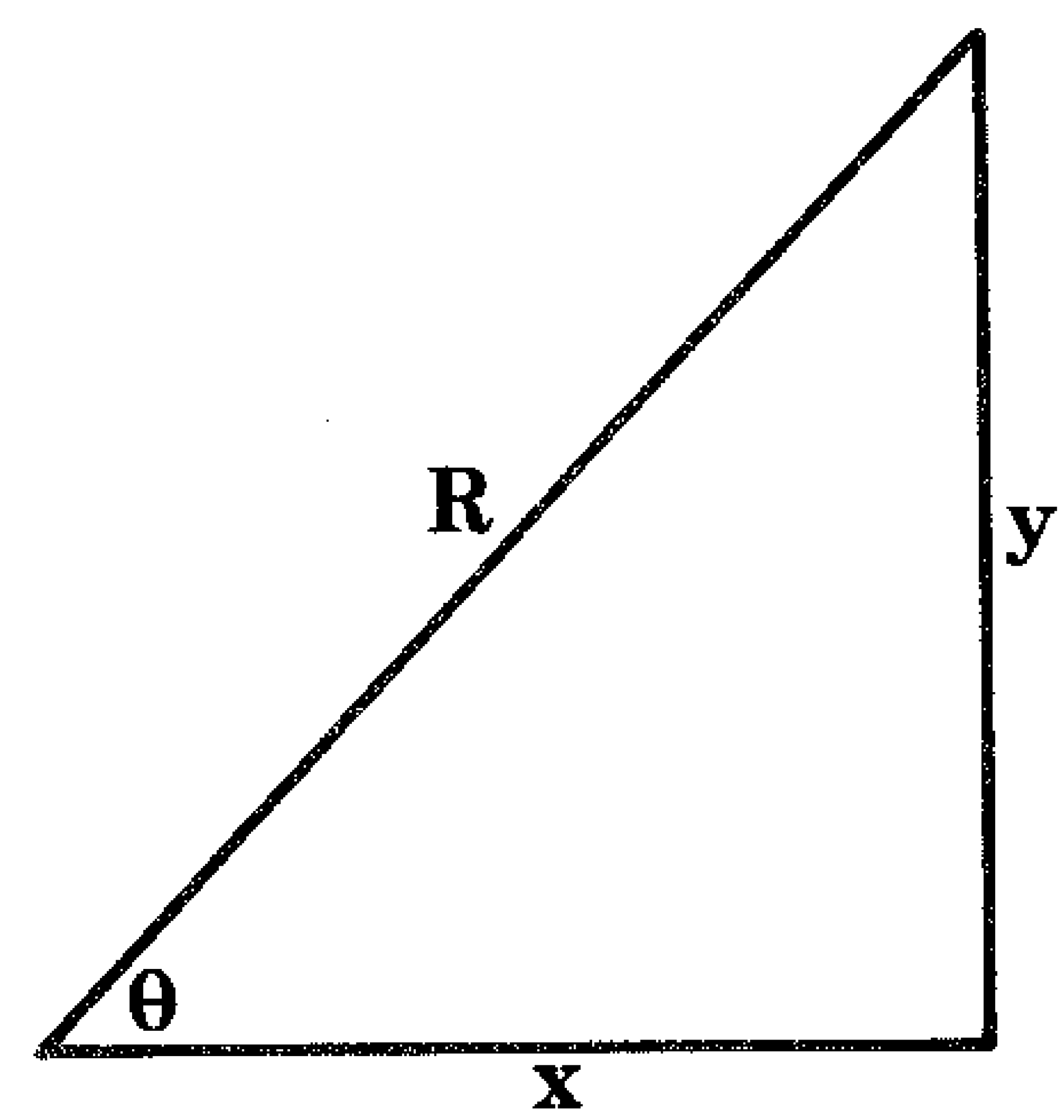
Polar to rectangular

Formulas:

$x = R \cos \theta$

$y = R \sin \theta$

(θ is in degrees)



Example:

If $\theta = 40°$, and R = 5.1, then

x = 3.907, and

y = 3.278.

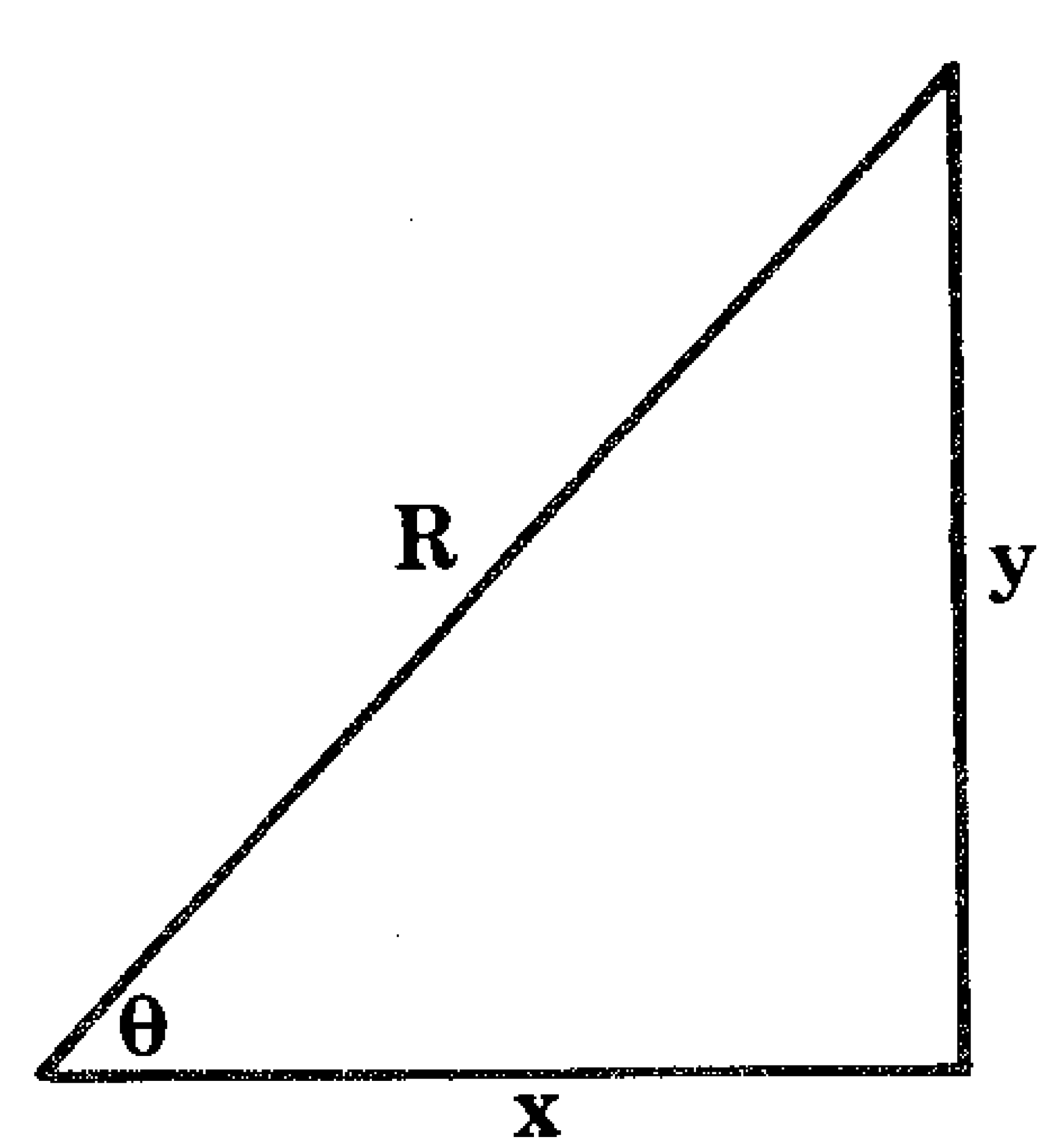
LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	θ	↑		
2	R	STO x⇄y ↑ cos RCL		
3		x	x	
4		x⇄y sin RCL x	y	

Rectangular to polar

Formulas:

$R = \sqrt{x^2 + y^2},$

$\theta = \text{arc tan}\left(\frac{y}{x}\right)$



Example:

If x = 5.7, and y = 4.2, then

R = 7.08, and

θ = 36.38.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	y	↑						
2	x							If x ≠ 0, go to 4
3		CLX	EEX	CHS	9	9		
4		STO	x⇄y	↑	↑	x		
5		RCL	↑	x	+	√x	R	
6		x⇄y	RCL	÷	arc	tan		If x ⩾ 0, go to 8
7		1	8	0	+			
8							θ	

Correlation Coefficient

See page 144

Cosecant

See page 156

Cotangent

See page 155

Coversine

See page 157

Cross Product

See page 160

Cross Product Sum

See page 144

Degrees, Minutes, Seconds to Decimal Degrees

See page 11

Decimal Degrees to Degrees, Minutes, Seconds

See page 11

Degrees to Grads

See page 15

Degrees to Mils

See page 14

Degrees to Radians

See page 14

Discount

See page 95

Dot Product

See page 161

Double Precision Product

Terms used:

A and B are the two numbers to be multiplied (each having up to 10 significant digits).

A_{L5} and B_{L5} are the left most 5 significant digits, of A and B.

A_{R5} and B_{R5} are the remaining significant digits, (zero filled at the right if the number of significant digits is less than 10).

A_I and B_I are integers with the same significant digits as A and B, respectively.

A_n and B_n are the number of significant digits each in A and B.

X_{R5} is the rightmost portion (5 digits) of the display, regardless of how many digits are displayed.

X_{L1} is the leftmost digit of the display.

X_{Exp} is the value of the displayed exponent (=0, if none visible).

X_{LD} is the number of displayed digits to the left of the decimal point (= −1 if the number is of the form .od - - -).

X_{ND} is the total number of displayed digits.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	A	↑						
2	B	x	STO					
3	A_{L5}	↑						
4	B_{R5}	x						
5	B_{L5}	↑						
6	A_{R5}	x	↑	↑	CLX	+		
7		R↓	+				D1	If D1 < 10^{10}, go to 11
8		CLX	1	↑	EEX	1		
9		0	–	+	+	1		
10		–						
11	X_{R5}	0	0	0	0	0		
12		↑						
13	A_{R5}	↑						
14	B_{R5}	x	↑	↑	CLX	+		
15		R↓	+				D2	If D2 < 10^{10}, go to 19
16		CLX	1	↑	EEX	1		
17		0	–	+	+	1		
18		–						
19	A_n	↑						
20	B_n	+						
21	A_I						A_I	Read note 1
22		CLX						
23	999999997							Go to 26
24		CLX						
25	9999999949							
26		log						
27	B_I						B_I	Read note 1
28		CLX						
29	999999997							Go to 32
30		CLX						
31	9999999949							

Continued

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
32		log +	D3	If D3 is an integer, go to 35;
				otherwise, let f be the integer
				part of D3.
33		CLX		
34	f			
35		↑ 1 + −	D4	If D4 = 0, go to 37
36		R↓		If X_{ND} = 10, go to 39;
				otherwise, go to 41.
37		R↓		If $x_{ND} < 10$, go to 41;
				if $X_{L1} > 5$, go to 42
38				Go to 40
39	X_{L1}	EEX 9 −		
40		1 0 ×		If $X_{ND} < 10$, go to 41,
				if $X_{L1} > 5$, go to 42
41		RCL		Go to 47
42		RCL		Let E = X_{Exp}, D = X_{LD}
43	E	↑ 1 0 −		
44	D	+	P	
45		CLX EEX		
46	P	−		
47				Read note 2
48		x⇄y		Read note 3

Note 1: If A_I (B_I) contains 9 digits of which the first 8 are all 9's and the last digit is 8 or 9, go to 22 (28).

If A_I (B_I) contains 10 digits of which the first 8 are all 9's and the last 2 are 50 or more, go to 24 (30).

Otherwise, go to 26 (32).

Note 2: Read the sign, the leftmost 10 significant digits of the product, the decimal point, and the exponent. If the number starts with .0, the tenth significant digit is hidden. Multiply by 10 to read it. If less than 10 digits are displayed, the balance of the rightmost blank positions are significant zeros (providing the Y register is non-zero).

Note 3: Read the remaining significant digits. If less than 10 digits are displayed, fill in enough preceding zeros on the left to make 10 digits.

Example:

If $A = 349.27186$, $B = -2.431985517 \times 10^{12}$,

then $A_{L5} = 34927$, $B_{L5} = 24319$

$A_{R5} = 18600$, $B_{R5} = 85517$

$A_I = 34927186$, $B_I = 2431985517$

$A_n = 8$, $B_n = 10$

and $A \times B = -8.4942410501565162 \times 10^{14}$.

(Note that $A_{L5}, B_{L5}, A_{R5}, B_{R5}, A_I, B_I, A_n, B_n$ are all positive integers that contain neither signs nor decimal points.)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	349.27186	↑	CHS					
2	2.431985517	EEX	1	2	x	STO		
3	34927	↑						
4	85517	x						
5	24319	↑						
6	18600	x	↑	↑	CLX	+		
7		R↓	+				3439185659.	← D1 < 10^{10}
8	8565900000	↑						
9	18600	↑						
10	85517	x	↑	↑	CLX	+		
11		R↓	+				1.01565162 10	← D2 ≥ 10^{10}
12		CLX	1	↑	EEX	1		
13		0	–	+	+	1		
14		–					156516200.	
15		8	↑	1	0	+		
16	34927186	log						
17	2431985517	log	+				16.92912458	← D3
18		CLX	1	6	↑	1		
19		+	–				1.	← D4
20		R↓					156516200.	$X_{ND} = 9$
21		1	0	x			1565162000.	$X_{ND} = 10$, $X_{L1} = 1$
22		RCL					-8.49424105 14	Note: only 9 digits are
								displayed.
23		x⇄y					1565162000.	

Equation Solving (Iterative Techniques)

We will deal here with equations of the form

$$x = f(x)$$

for cases where it is difficult to separate all x's to one side of the equal sign. The iterative approach is illustrated through the solution of selected equations.

Example 1. Find x such that $x = e^{-x}$

Method:

Choose $X_a = 5$ as an approximation for the solution. Then after 44 iterations, the answer is X = .5671432904.

Note: The algorithm will converge to .5671432904 after about 50 iterations for any value of X_a.

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	5	[CHS] [e^x]	6.737946999 -03	
2		[CHS] [e^x]		Perform line 2 forty-three
				times
3			.5671432904	

Example 2. $4 = X - \frac{1}{X}$

Method:

Rewrite the equation as

$$X = \frac{1}{x} + 4$$

choose an approximate solution for X (e.g., $X_a = 4$).

Answer:

4.236067978

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	4	[1/x] [4] [+]	4.25	
2		[1/x] [4] [+]		Perform line 2 seven times
3			4.236067978	

Example 3. $X^X = 1000$

Method:

Rewrite the equation in the form

$$x = \ln 1000/\ln x$$

Pick an approximation x_a for x (e.g., $x_a = 4$). If we use the following sequence, convergence is from both sides and will take a long time.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	1000	ln	↑	↑	↑			
2	4	ln	÷				4.982892143	
3		ln	÷				4.301189432	
4		ln	÷				4.7349339	
5		ln	÷				4.442378437	
6		ln	÷				4.632377942	
7		ln	÷				4.505830645	
8		ln	÷				4.588735608	
9		ln	÷				4.533824354	
10		ln	÷				4.569933525	etc.

To hasten the convergence, we modify the loop and use the average of the last two approximations as the new approximation.

Answer:

4.555535705

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	1000	ln	↑	↑	↑			
2	4	STO	ln	÷	RCL	+		
3		2	÷	STO			4.491446072	
4		ln	÷	RCL	+			
5		2	÷	STO			4.544974651	
6		ln	÷	RCL	+			
7		2	÷	STO			4.553746975	
8		ln	÷	RCL	+			
9		2	÷	STO			4.555231426	
↑							↑	
│							│	7 iterations
↓							↓	
24		ln	÷	RCL	+			
25		2	÷	STO			4.555535705	
26		ln	÷	RCL	+			
27		2	÷	STO			4.555535705	Display converges

Example 4. Largest x^{x^x}

What is the largest value of x such that x^{x^x} does not overflow in the HP-35 (i.e. $x^{x^x} < 9.999999999 \times 10^{99}$)?

Method:

Since $9.999999999 \times 10^{99}$ is only slightly less than a *googol* (10^{100}), let us call this constant G. Then,

$$x^{x^x} = G$$

$$x^x \ln x = \ln G$$

$$x^x = \ln G/\ln x$$

$$x \ln x = \ln (\ln G/\ln x)$$

$$x = [\ln (\ln G/\ln x)]/\ln x$$

Use 3 as an initial approximation for the solution.

The loop in the following routine alternates between the two answers, 3.830482865 and 3.830482864. Upon substitution of both values in x^{x^x}, 3.830482865 gives an overflow while 3.830482864 does not. Thus, the answer is 3.830482864.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		9	e^x	e^x	ln	↑		
2		↑	↑				230.2585093	← ln G
3		3	ln	÷	ln	3		
4		ln	÷	STO			4.865369574	
5		ln	÷	ln	RCL	ln		
6		÷	RCL	+				
7		2	÷	STO			4.006633409	
8		ln	÷	ln	RCL	ln		
9		÷	RCL	+				
10		2	÷	STO			3.84465425	
↑							↑	
								6 iterations
↓							↓	
29		ln	÷	ln	RCL	ln		
30		÷	RCL	+				
31		2	÷	STO			3.830482865	
32		ln	÷	ln	RCL	ln		
33		÷	RCL	+				
34		2	÷	STO			3.830482864	etc.

Example 5. $x^2 + 4 \sin x = 0$

(x is in radians).

Method:

Rewrite the equation as

$$x = \pm\sqrt{-4 \sin x}.$$

Since x is in radians, we convert to degrees before using it with the trigonometric function keys.

Upon plotting this curve, we see that there is a root near −2, so take $x_a = -2$ as an approximation of the solution, and substitute in

$$x = -\sqrt{-4 \sin x}.$$

Note: This algorithm converges from both sides, to hasten convergence, the following routine averages two approximations to get the next approximation.

It is only necessary to do the loop 5 times to get 4 digits of accuracy.

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	2	CHS STO		
2		↑ 1 8 0 ×		Perform lines 2–5
3		π ÷ sin CHS 4		until A_i does not change.
4		× √x RCL − 2		
5		÷ CHS STO	A_i	
6			−1.933753764	After 17 iterations

A method for faster convergence

Some equations for f(x) = 0 converge very slowly by the above methods. The following method gives faster convergence.

Formula:

$$x_{i+1} = x_i - \frac{E_i (x_i - x_{i-1})}{E_i - E_{i-1}}$$

where $i = 1, 2, 3, ...$

x_i = current trial value

x_{i+1} = next trial value

x_{i-1} = previous trial value

E_i = current error = $f(x_i)$

E_{i-1} = last error = $f(x_{i-1})$

L = lower bound for the solution

U = upper bound for the solution.

The keystrokes evaluating the equation f(x) may not modify the S register, because the trial value is stored there.

Example 6. Solve $x^3 = 3^x$ where $1 < x < e$

Method:

Rewrite the equation in the form

$$x^3 - 3^x = 0.$$

Replace f(x) in the routine (shown after example 7) by the key sequence:

[↑] [↑] 3 [x⇄y] [x^y] [x⇄y] 3 [x^y] [−]

Let L = 1
U = e.

Answer:

2.478052685 (E_0 = −2.000000001, E_1 = .27254617, E_2 = .05608461, E_3 = −.03342142, $E_4 = 1.19176 \times 10^{-3}$, $E_5 = 2.254 \times 10^{-5}$, $E_6 = E_7$)

Example 7. Find a root of a polynomial

$$f(x) = x^4 - 4x^3 + 8x^2 + 20x - 65.$$

Method:

Replace f(x) in the program by

[↑] [↑] [↑] 4 [−] [×] 8 [+] [×] 20 [+] [×] 65 [−]

Note that $f(2) = -9 < 0$, $f(3) = 40 > 0$, so there is a root between 2 and 3.

Let $L = 2$
$U = 3$

Answer:

2.236067977 (or $\sqrt{5}$) ($x_0 = 2$, $x_1 = 3$, $x_2 = 2.183673469$, $x_3 = 2.224244398$, $x_4 = 2.236236914$, $x_5 = 2.236067428$, $x_6 = x_7 = 2.236067977$)

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	L	[STO] [ ] [ ] [ ] [ ]		
2		[f(x)] [ ] [ ] [ ] [ ]	E_0	Replace f(x) by proper
		[ ] [ ] [ ] [ ] [ ]		keystrokes
3	U	[↑] [ ] [ ] [ ] [ ]		
4		[f(x)] [ ] [ ] [ ] [ ]	E_i	Perform lines 4–8 for i = 1,
5		[RCL] [R↓] [R↓] [STO] [x⇄y]		2, ..., until either the last
6		[CLX] [+] [x⇄y] [−] [×]		two trial values are the same
7		[RCL] [x⇄y] [R↓] [R↓] [R↓]		or the last two errors are
8	E_{i-1}	[−] [÷] [−] [↑] [ ]	X_{i+1}	the same.

Euler Numbers

Compute the n^{th} Euler number

Formula

$$E_n = \frac{2^{2n+2}(2n)!}{\pi^{2n+1}}\left[1 - \frac{1}{3^{2n+1}} + \frac{1}{5^{2n+1}} - \frac{1}{7^{2n+1}} + \ldots\right]$$

Note: The Euler numbers are 1, 5, 61, 1385, 50521, ...

Example:

The 5^{th} Euler number = 50521.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	n	↑	2	x	↑	↑		
2		↑					D	If D ⩽ 2, go to 4
3	i	x						Perform line 3 for i = D−1,
								D−2, ..., 2
4		x⇄y	2	+	2	x^y		
5		x	x⇄y	1	+	π		
6		x^y	÷	EEX	9	+		
7		EEX	9	−				

Exponentiation

Multiple successive power operations

As written, these terms must be executed from right to left. For example,

$$e^{x^{1.5}}$$

means

$$e^{(x^{1.5})}, \text{ not } (e^x)^{1.5}$$

$$a^{b^c}$$

means

$$a^{(b^c)}, \text{not } (a^b)^c.$$

Example:

Evaluate $t^{t^{t^{t^{\cdot^{\cdot^{\cdot}}}}}}$ where $t = e^{\frac{1}{e}}$.

Answer:

1.98957

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		[1] [e^x] [1/x] [e^x] [↑]		
2		[STO] [x^y] [RCL] [x^y] [RCL]		
3		[x^y]		

Example:

Find the limit of $t^{t^{t^{\cdot^{\cdot^{\cdot}}}}}$ where $t = \sqrt{3}$.

Answer:

∞

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		[3] [$\sqrt{x}$] [↑] [STO]		
2		[RCL] [x^y]		Perform line 2 six times
3			9.999999999 99	

Example:

Find L, the limit of $t^{t^{t^{\cdot^{\cdot^{\cdot}}}}}$ where $t = \sqrt{2}$.

Answer:

2 (rounded)

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		[2] [$\sqrt{x}$] [↑] [STO] [ ]		
2		[RCL] [x^y] [ ] [ ] [ ]	L_i	Perform line 2 until $L_i = L_{i+1}$
3		[ ] [ ] [ ] [ ] [ ]	1.999999995	Unchanging L after 54 iterations

e^x for large positive x (x > 230)

Formula:

Suppose $e^x = a \times 10^b$

where $a < 10$

Find a, b for a given x.

Example:

$$e^{300} = 1.942426525 \times 10^{130}$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	[↑] [1] [0] [ln] [STO]		
2		[÷] [↑] [↑] [↑] [.]		
3		[5] [−] [EEX] [9] [+]		
4		[EEX] [9] [−] [↑] [R↓]		
5		[−] [RCL] [x] [e^x] [ ]	a	
6		[R↓] [R↓] [ ] [ ] [ ]	b	

x^y for large x and/or y ($y \ln x > 230$)

Formula:

Suppose

$$x^y = a \times 10^b$$

where

$$a < 10,$$

find a, b for a given x, y.

Example:

$$75^{100} = 3.207202635 \times 10^{187}$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	y	[↑] [ ] [ ] [ ] [ ]		
2	x	[ln] [x] [1] [0] [ln]		
3		[STO] [÷] [↑] [↑] [↑]		
4		[.] [5] [−] [EEX] [9]		
5		[+] [EEX] [9] [−] [↑]		
6		[R↓] [−] [RCL] [x] [e^x]	a	
7		[R↓] [R↓] [ ] [ ] [ ]	b	

Example:

The largest number that can be written with three digits and no other symbols is 9^{9^9}. How big is this number?

Answer:

$$9^{9^9} = 3.981071706 \times 10^{369693099}$$

(Due to machine accuracy limitations, this is only an approximate answer.)

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		[9] [↑] [↑] [↑] [x]		
2		[x] [x] [x] [x] [x]		
3		[x] [x] [9] [ln] [x]		
4		[1] [0] [ln] [STO] [÷]		
5		[↑] [↑] [↑] [.] [5]		
6		[−] [EEX] [9] [+] [EEX]		
7		[9] [−] [↑] [R↓] [−]		
8		[RCL] [x] [e^x] [ ] [ ]	3.981071706	
9		[R↓] [R↓] [ ] [ ] [ ]	369693099.	

Converging $u^{u^{u^{\cdot^{\cdot^{\cdot}}}}}$

Formula

If $0 < x < e$ and $u = x^{\frac{1}{x}}$, then $u^{u^{u^{\cdot^{\cdot^{\cdot}}}}}$ will converge at x.

Example:

Let $x = 1.5$, $u = x^{\frac{1}{x}}$

Answer:

$u^{u^{u^{\cdot^{\cdot^{\cdot}}}}}$ converges at 1.5 in 21 iterations. (Display = 1.499999999).

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	[↑] [1/x] [x⇄y] [x^y] [STO]	u	
2		[↑]		
3		[RCL] [x^y]		Perform line 3 as many times
				as necessary to see convergence

Diverging $u^{u^{u^{\cdot^{\cdot^{\cdot}}}}}$

Formula:

If $u > e^{\frac{1}{e}}$ (= 1.444667861), $u^{u^{u^{\cdot^{\cdot^{\cdot}}}}}$ will diverge.

Example:

Let $u = 1.45$

Answer:

$u^{u^{u^{\cdot^{\cdot^{\cdot}}}}}$ diverges (overflows) after 43 iterations.

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	u	[STO] [↑]		
2		[RCL] [x^y]		Perform line 2 as many times
				as necessary to see divergence

Calculator limits for x^y

1. For a given positive value of x, how big can y be without causing x^y to overflow?

Example:

If x = 50, then y can be as large as around 58 (50^{59} will overflow).

Note: The following gives an approximate, not exact answer.

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		[2] [3] [0] [↑] [ ]		
2	x	[ln] [÷] [ ] [ ] [ ]	D	Answer is the largest
		[ ] [ ] [ ] [ ] [ ]		integer ≤ D.

2. For a given positive value of y, how big can x be without causing x^y to overflow?

Example:

If y = 50, then we can take x as large as around 99 (100^{50} will overflow).

Note: The following gives an approximate, not exact answer.

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		[2] [3] [0] [↑] [ ]		
2	y	[÷] [e^x] [ ] [ ] [ ]	D	Answer is the largest
		[ ] [ ] [ ] [ ] [ ]		integer ≤ D.

Factorial

Formula:

$$x! = x \cdot (x-1) \cdot \ldots \cdot 2 \cdot 1$$

(x is a positive integer).

Example:

$$9! = 362880.$$

Note: 0! = 1.

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	↑		If x ⩽ 2, Stop
2	i	x		Perform line 2 for i = x−1,
				x−2, ..., 2

Factorial and Gamma Function

Stirling's approximation

Notes:

This approximation can be used for positive $x < 69$ (otherwise the answer is $> 10^{100}$).

This approximation is good for large x.

For $x < 1$, use polynomial approximation.

To compute Gamma function:

$$\Gamma(x) = (x-1)!$$

Formula:

$$x! \cong \sqrt{2\pi x}\, x^x\, e^{-x} \left(1 + \frac{1}{12x} + \frac{1}{288x^2} - \frac{139}{51840x^3} - \frac{571}{2488320x^4}\right)$$

Example:

$$4.25! \cong 35.21159$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x	STO	↑	↑	2	x		
2		π	x	√x	RCL	2		
3		÷	RCL	x^y	x	RCL		
4		e^x	1/x	x	RCL	2		
5		÷	RCL	x^y	x	STO		
6		x⇄y	1/x	↑	↑	↑		
7		CHS	5	7	1	↑		
8	2488320	÷	x	1	3	9		
9		↑	5	1	8	4		
10		0	÷	–	x	2		
11		8	8	1/x	+	x		
12		1	2	1/x	+	x		
13		1	+	RCL	x			

Polynomial approximation

Notes:

This approximation can be used for positive $x < 69$.

This is a more accurate method (to 6 or 7 decimal places), but longer than Stirling's approximation.

Formula:

$$x! \cong 1 + b_1 x + b_2 x^2 + \dots + b_8 x^8$$

for $0 \leqslant x \leqslant 1$

where $b_1 = -.577191652$, $b_2 = .988205891$

$b_3 = -.897056937$, $b_4 = .918206857$

$b_5 = -.756704078$, $b_6 = .482199394$

$b_7 = -.193527818$, $b_8 = .035868343$

Example:

$$4.25! \cong 35.21161962$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x	↑						If x > 1, go to 3
2		1	STO	R↓				Go to 8
3								If x > 2, go to 5
4		STO	↑	1	−			Go to 8
5		↑	1	−				
6		STO	x	RCL	1	−	D	Perform line 6 until D ⩽ 1
7		x⇄y	STO	x⇄y				
8		↑	↑	↑				
9	.035868343	x						
10	.193527818	−	x					
11	.482199394	+	x					
12	.756704078	−	x					
13	.918206857	+	x					
14	.897056937	−	x					
15	.988205891	+	x					
16	.577191652	−	x	1	+	RCL		
17		x						

Factoring Integers and Determining Primes

Prime Numbers under 100

2	13	31	53	73
3	17	37	59	79
5	19	41	61	83
7	23	43	67	89
11	29	47	71	97

With the list memorized or in sight, it is easy to factor any integer x less than 10000 (and many other integers even greater). In the following program, omit the numbers 2 and 5 from the list of primes if the integer ends in 1, 3, 7 or 9.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x							If x is even, let P = 2;
								otherwise P = 3.
2		STO	$\sqrt{x}$				Max	
3								If P ⩾ Max, stop
4		RCL						
5	P	÷					Q	Read note

Note: If Q is not an integer, let P = next prime number, go to 3. If Q is a prime then both P and Q are factors, stop. Otherwise P is a factor, let P = current prime, go to 2.

Example: Factor 4807.

Answer: 4807 = 11 x 19 x 23

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	4807	STO	$\sqrt{x}$				69.33253205	← Max, P = 3
2		RCL	3	÷			1602.333333	P = 7
3		RCL	7	÷			686.7142857	P = 11
4		RCL	1	1	÷		437.	← Q is an integer, so 11
								is a factor
5		STO	$\sqrt{x}$				20.90454496	← Max, P = 11
6		RCL	1	1	÷		39.72727273	P = 13
7		RCL	1	3	÷		33.61538462	P = 17
8		RCL	1	7	÷		25.70588235	P = 19
9		RCL	1	9	÷		23.	Q = 23 is a prime; 19 and
								23 are factors

Example: Factor 2909.

Answer: 2909 is a prime.

Fahrenheit to Centigrade

See page 55

Feet, Inches to Centimeters

See page 56

Fibonacci Numbers

Fibonacci sequence

Formula:

In a Fibonacci sequence, each term is the sum of the two preceding terms.

$$f_i = f_{i-1} + f_{i-2}$$

f_i represents the i^{th} term in the sequence.

Example:

Develop the Fibonacci sequence with $f_1 = 1$, $f_2 = 1$

Answer:

1, 1, 2, 3, 5, 8, 13, 21, . . .

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	f_1	↑		
2	f_2			
3		↑ ↑ R↓ R↓ +	f_i	Perform line 3 for i = 3, 4, ...

Finding the n^{th} Fibonacci number

Example:

Find the 12^{th} Fibonacci number in the sequence 1, 1, 2, 3, 5, 8 . . .

Answer: 144

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	n	↑ 5 √x STO ↑		
2		1 + 2 ÷ x^y		
3		RCL ÷ EEX 9 +		
4		EEX 9 −		

Fractions

Improper fractions to mixed numbers

Formula:

$$\frac{a}{b} = p + \frac{q}{b}$$

where $a > b$

$q < b$

Example:

$$\frac{517}{13} = 39\,\frac{10}{13}$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑	↑					
2	b	↑	R↓	÷			D	Let p = integer part of D
3		CLX						
4	p	R↓	R↓	R↓	x	−	q	

Reduce fraction to lowest terms

Formula:

$$\frac{a}{b} \to \frac{p}{q}$$

Note: If $p > q$, we can simplify the fraction further, see "Improper fractions to mixed numbers", above.

Example:

$$\frac{138}{391} = \frac{6}{17}$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	STO						
2	b							
3		↑	↑	RCL	÷		D	Let f be the largest integer ⩽ D
4	f	x⇄y	CLX	RCL	x	−	E	If E = 0, go to 6
5		RCL	x⇄y	STO	CLX	+		Go to 3
6	a	RCL	÷				p	
7	b	RCL	÷				q	

Addition and subtraction of fractions

Formulas:

$$\frac{a}{b} + \frac{c}{d} = \frac{p}{q}$$

Since

$$\frac{a}{b} - \frac{c}{d} = \frac{a}{b} + \left(\frac{-c}{d}\right),$$

Only an addition routine is necessary.

Note: If the highest common factor HCF (b, d) = 1, start from Line 6 and skip the three "RCL ÷" sequences in the routine.

Example 1:

$$\frac{11}{12} + \frac{9}{34} = \frac{241}{204}$$

Example 2:

$$\frac{4}{11} - \frac{5}{31} = \frac{69}{341}$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	b	STO						
2	d							
3		↑	↑	RCL	÷		D	Let f be the largest integer ⩽ D
4	f	x⇄y	CLX	RCL	x	-	E	If E = 0, go to 6
5		RCL	x⇄y	STO	CLX	+		Go to 3
6	a	↑						
7	d	x	RCL	÷				
8	c	↑						
9	b	x	RCL	÷	+		p	
10	•b	↑						
11	d	x	RCL	÷			q	

Multiplication and division of fractions

Formulas:

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{db} = \frac{p}{q}$$

Since

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} \quad ,$$

Only a multiplication routine is necessary.

Note: The result is not necessarily reduced to lowest terms, (see "Reduce fraction to lowest terms", above).

Example:

$$\frac{3}{19} \div \frac{2}{7} = \frac{3}{19} \times \frac{7}{2} = \frac{21}{38}$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑						
2	c	x					p	
3	b	↑						
4	d	x					q	

Gamma Function

See page 75

Gaussian Probability Function

Formula:

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

Example:

$$f(2.23) = .03319$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x	↑	x	2	÷	CHS		
2		e^x	π	2	x	$\sqrt{x}$		
3		÷						

Geometric Progressions

See pages 114–118

Grads to Degrees

See page 15

Gudermannian Function

See page 85

Harmonic Mean

See page 139

Harmonic Numbers

Formula:

The Harmonic numbers $H_i (i = 1, 2, \ldots)$ are

$$1, 1 + \frac{1}{2}, 1 + \frac{1}{2} + \frac{1}{3}, 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}, \ldots$$

or

$$1, \frac{3}{2}, \frac{11}{6}, \frac{25}{12}, \ldots$$

Example:

Display the sequence in decimal form.

Answer:

1, 1.5, 1.833, 2.083, ...

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		CLR		
2		1 + ↑ 1/x RCL		Perform lines 2–3 for i = 1,
				2, 3, ...
3		+ STO	H_i	

n^th Harmonic number

Example:

Find the 7th Harmonic number

Answer:

2.59

Note: .5772156649 is Euler's constant.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	n	STO	1/x	↑	↑	↑		
2		1	2	0	1/x	x		
3		x	1	2	1/x	−		
4		x	2	1/x	+	x		
5	.5772156649	+	RCL	ln	+		H_n	

Harmonic Progressions

See page 114

Haversine

See page 157

Highest Common Factor

Definition:

The highest common factor (or greatest common divisor) of two positive integers a and b is the largest integer which divides both a and b. We write it as HCF(a, b).

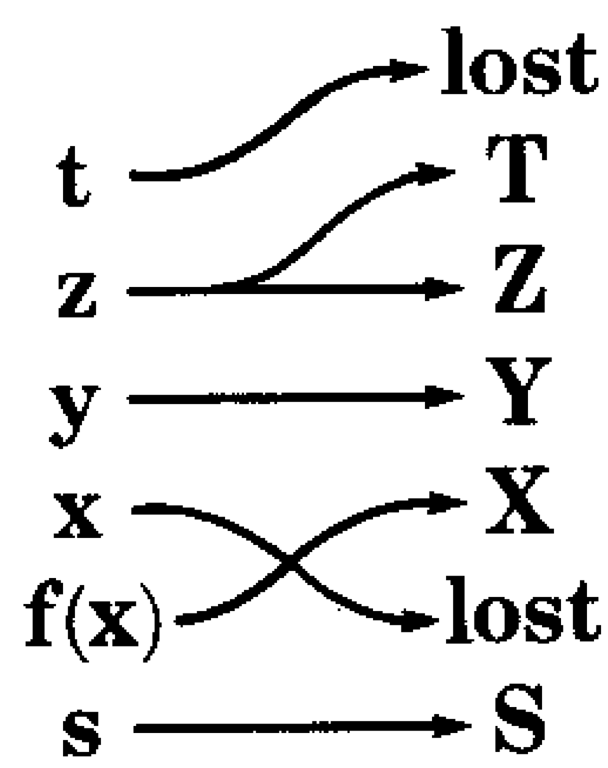
Example: HCF (51, 119) = 17.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	STO						
2	b							
3		↑	↑	RCL	÷		D	Let f be the largest integer ≤ D
4	f	x⇄y	CLX	RCL	x	−	E	If E = 0, go to 6
5		RCL	x⇄y	STO	CLX	+		Go to 3
6		RCL						

Hyperbolic and Inverse Hyperbolic Functions

Register usage for the following functions:

(Same as for keyboard trigonometric functions.)



Gudermannian function

Formula:

$$\text{gd } x = 2 \text{ arc tan } e^x - \frac{\pi}{2}$$

Example:

$$\text{gd } 0.345 = 19.386$$

Note: $\frac{\pi}{2} = 90°$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	[e^x] [arc] [tan] [2] [×]		
2		[9] [0] [–] [ ] [ ]		

Hyperbolic sine

Formula:

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

Example:

$$\sinh 3.2 = 12.25$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	[e^x] [↑] [$1/x$] [–] [2]		
2		[÷] [ ] [ ] [ ] [ ]		

Hyperbolic cosine

Formula:

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

Example:

$$\cosh 3.2 = 12.29$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x	e^x	↑	1/x	+	2		
2		÷						

Hyperbolic tangent

Formula:

$$\tanh x = \frac{\sinh x}{\cosh x} = \sin \text{gd } x$$

Example:

$$\tanh 3.2 = .99668$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x	e^x	arc	tan	2	×		
2		9	0	−	sin			

Hyperbolic cotangent

Formula:

$$\coth x = \frac{1}{\tanh x}$$

Example:

$$\coth 3.2 = 1.003$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x	e^x	arc	tan	2	×		
2		9	0	−	sin	1/x		

Hyperbolic cosecant

Formula:

$$\text{csch } x = \frac{1}{\sinh x}$$

Example:

$$\text{csch } 3.2 = .08166$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	e^x ↑ 1/x − 2		
2		÷ 1/x		

Hyperbolic secant

Formula:

$$\text{sech } x = \frac{1}{\cosh x}$$

Example:

$$\text{sech } 3.2 = .081$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	e^x ↑ 1/x + 2		
2		÷ 1/x		

Inverse Gudermannian function

Formula:

$$\text{gd}^{-1}\theta = \ln \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$$

Example:

$$\text{gd}^{-1}\ 30° = .549$$

Note: $\frac{\pi}{4} = 45°$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	θ	↑ 2 ÷ 4 5		
2		+ tan ln		

Inverse hyperbolic sine

Formula:

$$\sinh^{-1} x = \ln\left[x + (x^2 + 1)^{\frac{1}{2}}\right]$$
$$= \mathrm{gd}^{-1}(\tan^{-1} x)$$

Example:

$$\sinh^{-1} 51.777 = 4.64$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	arc tan 2 ÷ 4		
2		5 + tan ln [ ]		

Inverse hyperbolic tangent

Formula:

$$\tanh^{-1} x = \frac{1}{2}\ln\frac{1+x}{1-x} = \mathrm{gd}^{-1}(\sin^{-1} x)$$

Example:

$$\tanh^{-1} 0.777 = 1.038$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	arc sin 2 ÷ 4		
2		5 + tan ln [ ]		

Inverse hyperbolic secant

Formula:

$$\mathrm{sech}^{-1} x = \ln\left[\frac{1}{x} + \left(\frac{1}{x^2} - 1\right)^{\frac{1}{2}}\right] = \mathrm{gd}^{-1}(\cos^{-1} x)$$

Example:

$$\mathrm{sech}^{-1} 0.777 = .74$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	arc cos 2 ÷ 4		
2		5 + tan ln [ ]		

Inverse hyperbolic cosine

Formula:

$$\cosh^{-1} x = \text{sech}^{-1} \frac{1}{x}$$

Example:

$$\cosh^{-1} 51.777 = 4.64$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	[1/x] [arc] [cos] [2] [÷]		
2		[4] [5] [+] [tan] [ln]		

Inverse hyperbolic cotangent

Formula:

$$\coth^{-1} x = \tanh^{-1} \frac{1}{x}$$

Example:

$$\coth^{-1} 51.777 = .0193$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	[1/x] [arc] [sin] [2] [÷]		
2		[4] [5] [+] [tan] [ln]		

Inverse hyperbolic cosecant

Formula:

$$\text{csch}^{-1} x = \sinh^{-1} \frac{1}{x}$$

Example:

$$\text{csch}^{-1} 0.777 = 1.0705$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	[1/x] [arc] [tan] [2] [÷]		
2		[4] [5] [+] [tan] [ln]		

Hyperbolic Cosecant

See page 87

Hyperbolic Cosine

See page 86

Hyperbolic Cotangent

See page 86

Hyperbolic Secant

See page 87

Hyperbolic Sine

See page 85

Hyperbolic Tangent

See page 86

Improper Fractions

See page 80

Infinite Power Series Summation

Method:

To sum an infinite power series

$$\sum_{i=1}^{\infty} a_i x^i \, ,$$

1. accumulate the sum in the S register,
2. keep the value of x in the T register
3. retain the previous term in the Z register, and
4. calculate the coefficient modifier using only the X and Y registers.

Example:

Evaluate the sine integral SI(x) for $x = \frac{\pi}{2}$

$$SI(x) = \int_0^x \frac{\sin u}{u} du = x - \frac{x^3}{3 \cdot 3!} + \frac{x^5}{5 \cdot 5!} - \frac{x^7}{7 \cdot 7!} + \ldots$$

i takes the values 1, 3, 5, ... , 13 in the algorithm

Answer:

$$SI\left(\frac{\pi}{2}\right) = 1.37$$

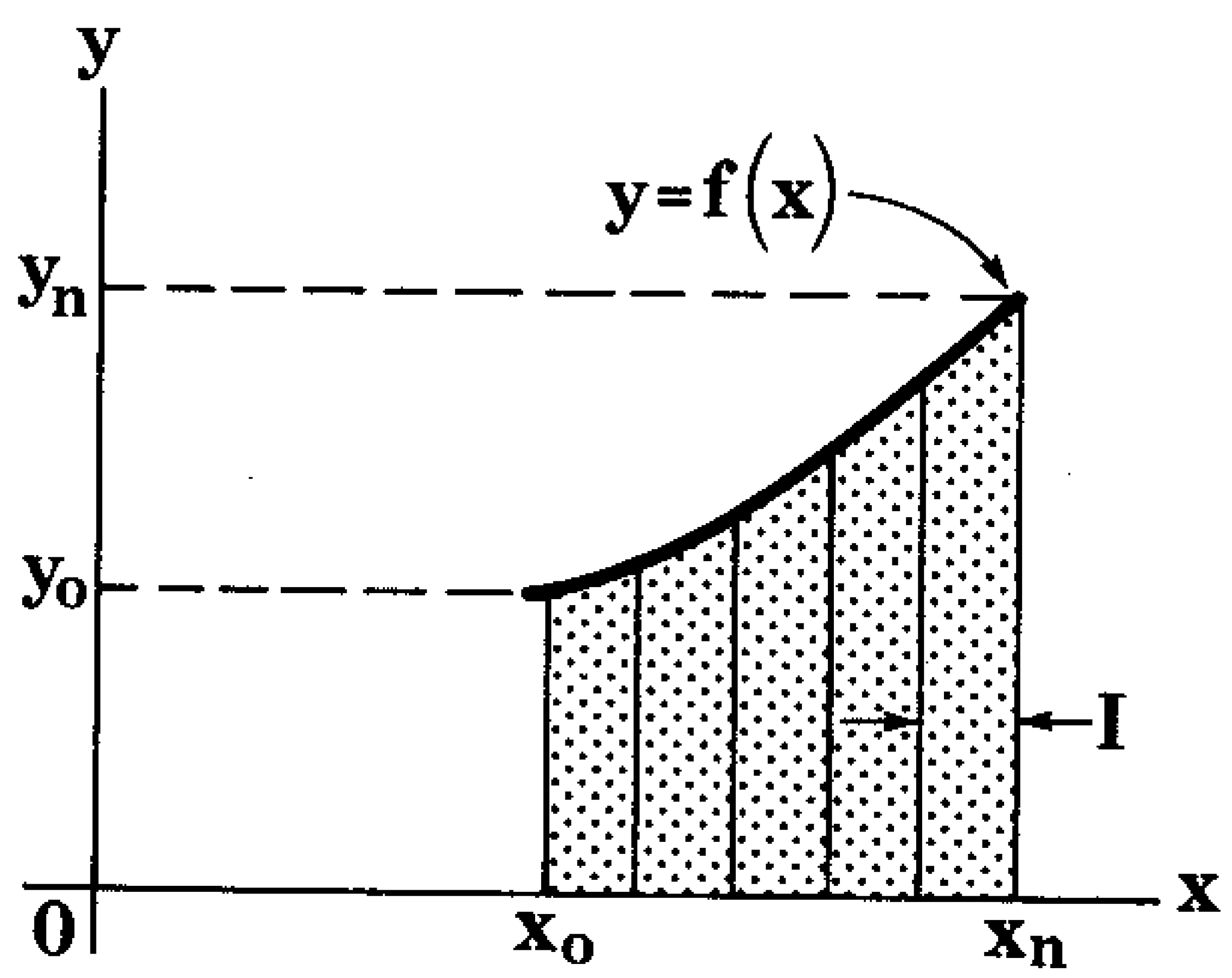
LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	↑ ↑ ↑ STO ☐		
2		CLX + x x ☐		Perform lines 2–7 for i = 1,
3	i	1/x ☐ ☐ ☐ ☐		3, 5, 7, ... until D_i does
4	i + 1	x ☐ ☐ ☐ ☐		not change.
5	i + 2	x ☐ ☐ ☐ ☐		
6	i + 2	x CHS ÷ ↑ ↑		
7		RCL + STO ☐ ☐	D_i	

Numerical Integration

Method:

To approximate the area A between a curve and the x-axis, sum the areas of the constituent trapezoids of width I. Each trapezoid has the area

$$I \times \frac{y_i + y_{i-1}}{2}$$



Example:

Find the area bounded by

$$y = x^2 + 2x - 3,$$

$$x = -2,$$

$$x = 0,$$

$$y = 0 \text{ (the x axis).}$$

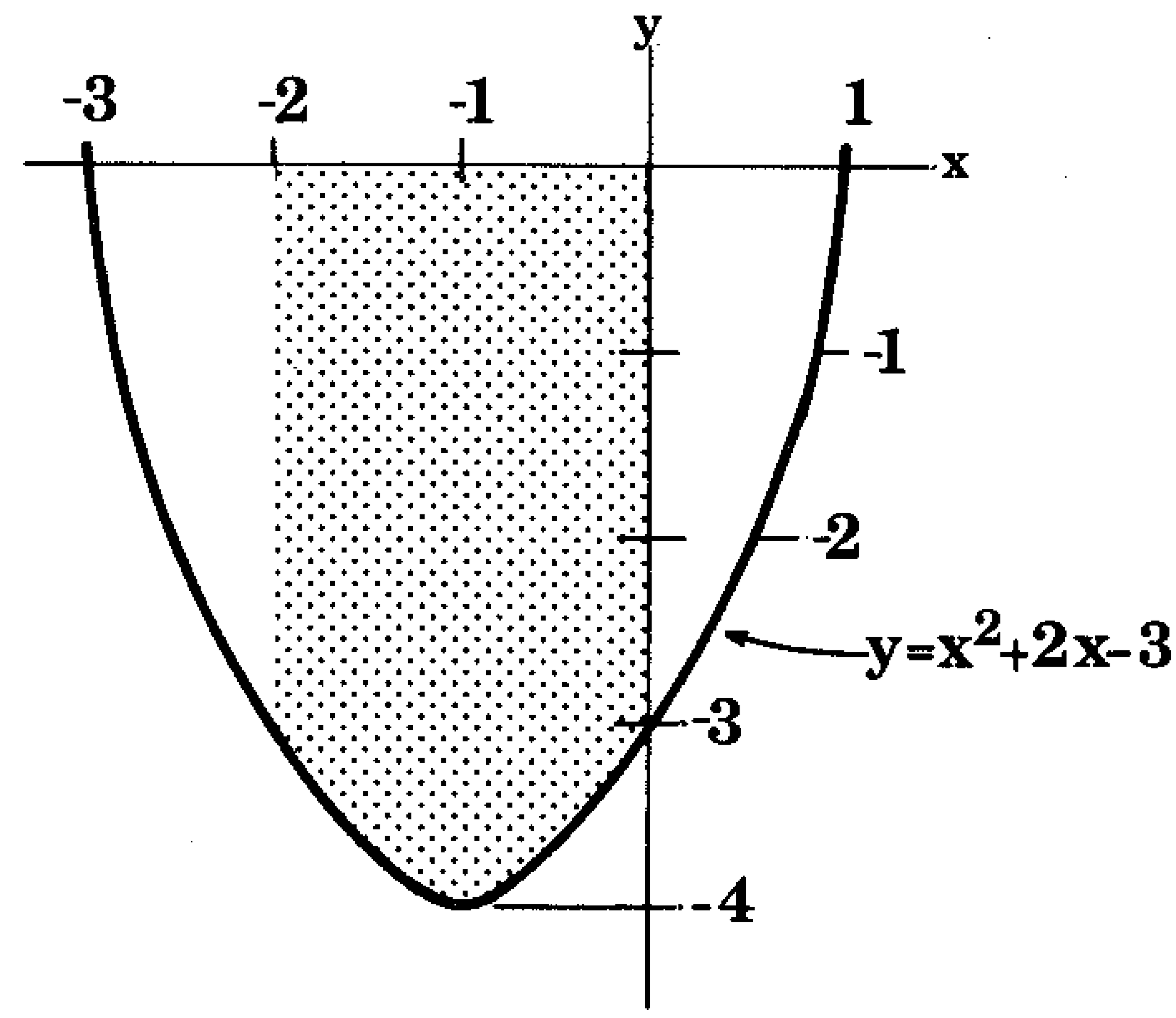
In this case

$$x_o = -2, x_n = 0, n = 4, I = .5$$

Replace f(x) by " [↑] [↑] [↑] 2 [+] [×] 3 [−] "

Answer:

7.25



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x_n	↑		
2	x_0	−		
3	n	÷	I	
4	x_0			
5		f(x)	y_0	Replace f(x) by proper
				keystroke(s)
6		STO		
7	x_0	↑		
8	I	+		
9		f(x)	y_1	
10		RCL +		
11	I	x 2 ÷	D	If D > 0, go to 13
12		CHS		
13		STO		
14	x_0	↑		Perform lines 14–21 for i = 2,
				..., n
15	I	↑		
16	i	x +		
17		f(x)	y_i	
18	y_{i-1}	+		
19	I	x 2 ÷	E_i	If E_i > 0, go to 21
20		CHS		
21		RCL + STO		

Interest and Discount (Simple)

Simple amount and interest

Formulas:

The simple interest I on a principal PV for n years at the annual interest rate i is given by

$$I = PV \cdot i \cdot n$$

The simple amount FV is

$$FV = PV + I = PV(1 + i\,n)$$

Example:

Find the simple amount and interest on $1000 at 5% interest for 2 years (i = 0.05).

Answers:

FV = $1100, I = $100

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	PV	↑ ↑		
2	i	x		
3	n	x EEX 7 + EEX		
4		7 −	I	
5		+	FV	

Simple add-on interest

The following key sequence finds the **monthly payments** and **finance charge** for an amount PV, to be paid over t years at interest rate i.

Example:

Find the monthly payments and finance charge for a loan of $2037.01 which will be paid over 3 years at 5%. (i = 0.05)

Answers:

TP (Total of payments) = $2342.57

F (Finance charge) = $305.56

n (Number of payments) = 36

PMT_1 (First payment) = $65.12

PMT (Each payment except first) = $65.07

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	PV	STO	↑	↑				
2	t	x						
3	i	x	+	.	0	0		
4		5	+	EEX	7	+		
5		EEX	7	–			TP	Total of payments
6		RCL	x⇄y	STO	x⇄y	–	F	Finance charge
7		RCL	↑	↑	1	2		
8		↑						
9	t	x					n	Number of payments
10		÷	.	0	0	5		
11		–	EEX	7	+	EEX		
12		7	–				PMT	Each payment except first
13	n	↑	1	–	x	RCL		
14		x⇄y	–				PMT_1	First payment

Simple discount

Formula:

The **simple discount** D on a sum S for n years at the discount rate i is given by

$$D = S\,n\,i$$

and the **present value** of S is given by

$$PV = S - D = S\,(1 - i\,n).$$

Example:

Find the simple discount on a debt of $1500 due in 9 months at a discount rate of 6%. What is the present value of the debt? (i = 0.06, n = 9/12).

Answers:

D (Simple discount) = $67.50

PV (Present value) = $1432.50

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	S	↑	STO					
2	i	x						
3	n	x	EEX	7	+	EEX		
4		7	–	RCL	x⇄y		D	
5		–					PV	

Interest (Compound)

Formula:

The formula for finding the future value FV of PV dollars invested for t years at interest i compounded q times a year is:

$$FV = PV \left(1 + \frac{i}{q}\right)^{qt}$$

Example 1.

If \$50 is deposited at 6% interest, compounded quarterly, what will be its value after 5 years? (i = 0.06)

Answer:

\$67.34

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	q	↑ STO		
2	t	x		
3	i	RCL ÷ 1 + x^y		
4	PV	x EEX 7 + EEX		
5		7 –	FV	

Example 2:

Suppose \$500 is to be initially deposited in a savings account and \$100 is added to it monthly. If the account grows at a rate of 6% compounded monthly, what is the balance at the end of each month over a 1-year period? (i = 0.06)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	.06	↑	1	2	÷	1		
2		+	↑	↑	↑		1.005	
3	500	x					502.5	After 1 month
4	100	STO	+	x			605.5125	After 2 months
5		RCL	+	x			709.0400625	After 3 months
6		RCL	+	x			813.0852628	After 4 months
7		RCL	+	x			917.6506891	After 5 months
8		RCL	+	x			1022.738942	After 6 months
9		RCL	+	x			1128.352637	After 7 months
10		RCL	+	x			1234.4944	After 8 months
11		RCL	+	x			1341.166872	After 9 months
12		RCL	+	x			1448.372706	After 10 months
13		RCL	+	x			1556.11457	After 11 months
14		RCL	+	x			1664.395143	After 12 months

Continuous compounding

Formula:

The future value FV of PV dollars invested at interest rate i for t years, compounded continuously is:

$$FV = PV \cdot e^{it}$$

Example:

Determine the value of $50 deposited at 6% for 5 years, compounded continuously. (i = 0.06)

Answer:

$67.49

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	i	↑						
2	t	x	e^x					
3	PV	x	EEX	7	+	EEX		
4		7	−				FV	

Interest Rates

True interest rate

Formula:

The true interest rate (approximate) on installment purchases may be calculated by using the direct ratio formula:

$$i_t = \frac{6\,mF}{3\,PV\,(n + 1) + F\,(n - 1)}$$

where n = number of payments

m = number of payments in one year

PV = amount financed

F = finance charge (carrying charge).

Example:

A financial corporation advertises consumer installment loans up to $1000 at 6% (add-on) rate. If such a loan is to be paid off in 12 equal monthly payments, what is the true interest rate? (n = m = 12, F = 60, PV = 1000)

Answer: 10.89%

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		6	↑					
2	m	x						
3	F	STO	x	RCL				
4	n	↑	STO	1	−	x		
5		RCL	1	+	3	x		
6	PV	x	+	÷	EEX	2		
7		x	EEX	7	+	EEX		
8		7	−					

Effective rate from nominal rate

Formula:

The effective rate equivalent of the nominal rate r compounded n times per year is:

$$\text{effective rate} = \left(1 + \frac{r}{n}\right)^n - 1$$

Example:

What is the effective rate if the nominal rate is 6% compounded monthly? (r = 0.06)

Answer:

6.17%

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	n	↑ 1/x		
2	y	× 1 + x^y 1		
3		− EEX 2 × EEX		
4		7 + EEX 7 −		Answer is in %

Find i (given PMT, PV, and n)

Formula:

$$i = \frac{PMT\left[1 - \left(\frac{1}{1+i}\right)^n\right]}{PV}$$

Example:

A $30,000 mortgage is paid at $179.86 per month for 360 months. Find the interest rate. (PV = 30,000; n = 360; PMT = 179.86).

Answer:

.5% after 8 iterations (monthly interest rate)

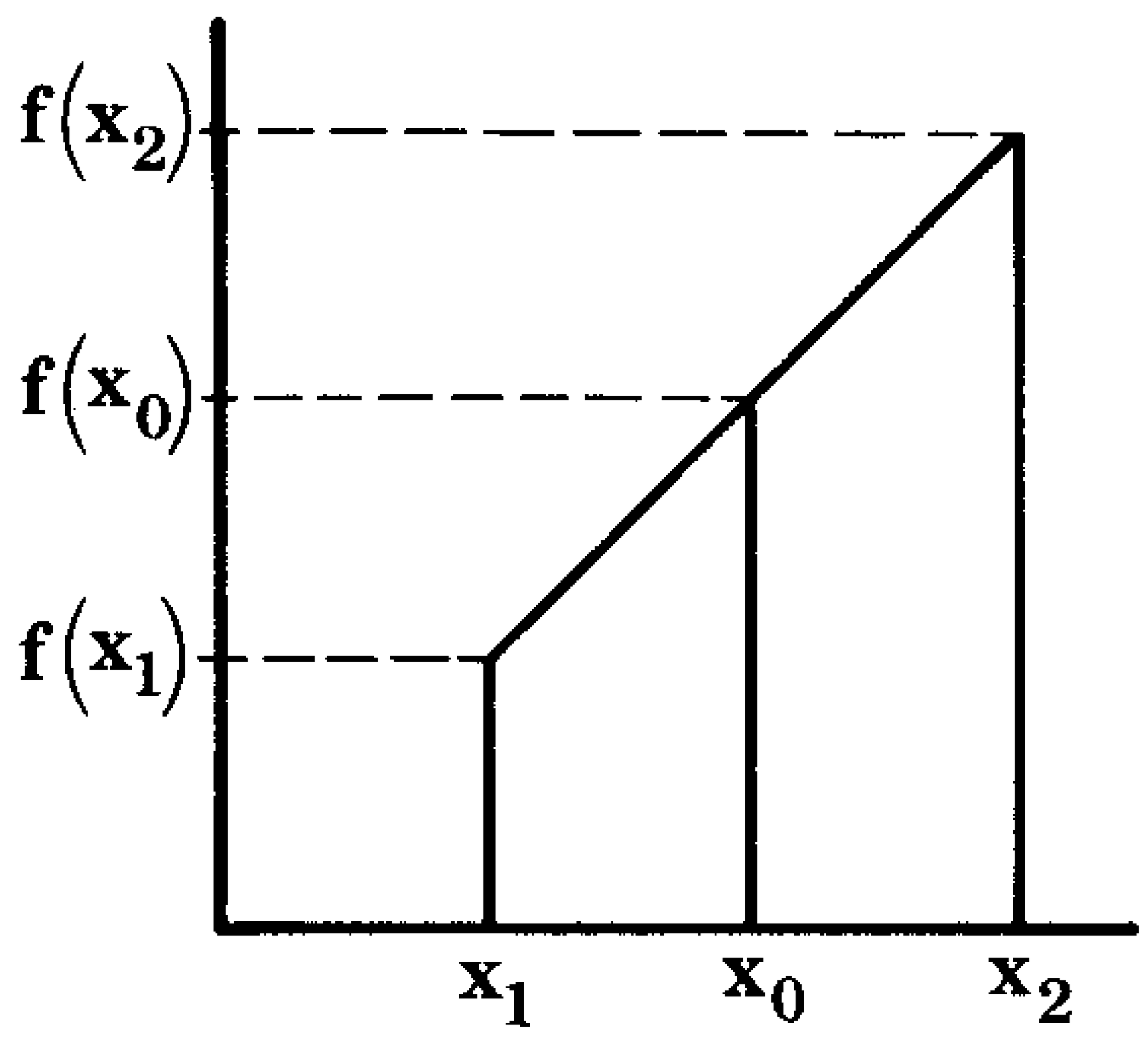
LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	n	↑ ↑		
2	PMT	↑		
3	PV	÷ STO		
4		1 + 1/x x^y 1		Perform lines 4–5 for k = 1, 2, ...
5		x⇄y − RCL ×	D_k	until D_k converges (to 2nd
				decimal place)
6		EEX 2 × EEX 6		
7		+ EEX 6 −		Answer is in %

Interpolation

Formula:

If f(x) is a function of x and $x_1 < x_0 < x_2$, we can approximate $f(x_0)$ by

$$f(x_0) \cong \frac{(x_2 - x_0)\, f(x_1) + (x_0 - x_1)\, f(x_2)}{x_2 - x_1}$$



Example:

Suppose a table shows

x	f(x)
1.2	0.30119
1.3	0.27253

Interpolate f(x) to 5 decimal places for x = 1.27.

Answer:

.28113 (Display = .281128)

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x_0	[↑] [ ] [ ] [ ] [ ]		
2	x_1	[STO] [−] [ ] [ ] [ ]		
3	x_2	[RCL] [−] [÷] [STO] [↑]		
4		[↑] [RCL] [−] [ ] [ ]		
5	$f(x_1)$	[x] [ ] [ ] [ ] [ ]		
6	$f(x_2)$	[RCL] [x] [+] [ ] [ ]		

Short-cut for interpolating tables

Example:

Find f (21.37)

where

x	f(x)
21.3	0.4283
21.4	0.4153

Answer:

Since 21.37 is 7/10 the distance to 21.4 and 3/10 the distance to 21.3,

$$f(21.37) = 0.7\ f(21.4) + 0.3\ f(21.3) = .4192$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	.7	↑		
2	.4153	x		
3	.3	↑		
4	.4283	x +	.4192	

Inverse Gudermannian

See page 87

Inverse Hyperbolic Functions

See pages 88–89

Iterative Solutions to Equations

See page 63

Least Common Multiple

Formula:

The least common multiple of two positive integers a and b is the smallest positive integer that both a and b can divide.

$$LCM\,(a, b) = \frac{a \cdot b}{HCF\,(a, b)}$$

Example:

LCM (51,119) = 357.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	STO						
2	b							
3		↑	↑	RCL	÷		D	Let f be the largest integer ≤ D
4	f	x⇄y	CLX	RCL	x	–	E	If E = 0, go to 6
5		RCL	x⇄y	STO	CLX	+		Go to 3
6		RCL	↑					
7	a	↑						
8	b	x	x⇄y	÷				

Linear Regression

See page 144

Loan Repayments

(See also, simple add-on interest page 94)

Down payment, monthly payments, and finance charge

Formulas:

The key sequence below computes the following values:

DP = down payment (= S x D)

PV = principal (= S – DP)

i = monthly interest rate (= r/12)

n = number of payments (= 12 t)

PMT = monthly payment

$$= PV \left[\frac{i}{1 - (1 + i)^{-n}} \right]$$

TP = total of payments (= n x PMT)

F = finance charge (= TP – PV)

T = total paid for items (= S + F)

Using the following given values:

S = sale price

D = % down payment/100

r = annual interest rate

t = number of years

Example:

Find the monthly payments to buy a tract of land priced at $1195.00 with annual interest 6¼% and 10% down, the balance to be paid over a 7-year period. (r = 0.625, D = 0.1)

Answers:

Down payment = $119.50

Amount financed = $1075.50

Monthly payment = $15.85

Total of payments = $1331.40

Finance charge = $255.90

Total paid for land = $1450.90

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	S	↑	↑					
2	D	x					DP	Down payment
3		–					PV	Amount financed
4	r	↑	1	2	÷	STO	i	Monthly interest rate
5		x						
6	t	↑	1	2	x		n	Number of payments
7		CHS	RCL	1	+	x^y		
8		1	x⇄y	–	÷	.		
9		0	0	5	+	EEX		
10		7	+	EEX	7	–	PMT	Monthly payment
11		1	2	↑				
12	t	x	x				TP	Total of payments
13	PV	–					F	Finance charge
14	S	+					T	Total paid for item

Monthly payments and finance charge

Objective

Find the monthly payments and finance charge if the amount financed is PV, which will be paid over t years at annual interest rate r.

Example:

Find the equal monthly payments and finance charge if the amount financed is $2037.01, which will be paid over 3 years at the rate of 9.25%. (r = .0925)

Answers:

Number of payments = 36

First payment = $65.15

Each payment except first = $65.01

Total of payments = $2340.50

Finance charge = $303.49

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	PV	↑						
2	r	↑	1	2	÷	STO		
3		x						
4	t	↑	1	2	x		n	Number of payments
5		CHS	RCL	1	+	x^y		
6		1	x⇄y	–	÷	STO		
7		↑	.	0	0	5		
8		–	EEX	7	+	EEX		
9		7	–				PMT	Each payment except first
10		RCL	1	2	↑			
11	t	x	x	.	0	0		
12		5	+	EEX	7	+		
13		EEX	7	–			TP	Total of payments
14		STO	↑					
15	PV	–					F	Finance charge
16		x⇄y						
17	n	↑	1	–	x	RCL		
18		x⇄y	–				PMT_1	First payment

Number of payments

Formula:

The number of monthly payments required to pay for an item costing PV dollars at annual rate r if payments are close to PMT dollars is:

$$n = \log_{1+i} \frac{1}{1 - \frac{PV \cdot i}{PMT}}$$

$$i = \frac{r}{12}$$

Example:

How many payments will pay off $4000 at 9.5% annual rate, if the payments are close to $150 per month?

Answer:

30 monthly payments.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		1	↑					
2	r	↑	1	2	÷	STO		
3	PV	RCL	x					
4	PMT	÷	−	1/x	ln	RCL		
5		1	+	ln	÷	EEX		
6		9	+	EEX	9	−		

Logarithms

Logarithm of x to the base y

Formula:

$$\log_y x = \frac{\ln x}{\ln y}$$

Example:

$$\log_7 5 = .827$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x	ln						
2	y	ln	÷					

Means

See page 138–139

Mils to Degrees

See page 14

Modulo: (Y Modulo X)

See page 129

Multiple

See page 102

Normal Function

See page 82

Odometer Adjustments

See page 136

Optimization of Register Use

See page 128

Percent

Percent change

Formula:

$$\% \text{ change} = \frac{V_N - V_O}{V_O} \times 100$$

where V_O = Original Value (base number)

V_N = New Value

Example:

If sales last year were $8.6 million, and this year were $9.3 million, what is the percent change?

Answer:

8.14%

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	V_0	STO		
2	V_N	RCL − RCL ÷ EEX		
3		2 x EEX 6 +		
4		EEX 6 −		

Markup percent

Formula:

To make a gross profit of G%, add A% to the cost price. To find A for a given G:

$$A = \frac{100G}{100 - G}$$

Example:

To make a profit of 30%, what is the percentage of markup?

Answer:

42.86%

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	G	↑ EEX 2 ÷ 1/x		
2		1 − 1/x EEX 2		
3		x EEX 7 + EEX		
4		7 −	A	

Percent profit

Formula:

If A% is added to the cost price, the profit will be G% of the selling price.

$$G = \frac{100A}{A + 100}$$

Example:

If we add 30% to our cost price, what percent of the selling price will be the profit?

Answer:

23.08%

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	A	↑ EEX 2 ÷ 1/x		
2		1 + 1/x EEX 2		
3		x EEX 7 + EEX		
4		7 -	G	

Permutations

Permutations of y objects taken x at a time

Formulas:

$${}_yP_x = P(y, x) = \frac{y!}{(y-x)!} = y \cdot (y-1) \cdot \ldots \cdot (y-x+1)$$

Example:

$${}_7P_5 = 2520$$

Notes:

1. ${}_yP_0 = 1$
2. ${}_yP_1 = y$
3. ${}_yP_y = y!$
4. The following routine applies for $y \geqslant x \geqslant 2$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	y	↑						
2	i	x						Perform line 2 for i = y – x + 1,
								..., y – 1

Plotting Curves

Objective:

The following two routines give values of y = f(x) in x increments of I for values of x between x_0 = a, and x = b where b > a. f(x) should be replaced by appropriate sequence of keystrokes.

Examples of f(x)

For	*Replace f(x) by*
$y = x^2$	↑ ×
y = ln sin x	sin ln
$y = 3\sqrt[x]{x}$	↑ 1/x x⇄y x^y 3 ×
$y = x^4 - 2x^2 + 3x - 7$	↑ ↑ ↑ × 2 − × 3 + × 7 −

Routine 1:

(Saves I in the stack.) When using this routine, f(x) must not destroy the stack value in the Y register.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	I	↑	↑					
2	x_0	↑						
3		f(x)					y_0	
4		CLX	+	+	↑		x_i	Perform lines 4–5 for i = 1,
5		f(x)					y_i	2, ..., k until x_k = b

Example:

Plot $y = \sqrt{\ln \frac{x}{2}}$ from x = 3 to x = 5 at intervals of 0.5. Replace f(x) in routine 1 by

[↑] 2 [÷] [ln] [√x]

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	.5	↑	↑					
2	3	↑					3.	← x_0
3		↑	2	÷	ln	√x	.6367614216	← $f(x_0)$
4		CLX	+	+	↑	↑	3.5	← x_1
5		2	÷	ln	√x		.7480747208	← $f(x_1)$
6		CLX	+	+	↑	↑	4.	← x_2
7		2	÷	ln	√x		.8325546111	← $f(x_2)$
8		CLR	+	+	↑	↑	4.5	← x_3
9		2	÷	ln	√x		.9005166385	← $f(x_3)$
10		CLX	+	+	↑	↑	5.	← x_4
11		2	÷	ln	√x		.957230762	← $f(x_4)$

Routine 2:

(Saves I in S register.) When using this routine, f(x) must not destroy the S register.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	I	STO						
2	x_0	↑						
3		f(x)					y_0	Replace f(x) by proper
								keystrokes
4		CLX	RCL	+	↑		x_i	Perform lines 4–5 for i = 1,
5		f(x)					y_i	2, ..., k, until x_k = b

Example:

Use routine 2 to plot $y = \sqrt{\ln \frac{x}{2}}$ from x = 3 to x = 5 at intervals of 1. Replace f(x) by

[↑] 2 [÷] [ln] [√x]

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	1	STO		
2	3	↑	3.	← x_0
3		↑ 2 ÷ ln √x	.6367614216	← $f(x_0)$
4		CLX RCL + ↑	4.	← x_1
5		↑ 2 ÷ ln √x	.8325546111	← $f(x_1)$
6		CLX RCL + ↑	5.	← x_2
7		↑ 2 ÷ ln √x	.957230762	← $f(x_2)$

Polar to Rectangular

See page 57

Polynomial Approximation

See page 76

Polynomial Evaluation

Formulas:

$$f(x) = c_0 x^n + c_1 x^{n-1} + \ldots + c_{n-1} x + c_n$$

write

$$f(x_0) = (\ldots (((c_0 x_0 + c_1) x_0 + c_2) x_0 + c_3) x_0 + \ldots) x_0 + c_n$$

The following algorithm may even be shortened further:

If $c_0 = 1$, both the c_0 and the **×** after it may be skipped.

If $c_i = 0$, both the c_i and the **+** after it may be skipped.

If $c_i < 0$, press **−** instead of **+** after c_i instead of using CHS .

Example:

If $f(x) = x^5 + 5x^4 - 3x^2 - 7x + 11$, find f(2.5)

Answer: 267.71875

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x_0	↑ ↑ ↑		
2	C_0			
3		×		Perform lines 3–4 for i = 1,
4	C_i	+		2, ..., n

Power Operations

See page 70

Powers

x^y —where x is any number, y is a positive integer

Examples:

$2^4 = 16$

$(3.14)^3 = 30.959$

$(-3.14)^3 = -30.959$

$(-2.34)^8 = 898.932$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	y	[↑] [ ] [ ] [ ] [ ]		
2	x	[ ] [ ] [ ] [ ] [ ]		If x > 0, go to 5
		[ ] [ ] [ ] [ ] [ ]		
3		[CHS] [x^y] [ ] [ ] [ ]		If y is even, stop
		[ ] [ ] [ ] [ ] [ ]		
4		[CHS] [x⇄y] [x⇄y] [ ] [ ]		Stop
		[ ] [ ] [ ] [ ] [ ]		
5		[x^y] [ ] [ ] [ ] [ ]		

Primes

See page 78

Progressions

Formulas:

Arithmetic Progression

$$a, a + d, a + 2d, \ldots, a + (n - 1)d$$

Geometric Progression

$$a, ar, ar^2, \ldots, ar^{n-1}$$

Harmonic Progression

$$\frac{a}{b}, \frac{a}{b + c}, \frac{a}{b + 2c}, \ldots, \frac{a}{b + (n - 1)c}$$

n = number of terms

a = first term in arithmetic and geometric progressions

l = last term

d = difference between two successive terms in an arithmetic progression

r = ratio between two successive terms in a geometric progression

S = sum of a progression

Step through an arithmetic progression

Formula:

$$a, a + d, a + 2d, \ldots, a + (n - 1)d$$

Example:

Display the progression with a = 0, d = 17.

Answer:

0, 17, 34, 51, 68, 85, 102, 119, ...

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	d	↑ ↑ ↑		
2	a			
3		+		Perform line 3 as many times
				as desired

Step through a geometric progression

Formula:

$$a, ar, ar^2, \ldots, ar^{n-1}$$

Example:

Step through the powers of 8.

Answers:

8, 64, 512, 4096, 32768, ...

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	r	↑	↑	↑				
2	a							
3		x						Perform line 3 as many times
								as desired

Step through an harmonic progression

Formula:

$$\frac{a}{b}, \frac{a}{b+c}, \frac{a}{b+2c}, \ldots, \frac{a}{b+(n-1)c}$$

Note: A harmonic progression can be obtained by multiplying the constant a by the reciprocals of the terms of the arithmetic progression b, b + c, b + 2c, ..., b + (n − 1) c. In the following algorithm, x_i (i = 1, 2, ...) represents the i^{th} term of the progression.

Example:

Step through the harmonic progression where a = 1, b = 2, and c = 3

Answers:

.5, .2, .125, .0909, ...

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	STO						
2	c	↑	↑	↑				
3	b	↑	1/x	RCL	x		x_1	
4		CLX	+	+	↑	1/x		Perform lines 4–5 for i = 2, 3, ...
5		RCL	x				x_i	

n^{th} term of arithmetic progression

Formula:

Given the number of terms, the last term of an arithmetic progression is given by

$$n^{th}\ \text{term} = a + (n - 1)\,d$$

Example:

Find the 25^{th} term of the arithmetic progression with $a = 2$, $d = 3.14$.

Answer: 77.36

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	n	↑ 1 –		
2	d	x		
3	a	+		

n^{th} term of geometric progression

Formula:

Given the number of terms, the last term of a geometric progression is given by

$$n^{th}\ \text{term} = ar^{n-1}$$

Example:

Find the 14^{th} term of the geometric progression with $a = 2$, $r = 3.14$.

Answer: 5769197.692

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	n	↑ 1 –		
2	r			If r > 0, go to 5
3		CHS x^y		If n is odd, go to 6
4		CHS x⇄y x⇄y		Go to 6
5		x^y		
6	a	x		

Arithmetic progression sum (knowing the last term)

Formula:

Given the last term, the sum of an arithmetic progression to n terms is

$$S = \frac{n}{2}(a + l)$$

Example:

If a = 3.5, l = 25, and n = 11, find the sum.

Answer:

S = 156.75

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	a	[↑] [ ] [ ] [ ] [ ]		
2	ℓ	[+] [ ] [ ] [ ] [ ]		
3	n	[x] [2] [÷] [ ] [ ]	S	

Arithmetic progression sum (knowing the difference)

Formula:

Given the first term and the difference between two successive terms, the sum of an arithmetic progression to n terms is:

$$S = na + \frac{n(n-1)d}{2}$$

Example:

If a = 3.5, n = 11, and d = 2.15, find the sum of 11 terms.

Answer:

S = 156.75

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	n	[↑] [↑] [1] [−] [ ]		
2	d	[x] [ ] [ ] [ ] [ ]		
3	a	[↑] [2] [x] [+] [x]		
4		[2] [÷] [ ] [ ] [ ]		

Sum of geometric progression (r < 1)

Formula:

The sum of a geometric progression to n terms with $r < 1$ is

$$S = \frac{a(1 - r^n)}{1 - r}$$

Example:

If $a = 1$, $r = -2.1$, and $n = 6$, find the sum of 6 terms.

Answer:

$S = -27.3439$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑	↑					
2	n	↑						
3	r	STO						If r > 0, go to 5
								If n is even, go to 6
4		CHS	x^y	CHS	x⇄y	x⇄y		Go to 7
5		x^y						Go to 7
6		CHS	x^y					
7		x	–	1	RCL	–		
8		÷						

Sum of geometric progression (r > 1)

Formula:

The sum of geometric progression to n terms with r > 1 is

$$S = \frac{a(r^n - 1)}{r - 1}$$

Example:

If a = 1, r = 2.1, n = 6, find the sum.

Answer:

S = 77.06

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑	↑					
2	n	↑						
3	r	STO	x^y	x	x⇄y	−		
4		RCL	1	−	÷			

Sum of infinite geometric progression (−1 < r < 1)

Formula:

$$S = \frac{a}{1 - r}$$

Example:

If a = 2 and r = .5, find the sum.

Answer:

S = 4

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑	1	↑				
2	r	−	÷					

Quadratic Equations

Formulas:

The roots of

$$Ax^2 + Bx + C = 0$$

are

$$\frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{-B}{2A} \pm \sqrt{\left(\frac{-B}{2A}\right)^2 - \frac{2C}{2A}}$$

If

$$D = (B^2 - 4AC)/4A^2$$

is positive, the roots are real; otherwise they are complex, being

$$\frac{-B}{2A} \pm \frac{\sqrt{4AC - B^2}}{2A}\, i\,.$$

Example 1:

Solve $3.142958x^2 - 6.987122x + 1.001976 = 0$

Answers:

$x = 2.069$ and $x = .154$

Example 2:

Solve $-7.231844x^2 + 2.665513x - 3.169115 = 0$

Answers:

Roots are $.1843 + .6358i$ and $.1843 - .6358i$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	A	[↑] [+] [STO] [ ] [ ]		
2	B	[RCL] [÷] [CHS] [↑] [↑]		
3		[x] [ ] [ ] [ ] [ ]		
4	C	[↑] [+] [RCL] [÷] [−]	D	If D < 0, go to 7
		[ ] [ ] [ ] [ ] [ ]		
5		[√x] [STO] [+] [ ] [ ]	Root 1	
6		[x⇄y] [RCL] [−] [ ] [ ]	Root 2	Stop
		[ ] [ ] [ ] [ ] [ ]		
7		[CHS] [√x] [x⇄y] [ ] [ ]	Real part	
8		[x⇄y] [ ] [ ] [ ] [ ]	Imag. part	

Questions and Answers

See page 163

Radians to Degrees

See page 13

Random Numbers

Objective:

This routine will use a "seed" S to generate a sequence of pseudo random numbers R_i in either of two ranges −1 to 0 (if $S < 0$) or 0 to 1 (if $S > 0$). For best results, the seed must be a ten digit decimal fraction containing all digits 0 through 9 in an arbitrary order.

Example:

If S = .2510637948, generate a random series.

Answers:

.280850049, .144651421, .194891209, .651845061
.90350677, .20169633, .84919357, etc.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		2	9	↑	↑	↑		
2	S							
3		x					D_i	Perform lines 3–4 for i = 1,
4	f_i	−					R_i	2, 3, ... Let f_i = integer
								part of D_i

Application

Suppose S is a positive seed, and L is any integer. The following program will simulate

(a) tossing a coin, if L = 2,

(b) throwing a dice, if L = 6,

(c) picking integers between 1 and 100, if L = 100.

Example:

Let S = .2510637948, the following sequences of numbers N_i will be generated:

(a) (L = 2) 1, 1, 1, 2, 2, 1, ...

(b) (L = 6) 2, 1, 2, 4, 6, 2, ...

(c) (L = 100) 29, 15, 20, 66, 91, 21, ...

Note: $1 \leqslant N_i \leqslant L$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	L	STO	2	9	↑	↑		
2	S	↑						
3		R↓	x				D_i	Perform lines 3–5 for i = 1,
4	f_i	–	↑	↑	RCL	x		2, 3, ... Let f_i = integer
								part of D_i
5		1	+				E_i	N_i = integer part of E_i

Rectangular to Polar

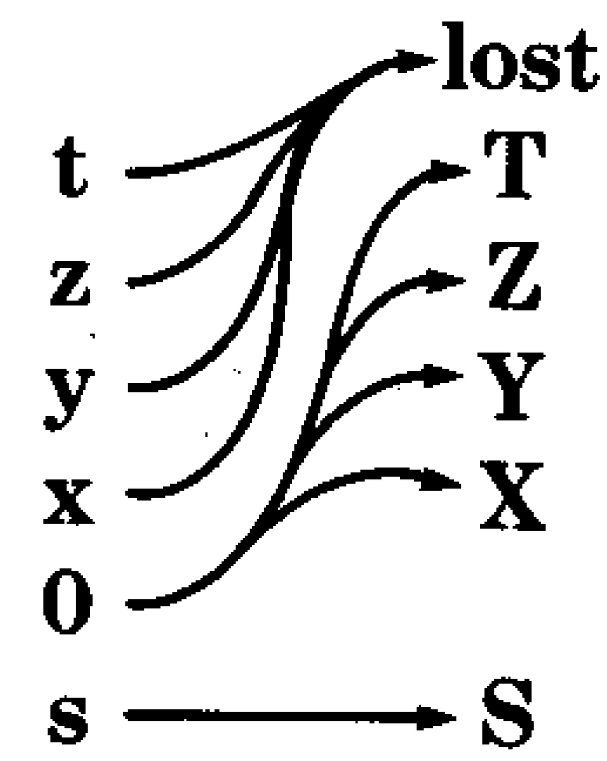
See page 57

Register Operations

In the following eighteen routines, x, y, z, t and s denote the contents of registers X, Y, Z, T and S, respectively.

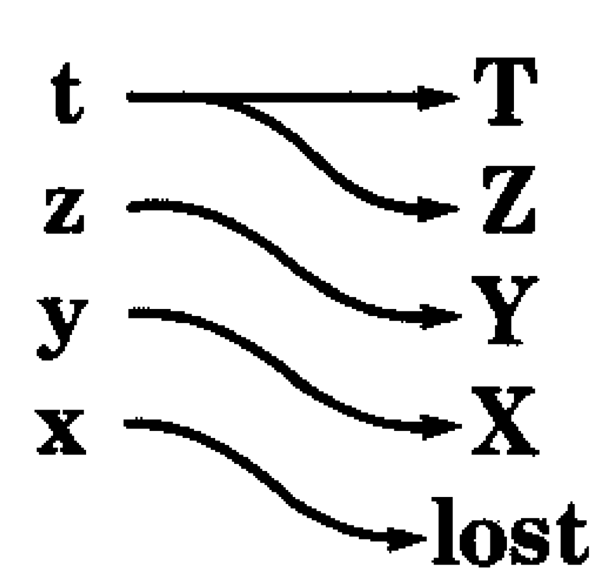
The following techniques may be useful when writing your own key sequence routines.

1. Clear stack; retain storage



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		[CLX] [↑] [↑] [↑] [ ]		

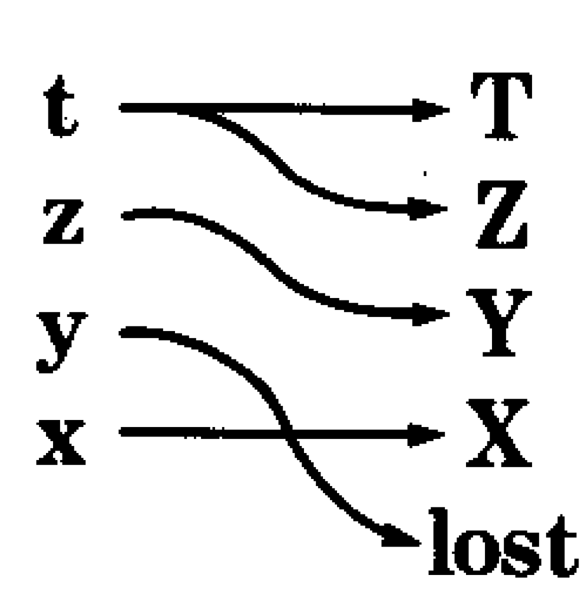
2. Delete x



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		[CLX] [+] [ ] [ ] [ ]		

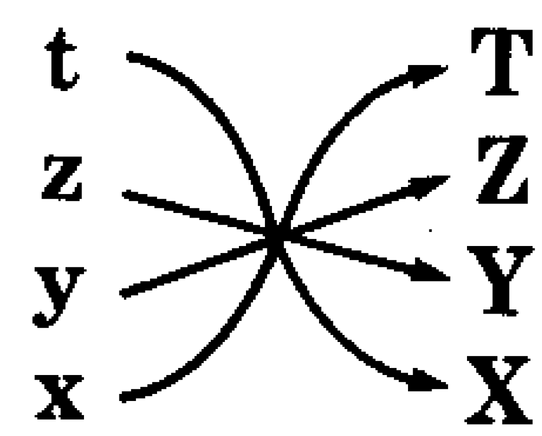
3. Delete y

(Lower that part of the stack above x)



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		[x⇄y] [CLX] [+] [ ] [ ]		

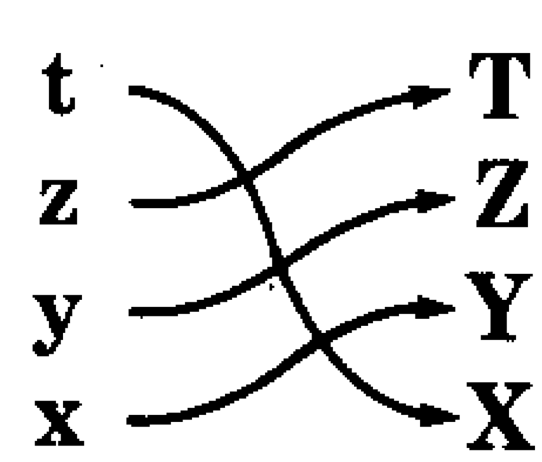
4. Reverse the stack



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		[x⇄y] [R↓] [R↓] [x⇄y] [ ]		

5. Fetch t or roll up

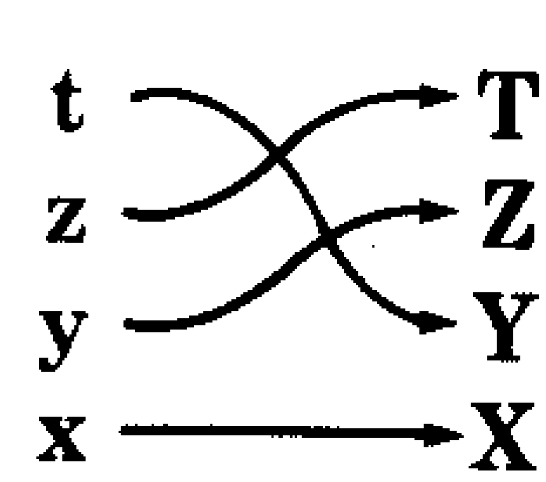
(Bring t to X, keeping the other operands in the same order)



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		[R↓] [R↓] [R↓] [ ] [ ]		

6. Fetch t to Y

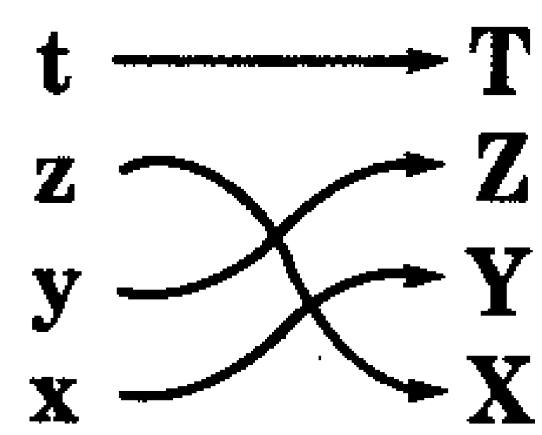
(Bring t to Y, keeping the other operands in the same order)



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		[R↓] [R↓] [R↓] [x⇄y] [ ]		

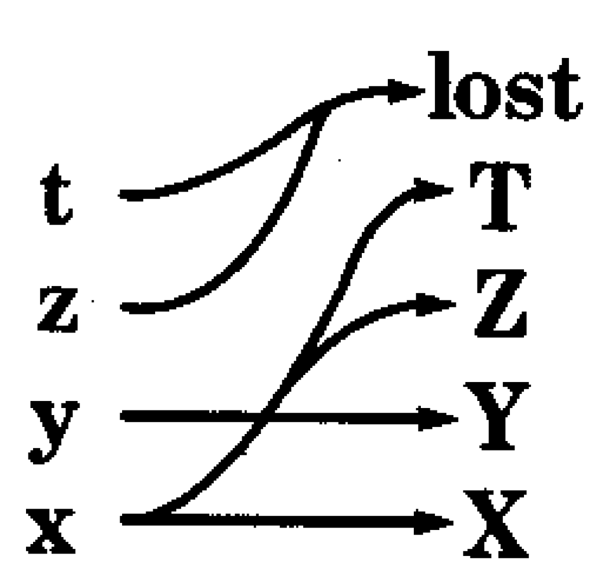
7. Fetch z to X

(Bring z to X, keeping the other operands in the same order)



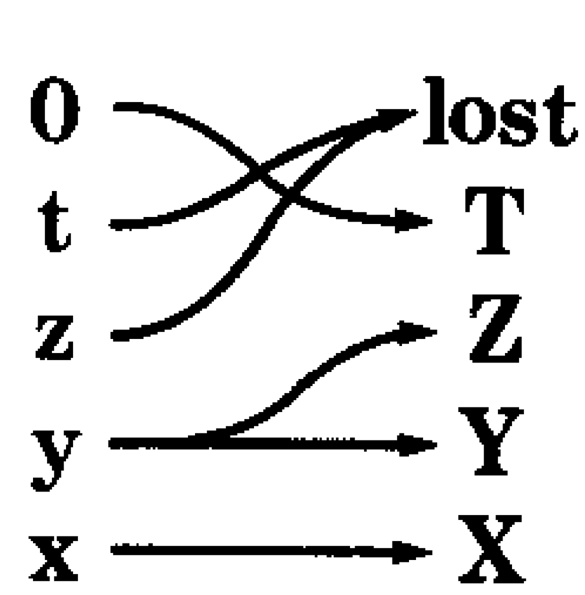
LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		R↓ R↓ x⇄y R↓		

8. Copy x into Z and T



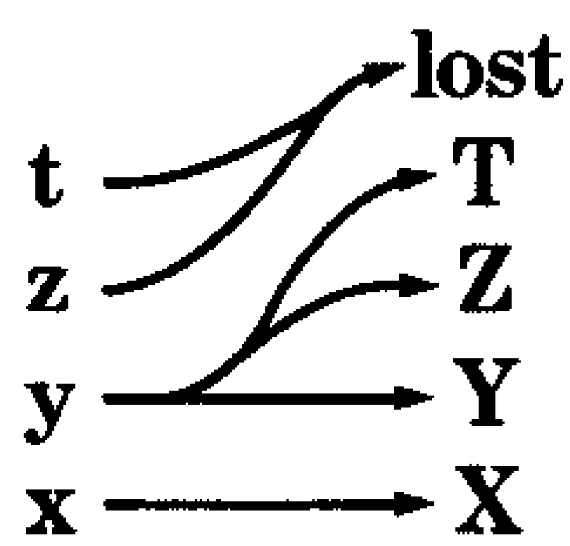
LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		↑ ↑ R↓ R↓		

9. Copy y into Z (T is cleared)



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		↑ ↑ – R↓		

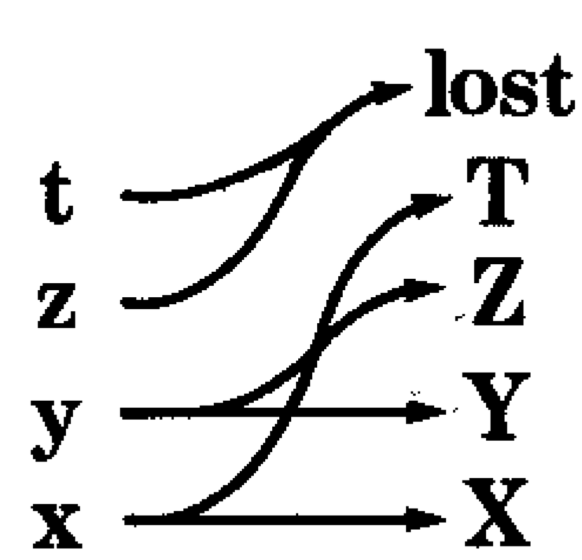
10. Copy y into Z and T



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		[x⇄y] [↑] [↑] [R↓] [R↓]		
2		[R↓] [ ] [ ] [ ] [ ]		

11. Copy y and x into Z and T respectively

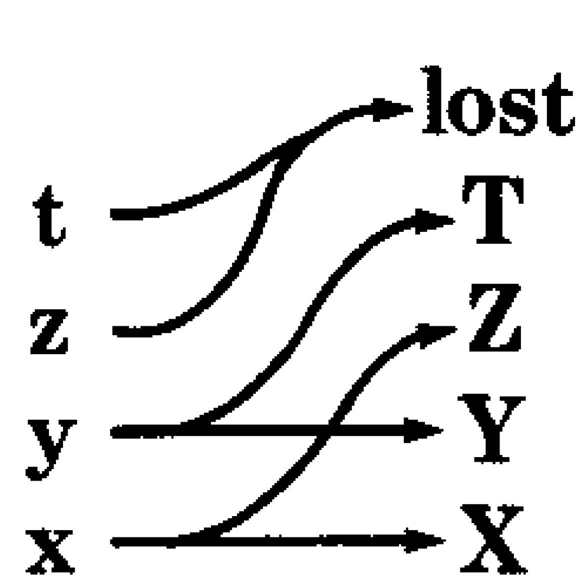
(Copy x and y in reverse stack order, but this is the shortest way to save both x and y in the stack)



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		[↑] [↑] [CLX] [+] [R↓]		

12. Copy y and x into T and Z respectively

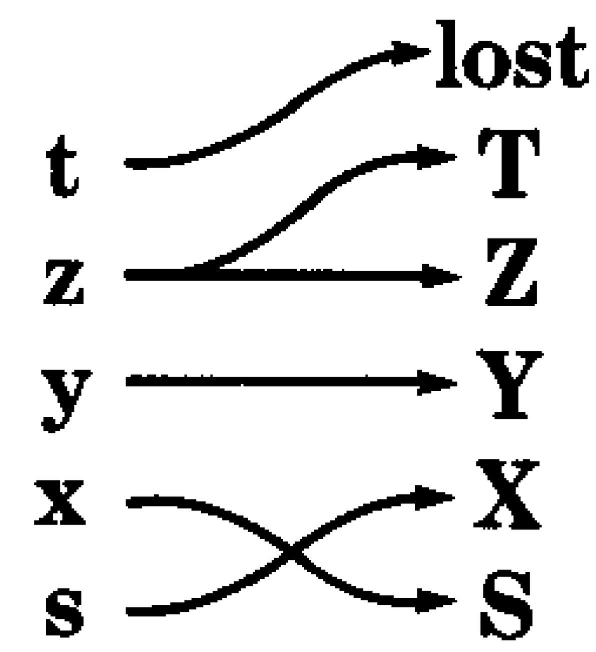
(Copy x and y in the same stack order to Z and T)



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		[x⇄y] [↑] [↑] [CLX] [+]		
2		[R↓] [x⇄y] [ ] [ ] [ ]		

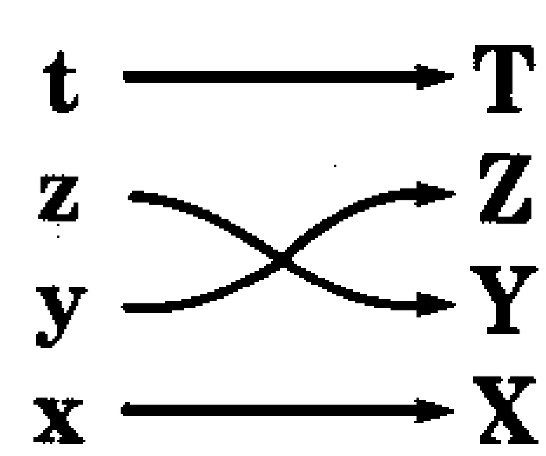
13. Swap x and s

(Exchange x and s, t is lost)



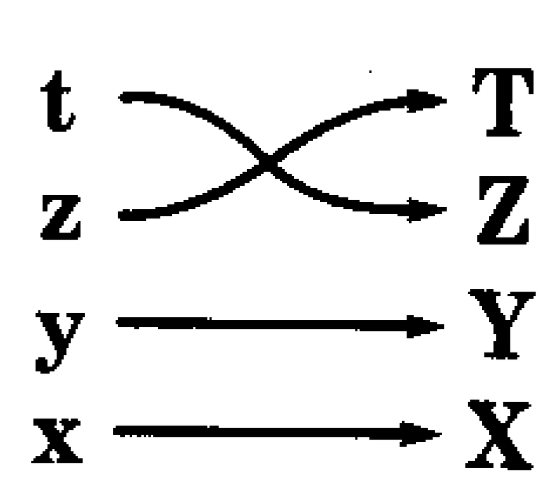
LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		RCL x⇄y STO CLX +		

14. Swap y and z



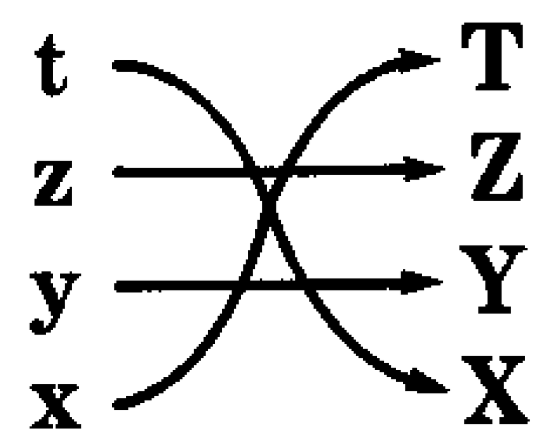
LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		R↓ x⇄y R↓ R↓ R↓		

15. Swap z and t



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		R↓ R↓ x⇄y R↓ R↓		

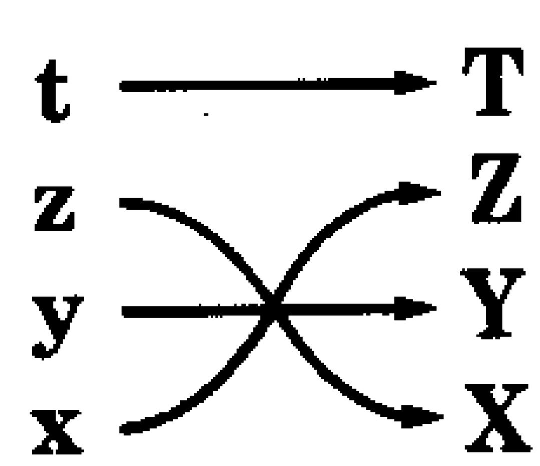
16. Swap x and t



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		[R↓] [R↓] [R↓] [x⇄y] [R↓]		

17. Swap x and z

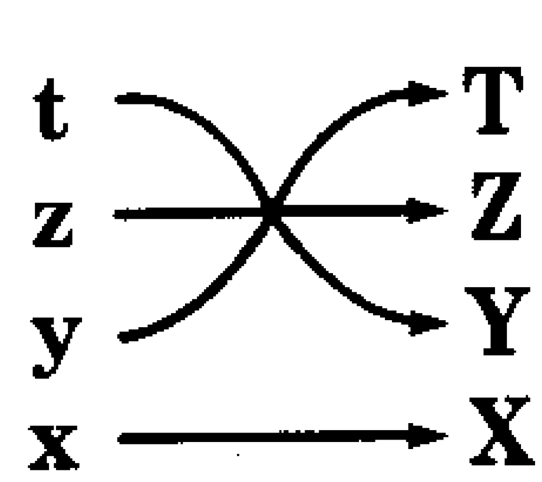
(Reverse contents of X, Y, Z)



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		[x⇄y] [R↓] [R↓] [x⇄y] [R↓]		

18. Swap y and t

(Reverse contents of Y, Z, T)



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		[R↓] [x⇄y] [R↓] [R↓] [x⇄y]		

Register Use Optimization

Illustrative routines to show the trade offs between the number of keystrokes and number of registers used.

Example 1:

Evaluate $\frac{A \times B}{B + C}$

5 keystrokes (+ data), 3 registers

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	B	↑	↑					
2	C	+	÷					
3	A	×						

Divide both the numerator and denominator by B.

$$\frac{A}{1 + \frac{C}{B}}$$

6 keystrokes (+ data), 2 registers

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	C	↑						
2	B	÷	1	+	1/x			
3	A	×						

Example 2:

Evaluate $\frac{A}{A + \frac{B}{C + D}}$

7 keystrokes (+ data), 4 registers

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	A	↑						
2	B	↑						
3	C	↑						
4	D	+	÷	+	÷			

Divide both the numerator and denominator by A.

$$\frac{1}{1 + \dfrac{B}{A(C + D)}}$$

8 keystrokes (+ data), 2 registers

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	C	[↑] [ ] [ ] [ ] [ ]		
2	D	[+] [ ] [ ] [ ] [ ]		
3	A	[x] [1/x] [ ] [ ] [ ]		
4	B	[x] [1] [+] [1/x] [ ]		

Remainder Function

y modulo x

Example:

7 modulo 5 = 2

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	y	[↑] [ ] [ ] [ ] [ ]		
2	x	[↑] [↑] [–] [R↓] [STO]		
3		[÷] [ ] [ ] [ ] [ ]	D	Let f be the largest integer ≤ D
4	f	[x⇄y] [CLX] [RCL] [x] [–]		

Resistance

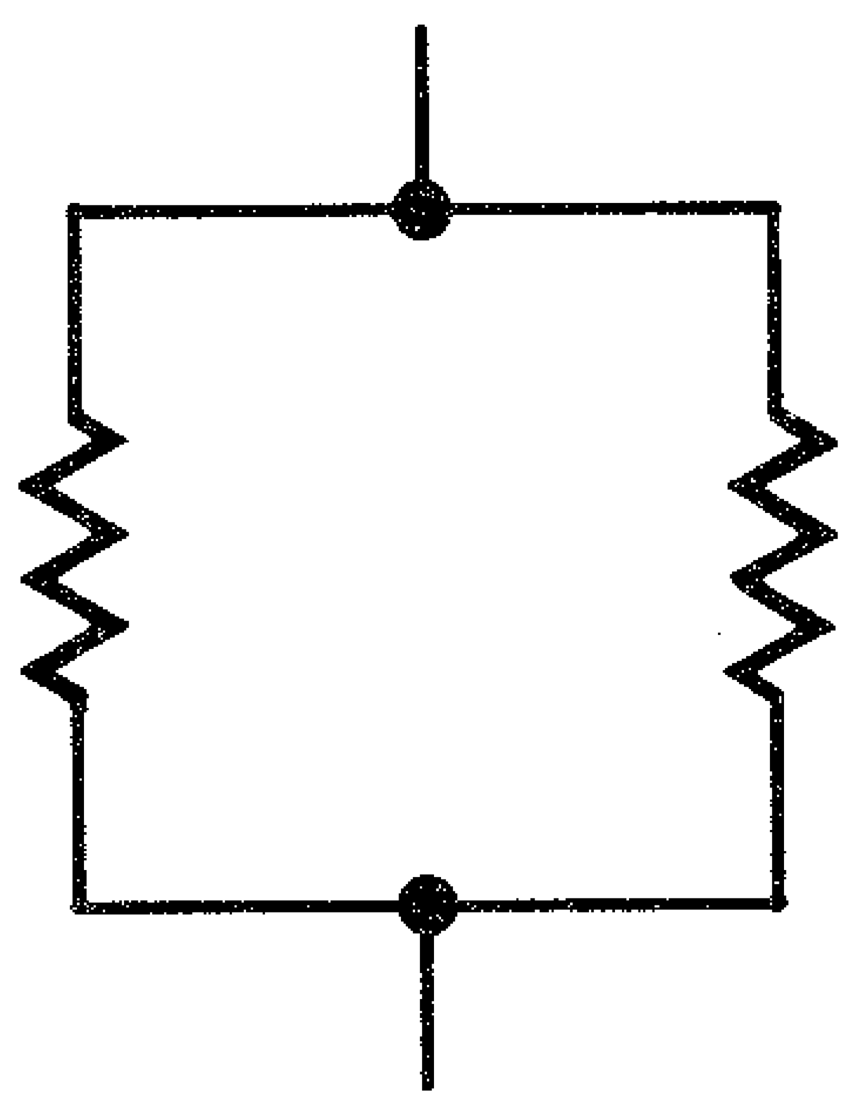
Formula:

The equivalent resistance of a parallel combination of resistors is

$$Req = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}}$$

Example:

Find Req.



Answer:

Req = 3.

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	R_1	[1/x] [ ] [ ] [ ] [ ]		
2	R_i	[1/x] [+] [ ] [ ] [ ]		Perform line 2 for i = 2, ..., n
3		[1/x] [ ] [ ] [ ] [ ]		

Roots

x^{th} root of y (> 0)

Example:

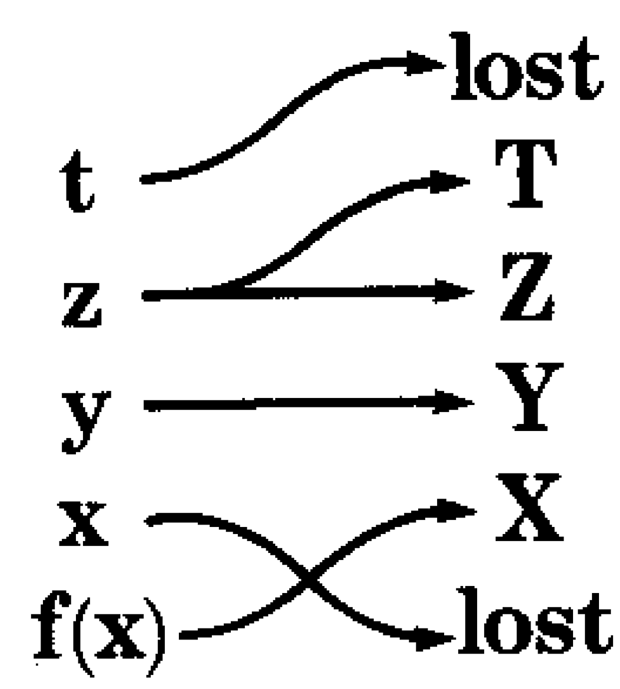
$$\sqrt[5]{7} = 1.47577$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	[1/x] [ ] [ ] [ ] [ ]		
2	y	[x^y] [ ] [ ] [ ] [ ]		

Rounding Numbers

Objective:

Round a number A to the n^{th} decimal place in the HP-35 for n = 0, 1, ..., 8 (if n = 0, round the number to an integer).



Examples:

1. Round 2.343678 to the 2^{nd} decimal place.
2. Round −4.3128057 to the 5^{th} decimal place.
3. Round −1.334455 to an integer.

Answers:

1. 2.34
2. −4.31281
3. −1.

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	A	↑ EEX		Let m = 9 − n
2	m			If A < 0, go to 5
3		+ EEX		
4	m	−		Stop
5		− EEX		
6	m	+		

Secant

See page 156

Sides of Triangle

See pages 147–151

Simultaneous Linear Equations

Formulas:

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

Determinant solution to those simultaneous equations:

$$x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{D}$$

$$y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{D}$$

$$z = \frac{d_1 - b_1y - a_1x}{c_1}$$

where

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$= a_1 b_2 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2 - a_3 b_2 c_1 - a_2 b_1 c_3 - a_1 b_3 c_2$$

$$\neq 0$$

Example:

$$\text{Solve} \begin{cases} 3x - 2y + z = 3 \\ 9x + 3y - 5z = -2 \\ 2x - y - 2z = -15 \end{cases}$$

Answers:

$$x = 2, y = 5, z = 7$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a_1	↑						
2	b_2	x						
3	c_3	x						
4	b_1	↑						
5	c_2	x						
6	a_3	x	+					
7	c_1	↑						
8	a_2	x						
9	b_3	x	+					
10	a_3	↑						
11	b_2	x						
12	c_1	x	−					
13	b_3	↑						
14	c_2	x						
15	a_1	x	−					
16	c_3	↑						
17	a_2	x						
18	b_1	x	−	STO			D	
19	d_1	↑						
20	b_2	x						
21	c_3	x						
22	b_1	↑						
23	c_2	x						
24	d_3	x	+					
25	c_1	↑						
26	d_2	x						
27	b_3	x	+					
28	d_3	↑						
29	b_2	x						
30	c_1	x	−					
31	b_3	↑						
32	c_2	x						
33	d_1	x	−					
34	c_3	↑						
35	d_2	x						
36	b_1	x	−	RCL	÷		x	
37	a_1	↑						
38	d_2	x						
39	c_3	x						

Continued

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
40	d_1	↑						
41	c_2	x						
42	a_3	x	+					
43	c_1	↑						
44	a_2	x						
45	d_3	x	+					
46	a_3	↑						
47	d_2	x						
48	c_1	x	–					
49	d_3	↑						
50	c_2	x						
51	a_1	x	–					
52	c_3	↑						
53	a_2	x						
54	d_1	x	–	RCL	÷		y	
55	b_1	x	CHS	↑				
56	d_1	+						
57	a_1	↑						
58	x	x	–					
59	c_1	÷					z	

Sinking Fund and Amortization

Sinking fund

Formula:

The sinking fund method of debt repayment is based on accumulating funds through periodic deposits which also earn interest. The periodic payment PMT required to accumulate a given future value FV is given by the formula

$$PMT = FV \frac{i}{(1+i)^n - 1}$$

where i = interest rate

n = number of payments

Example:

How much money must be deposited at the end of each year in a 15 year sinking fund to accumulate \$1000 if interest is to be paid at 5% annually? (i = 0.05)

Answer:

\$46.34

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	i	[↑] [STO] [ ] [ ] [ ]		
2	n	[↑] [1] [RCL] [+] [x^y]		
3		[1] [–] [÷] [ ] [ ]		
4	FV	[x] [EEX] [7] [+] [EEX]		
5		[7] [–] [ ] [ ] [ ]		

Amortization

To repay a loan by amortization, a payment amount is chosen that will (1) pay the current interest on the declining balance and (2) reduce the balance by whatever amount is not needed for interest.

$$PMT = PV \left[\frac{i}{1 - (1 + i)^{-n}} \right]$$

where n = number of payments

i = periodic interest rate

Example:

Find the uniform month-end payment that will repay a loan of \$1000 in 20 months at 6% annual interest. (i = 0.06/12)

Answer:

\$52.67

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	n	[↑] [1] [↑] [ ] [ ]		
2	i	[STO] [+] [x^y] [$1/x$] [1]		
3		[x⇄y] [–] [RCL] [x⇄y] [÷]		
4	PV	[x] [EEX] [7] [+] [EEX]		
5		[7] [–] [ ] [ ] [ ]		

Speedometer/Odometer Adjustments

Objective:

Assume that you are an automobile passenger approaching a speedometer test section with an HP-35 in your hand. You want to calculate

TS – true speed of the car

RS – what your speedometer will register at a posted speed

DS – distance traveled after a trip

RO – the reading of the odometer after a specific distance d

Notation:

a_1 = mileage read at beginning of trip

a_2 = mileage read at end of trip

b_1 = mileage read at beginning of test section (to nearest 20th of a mile, if possible)

b_2 = mileage read at end of test section

L = length of test section

s = speedometer value

p = posted speed

q = present milage

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	b_1	CHS ↑		
2	b_2	+		
3	L	÷ STO		
4				For TS, go to 5
				For RS, go to 6
				For DS, go to 7
				For RD, go to 9
5	s	RCL ÷	TS	Go to 4
6	p	RCL x	RS	Go to 4
7	a_2	↑		
8	a_1	– RCL ÷	DS	Go to 4
9	d	RCL x		
10	q	+	RD	Go to 4

Example:

Assume the following:

a_1 = 2185.2	Mileage at start of trip
b_1 = 2219.4	Mileage at start of test section
b_2 = 2224.15	Mileage at end of test section
L = 5 miles	Length of test section

1. How fast is the car actually traveling when the speedometer registers (a) 50 mph? (b) 70 mph? (c) 30 mph?
2. What will the speedometer register if the true speed is (a) 65 mph? (b) 55 mph?
3. When the odometer reads 2236.3, how many miles have you traveled since the start of your trip.
4. You see a sign saying "DOEVILLE 48". If your odometer now reads 2241.5, what will it register when you arrive at DOEVILLE?

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	2219.4	CHS ↑		
2	2224.15	+		
3	5	÷ STO		
4	50	RCL ÷	52.63157895	Answer for 1a
5	70	RCL ÷	73.68421053	Answer for 1b
6	30	RCL ÷	31.57894737	Answer for 1c
7	65	RCL x	61.75	Answer for 2a
8	55	RCL x	52.25	Answer for 2b
9	2236.3	↑		
10	2185.2	- RCL ÷	53.78947368	Answer for 3
11	48	RCL x		
12	2241.5	+	2287.1	Answer for 4

Square of x

$f(x) = x^2$

Example:

$(2.359)^2 = 5.564881$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	↑ x		

Stack Operations

See page 122

Standard Deviation

See page 141

Statistics

Arithmetic mean

Formula:

The arithmetic mean (average) of a set of numbers

$$\{a_1, a_2, \ldots, a_n\}$$

is

$$A = \frac{1}{n} \sum_{i=1}^{n} a_i$$

Example:

Compute the arithmetic mean of

2, 3.4, 3.41, 7, 11, 23

Answer:

8.3

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	a_1	↑		
2	a_i	+		Perform line 2 for i = 2, ..., n
3	n	÷		

Geometric mean

Formula:

The geometric mean of a set of numbers

$$\{a_1, a_2, \ldots, a_n\}$$

is

$$G = \sqrt[n]{a_1 \cdot a_2 \cdot \ldots \cdot a_n}$$

Example:

Compute the geometric mean of

$$2, 3.4, 3.41, 7, 11, 23$$

Answer:

5.87

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	a_1	[↑] [ ] [ ] [ ] [ ]		
2	a_i	[×] [ ] [ ] [ ] [ ]		Perform line 2 for i = 2, 3, ..., n
3	n	[1/x] [x⇄y] [x^y] [ ] [ ]		

Harmonic mean

Formula:

The harmonic mean of a set of numbers

$$\left\{a_1, a_2, \ldots, a_n\right\}$$

is

$$H = \frac{n}{\sum_{i=1}^{n} \frac{1}{a_i}}$$

Example:

Find the harmonic mean of

$$2, 3.4, 3.41, 7, 11, 23$$

Answer:

4.3968

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	a_1	[1/x] [ ] [ ] [ ] [ ]		
2	a_i	[1/x] [+] [ ] [ ] [ ]		Perform line 2 for i = 2, 3, ..., n
3	n	[1/x] [×] [1/x] [ ] [ ]		

Average and count simultaneously

Constraints

1. Averaged values must be positive integers.
2. Number of values may not exceed 999.
3. Sum of values may not exceed 10 million.

This technique is advantageous in that entries are counted as they are added. In the following routine, X_{L3} refers to the rightmost three digits visible in the display.

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		CLR						
2	n_i	0	0	1	+			Perform line 2 for i = 1, ..., n
3	X_{L3}	÷	1	-	EEX	3		
4		÷						

Example:

Assume that someone is calling out the following series of integers, and you don't know how many there will be, compute the average.

37, 16, 43, 18, 29, 11, 43, 2, 61, 73, 54, 19, 28

Answer:

33.38461538

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		CLR						
2	37001	+						
3	16001	+						
4	43001	+						
5	18001	+						
6	29001	+						
7	11001	+						
8	43001	+						
9	2001	+						
10	61001	+						
11	73001	+						
12	54001	+						
13	19001	+						
14	28001	+					434013	
15	13	÷	1	-	EEX	3		
16		÷					33.38461538	

Summing two columns simultaneously

Example:

Sum the following two columns simultaneously.

x_i	y_i
14	37.2
3	190.15
438	16.4
93	.005
1736	41.81
19	11.13

Answer: SUM x = 2303 SUM y = 296.695

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x_1	[R↓] [R↓] [R↓] [ ] [ ]		
2	y_1	[R↓] [R↓] [ ] [ ] [ ]		
3	x_i	[+] [R↓] [R↓] [ ] [ ]		Perform lines 3–4 for i = 2,
		[ ] [ ] [ ] [ ] [ ]		3, ..., n
4	y_i	[+] [R↓] [R↓] [ ] [ ]		
5		[x⇄y] [CLX] [+] [ ] [ ]	SUM_x	
6		[x⇄y] [ ] [ ] [ ] [ ]	SUM_y	
		[ ] [ ] [ ] [ ] [ ]		

Mean, standard deviation, and sums (ungrouped data)

Formulas:

Mean:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Standard deviation:

$$S = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}{n-1}}$$

Sum of values:

$$Sx = \sum_{i=1}^{n} x_i$$

Sum of squared values:

$$SS = \sum_{i=1}^{n} x_i^2$$

Example:

Compute $\overline{x}$, S_n, Sx, SS for the set of numbers $\{2, 3.4, 3.41, 7, 11, 23\}$, we have Sx = 49.81, SS = 726.1881, $\overline{x}$ = 8.3, S = 7.9.

Note: This routine can be used to find Sx and SS only.

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		CLR		
2	x_i	STO + x⇄y RCL ↑		Perform lines 2–3 for i = 1,
				2, ..., n
3		× + x⇄y		
4			Sx	
5		x⇄y	SS	Stop if $\overline{x}$ and S are not needed
6		x⇄y ↑ ↑		
7	n	STO ÷	$\overline{x}$	
8		R↓ ↑ × RCL ÷		
9		− RCL 1 − ÷		
10		√x	S	

Mean, standard deviation, and sums (grouped data)

Formulas:

Given a set of data points

$$x_1, x_2, \ldots, x_n$$

with respective frequencies

$$f_1, f_2, \ldots, f_n$$

Let $k = \sum_{i=1}^{n} f_i$.

Then

$$\overline{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

$$S = \sqrt{\frac{\sum_{i=1}^{n} f_i x_i^2 - \frac{\left(\sum_{i=1}^{n} f_i x_i\right)^2}{k}}{k-1}}$$

$$Sx = \sum_{i=1}^{n} f_i x_i$$

$$SS = \sum_{i=1}^{n} f_i x_i^2$$

Example:

Compute mean, standard deviation, and sums for

f_i	3	3	1	2	1
x_i	1	2	3	4	5

Answer:

$k = 10$

$Sx = 25$

$SS = 81$

$\bar{x} = 2.5$

$S = 1.43$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1		CLR						
2	f_i	↑						Perform lines 2–5 for i = 1, ..., n
3	x_i	STO	x	↑	R↓	+		
4		R↓	R↓	R↓	RCL	x		
5		R↓	R↓	R↓	+	x⇄y		
6							Sx	
7		x⇄y					SS	Stop if $\bar{x}$ and S are not needed
8		x⇄y	↑	↑				
9	k	STO	÷				$\bar{x}$	
10		R↓	↑	x	RCL	÷		
11		–	RCL	1	–	÷		
12		$\sqrt{x}$					S	

Cross product sum

Formula:

$$\sum_{i=1}^{n} x_i y_i$$

Example:

Find the cross product sums for the following:

x_i	1	2	3	4	5
y_i	2	3.4	3.41	7	11

Answer:

$$\sum_{i=1}^{5} x_i y_i = 102.03$$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x_1	↑						
2	y_1	×						
3	x_i	↑						Perform lines 3–4 for i = 2,
4	y_i	×	+					3, ..., n

Linear regression and correlation coefficient

Objective:

1. For a set of data points

$$\left\{(x_i, y_i), i = 1, \ldots, n\right\}$$

fit a straight line to them having minimum least squares deviation.

2. Find the correlation of x and y.

Formulas:

$$y = ax + b$$

$$a = \frac{\Sigma x_i y_i - \dfrac{\Sigma x_i \, \Sigma y_i}{n}}{\Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n}}$$

$$b = \bar{y} - a\bar{x}$$

where

$$\overline{x} = \frac{\Sigma x_i}{n}$$

$$\overline{y} = \frac{\Sigma y_i}{n}$$

Coefficient of determination

$$r^2 = \frac{\left[\Sigma x_i y_i - \frac{\Sigma x \, \Sigma y}{n}\right]^2}{\left[\Sigma x_i{}^2 - \frac{(\Sigma x_i)^2}{n}\right]\left[\Sigma y_i{}^2 - \frac{(\Sigma y_i)^2}{n}\right]}$$

Correlation coefficient

$$r = \sqrt{r^2} \text{ (using the sign of a)}$$

Example:

x_i	26	30	44	50	62	68	74
y_i	92	85	78	81	54	51	40

Answers:

Sx = 354

SSx = 19956

Sy = 481

SSy = 35451

Sxy = 22200

y = −1.03x + 121.04 = ax + b

$r^2 = 0.92$

r = −.957

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		CLR		
2	x_i	STO + x⇄y RCL ↑		Perform lines 2–3 for i = 1,
				2, ..., n
3		x + x⇄y		
4			Sx	
5		x⇄y	SSx	
6		CLR		
7	y_i	STO + x⇄y RCL ↑		Perform lines 7–8 for i = 1,
				2, ..., n
8		x + x⇄y		
9			Sy	
10		x⇄y	SSy	
11		CLR		
12	x_i	↑		Perform lines 12–13 for i = 1,
13	y_i	x +		2, ..., n
14			Sxy	
15	Sx	STO ↑		
16	Sy	x		
17	n	÷ – ↑ ↑ ↑		
18	SSx	↑ RCL ↑ x		
19	n	÷ – ÷ ↑ STO	a	
20	Sy	x⇄y		
21	Sx	x –		
22	n	÷	b	Stop if r^2 is not needed
23		R↓ RCL x		
24	SSy	↑		
25	Sy	↑ x		
26	n	÷ – ÷	r^2	
27		$\sqrt{x}$	r	If $a > 0$, stop
28		CHS	r	

Stirling's Approximation

See page 75

Sum Two Columns Simultaneously

See page 141

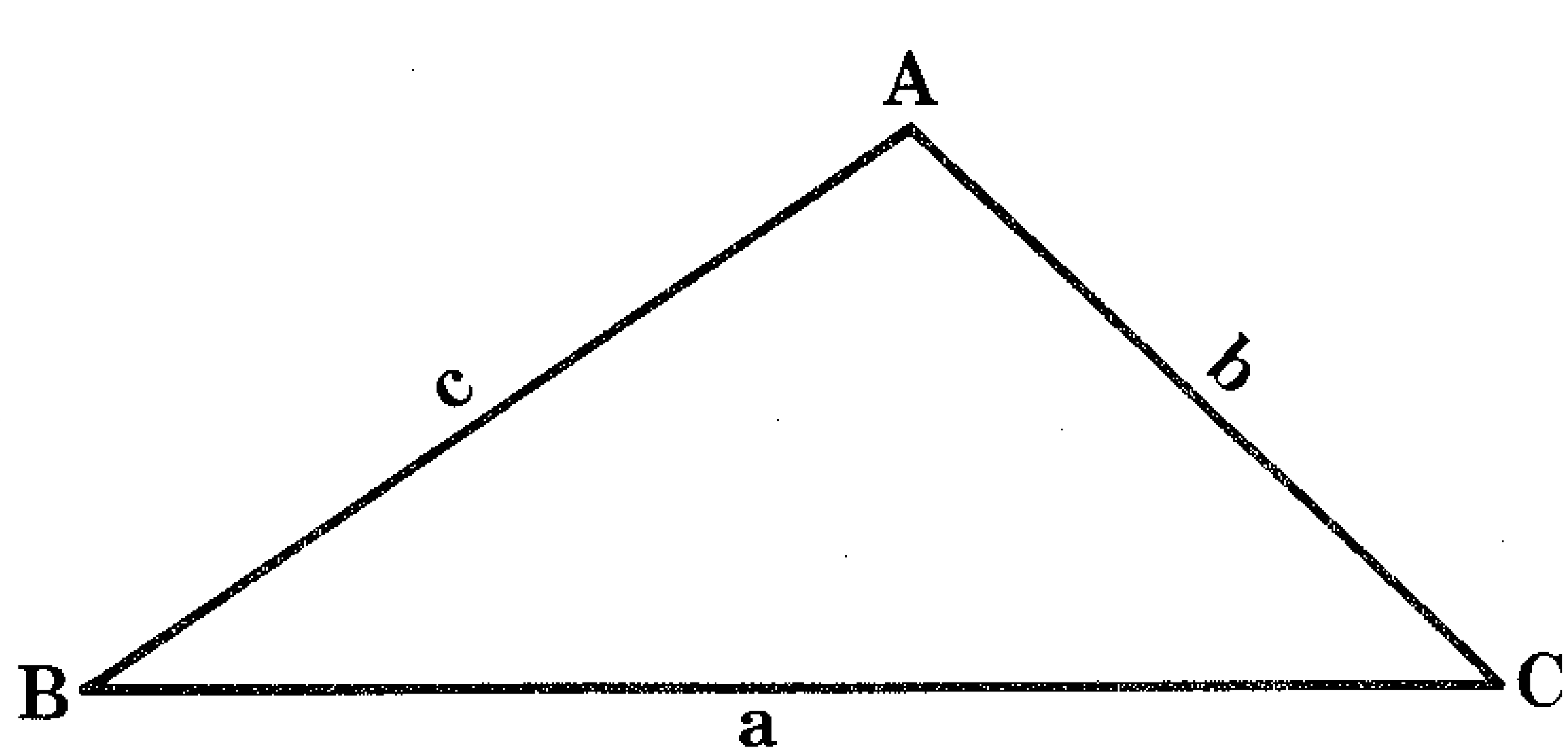
Triangles (Oblique)

Law of sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of cosines

$$c^2 = a^2 + b^2 - 2ab \cos C$$

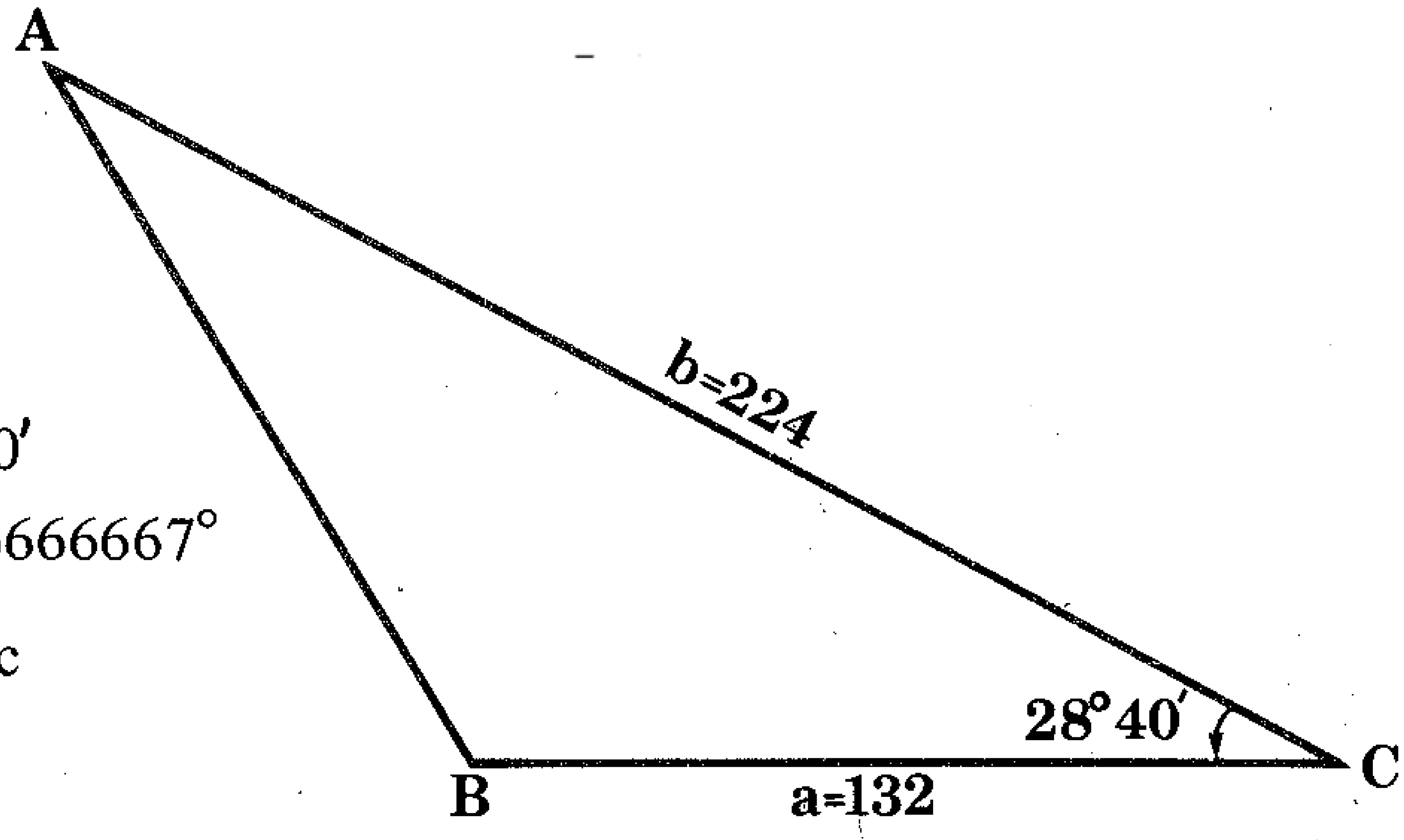


Notes:

1. When [arc] [sin], [arc] [cos], [arc] [tan] are used in programs, secondary values may be required instead of the principal values.

2. A . B . C are in degrees

Given a, b, C, find A, B, c.



Example:

Given a = 132
b = 224
C = 28°40′
= 28.66666667°

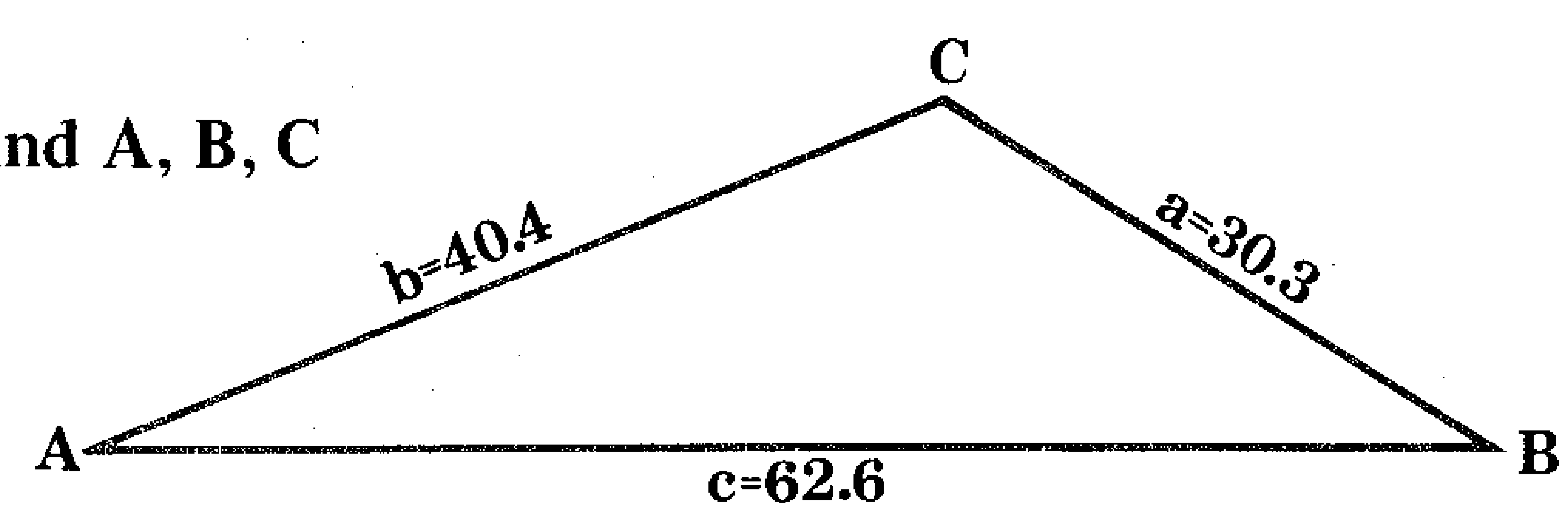
Find A, B, and c

Answer:

c = 125.35
A = 30.34°
B = 120.99° (secondary value)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑	↑	x				
2	b	↑	STO	x	+	x⇄y		
3		2	x	RCL	x			
4	C	STO	cos	x	−	√x	c	
5		↑	↑	↑	RCL	sin		
6		STO	↑					
7	a	x	x⇄y	÷	arc	sin	A	
8		R↓	RCL					
9	b	x	x⇄y	÷	arc	sin	B	

Given a, b, c; find A, B, C



Example:

Given a = 30.3
b = 40.4
c = 62.6

then find A, B, and C.

Answer:

A = 23.66°

B = 32.35°

C = 123.99°

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	a	↑ ↑ x		
2	b	↑ STO x +		
3	c	↑ x – x⇄y RCL		
4		x 2 x ÷ arc		
5		cos	C	
6	a	↑ ↑ x		
7	c	↑ STO x +		
8	b	↑ x – x⇄y RCL		
9		x 2 x ÷ arc		
10		cos ↑	B	
11	C	+ 1 8 0 x⇄y		
12		–	A	

Given a, A, C; find B, b, c.

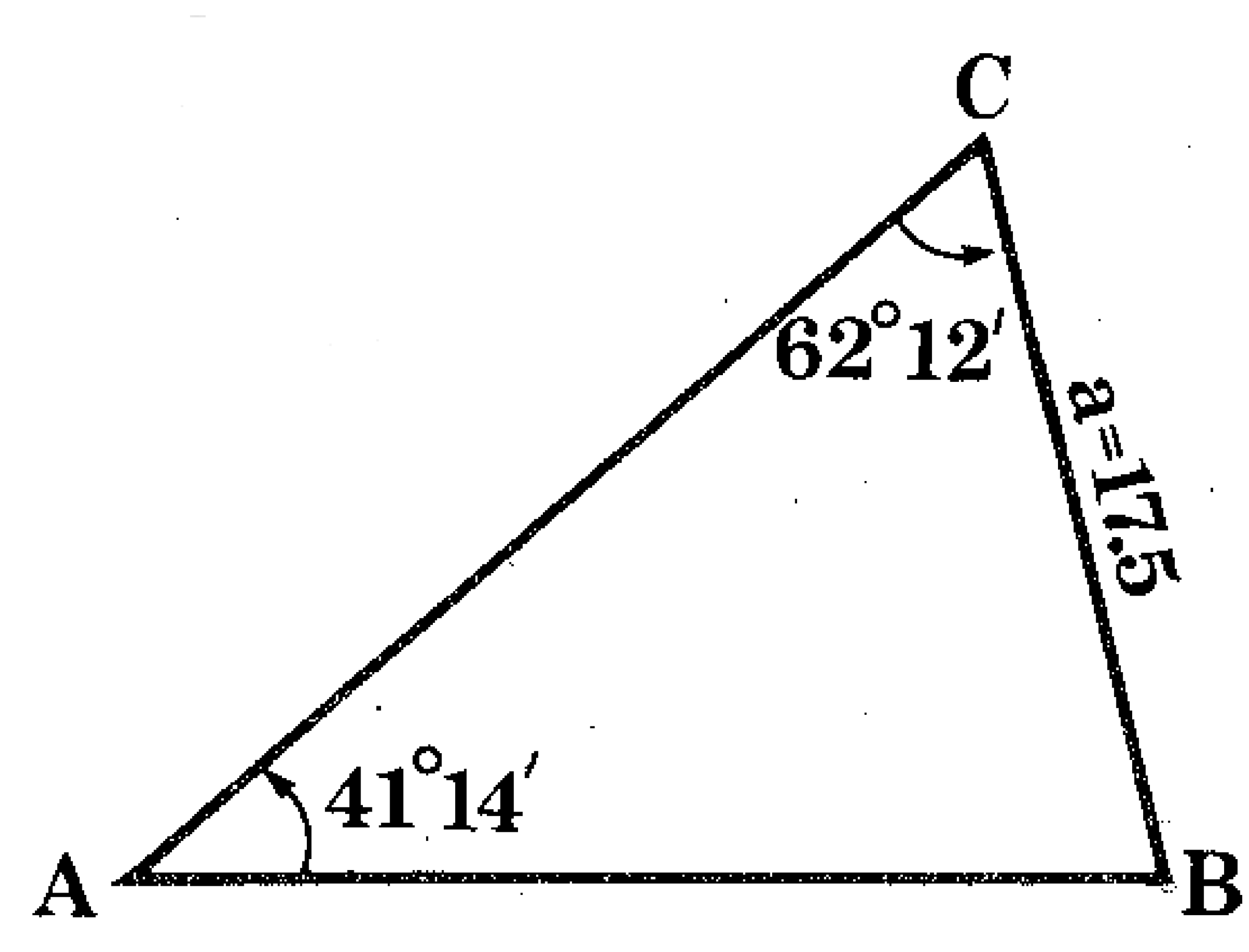
Example:

Given a = 17.5

A = 41° 14′ = 41.23333333°

C = 62° 12′ = 62.2°

Find B, b, and c



Answer:

B = 76.57°

b = 25.82

c = 23.486

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	C	↑ ↑		
2	A	STO + 1 8 0		
3		x⇄y – ↑	B	
4		R↓ sin		
5	a	x RCL sin STO ÷	b	
6		R↓ sin		
7	a	x RCL ÷	c	

Given a, B, C; find A, b, c

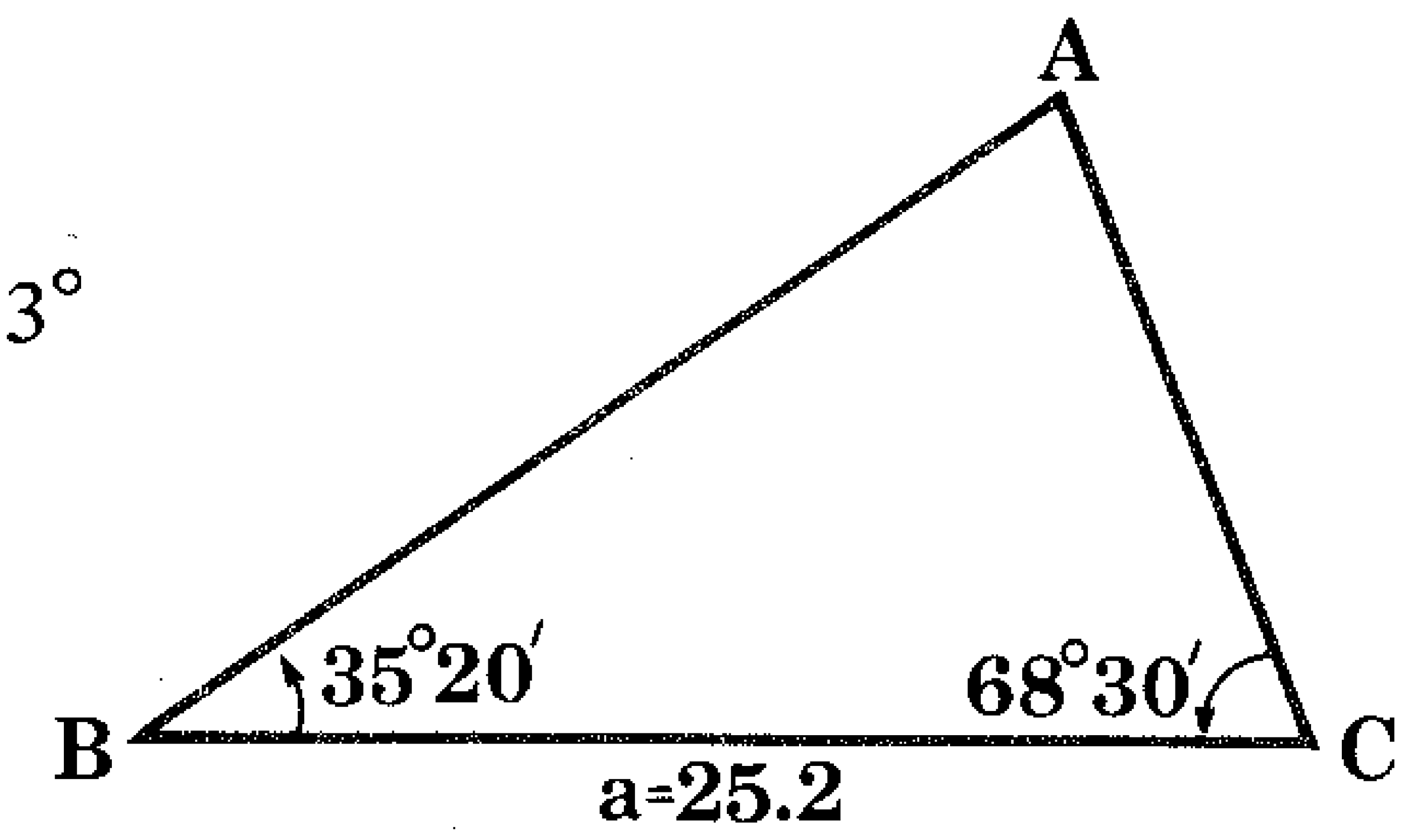
Example:

Given a = 25.2

B = 35°20′ = 35.33333333°

C = 68°30′ = 68.5°

Find A, b, and c



Answer:

A = 76.17°

b = 15.009

c = 24.1469

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	C	↑ ↑		
2	B	+ 1 8 0 x⇄y		
3		− STO	A	
4		R↓ sin		
5	a	× RCL sin STO ÷	c	
6	B	sin		
7	a	× RCL ÷	b	

Given B, b, c; find a, A, C.

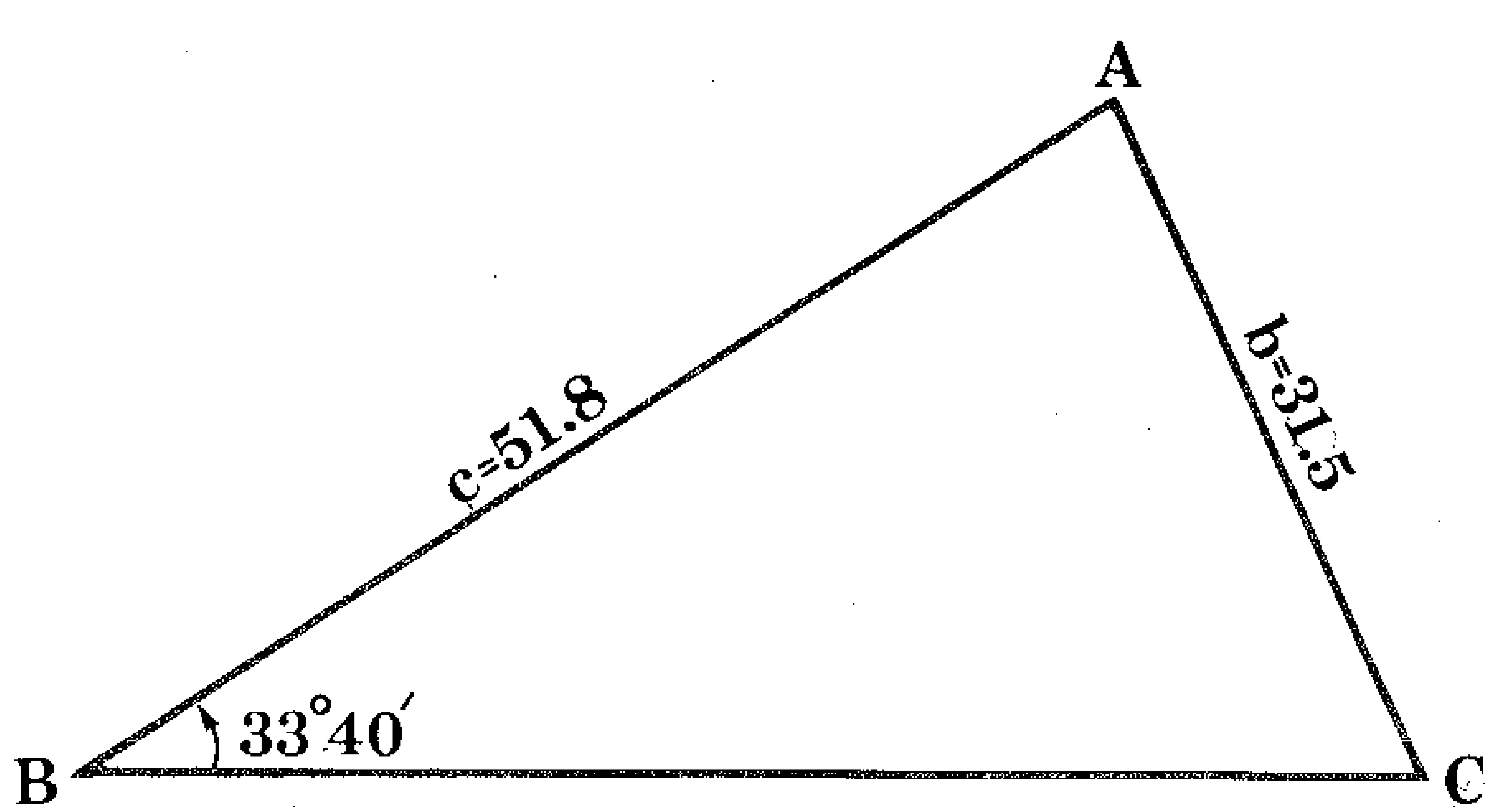
Example:

Given B = 33°40′ = 33.66666667°

b = 31.5

c = 51.8

Since B is acute and b < c, there are two solutions. Find a, A, and C.



Answer:

a = 56.06

A = 80.6°

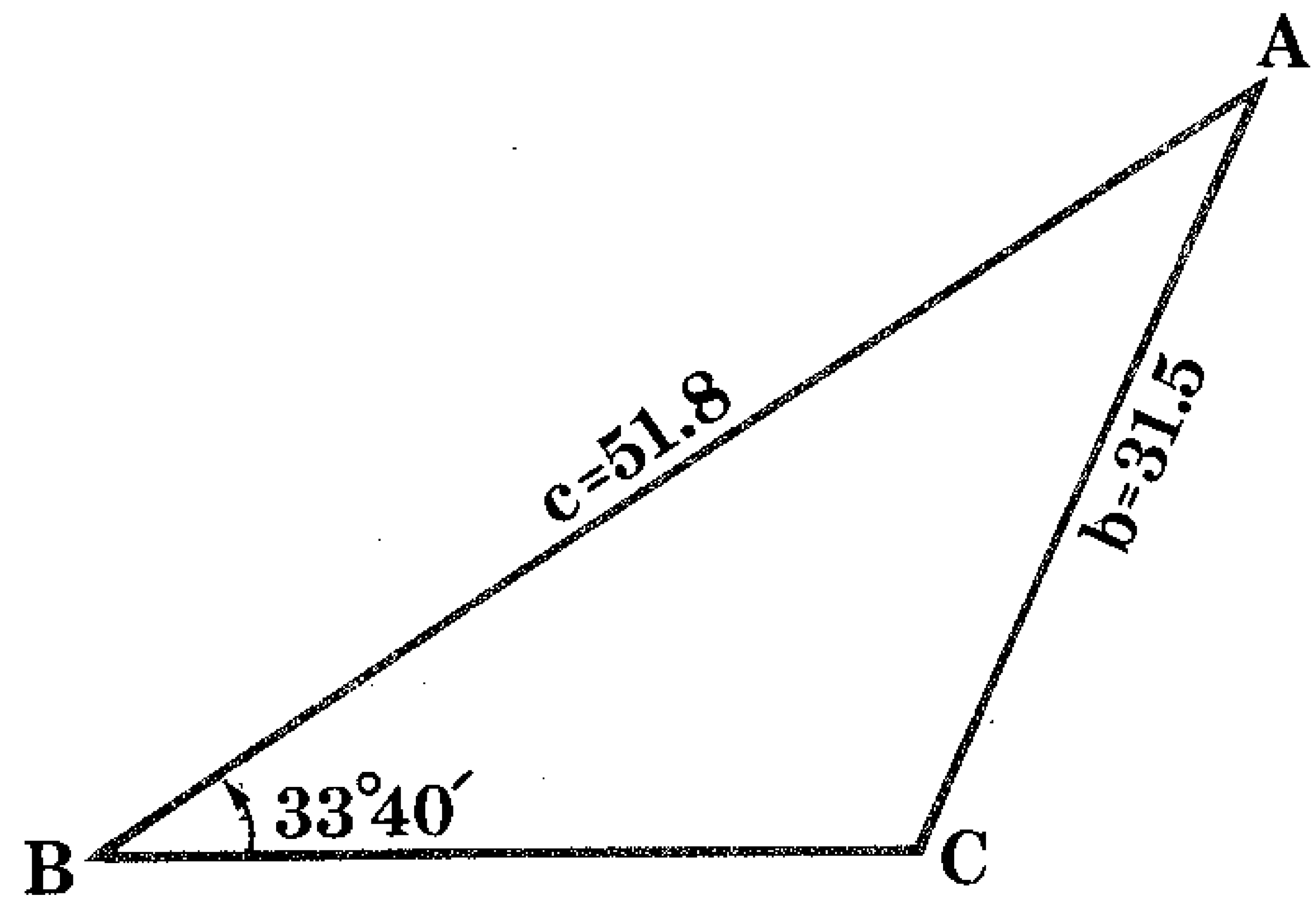
C = 65.729°

Alternate Answer:

$a_1 = 30.16$

$A_1 = 32.06°$

$C_1 = 114.27°$



LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	c	↑		
2	B	sin ×		
3	b	÷	D	If D > 1, no solution, stop
4		arc sin STO ↑	C	
5	B	+ 1 8 0 x⇄y		
6		−	A	
7		sin		
8	c	× RCL sin ÷	a	If b ≥ c, stop
9		1 8 0 RCL −		b < c, there are two solutions.
10		STO ↑	C_1	
11	B	+ 1 8 0 x⇄y		
12		−	A_1	
13		sin		
14	c	× RCL sin ÷	a_1	

Given a, b, and c; find the area.

Formulas:

Given the lengths of a triangle's three sides a, b, and c

$$\text{area} = \sqrt{S(S-a)(S-b)(S-c)}$$

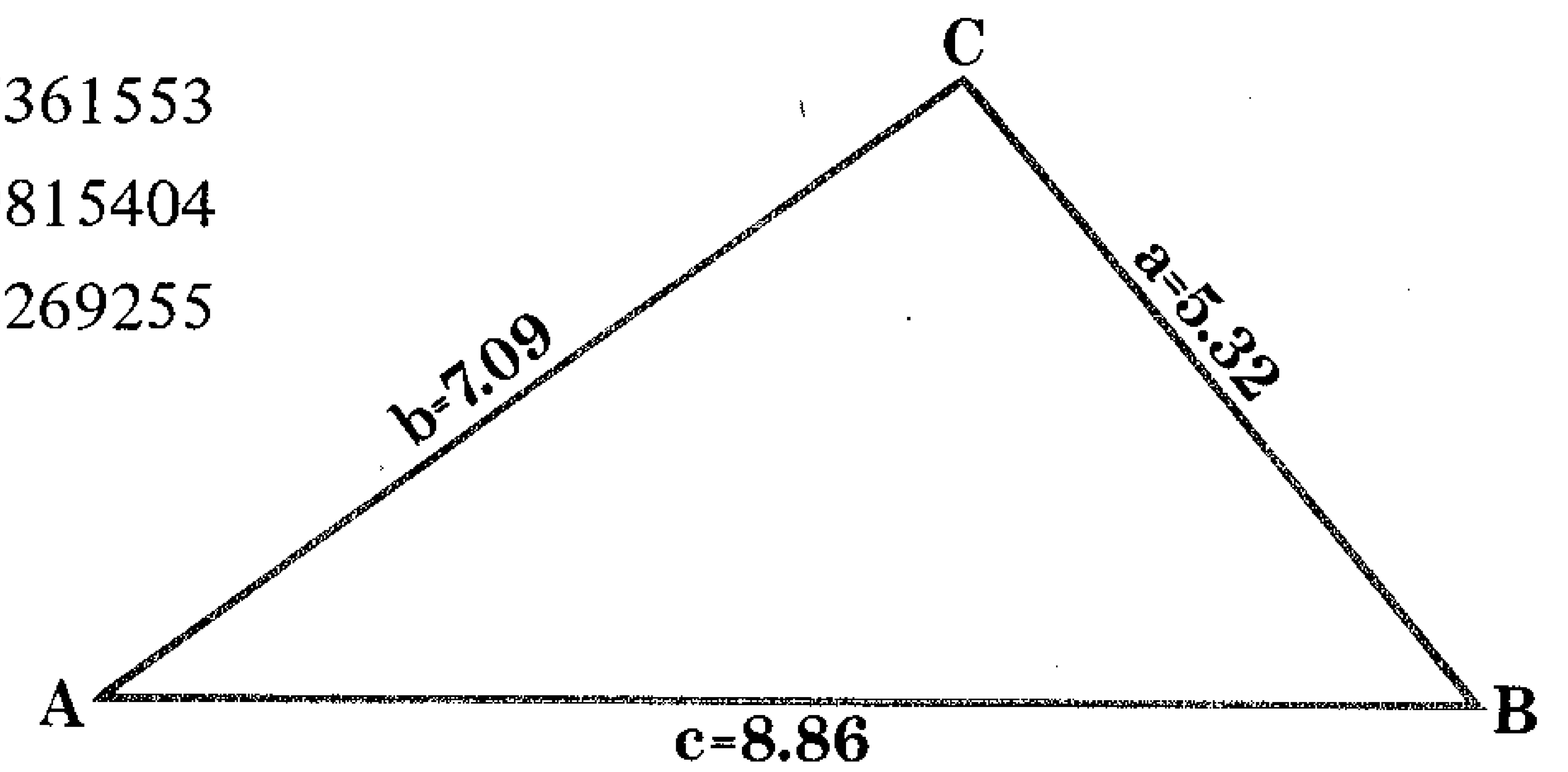
where

$$S = \frac{1}{2}(a + b + c)$$

Example:

Given a = 5.317361553
b = 7.089815404
c = 8.862269255

Find the area.



Answer:

Area = 18.85 (S = 10.63)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑						
2	b	↑	↑					
3	c	STO	+	R↓	x⇄y	R↓		
4		R↓	+	2	÷		S	
5		R↓	R↓	R↓	CLX	RCL		
6		R↓	STO	x⇄y	−	x⇄y		
7		RCL	x⇄y	−	x	x⇄y		
8		RCL	x⇄y	−	x	RCL		
9		x	$\sqrt{x}$				Area	

Given a, b, and C; find area

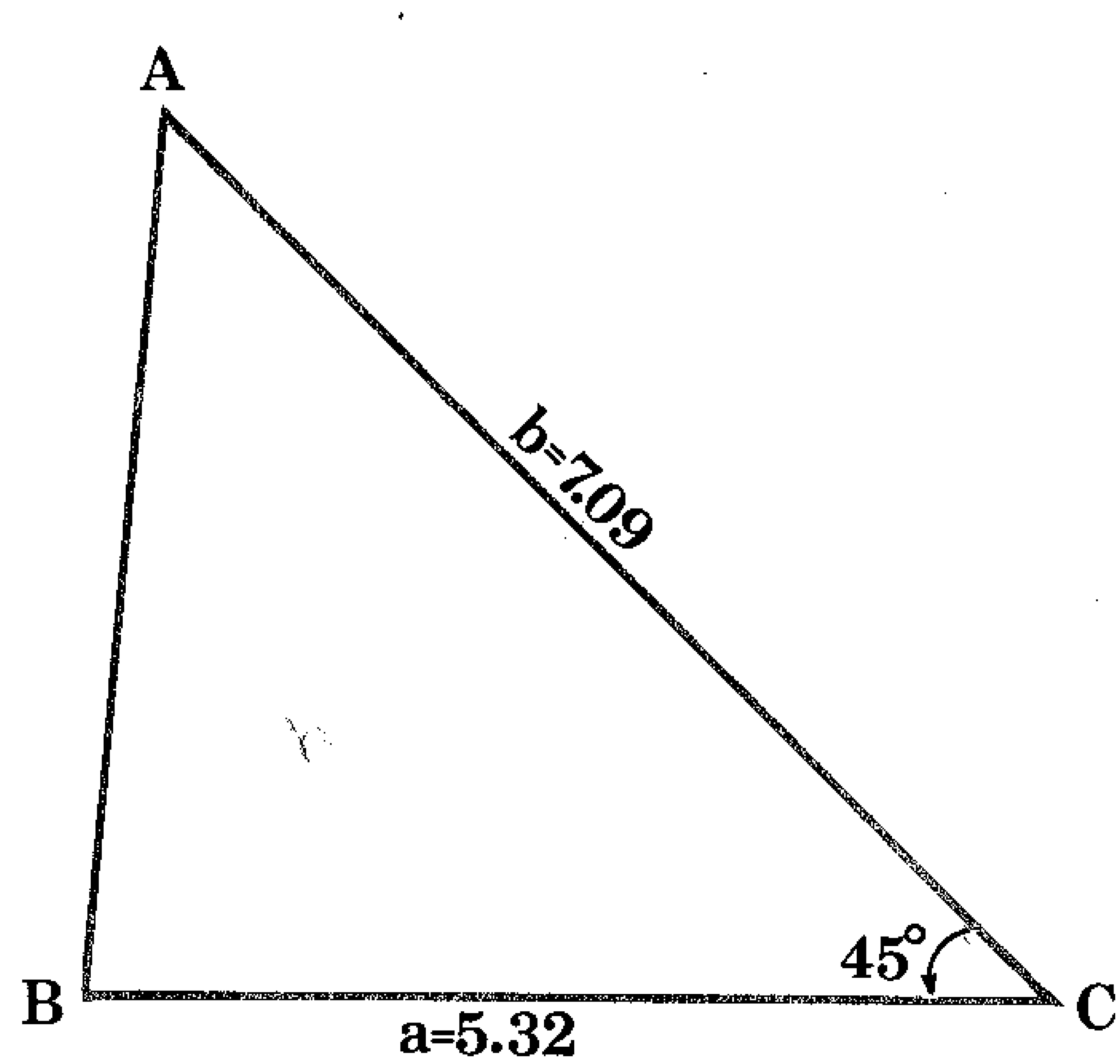
Formula:

$$\text{Area} = \frac{1}{2}\, a\, b \sin C$$

Example:

Given a = 5.3174
b = 7.0898
C = 45°

Find the area.



Answer:

Area = 13.33.

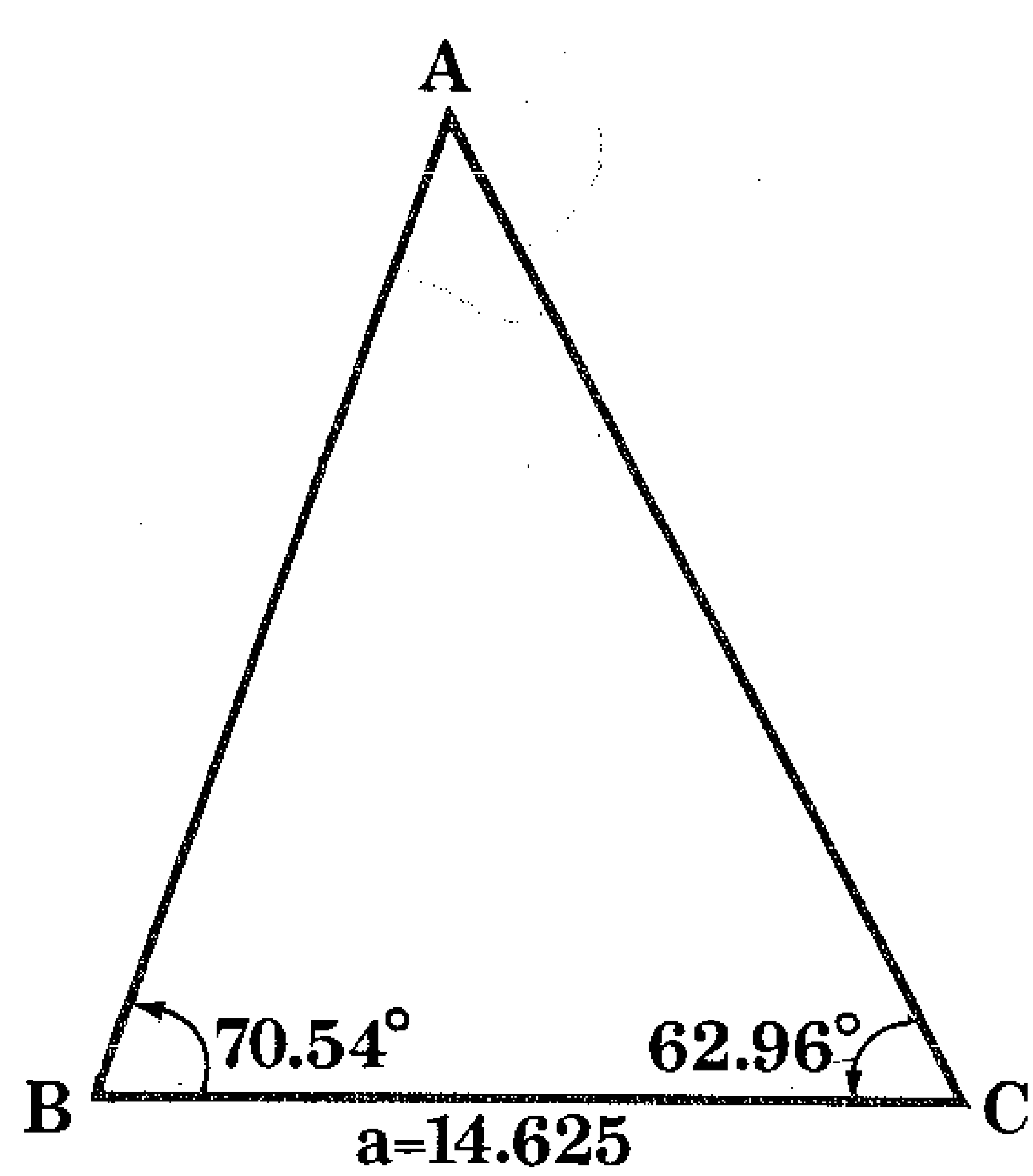
LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	a	↑						
2	b	x	2	÷				
3	c	sin	x					

Given a, B, and C; find the area

Formula:

$$\text{Area} = \frac{a^2}{2} \cdot \frac{\sin B \sin C}{\sin (B + C)}$$

Example:



Given $a = 14.625$

$B = 70.54°$

$C = 62.96°$

Find the area.

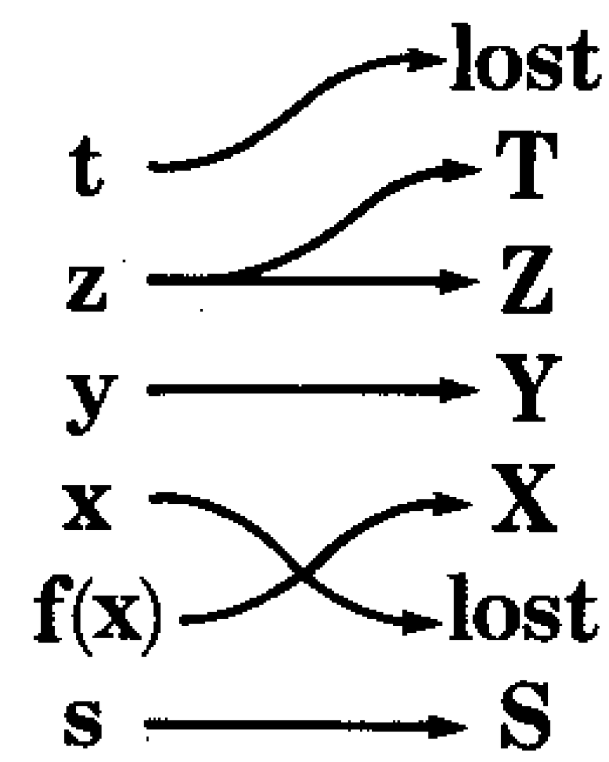
Answer:

Area = 123.8

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	B	↑	↑					
2	C	STO	+	sin	x⇄y	sin		
3		RCL	sin	x	x⇄y	÷		
4	a	↑	x	x	2	÷		

Trigonometric Functions

Register usage for trigonometric functions



Secondary value of arc sin x

Let p = principal value
q = secondary value

Example:

Find secondary value for arc sin (–.7660444436)

Answer:

230°

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	[arc] [sin] [ ] [ ] [ ]	p	
2		[1] [8] [0] [x⇄y] [–]	q	

Secondary value of arc cos x

Let p = principal value
q = secondary value

Example:

Find the secondary value of arc cos (.76).

Answer:

319.46°

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	arc cos	p	
2		3 6 0 x⇄y −	q	

Secondary value of arc tan x

Let p = principal value

q = secondary value

Example:

Find the secondary value of arc tan (2).

Answer:

243.43°

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	arc tan	p	
2		1 8 0 +	q	

Cotangent

Formula:

$$\cot x = \frac{1}{\tan x}$$

Example:

$$\cot 37° = 1.327$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	tan 1/x		

Cosecant

Formula:

$$\csc x = \frac{1}{\sin x}$$

Example:

$$\csc 30^\circ = 2$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	[sin] [1/x] [ ] [ ] [ ]		

Secant

Formula:

$$\sec x = \frac{1}{\cos x}$$

Example:

$$\sec 45^\circ = 1.41$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	[cos] [1/x] [ ] [ ] [ ]		

Versine

Formula:

$$\text{vers } x = 1 - \cos x$$

Example:

$$\text{vers } 38^\circ = .21$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	[cos] [1] [x⇄y] [–] [ ]		

Coversine

Formula:

$$\text{covers } x = 1 - \sin x$$

Example:

$$\text{covers } 38^\circ = .38$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	[sin] [1] [x⇄y] [−] [ ]		

Exsecant

Formula:

$$\text{exsec } x = \sec x - 1 = \frac{1}{\cos x} - 1$$

Example:

$$\text{exsec } 52^\circ = .624$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	[cos] [1/x] [1] [−] [ ]		

Haversine

Formula:

$$\text{hav } x = \frac{1 - \cos x}{2}$$

Example:

$$\text{hav } 42.3^\circ = .13$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	[cos] [1] [x⇄y] [−] [2]		
2		[÷] [ ] [ ] [ ] [ ]		

Arc cotangent

Formula:

$$\text{arc cot } x = \text{arc tan } \frac{1}{x}$$

Example:

$$\text{arc cot } 0.35 = 70.71^\circ$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	[1/x] [arc] [tan] [ ] [ ]		

Arc cosecant

Formula:

$$\text{arc csc x} = \text{arc sin}\ \frac{1}{x}$$

Example:

$$\text{arc csc } 3.45 = 16.849^\circ$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	[1/x] [arc] [sin] [ ] [ ]		

Arc secant

Formula:

$$\text{arc sec x} = \text{arc cos}\ \frac{1}{x}$$

Example:

$$\text{arc sec } 1.1547 = 30^\circ$$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x	[1/x] [arc] [cos] [ ] [ ]		

T-Statistic (Paired)

Formulas:

Given a set of paired observations:

x_i	x_1	x_2	x_3	...	x_n
y_i	y_1	y_2	y_3	...	y_n

$$D_i = x_i - y_i$$

$$\overline{D} = \frac{1}{n} \sum_{i=1}^{n} D_i$$

$$S_D = \sqrt{\frac{\Sigma D_i^2 - \frac{(\Sigma D_i)^2}{n}}{n-1}}$$

$$S_{\bar{D}} = \frac{S_D}{\sqrt{n}}$$

and test statistic

$$t = \frac{\bar{D}}{S_{\bar{D}}}$$

Example:

Compute t for the following:

x_i	14	17.5	17	17.5	15.4
y_i	17	20.7	21.6	20.9	17.2

Answer:

t = −7.1554

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1		CLR		
2	x_i	↑		Perform lines 2–4 for i = 1,
				2, ..., n
3	y_i	− STO + x⇄y RCL		
4		↑ x + x⇄y		
5		↑ ↑		
6	n	÷ STO	$\bar{D}$	
7		R↓ ↑ x		
8	n	↑ R↓ ÷ − R↓		
9		R↓ 1 − x ÷		
10		$\sqrt{x}$	$S_{\bar{D}}$	
11		RCL x⇄y ÷	t	

Vector Cross Product

Formula:

If $\vec{x} = (x_1, x_2, x_3)$ and $\vec{y} = (y_1, y_2, y_3)$ are two vectors, then cross product $\vec{z}$ is also a vector.

$$\vec{z} = \vec{x} \times \vec{y}$$

$$= (x_2y_3 - x_3y_2, x_3y_1 - x_1y_3, x_1y_2 - x_2y_1)$$

$$= (z_1, z_2, z_3)$$

Example:

If $\vec{x} = (2.34, 5.17, 7.43)$

$\vec{y} = (.072, .231, .409)$

Find $\vec{x} \times \vec{y}$

Answer:

$\vec{x} \times \vec{y} = (.3982, -.4221, .1683)$

LINE	DATA	OPERATIONS	DISPLAY	REMARKS
1	x_2	↑		
2	y_3	STO x		
3	y_2	↑		
4	x_3	↑ R↓ x –	z_1	
5		R↓		
6	y_1	x		
7		RCL		
8	x_1	STO x –	z_2	
9		RCL		
10	y_2	x		
11	x_2	↑		
12	y_1	x –	z_3	

Vector Dot Product

Formulas:

Given two vectors $\vec{x}$, $\vec{y}$ in an n-dimensional vector space

$$\vec{x} = (x_1, x_2, ..., x_n)$$

$$\vec{y} = (y_1, y_2, ..., y_n)$$

the dot product is

$$\vec{x} \cdot \vec{y} = x_1 y_1 + x_2 y_2 + ... + x_n y_n$$

Example:

If $\vec{x} = (2.34, 5.17, 7.43, 9.11, 11.41)$

$\vec{y} = (.072, .231, .409, .703, .891)$

then $\vec{x} \cdot \vec{y} = 20.97226$

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	x_1	↑						
2	y_1	x						
3	x_i	↑						Perform lines 3–4 for i = 2,
								3, ..., n
4	y_i	x	+					

Versine

See page 156

Weekday

DAY OF THE WEEK FOR ANY DATE (Since September 14, 1752)

d = day of month

m = month, with January and February being the 13th and 14th months of the previous year.

y = year (4 digits)

Output is read as follows:

0 – Saturday
1 – Sunday
2 – Monday
3 – Tuesday
4 – Wednesday
5 – Thursday
6 – Friday

Example:

On what day was February 29, 1972?

Answer:

Tuesday (d = 29, m = 14, y = 1971)

LINE	DATA	OPERATIONS					DISPLAY	REMARKS
1	d	↑						
2	m	↑						
3	y	STO	R↓	1	+	1		
4		3	x	5	÷		E_1	Let e_1 = integer part of E_1
5		CLX						
6	e_1	+	RCL	x⇄y	STO	x⇄y		
7		↑	↑	↑	5	x		
8		4	÷				E_2	Let e_2 = integer part of E_2
9		CLX						
10	e_2	RCL	+					For 20th century date, go to 17
11		STO	1	0	0	÷	E_3	Let e_3 = integer part of E_3
12		CLX						
13	e_3	CHS	RCL	+	STO	4		
14		0	0	÷			E_4	Let e_4 = integer part of E_4
15		CLX						
16	e_4	RCL	+					Go to 18
17		6	+					
18		↑	↑	7	STO	÷	E_5	Let e_5 = integer part of E_5
19		CLX						
20	e_5	RCL	x	–				

Appendix

Questions you may have wanted answered about your HP-35.

1. **Question:** What are the maximum and minimum light displays on the HP-35?

   Answer: Maximum: -8.888888888-88 after all decimal points light up on low battery power!

   Minimum: •

2. **Question:** A googol is 10^{100}. We can key this in as 10×10^{99} on the HP-35, right? Its reciprocal can be keyed in as $.1 \times 10^{-99}$, right?

   Answer:

   Wrong. 10 [EEX] 99 → 9.999999999 99

   .1 [EEX] [CHS] 99 → 0.

3. **Question:** The display is blanked out during computations. Is the keyboard also locked out?

   Answer: Yes

4. **Question:** What is the longest response on the HP-35?

   Answer: tan x where x = 9.9999999999 99 (about 2 seconds).

5. **Question:** Do:

   1 [↑] 2 [↑] 3 [↑] [CLx]

   Now, if we press [÷] or [x^y] (that is $\frac{3}{0}$ or 0^3) a flashing zero will be displayed. Will the stack be dropped?

   Answer: Yes, if we divide!:

   [÷] [R↓] → 2. [R↓] → 1. [R↓] → 1.

   No, if we do a x^y!:

   [x^y] [R↓] → 3. [R↓] → 2. [R↓] → 1.

6. **Question:** Pressing which key will reset the arc prefix?

Answer: Any key except [arc], [sin], [cos], or [tan].

7. **Question:** Do: 6 [EEX] 16. How would you make the whole number negative?

Answer: [$x \rightleftarrows y$] [$x \rightleftarrows y$] [CHS].

8. **Question:** When is [ENTER↑] desirable before entering data, but **NOT** just after a [CLx], [STO], or [ENTER↑]?

Answer: Sometimes after negating a result by [CHS]:

$-\sqrt{5}+6$ is 5 [$\sqrt{x}$] [CHS] [↑] 6 [+]

9. **Question:** How can three successive [tan] presses be used to calculate the tangent of 30° (accurate to about 7 decimal places)?

Answer: 30 [arc] [tan] [tan] [tan] → .5773502648

10. **Question:** When may it be desirable to press [CLx] if the display is already zero?

Answer: When we get zero as the result of an operation, but don't want to raise it in the stack upon entering data.

11. **Question:** When may it be desirable to enter 0 if x is already zero?

Answer: If an algorithm requires the integer part of a number (already in the X register) such as .4, we would do: [CLx] 0, which would insure that this zero would be raised in the stack if the algorithm requires it.

12. **Question:** Do: [π] [sin] [π] [cos] [÷] → .0548861507
[π] [tan] → .0548861507
[−] → 1. −11; why not 0?

Answer: See problem 20.

13. **Question:** The calculator gives overflow for the tangent of 90°, right?

Answer: Wrong, but nearly so:

90 [tan] → 9.99999999 99

We can still add to it:

[EEX] 90 [+] → 9.999999991 99

14. **Question:** Enter any three digits in the form d.dd x 10^{-7}:

d . d d [EEX] [CHS] 7.

How do you add 1 to this number with one keystroke? Your number now has the form: 1.000000ddd, how do you subtract 1 from it with one keystroke?

Answer: [e^x] and [ln]
(All this means that ln(1 + x) approaches x as x approaches 0.)

15. **Question:** Display .00026 x 10^{-2} in correct scientific notation.

Answer: .00026 [EEX] [CHS] 2 [x⇄y] [x⇄y] → 2.6 −06

16. **Question:** If we do "5 [EEX] [arc] 8", does 8 appear as an exponent or does it push up the stack?

Answer: Exponent.

17. **Question:** For which numbers will the HP-35 **not** give reciprocals?

Answer: 0. and any number with an exponent of +99 having a value other than ±1.

18. **Question:** How can you insure an integer result for [x^y], if x and y both contain integers?

Answer: [x^y] [EEX] 9 [+] [EEX] 9 [−]

19. **Question:** X contains 26.56505118. You wish to get tan x, but accidentally you hit the [arc] key. How can you cancel the [arc] prefix, and get tan x?

Answer: [$x \rightleftarrows y$] [$x \rightleftarrows y$] [tan] → .4999999999

20. **Question:** Numbers which start with .0d, where d is any digit other than zero, are displayed with only 9 significant digits, although ten significant digits are used in calculations. (This includes numbers .01 through .09999999999). The number is not rounded to 9 significant digits. Can you give some examples to show this fact?

Answer: Example 1:

23 [1/x] → .0434782608 10 [×] → .4347826087

Example 2:

1.000000001 [EEX] [CHS] 2 → 1.000000001 −02

[$x \rightleftarrows y$] [$x \rightleftarrows y$] → .01 10 [×] → .1000000001

Example 3:

[EEX] 9 [↑] 9 [+] [EEX] 11 [÷] [STO] .01

[1/x] → 100 [RCL] → .01 [1/x] → 99.9999991

Example 4:

1.000000001 [EEX] [CHS] 2 [↑] 23 [1/x] [+]

→ .0534782608 10 [÷] → .5347826088−03

21. **Question:** Compute $e^\pi - \pi^e$.

Answer: [π] [e^x] 1 [e^x] [π] [x^y] [−] → .68153493

22. **Question:** [EEX] [CHS] 8 [STO] 30

[RCL] [−] [↑] → 29.99999999 [sin] → .4999999998

[↓] [RCL] [+] [↑] → 30. [sin] → .5

[↓] [RCL] [+] [↑] → 30.00000001 [sin] → .4999999998

[↓] [RCL] [+] → 30.00000002 [sin] → .5000000002

Answer: Can be explained by the accuracy statement on page 21 of the operating manual.

23. **Question:** 2050667999 [arc] [tan] → 89.99999995

[EEX] [CHS] 8 [+] → 89.99999996 [tan]

→ 861719914.7

Answer: Can be explained by the accuracy statement on page 21 of the operating manual, an extreme "end case".

24. **Question:** For values:

89.99999997 to 90.	[tan]	→ 9.99999999	99
89.99999992 to 89.99999996	[tan]	→ 861719914.7	
89.99999987 to 89.99999991	[tan]	→ 457788704.7	
89.99999980 to 89.99999986	[tan]	→ 311685926.6	
89.99999975 to 89.99999979	[tan]	→ 240151451.6	

Answer: Can be explained by the accuracy statement on page 21 of the operating manual, an extreme "end case".

25. **Question:** How many decimal points light up on low battery power?

Answer: Only 14—the point following the true decimal point does not come on!

26. **Question:** [EEX] 9 [↑] .5 [+] .5 [−] → 1000000001. Why?

Answer: [EEX] 9 or 1000000000 [+] .5 = 1000000000.5 which is rounded to 1000000001, and 1000000001 [−] .5 = 1000000000.5 which is again rounded to 1000000001.

27. Question: How would you display all 1's, 2's, ... 9's?

Answer:

1's: 9 [e^x] [e^x] 9 [EEX] 88 [÷]

2's: 9 [e^x] [e^x] 4.5 [EEX] 77 [÷]

3's: 9 [e^x] [e^x] 3 [EEX] 66 [÷]

4's: 9 [e^x] [e^x] 2.25 [EEX] 55 [÷]

5's: 9 [e^x] [e^x] 1.8 [EEX] 44 [÷]

6's: 9 [e^x] [e^x] 1.5 [EEX] 33 [÷]

7's: 7.777777777 [EEX] 77

8's: 9 [e^x] [e^x] 1.125 [EEX] 11 [÷]

9's: 9 [e^x] [e^x]

28. Question: To divide the displayed value by 2: [↑] 2 [÷]. What other three keystrokes will do the same?

Answer: [e^x] [$\sqrt{x}$] [ln], for $|x| \leqslant 227$.

29. Question: [CHS] 9 [↑] 8 [÷] [EEX] 97 [×] [1/x] → −8.888888889−98

Change this number to −8.888888888−98 in a shorter number of keystrokes than keying in the new value.

Note: we must add 1×10^{-107} to the number in the display. If we try to add $.000000001 \times 10^{-98}$, we also get into trouble.

Answer: [EEX] 98 [STO] [×] [EEX] [CHS] 9 [+] [RCL] [÷]

30. Question: How would you display the number e (2.718281828):

(a) in the shortest number of keystrokes? (2 keystrokes)

(b) as the limit of $\left(1 + \frac{1}{n}\right)^n$ as n gets large? (7 keystrokes)

(c) as the function of a digit? (5 keystrokes)

(d) using the [π] key, but not [e^x] or [x^y]? (13 keystrokes)

(e) as a continued fraction? (21 keystrokes)

Answer: (a) 1 [e^x] or if x = 0: [e^x] [e^x]

(b) [EEX] 6 [↑] [1/x] 1 [+] [x^y]

(c) 5 [1/x] 5 [e^x] [x^y]

(d) [π] [↑] [ln] [÷] [↑] [ln] [÷] [↑] [ln] [÷] [↑] [ln] [÷]

(e) 18 [1/x] 14 [+] [1/x] 10 [+] [1/x] 6 [+] [1/x] 1 [+] [1/x] 2 [×] 1 [+]

31. Question: Can you display all ten L.E.D. digits in an orderly sequence? (5 keystrokes):

Answer: 8 [↑] 81 [÷] → .0987654321

32. Question: Determine the Golden Ratio ϕ to 11 significant digits, given $\phi = (\sqrt{5} + 1)/2$ and $\phi - \frac{1}{\phi} = 1$.

Answer: 5 [$\sqrt{x}$] 1 [+] 2 [÷] → 1.618033989 [↑] [1/x] → .6180339887 [−] → 1.

$\phi = 1.6180339887$

HP-35 Key Sequence Form

Date: Page:

Routine:	Author:

LINE	DATA	OPERATIONS	DISPLAY	REMARKS

LINE	DATA	OPERATIONS	DISPLAY	REMARKS

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